Efficiency of Thin and Thick Markets[1]

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Abstract

In this paper, we propose a matching model to study the efficiency of thin and thick markets. Our model shows that the probabilities of matches in a thin market are significantly lower than those in a thick market. When applying our results to a job search model, it implies that, if the ratio of job candidates to job openings remains (roughly) a constant, the probability that a person can find a job is higher in a thick market than in a thin market. We apply our matching model to the U.S. academic market for new PhD economists. Consistent with the prediction of our model, a field of specialization with more job openings and more candidates has a higher probability of matching.

Key words: matching function, market efficiency, empirical test.
JEL Classification: J6, J4

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1. Introduction

In this paper, we are interested in the following question: Compare two markets, one of which has five candidates and five openings in five firms (each firm has one opening), and the other of which has fifty candidates and fifty openings. Which market has a lower average unemployment rate or a higher probability of successful match? The market with a lower unemployment rate is said to be more efficient than the one with a higher unemployment rate.

To answer this question, we set up the following model. Firms are heterogeneous in types, and job candidates have heterogeneous productivities. A match between a firm and a candidate is assortive such that the firm is willing to hire any candidate with a productivity higher than its type, but prefers the candidate with a higher productivity. A candidate is willing to accept any offer but prefers a higher-type firm. Further, the types and productivities are randomly drawn from a common distribution, and thereupon to become public information.

For such a market, we prove: (1) the matching probability does not depend on the underlying distribution that types and productivities are drawn upon. (2) When the numbers of firms and candidates increase or the market becomes thicker, the unemployment rate decreases or the matching probability increases. As the number of firms and candidates approach to infinity, the matching probability approaches to 1. (3) The matching probabilities decreases when numbers of firms and candidates both increase. (4) The conclusion that a thicker market has a larger matching probability than a thinner market continues to hold for the following three more general cases: first, the number of openings does not equal the number of candidates; second, the firm type and the productivity of a candidate are random draws from different distributions; and finally, the firms and candidates arrive at the market sequentially.

Information on the U.S. academic market for new PhD economists is used to test our model. We gather the American academic job openings listed for each field in the September, October, November and December issues of *Job Openings for Economists* (JOE), in both 1999 and 2000. In the following year we find out how many of these openings are filled. The ratio of the total number of filled jobs divided by the openings in each field is the measure of the probability of job matching. CVs of all job candidates from the top 50 departments of economics in the U.S. universities in year 2000 and 2001 are also collected from each university's placement website or candidates' own websites.

The empirical estimates support our theoretical hypothesis: a thicker market does have a higher matching probability than a thinner market. In particular, in the new PhD markets in economics, when the numbers of openings and candidates are five in one field, the matching probability is .361. When the numbers of openings and candidates are fifty, the matching probability is .523.

Previous literature often uses changes in unmatched probabilities such as vacancy rates or unemployment rates to empirically estimate matching functions.[2] Using the market data for PhD economists offers several advantages over regular job markets. First, there is less of an information problem in this market in the sense that each institute receives applications from almost all potentially qualified job candidates, and almost all job openings are well known to all candidates, as they are published in a single magazine JOE. Second, there is a reasonable consensus in terms of the ranking of a job, i.e., a job in a better-ranked department is considered by most to be a better job. Third, there is some consensus in terms of the ranking of candidates, although significant heterogeneity still exists.

Although intuitively a thicker market with more openings and candidates has a larger probability of matching, there is no consensus in the literature regarding this intuition. For example, a thicker market has an adverse effect in Burdett, Shi and Wright (2001), has no effect in Lagos (2000), and has a positive effect in Coles and Smith (1998). The different conclusions of these papers result from different matching technology. In Diamond (1982) and Howitt and McAfee (1987), the thickness of a market has externality on itself. If buyers expect that fewer sellers exist in the market, the expected higher cost of transaction discourages buyers from entering into the market. When the same conditions are applied to sellers, low economic activities are expected. Howitt and McAfee (1988) show that it is possible to have multiple externalities (one positive and one negative. These discussions assume that the transaction cost is higher or the probability of matching is smaller in a thinner market. This assumption is intuitively appealing but was neither formally modeled nor empirically verified.

The effect of "thickness" in the market has been studied extensively in the microstructure literature in finance under the term "liquidity." For example, in Lippman and McCall (1986), a thicker market indicates that more transactions of a homogeneous good take place in a unit of time. In their paper, liquidity is defined in terms of the time elapsed between transactions. This length of

[2] See, for example, Blanchard and Diamond (1989) and Berman (1997).

time is a function of a number of factors, including the frequency of offers and the flexibility of prices, among others. In an empirical study on common factors that affect liquidity, Chordia, Roll, and Subrahmanyam (2000) use five liquidity measures including the difference in prices offered by buyers and sellers, and in quantities offered by buyers and sellers in a period of time. In their approach, the smaller the difference between the prices and the larger the quantities offered by the buyers and sellers of a homogeneous good (an equity), the more liquid a market is. One distinguishing feature in financial markets is that buyers and sellers often arise endogenously. If prices are low, potential sellers easily become buyers. In the labor market, it is hard for workers to become employers or vice versa. Therefore, the pool of employers and employees is often exogenously determined.

Since our model relates the matching probability with the thickness of the market, it provides a matching function with a microfoundation. The importance of the matching function has been discussed in a survey paper by Petrongolo amd Pissarides (2001). They claim that both the matching function and the demand-for-money function are as important as the production function as a tool kit for macroeconomists. However, as stated in Petrongolo and Pissarides (2001), most of the existing matching functions lack well-received microfoundation. This paper provides a matching function with microfoundations and the matching function is shown to perform reasonably well for the empirical data we collected.

This rest of the paper is organized as follows: Section 2 introduces the matching model and the basic implications of the model. Section 3 presents the empirical test of the model using the data collected from the U.S. academic job market for new PhD economists. Section 4 concludes the paper.

2. The Model

2.1. The matching mechanism

Let u be a measure of the productivity of a job candidate, and v be a firm's type. The match between the firm and the candidate is assortive, with a production function given by $f(u,v)=uv$, as suggested in Lu and McAfee (1996). The cost of hiring a worker consists of wage and other costs, such as the cost of capital stock that is associated with this opening. Both are assumed to be proportional to the firm type v. The total cost is given by kv^2, where k is a constant. Therefore, the firm's profit function is:

$$\pi(u,v) = \max\{0, uv - kv^2\}. \quad (1)$$

In this simple model, the firm will hire a candidate if $u > kv$, and it prefers a candidate with a higher productivity than one with a lower productivity.

Without loss of generality we assume that v takes value in $[0, C]$ for some $C > 0$. We assume a candidate's utility function is:

$$w(u,v) = \max\{\delta u, v\}. \quad (2)$$

The candidate will accept all job offers as long as $v > \delta u$ but prefers the firm with higher v than one with a lower v. Here we let the candidate's reservation wage δu be dependent on his ability index u. Equations (1) and (2) present necessary conditions for a match:

$$\frac{1}{k}u > v > \delta u$$

In the benchmark case, we let $k = 1$ and δ to be small that $v > \delta u$ holds. In this case, the firm will only hire a worker if $u > v$. The firm type v can be thought as the firm's minimum quality requirement. In Section ???, we will discuss variations of the benchmark case.

The matching technology between firms and candidates defined in (1) and (2) is similar to the matching mechanism between hospitals and medical interns or residents as described in Roth (1984). In Roth (1984), almost all medical interns would find jobs because of a large demand for their services. Therefore, Roth (1984) and the subsequent literature do not put much emphasis on matching probabilities.[3]

Our primary goal here is to examine how the matching probability varies with the number of job candidates and the number of openings. We consider the problem of V firms and U job candidates. Let the firms' types be $v_1,\ldots,v_V$, and job candidates' productivity be $u_1, \ldots, u_U$. We assume that all productivities and job types are randomly drawn from a common continuous distribution $F(\cdot)$ so that no productivity is exactly the same with probability 1 as any job types. All candidates' u_i's and all firm types v_j's are assumed to be known after they are drawn.

According to our model described by equations (1) and (2), a candidate i may be hired by firm j only if the productivity of the applicant u_i is higher than the type of the firm v_j. If more than

[3] In a more recent paper, Niederle and Roth (2003) study the matching probability of the gastroenterologists market. They find matching probability increases when the market becomes thicker (through a centralized clearinghouse).

one applicant has a higher productivity than the firm type v_j, firm j hires the candidate with the highest productivity. Similarly, if more than one firm has lower types than applicant i's productivity u_i, the applicant prefers the firm with the highest type. After a match occurs, both the firm and the candidate are out of the market. The process continues until no candidate has a higher productivity than any of the remaining firm types.

In the academic market, a better department is preferred by all new PhDs. The constraint in the market is that each department is different in its type and a better department has a higher v. Each department prefers the candidate with the highest quality that satisfies their type. In this case, a necessary condition for a trade to occur between candidate i and department j is $u_i > v_j$.

In this matching mechanism, the job candidate who has the highest productivity matches with the firm with the highest type, provided that this highest ranked candidate meets the type of the firm. Otherwise, the firm leaves the market without filling its opening. However, the applicant who does not match with the highest-type firm has additional chances to match with other firms. This matching process repeats in the remaining pool of the applicants and firms.

An alternative way to describe our matching technology is as follows. First we sort all the randomly drawn productivities and types. Then a job candidate with the highest productivity matches with the firm with the highest type, as long as the productivity is higher than the type. If a match occurs, both the candidate and the firm leave the market. This process is repeated until no firm's types are lower than any remaining candidates.

Consider order statistics $v_{(1)} < v_{(2)} < ... < v_{(V)}$ and $u_{(1)} < u_{(2)} < ... < u_{(U)}$ that are obtained from $v_1,...,v_V$ and $u_1,...,u_U$, respectively. We are interested in the probability that a randomly chosen candidate can find a job. Let r be the number of people that find jobs, $0 \le r \le n = \min\{V, U\}$. Let $\Pr(r)$ be the probability that exactly r candidates find jobs. The average or expected value of r is given by:

$$M_{U,V} \equiv E(r) = \sum_{r=0}^{n} r \Pr(r) \tag{3}$$

In *Lemma* 1 below, we give the exact probability for a particular order statistic. This lemma is useful for us to obtain matching probabilities.

Lemma 1: Let $u_{(1)} < u_{(2)} < ... < u_{(U)}$ be the order statistic obtained from i.i.d. data $u_1, u_2, .., u_U$, and $v_{(1)} < v_{(2)} < ... < v_{(V)}$ be the order statistic obtained from i.i.d. data $v_1, v_2, ..., v_V$. The productivity index u_i and firm type v_j have the common distribution $F(\cdot)$ with pdf $f(\cdot)$. Let $z_{(1)} < z_{(2)} < \ldots < z_{(U+V)}$ denote the order statistic obtained from $u_{(1)}, \ldots, u_{(U)}, v_{(1)}, \ldots, v_{(V)}$. There are $(U+V)!/(V!U!)$ such orderings. Let $\boldsymbol{Z_n}$ denote the random variable $z_{(1)}, \ldots, z_{(U+V)}$. Then for any one particular order z, we have

$$\Pr(\boldsymbol{Z_n} = z) = \frac{U!V!}{(U+V)!}. \tag{4}$$

The proof of *Lemma* 1 is given in Appendix A1. In *Lemma* 1, the probability of any particular order statistic z does not depend on the underlying distribution of candidates and openings. Since the overall matching probability involves accounting the number of appropriate orderings, it does not depend on underlying distributions. In the following sections, we apply *Lemma* 1 to study how the matching probabilities vary with the number of openings and the number of candidates. We first discuss the case where the number of openings and the number of candidates are the same, and then we proceed with the case where they are different. Note a critical assumption of the lemma is the independence between u and v.

2.2 When the number of openings equals the number of candidates

Our primary interest in this paper is to study how the matching probability changes when the number of openings and the number of candidates change. Our discussion starts with the case where the number of applicants is the same as the number of openings. Let $n = V = U$, and we write $M_{n,n} = M_n$. We denote by A_n the probability that a randomly selected person can find a job (when $V = U$), i.e.,

$$A_n = \frac{1}{n} M_n = \frac{1}{n} \sum_{r=1}^{n} r \Pr(r).$$

We investigate below how A_n changes as n changes. We build our model from the simplest case where there is one firm and one job candidate.

The case of n = 1:

Let u and v be randomly drawn from the same distribution $f(\cdot)$. A match occurs if and only if $u > v$.

$$A_1 = \int_{v<u} f(v)f(u)dvdu = \int_{-\infty}^{\infty}\left[\int_a^u dF(v)\right]dF(u) = \int_{-\infty}^{\infty}\left[F(u)dF(u)\right] = 1/2.$$

In this simple the case of one applicant and one job opening, given that both u and v are randomly drawn from the same distribution the probability that one random draw is larger than the other is 1/2.

The case of *n* = 2:

Let (u_1, u_2) be random draws of the two candidates' productivities, and let (v_1, v_2) be random draws of the types of two job openings. All are from the same distribution. Let $u_{(1)} < u_{(2)}$ be the order statistic of (u_1, u_2) and $v_{(1)} < v_{(2)}$ be the order statistic of (v_1, v_2). Using *Lemma* 1 we have:

$$\begin{aligned}
\Pr(0) &= \Pr\left(u_{(1)} < u_{(2)} < v_{(1)} < v_{(2)}\right) = (2!)^2(1/4!) = 1/6 \\
\Pr(2) &= \Pr\left(u_{(2)} > v_{(2)}, u_{(1)} > v_{(1)}\right) \\
&= \Pr\left(u_{(2)} > v_{(2)} > u_{(1)} > v_{(1)}\right) + \Pr\left(u_{(2)} > u_{(1)} > v_{(2)} > v_{(1)}\right) \\
&= 2\left[(2!)^2(1/4!) = 1/6\right] = 1/3 \\
\Pr(1) &= 1 - \Pr(0) - \Pr(2) = 1 - 1/6 - 1/3 = 1/2
\end{aligned}$$

Therefore we have:

$$A_2 = \frac{1}{2}\sum_{r=1}^{2} r\Pr(r) = [1/2 + 2\times 1/3]/2 = 7/12$$

We observe that $A_2 = 7/12 > 1/2 = A_1$. That is, when the market becomes thicker (n increases from 1 to 2), the probability that *each* person can find a job is increased from 1/2 to 7/12. To understand the intuition of this result, note that since u_1, u_2 and v_1, v_2 are from the same distribution, the order statistics also have the distribution: $F_{u_{(1)}}(\cdot) = F_{v_{(1)}}(\cdot)$ and $F_{u_{(2)}}(\cdot) = F_{v_{(2)}}(\cdot)$. Given this, we have:

$$\Pr\left(u_{(1)} > v_{(1)}\right) = 1/2 \text{, and } \Pr\left(u_{(2)} > v_{(2)}\right) = 1/2, \tag{5}$$

If (5) were the only cases that candidates and openings match, we would still end up with a matching probability of 1/2. However, an additional chance exists even when $u_{(1)} < v_{(1)}$ and $u_{(2)} < v_{(2)}$ since it is still possible to have $u_{(2)} > v_{(1)}$. This additional chance of matching is the source of the effect of a thicker market.

In Appendix A3 and A4, we calculate the matching probabilities for $n = 3$ and $n = 4$. Although the same approach can be applied to compute A_n for any $n > 4$, computation is increasingly burdensome and tedious as n increases. A simple alternative is to use simulations to numerically compute A_n. Let $A_{n,j}$ be the estimated value of A_n based on the jth simulation draw of (u_1,...,u_n) and (v_1,...,v_n), i.e., $A_{n,j}$ equals the number of people finding jobs in the jth random draw. We estimate A_n by $\overline{A}_n = \sum_{j=1}^{J} A_{n,j} / J$. Provided that J is sufficiently large, we can obtain an estimated value of A_n with any desired accuracy. We use J = 100,000 in our simulation. We also compute the sample standard error of $\{A_{n,j}\}_{j=1}^{J}$ by $\left[\sum_{j=1}^{J} \left(A_{n,j} - \overline{A}_n\right)^2 /(J-1)\right]^{1/2}$. The results are given in Table 1.

We have already shown that $A_1 = 0.5$, $A_2 = 7/12 \approx 0.5833$. In the appendix we also compute the exact values of A_n for $n = 3, 4$; they are $A_3 = 19/30 \approx 0.6333$ and $A_4 = 187/280 \approx 0.6679$. Comparing these results with the simulation results of Table 1, we see that the simulation results differ from the theoretical results only in the fourth decimal.

From Table 1, we observe that A_n increases as n increases, while the standard error decreases as n increases. The monotonically increasing relationship between matching probabilities and the thickness of the market can also be clearly seen in Figure 1. The solid line in Figure 1 illustrates the matching probabilities as a function of the number of candidates. As $n \to \infty$, both the candidates and openings become dense in the support of $f(\cdot)$. Therefore, the probability of matching is expected to converge to one as $n \to \infty$. This is indeed the case as the next lemma shows.

Lemma 2: The employment rate or the matching probability A_n converges to one as $n \to \infty$.

The proof of *Lemma* 2 is given in the appendix. Note that *Lemma* 2 does not mean that as $n \to \infty$, every individual will find a match. In fact the total number of unmatched candidates, calculated by $n(1- A_n)$, also goes up as n increases. For example, when n = 10, 100 and 1000, the average numbers of unemployed workers are roughly 2, 8, and 30, respectively. It is $1 - A_n$, the *percentage* of unemployed (the unemployment rate) that goes down as n increases.

Our theoretical analysis and simulation results show that: (1) A thicker market provides a larger chance of matching; (2) the probability of matching varies less in a thicker market than in a thinner market.

Previous results are obtained by assuming that the types of firms and candidates' productivities have the same distribution. Next, we briefly discuss the case that they have different distributions. We show that in this case the matching probability will depend on the specific distribution functions, but a thicker market still has a larger probability of matching.

We consider the simple case where candidates are randomly drawn from *Uniform*[0,1], and the openings are randomly drawn from *Uniform* [δ, 1+δ], $0 \le \delta \le 1$. We will only consider the case of $V = U = n$. In Section A.5 we show that:

$$A_1 = \frac{1}{2}(1-\delta)^2,$$
$$A_2 = \frac{7}{12}(1-\delta)^2 + \frac{1}{12}\delta(1-\delta)^2(2+3\delta). \tag{6}$$

Obviously, $A_2 > A_1$ for all $\delta \in [0,1]$. A thicker market still has a larger probability of matching. For $n > 2$, the computation becomes quite tedious. However, one can use simulations to compute A_n easily for any value of n. Figure 2 illustrates how the simulated matching probabilities vary with n and with δ. Two patterns emerge from Figure 2. First, as expected, a larger difference in means results in lower matching probabilities. Second, for a fixed value of δ, the matching probabilities increase as the market becomes thicker. By exactly the same argument as in the proof of *Lemma* 2, one can show that as $n \to \infty$, $A_n \to 1- \delta$, where $0 < \delta < 1$.

Note that when $\delta \ge 1$, $A_n = 0$ for all n, because in this case the highest seller's productivity is lower than the lowest firm's type. However, if the two distributions are $N(\mu, \sigma^2)$ and $N(\mu + \delta, \sigma^2)$ where the two means also differ by δ, then $A_n > 0$ for all values of δ. This simple example shows that when the two distributions are different, the matching probability will depend on the specific distributions.

2.3 The number of firms is different from the number of candidates

In the previous section, we only focus on the case where the number of candidates equals the number of openings. In a real market, it is unlikely that there will be exactly the same number of candidates and openings. In this section, we consider cases where the number of candidates is different from the number of openings. They are still random draws from a common distribution.

Let U be the number of candidates and V be the number of openings. The number of people who find jobs, r, must satisfy $0 \le r \le n = \min\{U,V\}$. Recall that the expected value of r is:

$$M_{U,V} = E(r) = \sum_{r=0}^{n} r \Pr(r). \quad (7)$$

Here, we summarize some properties of matching functions. (i) $M_{U,V} = M_{V,U}$ is symmetric in V and U, (ii) $M_{U,V}$ increases as either V or U increases, (iii) if both V and U increase with $V/U = a$, a fixed positive constant, then $B_{U,V} = M_{U,V}/V$ increase as V ($U=V/a$) increases.

(i) follows from a simple symmetry argument. (ii) is true because adding more candidates or openings to a market obviously cannot reduce the number of matching; in fact, there is a positive probability of increasing the number of matching, thus the average matching of $M_{U,V}$ will be larger (for any finite values of V and U. (iii) is the most interesting result: it says that when the market becomes thicker, the probability of matching success increases for both candidates and openings. The intuition behind (iii) is quite simple. We have already seen that this is true for the case of $V = U = n$. In Appendix A6 we show how to compute $M_{U,V}$ (or $B_{U,V}$) for the general (U, V) case. For example, for (U, V) = (1, 2) (or (2, 1)), $M_{U,V}$ = 2/3; for (U, V) = (1, 3), $M_{U,V}$ = 3/4; and for (U, V) = (2, 4), $M_{U,V}$ = 23/15. First we note that $B_{1,2} = 2/3 < B_{2,4} = (1/2)(23/15) = 23/30$, so that as the number of V and U doubles (the market becomes thicker), the matching probability increases. Next we compare the case of (U, V) = (1, 3) and (2, 2), where we have $M_{2,2} = 7/6 > 3/4 = M_{1,3}$. With the *same* total number of openings and candidates, the closer the ratio of V/U is to 1, the higher the averaging number of people that can find jobs.

Again, a simple alternative is to use simulations to estimate $M_{U,V}$ ($B_{U,V}$). We will use the simulation method to help us evaluate some of our proposed matching functions in the next section.

2.4 A matching function

Because our model relates the matching probability with the thickness of the market, it can provide a matching function with a microfoundation. A series of matching functions has already been introduced in the literature; here we briefly discuss some of the existing matching functions and compare them with our matching function.

In a typical matching model with constant return to scale, the thickness of the market does not enter the matching probability. The relationship between the number of people who are looking for jobs and the number of people who find jobs is different from our claim that market thickness has a positive effect on the job matching ratio. For example, consider a typical matching model with constant return to scale,

$$M = m(U,V) = Vm\left(\frac{U}{V},1\right),$$

where *m(U,V)* is the matching function, *M* is the number of people who find jobs, *V* and *U* are numbers of job openings and job searchers. The second equality of the previous equation is due to the assumption of the constant return to scale. Rearranging the previous equation, we get:

$$B_{U,V} = \frac{M}{V} = m\left(\frac{U}{V},1\right), \tag{8}$$

where $B_{U,V}$ is firms' matching probability. If the ratio of candidates to openings is fixed, so is the matching probability *M/V*. A particular form of constant return to scale function is $M/V = 1 - e^{-aU/V}$, which is used in Blanchard and Diamond (1994) where *a* is the intensity of the search. Other interesting works related to our matching model include Burdett, Shi and Wright (2001) and the stock-flow matching of Coles and Smith (1998).

It would be ideal if one could derive an explicit functional form to relate matching probabilities with the thickness of the market. While this goal may be quite difficult to accomplish, we are able to propose a parsimonious approximate matching function which satisfies some basic properties of the theoretical matching function. We will show that this approximate matching function can fit the theoretical matching probabilities very well. We are interested in obtaining a probability matching function, say $B_{U,V} = M_{U,V}/V$. However, it is easier to impose restrictions on the matching function $M_{U,V}$. We first list some of the properties that $M_{U,V}$ should preserve.

(i) $M_{U,V}$ is symmetric on (*U*, *V*).

(ii) For any finite values of (*U*, *V*), $M_{U,V} < \min\{U, V\}$, and $M_{U,V}$ is an increasing function in *U*(*V*) for a fixed value of *V*(*U*).

(iii) Let $d = \sqrt{U^2 + V^2}$ denote the distance of (*U*, *V*) to the origin. For $(U,V) \in R_+^2$ with d = c, a constant, $M_{U,V}$ is monotonically decreasing as d (*U*, *V*) moves away from the middle point of *V* = *U* (along the arc of *d* = *c*).

The following simple matching function satisfies the above three conditions:

$$M_{U,V}^{(0)} = \alpha_0 + \alpha_1 \min\{U,V\} + \frac{\alpha_2}{d}, \tag{9}$$

where $d = \sqrt{U^2 + V^2}$ and α_0, α_1 and α_2 are parameters (α_1 is positive and α_2 is negative.

It is obvious that $M_{U,V}^{(0)}$ in (9) satisfies properties (i) and (ii) above. To see that it also satisfies (iii), note that when d = c is a constant,

$$M_{U,V}^{(0)} = \alpha_1 \min\{U,V\}\big|_{d=c} + \frac{\alpha_2}{c},$$

which decreases monotonically as (U, V) moves away from the middle point of $U = V$ (along the arc of $d = c$).

By rearranging (9) in terms of matching probability, we get

$$\frac{M_{U,V}^{(0)}}{V} = \alpha_0 + \alpha_1 \min\left\{\frac{U}{V},1\right\} + \frac{\alpha_2}{Vd}. \tag{10}$$

If we replace $\alpha_2/(Vd)$ by α_2/d (removing the $1/V$ factor) in (10), we obtain the following alternative approximate matching function:

$$\frac{M_{U,V}^{(0)}}{V} = \alpha_0 + \alpha_1 \min\left\{\frac{U}{V},1\right\} + \frac{\alpha_2}{d}. \tag{11}$$

Interestingly we observe that (11) performs better that model (10) using both theoretical (simulated) matching probabilities and the empirical data. A more flexible model than (11) is

$$\frac{M_{U,V}^{(0)}}{V} = \alpha_0 + \alpha_1 \min\left\{\frac{U}{V},1\right\} + \frac{\alpha_2}{d^{\alpha_3}}. \tag{12}$$

When $\alpha_3 = 1$, (12) reduces back to (11). To examine how well our proposed matching functions approximate the theoretical (simulated) matching function, we carry out a least squares regression, using (simulated) theoretical values of $B_{U,V} = M_{U,V}/V$ as the dependent variable, and estimate models (10), (11), and (12). The regression results for models in (10) to (12) are reported in Table 2.

As can be seen in Table 2, our specifications can explain the (simulated) theoretical matching probability well, with R^2 being at least .937. The R^2 is .951 for model (11) and .957 for model (12). The results show that all of the proposed models fit the theoretical model very well. However, this does not imply that one should expect that all of them should fit empirical data equally well. As we will see shortly, model (12) provides the best fit for the empirical data we collected.

2.5 A sequential matching mechanism

Up to now we have only considered a static model where all candidates and openings arrive at the market simultaneously. In this section we briefly discuss the case of a sequential matching model. Our approach follows closely that of Coles and Smith (1998) who capture a realistic feature of market search, that if a job seeker cannot match with the exiting pool of vacancies, he/she will wait for the arrivals of new job vacancies. We consider two extreme cases: (i) All matched pairs can break up an earlier match and re-match in a later period without a cost. The time discount rate is zero. (ii) Both the re-match cost and the time discount rate are infinite.

A zero re-matching cost and a zero time discount rate

It is easy to see that in this case the results of sections 2.2 and 2.3 remain valid without changes. Suppose at period t, we have a cumulative of U_t job candidates, and a cumulative of V_t vacancies, the number of matches will be exactly the same as in the static case with a total number of U_t candidates and V_t vacancies. This is because all matched pairs can freely break up with earlier matches and to find the best match available to them. The highest quality individual will match with the best job available provided her quality meets the type of that job. The second highest quality individual will match with the next best available job. Consequently, the matching results will be identical as in the static case with the same total numbers of candidates and vacancies.

An infinite re-matching cost and an infinite time discount rate

When both the cost of re-entering the market and the time discount rate are infinite, all firms and individuals will try to find a match as soon as possible, and when a match is found, the matched pair will exit the market. Although these assumptions are not realistic, they serve as a benchmark case and from which we can deduct useful information on the more realistic finite re-matching cost/discount factor cases.

We will only consider the case where the number of candidates equals the number of openings. This will be the case if candidates and openings arrive at the market in pairs so that the total numbers of candidates and openings equal each other at all times. We assume that different pairs arrive at the market sequentially. If the first pair of candidate and opening matches with each other, they will sign a contract and exit the market. If not, they become stock and wait for matching opportunities among future arrivals. When a pair (a candidate and an opening) arrives at the market, the two can match with each other or match with the existing stock, according to whichever gives the higher utility. If no match is found, they become stock.

Let $n = V = U$ denote the total number of openings and candidates. We use $\overline{P}(r)$ to denote the probability that exactly r people find jobs, and use $\overline{A}_n = n^{-1}\sum_{r=0}^{n} r\overline{P}(r)$ to denote the mean value of r (the bar notation is used to emphasize a sequential matching process).

The case of *V* = *U* = 1.

In this case we have $\overline{P}(U > V) = 1/2$ as before, which gives $\overline{A}_1 = \overline{P}_1(1) = 1/2$.

The case of *V* = *U* = 2.

Let $u_j(v_j)$ be the *j*th arrival of candidates(openings), j=1, 2. Because candidates and openings arrive sequentially, we cannot use the order statistics to compute $\overline{P}(r)$. However, the result obtained earlier can help calculate the matching probabilities.

The total number of different rankings of u_1, u_2, v_1 and v_2 is 4! = 24 (four times that of the order statistic case). We can use the calculation of Pr(r) to help us to obtain $\overline{P}(r)$. For example, in the market of simultaneous arrival, the order statistic that no one finds a job is: $u_{(1)} < u_{(2)} < v_{(1)} < v_{(2)}$, and the probability is Pr(0) = 1/6. In the market of sequential arrival, there are four cases that no one finds a job: (i) $u_1 < u_2 < v_1 < v_2$, (ii) $u_2 < u_1 < v_1 < v_2$, (iii) $u_1 < u_2 < v_2 < v_1$, and (iv) $u_2 < u_1 < v_2 < v_1$, giving $\overline{P}_1(0) = 4/24 = 1/6$. So the probability that no one finds a job remains unchanged.

There is only one case that results in different matching probabilities between a sequential market and a simultaneous market. In the case of $v_1 < u_2 < v_2 < u_1$, u_1 will match with v_1 and then (u_1, v_1) exit the market. In the second period, v_2 and u_2 arrive at the market but they cannot match because $u_2 < v_2$. If the two pairs had arrived simultaneously, there would be two matched pairs, u_1 with v_2, and u_2 with v_2. So we see that when arrivals are sequential, the matching probability decreases and the market becomes less efficient. Using (4), we obtain:

$\overline{P}(0) = 4/24 = 1/6$ (=4/24 as in the simultaneous arrival case).

$\overline{P}(1) = (12+1)/24 = 13/24$ (it was 12/24=1/2 in the simultaneous arrival case).

$\overline{P}(2) = (8-1)/24 = 7/24$ (it was 8/24=1/3 in the simultaneous arrival case).

Thus, $\overline{A}_2 = (1/2)\sum_{r=0}^{2} r\overline{P}(r) = [(13/24) + 2*(7/24)]/2 = 27/48 > 1/2 = \overline{A}_1$.

We still observe that as the total number of candidates and openings goes up, the average matching probability increases. However, $\overline{A}_2 = 27/48 < 28/48$, the market of sequential arrival is less efficient compared to the case that all the candidates and openings arrive simultaneously, which is an expected result since sequential trading may lead to a very high quality candidate to match with a vacancy with a very low type, resulting in a less efficient market.

Let $\Pr[(u_i, v_j)]$ denote the probability that u_i matches v_j. Then conditional on $u_1 < v_1$ (so that u_1 and v_1 become stock), it is easy to show that $\Pr[(u_2, v_2)] = 1/3 > \Pr[(u_2, v_1)] = 1/4$. Thus, our matching mechanism implies that u_2 has a lower matching probability, or higher rejection rate, when meeting with v_1 (an opening from the stock) than when meeting with v_2 (a random draw from the distribution of job openings). This is because the openings from the stock have a higher mean value than those drawn from the population. As is easily shown, our model implies that the rejection rate between a candidate from the flow and an opening from the stock increases as the size of the stock increases, or equivalently as the averaging matching probability increases (as argued by Petrongolo and Pissarides (2001, p.406)). This is because the mean of the stock of openings goes up as its size goes up, resulting in a higher rejection rate for a given pair. Nevertheless, the average matching probability still goes up since there are more matching opportunities as the market gets thicker. It differs from the matching mechanism of Coles and Smith (1998) who assume that a firm has a constant probability to match a candidate.

Table 3 reports the simulated values of $\overline{A}_n$ for n from 1 to 1,000 (based on 100,000 replications). Since $\overline{A}_2 = 27/48 = .5625$, we see again that the simulated value matches the true value in the first three decimals.

In Table 3, we observe similar phenomena in the case of simultaneous trading, i.e., $\overline{A}_n$ increases (with a decreasing rate) while the standard deviation of $\overline{A}_n$ decreases as n increases. The dashed line in Figure 1 shows that, as expected, the $\overline{A}_n$ curve is lower than the solid line of the A_n curve. A larger friction exists in a market of sequential arrivals.

Even though sequential arrival has higher friction, the conclusion that a thick market is more efficient than a thin market remains the same as in the case of a simultaneous arrival. Further, it can be shown that $\overline{A}_n \to 1$ as $n \to \infty$.

So far we consider two extreme cases: zero time discount rate and re-match cost *vs*. infinite time discount rate and re-match cost. Complete discussions of more realistic cases where the time discount rate and the re-match cost are some finite positive numbers are left for future research.

We do not yet consider searching cost in our model. Adding a fixed searching cost will not alter any of the conclusions obtained earlier. A variable searching cost may reduce matching probability. Given the rapid improvement of internet searching, it seems that a fixed search cost is appropriate for most situations such as economics new PhD's market. We leave more detailed discussions on extensions such as strategic trading behavior and variable searching cost to future research work.

3. Data Collection and an Empirical Test

Empirical study of this issue can be very difficult. It is relatively simple to collect information about successfully completed transactions in a particular market. However, gathering data about the participants who failed to complete transactions is often rather difficult.

The job market for new PhD economists, therefore, provides an excellent opportunity for exactly such an empirical study of thin and thick markets' performances. First, we must identify the levels of supply and demand for this market. The information we require to determine market demand is available through the journal *Job Openings for Economists* (JOE). The problem of information asymmetry is minimized when we consider the job market in economics because JOE provides virtually complete information sets for the supply of the academic jobs in the U.S. In other labor markets, we often do not know what specific information sets job applicants can access; but in this case we do because the journal is widely available to candidates going on the job market. In addition, we may determine the level of market supply by contacting graduate programs in economics regarding their PhDs who have gone on the job market in the past several years.

Our data is organized by field. The definition of the field can be found in the "Classification System of Journal Articles" by the *Journal of Economic Literature*. In particular, we use the field consisting of a capital letter and a numeral. For example, E0 means “Macroeconomics and Monetary Economics.”

We collect the American academic job openings listed in the September, October, November, and December issues of JOE in 1999 and 2000. In JOE, m job openings are listed with *n* fields where *m* and *n* are integers. We determine that each field has average openings of m/n. For example, in 2000, the Department of Economics at the Texas A&M University had five openings in nine different fields. We assign each field with 5/9 openings. We sum all the average openings for all American universities by each field. We then find out how many of these openings are filled by going to each institute's website and/or by contacting relevant people. The ratio of the total number of filled jobs divided by the average openings in each field is the measure of the probability of job matching.

In addition, we collect the job candidate information. We search the links of job candidates in each of the top-50 departments in the U.S. defined in Dusansky and Vernon (1998). We use the first field listed in each candidate's CV or in the brief introduction of a candidate if no CV is available.

The summary information of the markets is listed in Table 4. In addition to showing the ten fields with the most job openings in the table, we include any field (AF), the mean of the remaining fields not listed in the table, and the whole market. In both years, AF is by far the largest ``field." Macroeconomics (E0), Microeconomics (D0), and International Economics (F0) were the top three fields other than AF in both years. In 1999, the mean of the matching probabilities in the ten fields with the most job openings is .501, while the mean of the matching probabilities in the remaining fields is .305. Thicker fields do have larger matching probabilities than thinner fields. The same pattern repeats in 2000 where the mean of the matching probabilities for the ten fields with the largest demand is .451, while the rest of the fields have an average matching probability of .268.

We estimate our proposed matching models, given in (10), (11), and (12), using the collected data. We are primarily interested in the sign of the coefficient for the variable of thickness, measured by the variable $d = \sqrt{U^2 + V^2}$.

Table 5 gives the estimation results of models (10), (11), and (12), in the same format as that of Table 2. It is clear that the regressions based on model (12) have the best fit, followed by model (11). In all these different specifications using different sample data, the parameter estimates of α_2, the coefficient of the inverse of the thickness variable *d*, are negative. Moreover, they are significant at the 5% level for eight out of nine cases, and are all significant at the 10% level (note that it is a one-sided test). Thus our estimation results predict that the matching probability increases as the market becomes thicker, consistent with the main prediction of our theoretical model. In other words, a thicker market produces a better probability of matching. The statistically insignificant estimates of α_1 reflects the fact that the number of candidates in top-50 schools in each field is a noisy measure. One source of noise comes from the classification of candidates' fields. For example, students often indicate their fields to be one of the thicker fields. In the two year period we have data, among the 126 fields that have academic openings, only 45% of them have candidates, although 62% of those fields have some success to hire at least one candidate.

Figure 3a gives the estimated curves using 2000 job market data, Figure 3b uses 2001 job market data, and Figure 3c uses the pooled sample. These figures graph the observed and predicted matching probabilities for models (11) and (12). Each point in these figures represents one field. The dotted line in each figure represents the predicted probability based on model (11); the solid line plots the predicted probability based on model (12). The prediction is carried at the sample mean of $\min\{U/V,1\}$. Comparing model (11) with model (12) we observe that the nonlinear model shows more pronounced thickness effects. Within the same model (say model (12)), all three graphs are similar, reflecting the fact that estimates from different sample are similar. From all the graphs, we clearly see that matching probability is an increasing and concave function of the thickness (d) of the market.

To understand the magnitude of the effect of thickness on the matching probability, consider model (12) where the number of candidates equals the number of job openings; we have that (when $U = V$) the matching probability is

$$\hat{B} = \hat{\alpha}_0 + \hat{\alpha}_1(1) + \hat{\alpha}_2 / d^{\hat{\alpha}_3},$$

where $d = \sqrt{2}V$ (since $U = V$). For $U = V = 5$, 10, and 50, and using the pooled sample estimation result, our empirical model predicts the matching probabilities of .361, .421, and .523, respectively.

Finally, in order to check whether fields such as "any field" (AF) and "general economics" (A1) contaminate our estimation results, we conduct estimates removing fields "AF" and "A1". These two fields have large numbers of openings while there are no candidates labeled as "any field" or "general economics." Estimation results not reported here (they are available upon request) show virtually identical parameters estimates as well as the goodness-of-fit R^2 for the results given in Table 5. Thus the fact that there are zero candidates in the thick fields “AF” and “A1” does not affect our estimation results nor the conclusions derived from them.

4. Conclusions

In this paper we propose a matching model with the matching probability depending on the thickness of a market. In our model, the types of firms and the productivities of job candidates are randomly drawn from a common distribution. A firm employs a job applicant only if the job applicant's productivity is higher than its type.

All firms prefer a higher productivity applicant to a lower one, and all applicants prefer a higher minimum standard firm to a lower one. In this hypothetical market, we show that the probabilities of matches in a thin market differ significantly from those in a thick market.

We also characterize the case where firms and candidates have different distributions, the case where the number of openings does not equal the number of applicants, and the case where openings and candidates arrive at the market sequentially. In all these cases, the matching probability still increases with the thickness of the market. In addition, we propose a parsimonious matching function which is fairly close to the (simulated) theoretical matching function.

The implications of our model are consistent with the liquidity literature in the financial market where more trading occurs in a thicker market than in a thinner market. We apply our matching model to the U.S. academic market for junior PhD economists. Consistent with the prediction of our model, a field with more job openings and more candidates has a higher probability of matching. In particular, according to our model, the matching probability increases from .361 for 5 candidates and openings to .523 for 50 candidates and openings.

The model above can be extended in many directions, such as to the regular labor or housing markets. It may require more serious effort to relax some assumptions made in this paper. For example, job candidates may have different preference orderings of employers, as may employers. More general transferable utility functions may be more appropriate for a general labor market because one may enjoy a higher utility if he/she moves to a better quality firm. Buyers in the housing market have several aspects to consider, while sellers may only care about the selling prices.

Appendix

A1. Proof of *Lemma* 1

Proof: The joint distribution of ($u_{(1)}, u_{(2)}, \ldots, u_{(U)}$) is $U!\prod_{r=1}^{U} f\left(u_{(r)}\right)$and the joint distribution of ($v_{(1)}, v_{(2)}, \ldots, v_{(V)}$) is $V!\prod_{r=1}^{V} f\left(v_{(r)}\right)$. Therefore, the joint distribution of ($u_{(1)}, \ldots, u_{(U)}, v_{(1)}, \ldots, v_{(V)}$) is $U!V!\prod_{r=1}^{U} f\left(u_{(r)}\right)\prod_{r=1}^{V} f\left(v_{(r)}\right)$.

$$
\begin{aligned}
\Pr\left(\mathrm{Z}_n = z\right) &= \Pr\left(z_{(1)} < z_{(2)} \cdots z_{(U+V)}\right) \\
&= V!U!\int_{\left\{z_{(1)}<z_{(1)}\cdots z_{(U+V)}\right\}} \prod_{r=1}^{U+V} dF\left(z_{(r)}\right) \\
&= V!U!\int_{\left\{z_{(2)}<z_{(3)}\cdots z_{(U+V)}\right\}} F\left(z_{(2)}\right)\prod_{r=2}^{U+V} dF\left(z_{(r)}\right) \\
&= V!U!\int_{\left\{z_{(3)}<\cdots<z_{(U+V)}\right\}} (1/2)F^2\left(z_{(3)}\right)\prod_{r=3}^{U+V} dF\left(z_{(r)}\right) \\
&= V!U!\int_{\left\{z_{(3)}<\cdots<z_{(U+V)}\right\}} (1/2)\cdots(1/(U+V-1))\int_a^b F^{U+V-1}\left(z_{(U+V)}\right)F\left(z_{(U+V)}\right) \\
&= \frac{V!U!}{(U+V)!}
\end{aligned}
$$

A2. Proof of *Lemma* 2

We know that A_n is independent of the distribution *f*. Therefore, without of loss of generality, we assume that *f* is a uniform distribution in the unit interval. For any small $\eta > 0$, we choose $m = [1/\eta] > 1/\eta$, where [•] denotes the integer part of •.

The unit interval is divided into m intervals with equal length 1/m for each. That is: $[0,1] = \bigcup_{l=1}^{m} I_l$, where I_l=[(l-1)/m, l/m) (l = 1, …, m, with I_m = [(m-1)/m, 1]). Let $n_{u,l}$ and $n_{v,l}$ denote the number of observations from $\{u_i\}_{i=1}^{n}$ and $\{v_i\}_{i=1}^{n}$ that fall inside interval I_l. We know that on average there are n/m observations from both $\{u_i\}_{i=1}^{n}$ and $\{v_i\}_{i=1}^{n}$ that fall inside interval Il for all l = 1, …, m. In fact, by the strong law of large number (Billingsley 1986 (p. 80)) we have $\Pr\left(\lim_{n\to\infty} n_{s,l}/n = 1/m\right) = 1$ for all l =1, …, m (s = u, v).

Note that candidates whose u_i's fall inside the interval I_l can match with any job openings with v_j's falls in I_l-1 (l = 2, …, m). Given that with probability one that $n_{u,l}/n \to 1/m$, and $n_{u,l-1}/n \to 1/m$, with probability approaching to one as $n \to \infty$, there can be n/m matches for $u_i{}'s \in I_l$ matching with $v_i{}'s \in I_{l-1}$. Sum over l from 2 to me we get, with probability one, that the number of matched candidates is at least (since we ignore the possibility that $u_i{}'s \in I_1$ may also find match) $n_{match}/n \geq (m-1)/m \geq 1-\eta$, or more formally, we have, as $n \to \infty$,

$$\Pr\left(\frac{n_{match}}{n} \geq 1-\eta\right) \to 1. \qquad (13)$$

Therefore we have:

$$1 \geq A_n = \frac{1}{n}\sum_{r=0}^{n} r\Pr(r) \geq \frac{1}{n}\sum_{r \geq n(1-\eta)} r\Pr(r) \to 1,$$

because for any $1 > \varepsilon > 0$, we can choose $\eta = \varepsilon/2$ and by (13), we have:

$$\frac{1}{n}\sum_{r\geq n(1-\eta)} r\Pr(r) \geq \frac{1}{n}n(1-\eta)\sum_{r\geq n(1-\eta)}\Pr(r) \geq (1-\eta)^2 \geq 1-\varepsilon .$$

Thus, $\frac{1}{n}\sum_{r\geq n(1-\eta)} r\Pr(r) \to 1$ as $n \to \infty$ which implies $A_n \to 1$, completing the proof of *Lemma* 2.

Let Pr ($\# \geq r$) denote the probability that at least r people find jobs. Then it is easy to see that $\Pr(\#\geq r) = \Pr\left(u_{(n)} > v_{(n-r+1)},\ u_{(n-1)} > v_{(n-r)},\ \cdots,\ u_{(n-r)} > v_{(2)},\ u_{(n-r+1)} > v_{(1)}\right)$. The next lemma shows that Pr ($\# \geq r$) can be used to compute E(r).

Lemma 3: Let # denote the number of people who find jobs ($0 \leq \# \leq \min\{V, U\}$), and denote by $\Pr(\#\geq r) = \sum_{m=r}^{n}\Pr(m)$ the probability that at least r people find jobs. Then:

$$E(r) = \sum_{r=1}^{n}\Pr(\#\geq r)$$

Proof of *Lemma* 3:

$$\begin{aligned}
E(r) &= \sum_{r=1}^{n} r\Pr(r) \\
&= \Pr(1) + 2\Pr(2) + \cdots + n\Pr(n) \\
&= \left[\Pr(1) + \Pr(2) + \cdots + \Pr(n)\right] + \left[\Pr(2) + \cdots + \Pr(n)\right] + \cdots + \left[\Pr(n-1) + \Pr(n)\right] + \Pr(n) \\
&= \sum_{r=1}^{n}\Pr(\#\geq r)
\end{aligned}$$

A3 The case of n = 3.

Let $u_3 > u_2 > u_1$ be the order statistic of candidates, and $v_3 > v_2 > v_1$ be the order statistics of types of the vehicle.

$$\Pr(0) = \Pr(u_3 < u_2 < u_1 < v_1 < v_2 < v_3) = (3!)^2 (1/6!) = 1/20$$

$$\Pr(\# \geq 1) = 1 - \Pr(0) = 19/20 \quad ,$$

$$\Pr(\# \geq 2) = \Pr(u_3 > v_2, u_2 > v_1) = \Pr(u_3 > v_2 > u_2 > v_1) + \Pr(u_3 > u_2 > v_2 > v_1) = 14/20$$

since

$$\begin{aligned}\Pr(u_3 > v_2 > u_2 > v_1) &= \Pr(u_3 > v_3 > v_2 > u_2 > v_1 > u_1) + \Pr(v_3 > u_3 > v_2 > u_2 > v_1 > u_1) \\ &\quad + \Pr(u_3 > v_3 > v_2 > u_2 > u_1 > v_1) + \Pr(v_3 > u_3 > v_2 > u_2 > u_1 > v_1) \\ &= 4[(3!)/(6!)] = 4/20.\end{aligned}$$

$$\begin{aligned}\Pr(u_3 > u_2 > v_2 > v_1) &= \Pr(u_3 > u_2 > v_3 > v_2 > v_1 > u_1) + \Pr(u_3 > v_3 > u_2 > v_2 > v_1 > u_1) \\ &\quad + \Pr(v_3 > u_3 > u_2 > v_2 > v_1 > u_1) + \Pr(u_3 > u_2 > v_3 > v_2 > u_1 > v_1) \\ &\quad + \Pr(u_3 > v_3 > u_2 > v_2 > u_1 > v_1) + \Pr(v_3 > u_3 > u_2 > v_2 > u_1 > v_1) \\ &\quad + \Pr(u_3 > u_2 > v_3 > u_1 > v_2 > v_1) + \Pr(u_3 > v_3 > u_2 > u_1 > v_2 > v_1) \\ &\quad + \Pr(v_3 > u_3 > u_2 > u_1 > v_2 > v_1) + \Pr(u_3 > u_2 > u_1 > v_3 > v_2 > v_1) \\ &= 10[(3!)/(6!)] = 10/20.\end{aligned}$$

$$\begin{aligned}\Pr(3) &= \Pr(u_3 > v_3, u_2 > v_2, u_1 > v_1) \\ &= \Pr(u_3 > v_3 > u_2 > v_2 > u_1 > v_1) + \Pr(u_3 > u_2 > v_3 > v_2 > u_1 > v_1) \\ &\quad + \Pr(u_3 > v_3 > u_2 > u_1 > v_2 > v_1) + \Pr(u_3 > u_2 > v_3 > u_1 > v_2 > v_1) \\ &\quad + \Pr(u_3 > u_2 > u_1 > v_3 > v_2 > v_1) \\ &= 5[(3!)/(6!)] = 5/20.\end{aligned}$$

Therefore, by *Lemma* 3, we have:

$$A_3 = \frac{1}{3}\sum_{r=1}^{3} \Pr(\# \geq r) = [9/20 + 14/20 + 5/20]/3 = 19/30$$

A4. The Case of n = 4

Pr(0) = (4!)2/8! = 1/70 by Lemma 1. ➔ Pr(#≥1) = 1- Pr(0) = 69/70
Pr(#≥2) = [(3+6)+(3+6+10)+(3+6+10+15)]/70 = 62/70.
Pr(#≥3) = [(2+3) + (2+3+4) + (2+3+4+5)]/70 = 42/70.
Pr(4) = [(2+3)+(2+3+4)]/70 = 14/70.

Therefore, by Lemma 3, we have:

$$A_4 = \frac{1}{4}\sum_{r=1}^{4} \Pr(\# \geq r) = [(69/70) + (62/70) + (42/70) + (14/70)]/4 = 187/280$$

A5. The case of difference means

We assume that sellers are randomly drawn from a uniform distribution in the unit interval [0, 1], while the types of buyers are random draws with uniform distribution in the interval of [δ, 1+ δ]. We only consider the case of $V = U = n$.

For $n = 1$, straightforward calculation shows that: $A_1 = \Pr(1) = (1/2)(1 - \delta)^2$.
For $n = 2$, a more tedious calculation shows that:

$\Pr(0) = (1+4\delta + 6\delta^2 - 4\delta^3 - \delta^4)/6$, and,

$\Pr(2) = (1- \delta)^4/3$.

Hence, using $\Pr(1) = 1 - \Pr(0) - \Pr(2)$, we have:

$$A_2 = (1/2)[\Pr(1) + 2\Pr(2)]$$
$$= (7/12)(1- \delta)^2 + (1/12)\delta(1- \delta)^2(2 + 3\delta).$$

A6. The case of $V \neq U$

Case (i) $(U, V) = (1, 2)$ or $(2, 1)$

Let $v_1 < v_2$ be the order statistics of openings. By *Lemma* 1, we have:

$$\Pr(1) = \Pr(v_2 > u > v_1 \text{ or } u > v_2 > v_2)$$
$$= \Pr(v_2 > u > v_2) + \Pr(u > v_2 > v_2)$$
$$= 2[1!2!/3!] = 2/3$$

Therefore, $M_{U,V} = M_{1,2} = \Pr(1) = 2/3$. From this one can compute $B_{U,V}$.

Case (ii) $(U, V) = (3, 1)$ or $(1, 3)$

Let $v_1 < v_2 < v_3$ be the order statistics of openings.

$\Pr(0) = \Pr(u < v_1 < v_2 < v_3) = 1!3!/4! = 1/4$

$\Pr(1) = 1 - \Pr(0) = 3/4$. Therefore, $M_{1,3} = \Pr(1) = 3/4$. Then one can compute $B_{U,V}$.

Case (iii) $(U, V) = (2, 4)$ or $(4, 2)$

Let $u_1 < u_2$ and $v_1 < v_2 < v_3 < v_4$ be the order statistics of candidates and openings, respectively.

$\Pr(0) = \Pr(u_2 < v_2) = \Pr(u_1 < u_2 < v_1 < v_2 < v_3 < v_4) = 2!4!/6! = 1/15$.
$\Pr(1) = \Pr(v_1 < u_1 < u_2 < v_2) + \Pr(v_1 < u_1 < v_2) = 1/15 + 4/15 = 1/3$.
$\Pr(2) = 1 - \Pr(0) - \Pr(1) = 3/5$.

Hence, $M_{2,4} = 1/3 + 2(3/5) = 23/15$.

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Table 1: Matching Probabilities Based on 100,000 Simulations if (V=U)

(Openings and Candidates drawn from the same distributions)

n	1	2	3	4	5	6	7	8	9	10
A_n	.5003	.5835	.6337	.6682	.6940	.7135	.7288	.7448	.7562	.7665
Std of A_n	.5000	.3434	.2765	.2379	.2116	.1932	.1786	.1661	.1561	.1481
n	20	30	40	50	60	70	80	90	100	1000
A_n	.8258	.8543	.8720	.8845	.8938	.9013	.9074	.9124	.9163	.9725
Std of A_n	.1041	.0853	.0734	.0657	.0599	.0554	.0523	.0494	.0459	.0144

Table 2: Regressions of Matching Functions using Simulated Matching Probabilities

		$1 \leq$ U, V ≤ 10	
Models	Model (10)	Model (11)	Model (12)
Constant	.015	.075	.060
	(.841)*	(4.28)	(1.96)
$\min\{U/V, 1\}$	.805	.787	.779
	(37.7)	(42.8)	(42.3)
$1/(U^2+V^2)^{1/2}$		-.423	-.456
		(-7.07)	(-4.55)
$1/[V(U^2+V^2)^{1/2}]$	-.281		
	(-3.20)		
α_3			1.19
			(3.05)
R^2	.937	.951	.957
Number of observations	100	100	100

* t-values are in parentheses.

Table 3: Matching Probabilities Based on 100,000 Simulations if (V=U)
(Openings and Candidates Arrive Sequentially)

N	1	2	3	4	5	6	7	8	9	10
$\overline{A}_n$	.5003	.5622	.5999	.6257	.6452	.6603	.6278	.6833	.6922	.7001
Std of $\overline{A}_n$	.5000	.3330	.2665	.2273	.2034	.1846	.1694	.1575	.1475	.1403
N	20	30	40	50	60	70	80	90	100	1000
$\overline{A}_n$	.7486	.7736	.7905	.8127	.8207	.8207	.8272	.8332	.8383	.9248
Std of $\overline{A}_n$	.0977	.0812	.0713	.0632	.0582	.0538	.0515	.0486	.0461	.0146

Table 4: Summary of Academic Markets for New PhD Economists

Fields with most openings	Average Openings	# of filled Positions	Probability of matching	number of candidates
		Year 2000		
Any Field (AF)	95.3	42	.441	0
Macro(E0)	49.3	30	.608	83
Micro(D0)	36.2	18	.498	42
International (F0)	34.9	25	.717	39
Econometrics (C1)	33.5	13	.388	43
Financial Econ (G0)	33.4	19	.568	39
Agric Econ (Q0)	25.6	13	.507	16
Public Econ (H0)	25.4	9	.354	37
General Econ (A1)	22.6	4	.177	0
Health Econ (I1)	21.2	9	.426	11
IO (L0)	19.6	15	.765	60
Mean of remaining fields	2.96	1.40	.305	2.16
Total	617	308	.499	529
		Year 2001		
Any Field	125.0	64	.512	0
Macro (E0)	54.8	31	.566	72
International (F0)	39.6	11	.277	29
Micro (D0)	38.2	20	.523	34
Agric Econ (Q0)	38.9	13	.343	11
Econometrics (C1)	36.0	13	.361	32
Health Econ (I1)	34.5	18	.521	14
Financial Econ (G0)	31.9	16	.501	24
Public Econ (H0)	20.7	11	.532	25
IO (L0)	20.7	15	.726	59
General Econ (A1)	18.3	3	.164	0
Mean of remaining fields	3.27	1.20	.268	1.92
Total	696	308	.443	445

Table 5: Regression of Matching Probabilities

(US Academic Market for New PhD Economists)

Models	Model (10)	Model (11)	Model (12)
		Job Market in January 2000	
Constant	.284	.362	.847
	(5.69)*	(6.67)	(1.98)
$\min\{U/V, 1\}$	.208	.142	.020
	(2.51)	(1.75)	(.237)
$1/(U^2+V^2)^{1/2}$		-.112	-.674
		(-4.03)	(-1.52)
$1/[V(U^2+V^2)^{1/2}]$			
α_3			.223
			(1.30)
R^2	.250	.339	.466
Number of observations	61	61	61
		Job Market in January 2001	
Constant	.370	.456	.658
	(8.29)	(8.97)	(3.36)
$\min\{U/V, 1\}$	-.023	-.094	-.155
	(-.29)	(-1.19)	(-1.84)
$1/(U^2+V^2)^{1/2}$		-.212	-.468
		(-4.78)	(-2.20)
$1/[V(U^2+V^2)^{1/2}]$	-.061		
	(-3.80)		
α_3			.411
			(1.70)
R^2	.192	.273	.315
Number of observations	65	65	65
		Pooled Sample of 2000 and 2001	
Constant	.314	.398	.775
	(9.43)	(10.88)	(3.36)
$\min\{U/V, 1\}$	.106	.035	-.068
	(1.82)	(.62)	(-1.15)
$1/(U^2+V^2)^{1/2}$		-.139	-.591
		(-5.97)	(-2.45)
$1/[V(U^2+V^2)^{1/2}]$	-.023		
	(-3.94)		
α_3			.274
			(2.03)
R^2	.160	.266	.369
Number of observations	126	126	126

* t-values are in parentheses.