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RISK AVERSION IN CONTESTS

Stergios Skaperdas and Li Gan

In a contest, participants spend money or effort to increase their chances of winning a prize. We examine primarily the following question: Could the timid (the more risk averse) have a better chance of winning in contests? Under limited liability the answer is always positive. In the absence of limited liability there is no single answer, whereas when the prize is shared as a function of effort the outcome is independent of the contestants’ risk aversion. We also relate our results to self-protection and examine some other implications of risk aversion.

Many economic and social situations can be modelled as contests in which the participants expend money or effort to increase their chances of winning a prize. Examples include rent-seeking and lobbying situations; patent, R&D or entrepreneurial races; tournaments; arms races; political campaigns; and athletic contests. Since the outcome of a contest is typically uncertain, the effort and the likelihood of winning of each participant will depend on the risk preferences of all the participants. In this paper we focus on the effect of risk preferences on the behaviour of the participants in several variations of contests: ordinary winner-take-all contests, others in which the contestants face limited liability and the type in which the prize is divisible. In addition to other issues, we examine the effects of differential risk attitudes and in all three variations we attempt to answer the following question. Do the less risk averse contestants expend lower or higher effort (and thus have higher or lower probability of winning) than the more risk averse contestants?

The exertion of effort by contest participants could be thought of as a form of gambling. Then, such a viewpoint would lead one to think that the most risk seeking or the least risk averse would be most willing to gamble, exert higher effort and have a higher probability of winning. It is often considered, for instance, that successful entrepreneurs are those who are more willing to take risks. In Frank Knight’s famous quote, ‘the confident and the venturesome “assume the risk” or “insure” the doubtful and the timid’ (Knight, 1957, p. 269). If we were to accept that at least one characteristic of ‘the confident and

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1 A literature on rent-seeking contests started with Tullock (1980); recent examples include Nitzan (1991) and Baye et al. (1992). Compensation tournaments developed as alternatives to individualised contracts in the principal–agent literature (see, for example, Nalebuff and Stiglitz, 1982 and Rosen, 1986). R&D races are usually modelled somewhat differently from the other cases, putting emphasis on the time dimension of the process (e.g. Loury, 1979 or Dasgupta and Stiglitz, 1981). Skaperdas and Grofman (1995) model political campaigns as contests. Dixit (1987) considers the effect of differential moves of the players in generalised contests. The effect of risk aversion in rent seeking has been considered by Hillman and Katz (1984), Long and Voudens (1987), and for the case of an endogenous prize by Skaperdas (1991), whereas Konrad and Schlesinger (1993) examine a model similar to that of Section I.
the venturesome’ is their lower risk aversion, the intuition about the least risk averse doing better in contests would seem reasonable.

Our results, however, show that there is a competing intuition. Risk seekers tend to ‘consume’ the risk itself and be less fearful of the possibility of ruin, whereas the risk averse are more fearful of losing. And thinking of the effort expended on the contest as a form of insurance against losing, we could then expect a more risk averse agent to put more effort and thus have a higher probability of success than a less risk averse agent would have. In the case of an entrepreneurial contest, a more risk averse entrepreneur would work harder to avoid losing.

Exertion of effort in a contest very much resembles a certain type of insurance, that of self-protection: the expenditure of resources to reduce the probability of a loss (see Ehrlich and Becker, 1972). Although it is known (Dionne and Eeckhoudt, 1985; McGuire et al. 1991) that increasing risk aversion has ambiguous effects on the resources expended on self-protection, a result that does not bode well for obtaining discriminating results for the case of contests, we obtain unambiguous results for two out of the three types of contests we examine.

We obtain the least discriminating results in an ordinary winner-take-all contest, examined in Section I. There is a prize and the agents spend costly effort, with the relative amount of effort determining the winner and the loser through a contest success function. For the contest success functions used in the rent-seeking literature and under constant absolute risk aversion the less risk averse always exert more effort than the more risk averse agents. For another leading class of contest success functions, under plausible conditions both contestants put the maximum effort they can put into the contest, and if they could go beyond that maximum level of effort by, for example, borrowing, they would.

The variation of Section II is partly motivated by this last result and assumes that each contestant has limited liability and the realised payoff in the event of a loss cannot go below a minimum level. Rather surprisingly, under general conditions the intuition which considers effort as insurance against loss dominates in the contest with limited liability: the more risk averse agent always exerts higher effort, and thus has a higher probability of winning, than the less risk averse agent.

Risk aversion along with the possibility of dividing the prize among the contestants renders some divisions of the prize strictly Pareto superior to engaging in a probabilistic contest. We examine this variation in Section III. Costly effort is still exerted because it enhances each agent’s bargaining position. For a large class of prize-sharing rules, both contestants behave as if they were risk neutral and exert the same amount of effort. The actual shares of the prize, though, may be unequal and they depend on the sharing rule.

Finally, we should mention another effect of risk aversion which, although of a more technical nature, is nevertheless relevant to the study of contests: that

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2 It is evident from the context of the quote that Knight’s conception of these terms had other dimensions which would be harder to formalise.
of the existence of pure-strategy equilibria. In the basic winner-take-all contest increasing risk aversion makes the conditions for the existence of a pure strategy equilibrium progressively more stringent, whereas in the two other variations the conditions are independent of the degree of risk aversion.

1. A WINNER-TAKE-ALL CONTEST

We start with the most common model of contests to which we add risk aversion. There are two agents, labelled 1 and 2, competing for a prize with value $T$. Each agent has an endowment $Y$ of resources that can be devoted to the contest or consumed. Let $x_i \in [0, Y]$ be the amount of effort put into the contest by agent $i = 1, 2$. The probability of each agent winning the prize is a function of the relative efforts the two agents exert.

For a given choice of efforts $(x_1, x_2)$, the winning probability of agent 1 is denoted by $p(x_1, x_2)$, with the winning probability of agent 2 represented by $1 - p(x_1, x_2)$. We assume the following properties for the function $p(\cdot, \cdot)$, usually referred to as contest success function (CSF): it is symmetric in the sense that when the efforts of the agents are interchanged, their winning probabilities are also interchanged; $p(x_1, x_2)$ is increasing in $x_1$ and decreasing in $x_2$; and it is twice continuously differentiable. For brevity, whenever possible, we will be denoting $p(x_1, x_2)$ by $p$. Two examples of CSF's satisfying these properties are the following:

$$p = (x_1 + \delta)^m / [(x_1 + \delta)^m + (x_2 + \delta)^m] \quad \text{where } m > 0 \quad \text{and} \quad \delta > 0. \quad (1)$$

$$p = \exp(kx_1) / [\exp(kx_1) + \exp(kx_2)] \quad \text{where } k > 0. \quad (2)$$

Let $U(\cdot)$ and $V(\cdot)$ denote the Von Neumann–Morgenstern utility functions of agents 1 and 2, respectively. We assume these functions are concave and twice continuously differentiable. The payoffs, as functions of the effort level pairs $(x_1, x_2)$, are the following:

$$\pi^1(x_1, x_2) = p(x_1, x_2) U(T + Y - x_1) + [1 - p(x_1, x_2)] U(Y - x_1) \quad (3)$$

$$\pi^2(x_1, x_2) = [1 - p(x_1, x_2)] V(T + Y - x_2) + p(x_1, x_2) V(Y - x_2). \quad (4)$$

For a fixed effort level of one's opponent, $x_2$ for agent 1 and $x_1$ for agent 2, these payoff functions are formally identical to the individual decision problem of self-protection (see Ehrlich and Becker, 1972), except that the function $P(\cdot, \cdot)$ has a particular structure here. As mentioned in the introduction, for the self-protection problem it is known that increasing risk aversion does not unambiguously increase or decrease the amount of effort expended on self-protection. Moreover, unambiguous results have not been obtained even with specific utility functions. Thus, finding unambiguous results, even for specific functional forms, would appear unlikely in this game-theoretic analogue of self-protection.

The rent-seeking literature, beginning with Tullock (1980), has almost exclusively employed (1), with $\delta = 0$ and usually with $m = 1$. Later on in this section we will use this CSF to obtain one unambiguous result about the effect of risk preferences. (2) is the logit probabilistic choice function, more familiar to econometricians. As Hirsheifer (1989) has shown, comparing the properties of (1) and (2), interior contest equilibria cannot exist under risk neutrality and (2). We will extend this result for one case of risk aversion, but also show one interesting case of boundary equilibrium. Skaperdas (1994) axiomatises (1), (2), as well as the general class of functions which both belong.
Before going on, though, we need to resolve the issue of existence of interior
pure-strategy Nash equilibrium.\footnote{For the derivation of a mixed strategy equilibrium under risk neutrality and (1), see Baye et al. (1992). As is demonstrated in that paper, the derivation or simply the characterisation of mixed strategy equilibria is rather difficult. The presence of risk aversion certainly could not reduce these difficulties. Hillman and Riley (1989) examine mixed strategy equilibria under the condition that the player exerting the higher effort wins the prize for sure, or under (1) with $m \to 0$.} Contrary to most other instances in economics in which increasing risk aversion tends to make optimisation easier and pure-strategy equilibria more likely, here we have the opposite effect taking place. We demonstrate this effect with the following utility functions which exhibit constant absolute risk aversion.

$$U(y) = -e^{-\gamma y}, \quad V(y) = -e^{-\gamma y} \quad \text{for all } y, \quad \text{with } \gamma \geq \beta > 0. \quad (5)$$

**Proposition 1:** Consider the utility functions in (5) and let $p_i \equiv \partial p/\partial x_i, p_{ii} \equiv \partial^2 p/\partial x_i^2$ for $i = 1, 2$. (a) If $-p_{11}/p_1 > \beta$ and $p_{22}/p_2 > \gamma$ for all $(x_1, x_2)$, then a pure-strategy equilibrium exists; (b) If an interior pure-strategy equilibrium exists, then we have $-p_{11}/p_1^* \geq \beta$ and $p_{22}^*/p_2^* \geq \gamma$ (where $^*$ indicates evaluation at the equilibrium point). For the Proof, please see the Appendix.

These necessary and sufficient conditions for the existence of a pure strategy equilibrium essentially state that the CSF must be more concave than the utility functions of both agents (since, under (5), $\beta$ and $\gamma$ are the measures of concavity/absolute risk aversion of the agents’ utility functions). Consider, for instance, the first condition in part (a). Since $\beta > 0$ and $p_1 > 0$, for the condition to hold it is necessary that $p_{11}$ is not just negative but sufficiently negative, with higher risk aversion (higher $\beta$) requiring a higher degree of concavity of the $p$ function. The second condition has the same interpretation and applies to agent 2 (note that $p_2 < 0$). Thus, Proposition 1 shows that greater risk aversion magnifies problems of existence of pure strategy equilibria.

We now turn to the issue of relative efforts: Do the less risk averse expend more or less effort? Proposition 2 provides an answer under the utility functions in (5) and the CSF in (1).\footnote{The conditions for existence of equilibrium in Proposition 1a ($-p_{11}/p_1 > \beta$ and $p_{22}/p_2 > \gamma$) are not globally satisfied under (1), thus making the proof of Proposition 2a rely on attributes of the specific functional form.}

**Proposition 2:** Assume the utility functions in (5) under the CSF in (1) with $m \leq 1$. (a) Then for $\beta$ and $\gamma$ sufficiently close to each other and $\delta$ small enough an interior pure-strategy equilibrium exists. (b) In such an equilibrium the less risk averse agent will put more effort, and thus have a higher probability of winning the prize, then the more risk averse agent.

(Proofs of this and other results not shown here are in a supplementary Appendix which is available on request from the first author.)

This unambiguous result contrasts with those obtained in the seemingly simpler case of self-protection. As we mentioned earlier, the contest situation, although more complicated than the non-interactive self-protection situation, has additional structure by the use of the CSF. Here we have used a specific CSF which has not been used in the literature on self-protection.

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The higher equilibrium probability of success of the less risk averse agent reported in Proposition 2 cannot be generalised to all functional forms. As shown below, under the logit CSF in (2), there are equilibria in which both agents have equal probability of success.

Proposition 3: Assume the utility functions in (5) and the CSF in (2). (a) Then, no interior pure-strategy equilibrium exists. (b) If \( \pi^i(o, o) \) \( = \partial \pi^i(o, o) / \partial x_i \) > 0 for \( i = 1, 2 \), \( \pi^1(Y, Y) \geq \pi^1(o, Y) \) and \( \pi^2(Y, Y) \geq \pi^2(Y, o) \) then \( x_1^* = x_2^* = Y \) is an equilibrium.

Part (b) of the Proposition states that if the first derivatives of the payoff functions with respect to each agent’s effort are positive at the point in which neither agent puts any effort, and if for both contestants it is better to exert maximum effort than putting no effort when the opponent puts maximum effort, then both agents will put the maximum allowable level of effort. Consequently, the differential attitudes towards risk do not lead to different levels of equilibrium effort in this case although they influence the conditions under which this equilibrium would obtain. Other things being equal, this equilibrium is more likely to obtain the higher is the effectiveness of the contest as measured by the parameter \( k \) in (2), the higher is the prize \( T \) and the lower is the ‘income’ \( Y \).

In the absence of the ‘hard’ budget constraint we have imposed here by not allowing the efforts to exceed the level \( Y \), under the conditions described in Proposition 3b there would exist an unambiguous tendency to increase the effort levels beyond \( Y \). But if there were a ‘soft’ budget constraint and \( Y \) could be exceeded, then the loser of the contest would have a negative payoff. In such a case, the loser could conceivably default on any borrowings or obligations that made a higher effort level possible in the first place. Thus, if we were to relax the assumption of the ‘hard’ budget constraint, we would encounter problems of limited liability.

II. Winner-take-all with Limited Liability

We now examine a variation of the conventional winner-take-all contest. As in the previous section, the probability of winning depends on the relative efforts of the two agents through a CSF. The difference is that here the loser’s payoff is independent of the effort expended by that agent. For example, the contestants may be entrepreneurs who borrow money to spend on the development of a new product. The winner is the contestant who receives exclusive rights to marketing the product while the loser is unable to repay the loan and goes bankrupt with his payoff independent of the loan amount and of the money (effort) put into the contest. Governments often provide loan guarantees for pursuing R&D projects\(^6\) or they tacitly guarantee minimum payoffs to the losers in weapons system contests, in ways that fit the model of this section.

There are several reasons for examining this case. First, there are contests, as in the examples just mentioned above, in which the agents’ payoffs in the event

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\(^6\) See, for example, Cohen and Noll (1991), especially Chapter 10.

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of a loss are independent of effort. Second, in some ordinary winner-take-all contests of the type examined in the previous section there are clear incentives to expand effort beyond the income or wealth of the contestants with concomitant limited liability problems. Finally, the results obtained for this case are both clear-cut and rather surprising at first sight: pure-strategy equilibria exist under general conditions and the more risk averse contestant always puts more effort than the less risk averse contestant regardless of the utility and contest success functions used. Thus, the winner-take-all with limited liability contest is revealed as an important benchmark which can be used to further our understanding of the other types of contests.

Denote by \( B \) the payoff in the event of a loss and without loss of generality let \( U(B) = V(B) = 0 \). Then, the payoff functions become:

\[
\pi^1(x_1, x_2; l) = p(x_1, x_2) U(T + Y - x_1) \tag{6}
\]

\[
\pi^2(x_1, x_2; l) = [1 - p(x_1, x_2)] V(T + Y - x_2). \tag{7}
\]

The \( 'l' \) inside the parentheses is used to denote limited liability and to distinguish these payoff functions from those in (3)–(4). We will also employ the shorter notation \( \pi^i(l) \) to denote these functions.

Under limited liability a pure-strategy equilibrium exists as long as the agents are risk averse and under general conditions on the CSFs.\(^7\) To examine the effect to risk aversion, suppose agent 2 is strictly more risk averse than agent 1 in the Arrow–Pratt sense. Then \( V(\cdot) = k[U(\cdot)] \) where \( k[\cdot] \) is an increasing, strictly concave function. Consider an interior equilibrium and let \( U_i = U(T + Y - x_i) \) for \( i = 1, 2 \). Using the contestants’ first-order conditions, rearranging and dividing, we obtain:

\[
\frac{p_1(1 - p)}{-p_2} \geq \frac{U_1' k(U_2)}{k'(U_2) U_2' U_1}. \tag{8}
\]

Since \( k[U(B)] = U(B) = 0 \) and \( k(\cdot) \) is strictly concave the following inequality holds:

\[
k'(U_2) < k(U_2)/U_2. \tag{9}
\]

Using (9) and (8) we obtain:

\[
\frac{p_1(1 - p)}{-p_2} \geq \frac{U_2'}{U_1'} U_1. \tag{10}
\]

Now suppose that the less risk averse agent 1 puts at least as great effort in equilibrium as agent 2, so that \( x_1 \geq x_2 \) and \( p \geq 1/2 \). Then, it can be shown that the left-hand-side of (10) is less or equal than one (see Lemma A5 in Skaperdas, 1992). Consequently, the right-hand side of (10) must be strictly less than one.

\(^7\) The condition on the CSF is:

\[
p_{11} \rho \leq p_{12} (\text{and, by symmetry of the CSF, } -p_{22}(1 - p) \leq p_{21}).
\]

which is satisfied by all CSFs that are concave in each contestant’s effort, but it is also satisfied by CSFs that in some sense are not ‘too convex.’ The two functional forms in (1) and (2) satisfy the condition above regardless of the value of their respective parameters. The proof of this result is available on request and roughly parallels the Proof of Theorem 1 in Skaperdas (1992).
Given, though, that \( x_1 \geq x_2 \) we have \( U_2 = U(T + Y - x_2) \geq U(T + Y - x_1) = U_1 \) and \( U'_2 = U'(T + Y - x_2) \leq U'(T + Y - x_1) = U'_1 \) and, consequently, \( \frac{U'_1 U_0}{U'_2 U_1} \geq 1 \) which is a contradiction. Thus, \( x_1 \geq x_2 \) is impossible and the more risk averse agent 2 must exert greater effort in equilibrium. We state this result as a Proposition.

**Proposition 4**: Consider any pure-strategy equilibrium under the limited liability contest. Then, the more risk averse agent will always exert greater effort than the less risk averse agent and thus have a higher probability of success in equilibrium.

One way of intuitively grasping this result is to think of the effort expended by the two contestants as a form of insurance against losing, with the more risk averse contestant buying more insurance. More accurately, the form of insurance bought is self-protection and as shown in Konrad and Skaperdas (1993) limited liability produces a similar unambiguous effect for self-protection as well (see also Briys and Schlesinger, 1990).

Additional intuition can be gleaned by considering the following equation, obtained from the first-order condition of agent 1:

\[
\frac{U'_1}{U_1} = \frac{p_1}{p}. \tag{11}
\]

The right-hand side of (11) can be thought of as the percentage change in the probability of winning (or, alternatively, the probability of not losing) as a result of an infinitesimal change in effort. The left-hand side of (11) is a measure of boldness, and its inverse a measure of the fear of ruin, as analysed and interpreted by Aumann and Kurz (1977) (see also Roth, 1979, 1989); it measures an agent’s attitude toward risking all her fortune (in this instance \( T + Y - x_1 \)) against a small possible gain (relative to a ‘status quo’ point like \( B \)). A less risk averse agent is always bolder than a more risk averse agent. (Given that \( U(B) = 0 \); if \( U(B) \neq 0 \) then the measure of boldness should have this term subtracted from the denominator.) An expression similar to (11) can be derived for agent 2 and, thus, in equilibrium each agent equates her boldness to the percentage change in the probability of winning. The one who exerts greater effort can also be shown to face a lower percentage change in the probability of winning. Consequently, since boldness is equalised to this percentage change by each agent, the bolder agent will also put less effort. Since the bolder agent is also less risk averse, the less risk averse will put the lower effort.

The logic behind Proposition 4 is behind the following Proposition as well.

**Proposition 5**: Consider two situations with limited liability and identical agents: One in which both agents have Von Neumann–Morgenstern utility function \( U(\cdot) \) and one in which both agents have Von Neumann–Morgenstern utility function \( k[U(\cdot)] \). Assume a unique interior symmetric equilibrium in both cases. Then, equilibrium effort will be higher when the two agents are more risk averse.\(^8\)

\(^8\) Uniqueness of equilibrium (and, thus, a symmetric equilibrium when agents are identical) can be guaranteed under the following assumption on \( p(\cdot) \):

\[ p = \frac{f(x_2)}{f(x_1) + f(x_2)} \]

where \( f(\cdot) \) is a nonnegative increasing function.

We do not prove this result here, but for a proof following the same principle see Skaperdas and Syropoulos (1992). Equations (1) and (2) satisfy this assumption.

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Since increased risk aversion increases the equilibrium level of effort, if two identical contestants could collude and ‘choose’ their utility functions, they would like them to be as less risk averse as possible. When the efforts of the contestants do not add any social value, as in the case of arms races, increasing risk aversion reduces social efficiency. But when the efforts of the contestants represent transfers to third parties, as in the case of lobbying, or they are valued in their own right, as in an athletic tournament, the effect of increased risk aversion on efficiency cannot be assessed without additional information.

III. DIVIDING THE PRIZE

Up to this point we have assumed that one contestant wins the prize and the other one receives nothing. When the prize, however, is physically divisible and the environment allows the contestants to divide it, division may be a mutually advantageous arrangement from the contestants’ viewpoint. For example, two entrepreneurs competing for rights to import quotas may agree on a formula for sharing these rights, instead of letting the trade ministry decide on an all-or-nothing basis who receives the import rights and who does not. Similarly, two settlers embroiled in a water-rights dispute could settle peacefully on an allocation of these rights, instead of fighting it out until one drives the other out by force.

An agreed division of the prize, though, does not imply the absence of effort on the contestants’ part. Each contestant would like to establish a good bargaining position and the best way to do that is to have a credible fallback position in case negotiations fail. This fallback position would be \( \pi^* \) if the agents engage in the conventional contest of Section I or \( \pi^*(l) \) in the contest under limited liability; to establish either fallback position would require the expenditure of effort. Thus, each of the two entrepreneurs in the import quota case can be expected to have lobbied the trade ministry at least as a credible negotiation tool and, similarly, the two settlers can be expected to have acquired enough arms to be able to defend themselves in case of conflict.

In the words of bargaining theory the fallback positions are the disagreement or threat points. These points influence the division of the prize, but for each pair of threat points there will be many possible Pareto-improving divisions. We assume that for each pair of expected utility functions and disagreement points for the agents there is a function \( \alpha[p(x_1, x_2)] \) which gives the share of the prize received by agent 1 with agent 2 receiving the remainder \( 1 - \alpha(p) \) share. The simplest sharing rule would be to divide the prize according to the probabilities

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9 Existence of a pure strategy equilibrium is guaranteed for risk averse and risk neutral agents, but such an equilibrium with risk loving agents could obtain when the CSF is sufficiently concave.

10 If those who receive the benefits of the contestants’ effort are also responsible for designing the contest which in our case would mainly involve the choice of the value of the prize \( T \), a socially efficient outcome would be possible if the designers can take account of the level of risk aversion of the prospective contestants.

11 For additional examples pertaining to rent-seeking see Long and Vossen (1987), who also examine the case with a divisible prize. Their modelling and their focus, though, is different from ours. They introduce a (homogeneous of degree zero) function giving the probability of receiving a certain share of the prize for any given combination of effort, whereas we examine deterministic sharing rules which could be derived from the threat of engaging in a probabilistic context. Long and Vossen are primarily interested in the rent dissipation issue and we are interested in the effects of differential risk aversion.

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of winning, that is \( \alpha(p) = p \) and \( 1 - \alpha(p) = 1 - p \). But \( \alpha(\cdot) \) does not have to be symmetric and thus \( \alpha(1/2) \) may be more or less than \( 1/2 \). The rule should be common knowledge, increasing in \( p \) and it may be an arbitrary one or based on an axiomatic bargaining solution (see Roth, 1979, for an overview).\(^{12}\) Then, given a rule of division \( \alpha(p) \), the payoff functions of the two contestants are:

\[
\begin{align*}
\pi^1(\alpha) & \equiv \pi^1(x_1, x_2; \alpha) = U[\alpha(p) \ T + Y - x_1] \\
\pi^2(\alpha) & \equiv \pi^2(x_1, x_2; \alpha) = V[(1 - \alpha(p)) \ T + Y - x_2].
\end{align*}
\]

A pure strategy can easily be shown to exist when \( \alpha(\cdot) \) is concave and \( p(\cdot) \) is concave in each agent’s strategy. (Note that the latter is a more stringent condition than that for the contest with limited liability – please see footnote 7.) Now, consider an interior pure strategy equilibrium. At such an equilibrium the following conditions hold:

\[
\begin{align*}
\frac{\partial \pi^1}{\partial x_1} & = U'[\alpha(p) \ T + Y - x_1] \ [\alpha'(p) \ p_1 \ T - 1] = 0, \\
\frac{\partial \pi^2}{\partial x_2} & = V'[1 - \alpha(p)] \ T + Y - x_2] \ [\alpha'(p) \ (-p_2) \ T - 1] = 0.
\end{align*}
\]

Note that both contestants behave as if they were risk neutral. Risk preferences could influence the equilibrium efforts only through their potential indirect effect on the sharing function \( \alpha(p) \). In equilibrium, by \((14)-(15)\), we have \( p_1 = -p_2 \). As long as \( p(\cdot) \) is strictly concave in each agent’s strategy, this equality in turn implies \( x_1^* = x_2^* \). Consequently, in equilibrium the contestants would have equal probability of winning if, instead of dividing the prize according to the rule \( \alpha(p) \), they were to engage in a contest. By assumption, though, division of the prize according to the agreed upon rule would be Pareto superior.

In the case of the rule \( \alpha(p) = p \), the contestants would split the prize in half. We summarise these results in Proposition 6.

**Proposition 6:** In the game with the payoff functions in \((12)-(13)\) consider a unique interior equilibrium. (a) Then both contestants expend the same amount of effort in equilibrium. Moreover, conditional on the sharing rule they employ, they both behave as if they were risk neutral. (b) Suppose \( \alpha(p) = p \). Then, the contestants receive equal shares of the prize.

Since the agents behave as if they were risk neutral, in the case of the simple rule \( \alpha(p) = p \) and by following the logic of Proposition 5 the effort that would be collectively expended here would be lower than that expended in the case of the winner-take-all contest with limited liability (provided, of course, the agents are strictly risk averse). This result apparently is not generalisable to comparisons with the conventional contests we examined in Section I. It appears that for the example analysed in Proposition 2 the result would be reversed (i.e. the conventional winner-take-all contest with two identical risk averse contestants would yield lower equilibrium efforts than that by the same agents under bargaining with \( \alpha(p) = p \)).

\(^{12}\) It can be shown that under \((5)\) the Nash and Kalai-Smorodinsky bargaining solutions yield rules that depend on \( p \). In general, though, axiomatic bargaining solutions would depend directly on the values \( x_1 \) and \( x_2 \), in addition to the effects these variables have through \( p \). The results of the rest of this section qualitatively follow through with rules that satisfy this added complication.

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IV. CONCLUDING COMMENTS

As children we spend much time competing in school, play and for adult as well as peer attention. As adults we compete for mates, in work and for different types of recognition. Organisations like firms and states also compete with one another on various fronts. Outside the rent-seeking and R&D literatures, however, there has been little or no interest in formalising and examining these settings as contests, which we believe is a natural way of formalising them. In this paper, we set out to examine the effects of risk aversion in three variations of contests, with emphasis on the effects of differential risk preferences.

Given that a contest can be thought of as a multi-agent version of the problem of self-protection, and comparative static results regarding risk aversion in the case of self-protection are ambiguous, it is surprising that we found unambiguous results in two out of the three types of contests we examined. Among them, the limited liability contest is the most surprising for the fact that, in general, the more risk averse agent has a higher probability of winning the contest. The intuition that works in this case is that the more risk averse are more fearful of ruin, bankruptcy, and disaster and they thus put more effort into avoiding it.

Given this advantage of the more risk averse in this setting, is it possible that the more risk averse could self-select to participate in such contests while the less risk averse pursue safer routes? Could it ever be that those who become entrepreneurs or rent seekers are more risk averse than those who pursue less turbulent occupations? Certainly an affirmative answer to these questions would make us think twice about who takes risks and who does not in society; we see this as an important topic for future study.

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References

APPENDIX

We first state and prove a result used in the proof of Proposition 1.

**Lemma A1:** Suppose a twice differentiable function $f: [0, Y] \to \mathbb{R}$ with $f''(y) < 0$ if $f'(0) \leq 0 \forall y \in [0, Y]$. Then, $f$ is a quasiconcave function of its argument.

**Proof:** Consider three cases. (i) $f'(0) \leq 0$. In this case, since $f''(y) < 0$, $f$ must be decreasing in its argument. (ii) $f'(0) > 0$ and $f'(y) = 0$ for at least one $y \in [0, Y]$. Let $y^* \equiv \min \{ y \in [0, Y] : f'(y) = 0 \}$. It then follows that $f'(y) > 0 \forall y < y^*$ and, since $f''(y^*) \leq 0$, that $f'(y) \leq 0 \forall y \geq y^*$. (iii) $f'(y) > 0$ for all $y$. Overall, $f(y)$ is either monotonic ((i) and (iii)) or first increasing in $y$ and then decreasing (ii). In either circumstance quasi-concavity is implied since the function is defined on only one dimension.

**Proof of Proposition 1:** (a) The payoff functions are continuous (and differentiable) given that $\rho(\cdot)$ and the utility functions in (5) are differentiable. Our objective is to show quasi-concavity of each agent's payoff function in that agent's strategy. By Lemma A1, we just need to show that $\pi_{h}^{*} < 0$ if $\pi_{i}^{*} \leq 0$ for both $i = 1, 2$ (where the subscripts $i$ denote derivatives with respect to $x_i$). We will just show this for agent 1; the proof for agent 2 follows the same steps. Note that the second derivative of agent 1's payoff function is

$$\pi_{1}^{*} = \frac{p_{11}}{2} (U_h - U_i) + 2p_{1} (U'_{i} - U_{h}'') + p U''_{h} + (1 - p) U_{1}'', \tag{A1}$$

where $U_h \equiv U(T + Y - x_1)$ and $U_i \equiv U(Y - x_1)$. By (5) we then have

$$U'_{h} = \beta e^{-\rho(T + Y - x_1)}, \quad U'_{i} = \beta e^{-\rho(Y - x_1)}, \quad U''_{h} = -\beta e^{-\rho(T + Y - x_1)} \quad \text{and} \quad U''_{i} = -\beta e^{-\rho(Y - x_1)}.$$ 

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Substituting these expressions into (A1), we obtain:

$$\pi_{11}^1 = e^{-\rho(y-z_i)} \left[ -p_{11}(1-e^{-\beta T}) - 2p_{11}(1-e^{-\beta T}) + p_{11}(pe^{-\beta T} + 1 - p) \right]$$

(A2)

Now let

$$\pi_{11}^1 = -e^{-\rho(y-z_i)} p_{11}(e^{-\beta T} - 1) - \rho e^{-\rho(y-z_i)} (pe^{-\beta T} + 1 - p) \leq 0,$$

(A3)

which, since $-e^{-\rho(y-z_i)} < 0$, is equivalent to:

$$\beta(pe^{-\beta T} + 1 - p) \geq p_{11}(1-e^{-\beta T}).$$

(A4)

Substituting (A4) into the last term inside the brackets of (A2), we obtain (note $-e^{-\beta T} < 0$):

$$\pi_{11}^1 \leq -e^{-\rho(y-z_i)} \left[ -p_{11}(1-e^{-\beta T}) - 2p_{11}(1-e^{-\beta T}) + p_{11}(1-e^{-\beta T}) \right]$$

(A2')

$$= -e^{-\rho(y-z_i)} (1-e^{-\beta T}) (-p_{11} - p_{11} \beta).$$

By assumption we have $(-p_{11}/p_{11}) > \beta$ which implies $-p_{11} - p_{11} \beta > 0$ and thus $\pi_{11}^1 < 0$ when $\pi_{11}^1 \leq 0$, as required to complete the proof.

(b) The necessity of $(-p_{11}/p_{11}) \geq \beta$ at an interior equilibrium point can be established by noting that, at such an equilibrium point, (A3), (A4), and (A2') hold as equalities. Therefore, having $\pi_{11}^1 \leq 0$ would necessitate $(-p_{11} - p_{11} \beta) \geq 0$ and thus $(-p_{11}/p_{11}) \geq \beta$ as required. A similar argument can establish the necessity of $p_{11}/p_{11} \geq \gamma$. 

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