$$V_{h-3} = V_{h-2} \oplus \tilde{N}_{h-2} \oplus L_{h-2} ,$$

where $\underline{A}$ maps L_{h-2} isomorphically onto $\tilde{N}_{h-1} \oplus L_{h-1}$. [Thus an element of L_{h-2} is either the second vector in a Jordan chain of length $h-1$, or the third vector in a chain of length h.] We continue in this way until we get to $V_0 \equiv \mathcal{V}$. Thus we have a decomposition

$$\begin{aligned}
\mathcal{V} &= V_1 \oplus \tilde{N}_1 \oplus L_1 \\
&= V_2 \oplus \tilde{N}_2 \oplus L_2 \oplus \tilde{N}_1 \oplus L_1 \\
&= \dots \\
&= \underbrace{\underbrace{V_h \oplus \tilde{N}_h}_{V_{h-1}} \oplus \tilde{N}_{h-1} \oplus L_{h-1}}_{V_{h-2}\ ,\ ,\ ,} \oplus \cdots \oplus \tilde{N}_1 \oplus L_1
\end{aligned}$$

And the action of $\underline{A}$ on these subspaces is

$$\begin{tikzcd}
0 & & \\
0 & \tilde{N}_h \arrow[ul] & V_h \arrow[loop right] \\
0 & \tilde{N}_{h-1} \arrow[ul] & L_{h-1} \arrow[ul] \\
 & \tilde{N}_{h-2} \arrow[ul] & L_{h-2} \arrow[ul] \\
0 & \vdots & \vdots \\
0 & \tilde{N}_3 \arrow[ul] & L_3 \\
0 & \tilde{N}_2 \arrow[ul] & L_2 \arrow[ul] \\
 & \tilde{N}_1 \arrow[ul] & L_1 \arrow[ul]
\end{tikzcd}$$

To get a Jordan basis, start by choosing a basis for each $\tilde{N}_j$; let the basis for L_{h-1} be the inverse images of the basis vectors for $\tilde{N}_h$; let the basis vectors for L_{h-2} be the inverse images of the basis vectors for $\tilde{N}_{h-1} \oplus L_{h-1}$; etc. The matrix of $\underline{A}$ with respect to this basis has the form claimed in the lemma (with $V_h = \operatorname{dom} \underline{B}$). Indeed, the basis vectors in each chain of inverse images belong to a single Jordan block

$$\begin{array}{c}
\tilde{N}_j \\ L_{j-1} \\ L_{j-2} \\ \vdots \\ \\ L_2 \\ L_1
\end{array}
\begin{pmatrix}
0 & 1 & & & & 0 \\
 & 0 & 1 & & & \\
 & & 0 & 1 & & \\
 & & & \ddots & \ddots & \\
 & & & & \ddots & 1 \\
0 & & & & & 0
\end{pmatrix} .$$