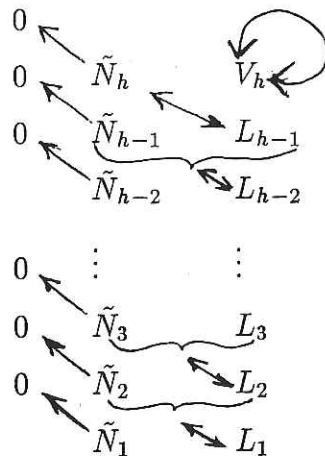


$$V_{h-3} = V_{h-2} \oplus \tilde{N}_{h-2} \oplus L_{h-2},$$

where \underline{A} maps L_{h-2} isomorphically onto $\tilde{N}_{h-1} \oplus L_{h-1}$. [Thus an element of L_{h-2} is either the second vector in a Jordan chain of length $h-1$, or the third vector in a chain of length h .] We continue in this way until we get to $V_0 \equiv \mathcal{V}$. Thus we have a decomposition

$$\begin{aligned} \mathcal{V} &= V_1 \oplus \tilde{N}_1 \oplus L_1 \\ &= V_2 \oplus \tilde{N}_2 \oplus L_2 \oplus \tilde{N}_1 \oplus L_1 \\ &= \dots \\ &= \underbrace{V_h \oplus \tilde{N}_h \oplus \tilde{N}_{h-1} \oplus L_{h-1}}_{V_{h-1}} \oplus \dots \oplus \tilde{N}_1 \oplus L_1 \\ &\quad \underbrace{\hspace{10em}}_{V_{h-2}} \dots \end{aligned}$$

And the action of \underline{A} on these subspaces is



To get a Jordan basis, start by choosing a basis for each \tilde{N}_j ; let the basis for L_{h-1} be the inverse images of the basis vectors for \tilde{N}_h ; let the basis vectors for L_{h-2} be the inverse images of the basis vectors for $\tilde{N}_{h-1} \oplus L_{h-1}$; etc. The matrix of \underline{A} with respect to this basis has the form claimed in the lemma (with $V_h = \text{dom } \underline{B}$). Indeed, the basis vectors in each chain of inverse images belong to a single Jordan block

$$\begin{matrix} \tilde{N}_j \\ L_{j-1} \\ L_{j-2} \\ \vdots \\ L_2 \\ L_1 \end{matrix} \begin{pmatrix} 0 & 1 & & & & & 0 \\ & 0 & 1 & & & & \\ & & 0 & 1 & & & \\ & & & \ddots & \ddots & & \\ & & & & \ddots & \ddots & \\ & & & & & \ddots & 1 \\ 0 & & & & & & 0 \end{pmatrix}.$$