Math. 640 Fall, 2018

# Homework 13, due December 5

1. Let $L_{\vec{\omega}}$ be the antisymmetric matrix corresponding to the vector $\vec{\omega} \in \mathbf{R}^3$ under the isomorphism defined by the vector cross product. Verify that (for orthogonal $O$) $OL_{\vec{\omega}}O^{-1} = (\det O)L_{O\vec{\omega}}$. (Use the hint in the notes.)

2. Define a new coordinate system in $\mathbf{R}^2$ by

$$x = \xi + \eta,$$
$$y = \eta.$$

Calculate the Cartesian components of the mutually reciprocal bases

$$\{\nabla\xi, \nabla\eta\} \qquad \text{and} \qquad \left\{\frac{d\vec{x}}{d\xi}, \frac{d\vec{x}}{d\eta}\right\}.$$

Sketch the results.

3. Fill in the details of the calculation of the mutually reciprocal bases for polar coordinates in the plane.

4. Exercise 30.4, pp. 200–201.

5. *(some unfinished business)* Let $\mathcal{V}$ and $\mathcal{U}$ be finite-dimensional inner-product spaces and $\underline{A}: \mathcal{V} \to \mathcal{U}$ be an operator. Return in your notes to the definition of the adjoint (not dual) operator $\underline{A}^*: \mathcal{U} \to \mathcal{V}$ and prove the assertion there that $\operatorname{dom} \underline{A}^* = \text{all of } \mathcal{U}$. Hint: Use the Riesz representation theorem (finite-dimensional version).

6. Show that a multilinear function on $\mathcal{V}_1 \times \cdots \times \mathcal{V}_r$ is equivalent (under a natural isomorphism) to a *linear* functional on $\mathcal{V}_1 \otimes \cdots \otimes \mathcal{V}_r$ (and totally different from a linear functional on $\mathcal{V}_1 \oplus \cdots \oplus \mathcal{V}_r \cong \mathcal{V}_1 \times \cdots \times \mathcal{V}_r$).