1. Consider a true model,
\[ y = X\beta + \gamma Z^* + u, \]
where \( Z^* \) is not observable. The observed \( Z \) can be represented as:
\[ Z = Z^* + \varepsilon, \]
where \( \text{Cov}(Z^*, \varepsilon) = 0, \text{Cov}(Z, \varepsilon) \neq 0 \)
(a) Show \( \hat{\gamma}_{OLS} \) is biased. How do you characterize the bias?
(b) Is \( \hat{\beta}_{OLS} \) bias?
(c) If we also observe \( W \), such that
\[ W = Z^* + \varepsilon_w \]
Can you consistently estimate the model and under what conditions if you can?
(Hint: use partitioned regression)

2. Consider a linear regression model,
\[ y = x_1\beta_1 + x_2\beta_2 + u, \text{ with } E(x_1)=0, E(x_2)=0, \text{ and } E(u) = 0. \]
Let \( \text{Cov}(x_1, u) = \theta, \text{ and } \text{Cov}(x_1, x_2) = \rho, \text{ and } \text{Cov}(x_2, u) = 0 \)
Show whether OLS estimates of \( \hat{\beta}_{1OLS} \) and \( \hat{\beta}_{2OLS} \) are consistent if:
(a) \( \theta \neq 0 \) and \( \rho = 0 \)
(b) \( \theta \neq 0 \) and \( \rho \neq 0 \)
(c) if \( \theta > 0 \) and \( \rho > 0 \), will plim \( \hat{\beta}_{1OLS} > \beta_1 \) and plim \( \hat{\beta}_{2OLS} > \beta_2 \) ?
(Hint: use portioned regression)

3. The Body Mass Index (BMI) measures obesity. Assume that we have a sample of individuals’ BMI and other information that is collected from various cities in the China. Consider a model of BMI:
\[ BMI_{ic} = \beta_0 + \beta_1 \times \text{age}_{ic} + \beta_2 \times \text{age}_{ic}^2 + \beta_3 \times \text{male}_{ic} + \beta_3 \times \text{exercise-time}_{ic} + u_{ic} \]
where subscript \( ic \) indicates individual \( i \) at city \( c \).
(a) Why might you concern about \( \text{exercise-time}_{ic} \) be correlated with the error term \( u_{ic} \)? Discuss within the framework of (i) omitted variables (ii) measurement error, and (iii) simultaneity. How would the OLS estimate of \( \beta_3 \) be biased?
(b) Suppose you can collect information on $income_{ic}$, age of kid, $kidage_{ic}$, and city-wide average commuting time $cmt_c$. Discuss if these variables are likely to be valid IV.

4. Wooldridge, page 80, #4.14

5. Wooldridge, page 109, #5.4