Estimating Labor Supply with Uncertain Non-Labor Income

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Abstract

In this paper, we introduce measurement error in non-labor income in the Hausman (1981) structural model of estimating labor supply. Such measurement errors are conceptually well founded. They naturally lead to the uncertain kink points, and hence uncertain budget segments, which seem to be in-line with Heckman (1983)’s critiques on the Hausman’s approach. We suggest a model that nests the Hausman’s original model and our measurement error model, and carry out the tests to which model fits the data better. The test results show that our model performs well and yields very different estimates of labor supply elasticities.

Key words: structural labor supply, non-labor income, measurement errors.

JEL Classification:

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1 Introduction

Progressive income tax and income transfer programs create piecewise-linear budget constraints that consist of budget segments and kink points. Pioneered by Hausman in 1980’s, a considerable body of work estimates the labor supply under such budget sets.\(^1\) Blundell and MaCurdy (1999) discuss several attractive features of this framework: recognizes the institutional features of the tax systems, and it readily incorporates the fixed cost of holding a job.

Major concerns of the Hausman approach were suggested in Heckman (1983) and later elaborated in MaCurdy, Green and Paarsch (1990). The key concerns of Heckman are that budget segments cannot be accurately measured in most cases, and that not many people are “assigned” to be at kink points by econometricians, contradictory to the suggestions of the model. MaCurdy et al. (1990) elaborate on Heckman’s concerns, arguing that the likelihood setup in Hausman’s framework may create artificial constraints on the parameter values.

The parameters of interest are the uncompensated (Marshallian) and compensated (Hicksian) elasticities. These parameters are the key in evaluating the effect of a large array of public policies, including tax and social welfare programs. Perhaps partly due the unsettled debate between Hausman and Heckman-MaCurdy, economists differ a great deal on the values of these key parameters. Table 2 in Blundell and MaCurdy (1999) summarize eleven papers using non-linear budget constraints for married women. The uncompensated wage elasticity vary from .28 (Triest 1990) to .97 (Hausman 1981). In a survey of prominent economists in labor and public finance, Fuchs, Krueger and Poterba (1997) report that the mean estimate of Marshallian elasticity of labor supply is .45 for women, with a standard deviation of .57.

There are some attempts in the literature to address the Heckman-MaCurdy concern within the Hausman framework. MaCurdy et al. (1990) suggest to smooth the budget segments so it is differentiable everywhere and hence only one solution for each individual is possible; however, the advantages of Hausman framework are lost in this approach. Heim and Meyer (2003) suggest using

\(^1\)See Hausman (1985) and Moffitt (1990) for surveys on this literature.
nonconvex utility function to solve this problem. Another promising attempt is to add measurement error in the Hausman framework. A typical way is to add measurement error in working hours. An early example is Triest (1990). Blundell and MaCurdy (1999) discuss this method in detail and questioned its conceptual standing. Since wages are often computed as average hourly earnings, the measurement error in working hours will often lead to measurement errors in wages, an issue not considered in this literature. In this paper, we point out that the Triest model is a special case of the random coefficient model in Hausman (1980).

In his comment on Hausman’s method, Heckman (1983) wrote, “That Hausman’s econometric procedures require that the budget set confronting the consumer be known to the econometrician.” In this paper, we introduce measurement errors in non-labor income. Such measurement errors are conceptually well founded. They naturally lead to the uncertain kink points, and hence uncertain budget segments, which seem to be precisely in-line with Heckman’s comments. We suggest a model which nests the two different models of measurement errors, and carry out the test to show which model fits the data better.

The importance of non-labor income is documented in Eklof and Sacklen (2000) where they compare estimates in MaCurdy et al. (1990) and Hausman (1981). The key reason for the large difference in the parameter estimates in these two studies is the construction of the non-labor income. In Hausman (1981), the non-labor income is 8 percent of the value of the financial assets where in MaCurdy et al. (1990) it is total labor earnings of the husband. As a result, the nonlabor income in Hausman (1981) is merely one third of the value in MaCurdy et al. (1990).

The main objective in our paper is to estimate and test various specifications of the model under piecewise-linear budget constraints, including (1) the MaCurdy characterization of Hausman model; (2) the Hausman random coefficient model and the Triest model as its special case; (3) the measurement error model in non-labor income. In Section 2, we discusses in detail the various models. Section 3 estimates the models. We conclude in Section 4.

\[2\] Italics is originally added in Heckman (1983).
2 Different Specifications on the Error Term

An individual \( i \) faces a piecewise-linear budget constraint. Let a tax bracket be characterized by \( \{t_j; Y_{j-1}, Y_j\} \), where \( t_j \) is the marginal tax rate for a person whose before-tax income lies within the interval \( (Y_{j-1}, Y_j] \). Information about \( \{t_j; Y_{j-1}, Y_j\} \) can be found from tax tables. Note the relevant budget set is based on after-tax income. Let the end points of the segment in a budget set that corresponds to bracket \( \{Y_{j-1}, Y_j\} \) be \( \{y_{a,j-1}, y_{a,j}\} \), where \( y_a \) refers to after-tax income. A complete characterization of budget segments requires information on working hours that correspond to the set \( [y_{a,j-1}, y_{a,j}] \), and we denote these hours as \( [H_{j-1}, H_j] \). To calculate the location of each budget segment, we start with the first budget segment and proceed through all budget segments. Besides the before-tax wage rate \( w \), another critical piece of information is \( Y^n \), the non-labor income this person may have. Let \( y^n \) be after-tax non-labor income, where the tax is calculated as if the person had no labor income. Then any labor income pushes the person into even higher tax brackets. We summarize information on budget segments in Table 1.

It is well known in the literature that a person’s optimal hours may be at a kink point instead of being on a segment, in the framework of piecewise-linear budget constraints.\(^3\) Let \( N_i \) be the total number of segments of the budget set of individual \( i \). Define

\[
S_{ij} = \begin{cases} 
1 & \text{if on segment } j, \\
0 & \text{otherwise;}
\end{cases} \quad j = 1, \ldots, N_i
\]

\[
K_{ij} = \begin{cases} 
1 & \text{if at kink } j, \\
0 & \text{otherwise;}
\end{cases} \quad j = 0, 1, \ldots, N_i.
\]

Following the tradition in this literature, we assume labor supply function is linear. Let \( h^*_ij \) be the optimal hours for person \( i \) if his budget constraint were segment \( j \), and \( w_{ij} \) be the wage rate for person \( i \) if his budget constraint were segment \( j \). Then,

\[
h^*_ij = \begin{cases} 
\alpha w_{ij} + \beta \tilde{y}_{ij} + Z_i \gamma + u_i, & \text{if positive} \\
0 & \text{otherwise}
\end{cases}
\]

\(^3\)We define “being on segment” as being in the interior of \( (H_{j-1}, H_j) \).
where $\tilde{y}_{ij}$ is virtual income, defined as the intercept of the line that extends this budget segment to the zero-hours axis (see Table 1), and $Z_i$ represents other socio-demographic variables that affect the labor supply, such as number of children in the household, the age of the worker, and the local unemployment rate. Since $Z_i$ does not vary across different segments, the term $Z_i\gamma$ will not be included in our equations hereafter to simplify the notations, although the term is included when the actual estimates are conducted. One important term in (2) is $u_i$, representing the approximation error of the linear specification. Given the labor supply function in (2), the condition for $S_{ij} = 1$ is:

$$S_{ij} = 1 \text{ if } H_{ij-1} < \alpha w_{ij} + \beta \tilde{y}_{ij} + u_i < H_{ij}, \quad j = 1, \cdots, N_i$$

(3)

The condition for $K_{ij} = 1$ is:

$$K_{i0} = 1 \text{ if } \alpha w_{i1} + \beta \tilde{y}_{i1} + u_i \leq 0$$

$$K_{iN_i} = 1 \text{ if } \alpha w_{iN_i} + \beta \tilde{y}_{iN_i} + u_i \geq H_{imax}$$

$$K_{ij} = 1 \text{ if } \alpha w_{ij+1} + \beta \tilde{y}_{ij+1} + u_i \leq H_{ij} \leq \alpha w_{ij} + \beta \tilde{y}_{ij} + u_i, \quad j = 1, \cdots, N_i - 1$$

(4)

where $H_{imax}$ is the allowable maximum working hours for person $i$. From (4), an implied inequality constraint for the kink point $K_{ij} = 1$ in is:

$$\alpha w_{ij} + \beta \tilde{y}_{ij} > \alpha w_{ij+1} + \beta \tilde{y}_{ij+1}$$

(5)

A sufficient and necessary condition for this inequality is:

$$\alpha > \beta H_{ij}, \quad \forall \ i \text{ and } j.$$  

(6)

We can rewrite (6) as

$$\alpha > \beta H_{max} \quad \text{and} \quad \alpha > \beta H_{min}.$$  

If $H_{max} = 5110$ in hours,\(^4\) i.e., the maximum working hours for a person in a year is 5110, and $H_{min} = 0$, a sufficient condition for the inequalities in (5) and (6) is

$$\alpha > 0 \quad \text{and} \quad \alpha > 5110/\beta$$

(7)

\(^4\)The number 5110 is calculated by 14 hours/day × 365 days = 5110.
Note inequalities in (6) and (7) are obtained when wages, hours and virtual income are in levels. Empirically, wages and virtual income are often in the form of logarithm. There are no explicit expressions as in (6) and (7) if log of values are used. Nevertheless, one can still obtain a sufficient condition as in (8) for inequality in (5).

$$\alpha > 0, \ \beta \leq 0$$ (8)

Apparently, (8) is also sufficient for (7). In the presence of other conditional variables, such as some social-demographic variables which do not vary across different segments for each individual, the inequalities in (6) and (7) remain the same.

Various specifications of labor supply have been suggested in the literature. They differ in their treatment of the error terms to Equation (2). Next, we first introduce two influential specifications, and we then introduce our specification.

### 2.1 The MaCurdy model and the potential problem of the Hausman approach

In an influential study, MaCurdy et al. (1990) suggest a slightly different model to help to disentangle the potential problems of the Hausman framework. Let $h_i$ be the observed working hours of person $i$.

$$h_i = \begin{cases} 
\text{h}_{ij}^* & \text{if on jth segment} \\
H_{ij} & \text{if at jth kink point}
\end{cases} \quad (9)$$

where $h_{ij}^*$ is defined in (2). Equations (3) and (4) determine which kink or segment that worker chooses. The log likelihood function can be written as

$$l = \sum_{i=1}^{N} \left\{ 1_{(K_{i0}=1)} \ln \left( 1 - F_u(\alpha w_{i1} + \beta \tilde{y}_{i1}) \right) + 1_{(K_{iN_i}=1)} \ln \left( 1 - F_u(H_{imax} - \alpha w_{iN_i} - \beta \tilde{y}_{iN_i}) \right) + \sum_{j=1}^{N_i-1} 1_{(K_{ij}=1)} \ln \left( F_u(H_{ij} - \alpha w_{ij+1} - \beta \tilde{y}_{ij+1}) - F_u(H_{ij} - \alpha w_{ij} - \beta \tilde{y}_{ij}) \right) + \sum_{j=1}^{N_i} 1_{(S_{ij}=1)} \ln f_u(h_i - \alpha w_{ij} - \beta \tilde{y}_{ij}) \right\} \quad (10)$$

where $F_u(\cdot)$ and $f_u(\cdot)$ are cdf and pdf for the random error $u_i$. The first three parts of (10) give the likelihood at kink points, and the fourth part in (10) gives the likelihood at segments. Apparently,
for any individual \( i \), he is either at segment or at kink points. Suppose he is at kink point \( k \), then all other kink points \( j \neq k \) will not contribute to the likelihood.

In order to implement MLE of (10), one must be able to assign the values of \( K_{ij} \) and \( S_{ij} \) precisely for each individual \( i \) at each segment \( j \). In order to have precise assignment of the values, we must have accurate knowledge on segments. As pointed in Blundell and MaCurdy (1999), this is unlikely to be true in practice due to measurement errors. We then have a computational problem, because if (7) is not satisfied, then without knowing the values of \( K_{ij} \) and \( S_{ij} \) precisely, Equation (10) may call for the log of a negative number. Thus, for computational feasibility, condition (7) and (8) become necessary. In other words, (7) or (8) is true not because it leads to a higher likelihood value; it is true because of the setup of the likelihood function itself. This is the basic critique of MaCurdy et al. (1990).

Although the MaCurdy model in (10) is useful to understand the potential problems of the Hausman approach, this model has not been used in most of the Hausman’s papers. Instead, Hausman often adopts a random-coefficient model which we now turn to.

### 2.2 The double error models

The second model we consider is the Hausman random coefficient model. Let \( h_i^* \) be the true working hours for individual \( i \), specified as:

\[
h_i^* = \alpha w_{ij} + (\beta_0 + \epsilon_i)\tilde{y}_{ij}
\]

where the coefficient \( \beta \) in Equation (2) becomes a random variable, \( \beta = \beta_0 + \epsilon_i \). The random error \( \epsilon_i \) is not observed by the econometrician. The decision rule is summarized in (12).

\[
\begin{align*}
S_{ij} = 1 & \quad \text{iff} \quad H_{ij-1} \leq \alpha w_{ij} + \beta \tilde{y}_{ij} + \epsilon_i \tilde{x}_{ij} \leq H_{ij} \\
K_{ij} = 1 & \quad \text{iff} \quad \alpha w_{ij+1} + \beta \tilde{y}_{ij+1} + \epsilon_i \tilde{x}_{ij+1} \leq H_{ij} \leq \alpha w_{ij} + \beta \tilde{y}_{ij} + \epsilon_i \tilde{x}_{ij} \\
K_{iN_i} = 1 & \quad \text{iff} \quad \alpha w_{iN_i} + \beta \tilde{y}_{iN_i} + \epsilon_i \tilde{x}_{iN_i} \geq H_{i\text{max}}
\end{align*}
\]

When \( \tilde{x}_{ij} = \tilde{y}_{ij} \), the model in (12) is exactly the Hausman random coefficient model. The model
of measurement error in working hours of Triest (1990) is a special case of the model in (11) when $\tilde{x}_{ij} = 1$.

In this framework, the true budget set for each individual is known to both the econometrician and the worker. However, the econometrician cannot assign the worker to a particular segment or kink point because of the individual heterogeneity $\epsilon_i$. The probabilities that the worker is at each segment or kink point can be computed by the econometrician from the decision rule in (12).

There is yet another error in this framework: the measurement error in working hours, denoted as $u_i$. The observed working hours $h_i$ deviates from the true working hours by $u_i$:

$$h_i = h_i^* + u_i$$  \hspace{1cm} (13)

The decision rules in (12) do not include those who do not work. Following Hausman (1981), the decision rule for people with zero hours of work can come from two sources: the optimal hour is zero regardless the values of $u_i$, i.e., $h_i^* \leq 0$, or $u_i$ is negatively enough that $h_i \leq 0$ when $h_i^* > 0$.

The decision rule for zero hours are summarized in (14):

\[
K_{i0} = 0 \text{ if } \begin{cases} 
\alpha w_i + \beta \tilde{y}_{i1} + \epsilon_i \tilde{x}_{i1} \leq 0 & S_{ij} = 1 \text{ and } u_i \leq - (\alpha w_{ij} + \beta \tilde{y}_{ij} + \epsilon_i \tilde{x}_{ij}) \\
K_{ij} = 1 \text{ and } u_i \leq -H_{ij} 
\end{cases}
\]  \hspace{1cm} (14)

The pdf of observed hours when $h_i > 0$ based on the decision rule in (12) and the measurement error in (13) is given by:

\[
f(h_i) = \sum_{j=1}^{N_i} \int_{(H_{ij} - \alpha w_{ij} - \beta \tilde{y}_{ij})/\tilde{x}_{ij}}^{(H_{ij} - \alpha w_{ij} - \beta \tilde{y}_{ij})/\tilde{x}_{ij}} f_u(h_i - \alpha w_{ij} - \beta \tilde{y}_{ij} - \epsilon_i \tilde{x}_{ij}) \, dF_\epsilon(\epsilon_i)
\]  + \sum_{j=1}^{N_i-1} \left[ F_\epsilon \left( \frac{H_{ij} - \alpha w_{ij} + 1 - \beta \tilde{y}_{ij} + 1}{\tilde{x}_{ij} + 1} \right) - F_\epsilon \left( \frac{H_{ij} - \alpha w_{ij} - \beta \tilde{y}_{ij}}{\tilde{x}_{ij}} \right) \right] f_u(h_i - H_{ij})

\[
+ \left[ 1 - F_\epsilon \left( \frac{H_{imax} - \alpha w_{iN_i} - \beta \tilde{y}_{iN_i}}{\tilde{x}_{iN_i}} \right) \right] f_u(h_i - H_{imax})
\]  \hspace{1cm} (15)

The probability that $h_i = 0$ based on the decision rule in (14) is given by:

\[
Pr(h_i = 0) = F_\epsilon \left( - \frac{\alpha w_{i1} + \beta \tilde{y}_{i1}}{\tilde{x}_{i1}} \right)
\]
Although the model in (15) and (16) which we henceforth refer to as the “Dual Additive Model” assumes perfect knowledge of the whole budget segments for each individual by the econometrician, assigning a person to a kink point or a budget segment is not perfect because of heterogeneity among individuals. The model does not have an implication of piling up observations at any point. But rather, each observed working hour may have positive probabilities at any segment or at any kink points.

As pointed out by Blundell and MaCurdy (1999), there are two problems in this model. (1) The model makes a rather suspicious assumption of perfect knowledge of the entire budget constraints by econometricians. (2) The measurement error in hours of work implies measurement error in wages, an issue which is not addressed in this setting. In the next section, we consider alternative specification, namely, measurement error in non-labor income. Such a measurement error does not have the two problems that the Dual Additive Model has.

2.3 Measurement error in non-labor income

In this section, we introduce an alternative measurement error in the Hausman framework: the measurement error in non-labor income $Y^n_i$. Let

$$Y^*_i = Y^{n*}_i - \epsilon_i,$$

(17)

where $Y^{n*}_i$ is the true non-labor income, known to individual $i$, but unobserved to the econometrician. The measurement error for an observed value $Y^n_i$ is $\epsilon_i$. Again, let $H^*_i$ and $\tilde{y}^*_i$ be the true

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5We “substract” the error in order to be comparable with (16).
values observed by individual $i$ but not to the econometrician. Obtained from Table 1, Equation (18) lists the relationships between observed values and the true values.

$$
H_{ij} = H^*_{ij} + \epsilon_i/w_i \\
\tilde{y}_{ij} = \tilde{y}^*_{ij} - \epsilon_i(1 - t_{i0} + t_{i1} - t_{ij}) \equiv \tilde{y}^*_{ij} - \epsilon_i m_{ij}
$$

(18)

Since individual $i$ observes true values $\tilde{y}^*_{ij}$ and $H^*_{ij}$, the optimal choice of segment $j$ or kink point $j$ is based on the true values. The true decision process by individual $i$ can be expressed as:

$$
S_{ij} = 1 \text{ iff } H^*_{ij-1} \leq \alpha w_{ij} + \beta \tilde{y}^*_{ij} \leq H^*_{ij}
$$

$$
K_{ij} = 1 \text{ iff } \alpha w_{ij+1} + \beta \tilde{y}^*_{ij+1} \leq H^*_{ij} \leq \alpha w_{ij} + \beta \tilde{y}^*_{ij}
$$

$$
K_{iN_i} = 1 \text{ iff } \alpha w_{iN_i} + \beta \tilde{y}^*_{iN_i} \geq H_{imax}
$$

(19)

Since the decision process in (19) is not perfectly observed by the econometrician due to measurement error in $Y^*_n$, it is only possible to assign values of $K_{ij}$ and $S_{ij}$ based on observed values of $Y^*_n$. In (20), we rewrite (19) in terms of observables $\tilde{y}_{ij}$ and $H_{ij}$. The decision rules when $h_i > 0$ are in (20).

$$
S_{ij} = 1 \text{ iff } H_{ij-1} \leq \alpha w_{ij} + \beta \tilde{y}_{ij} + \epsilon_i(1/w_i + \beta m_{ij}) \geq H_{ij}
$$

$$
K_{ij} = 1 \text{ iff } \epsilon_i(1/w_i + \beta m_{ij}) \geq H_{ij} - \alpha w_{ij} - \beta \tilde{y}_{ij} \text{, and}
$$

$$
\epsilon_i(1/w_i + \beta m_{ij+1}) \leq H_{ij} - \alpha w_{ij+1} - \beta \tilde{y}_{ij+1}
$$

$$
K_{iN_i} = 1 \text{ iff } \epsilon_i(1/w_i + \beta m_{iN_i}) \geq H_{imax} - \alpha w_{iN_i} - \beta \tilde{y}_{iN_i}
$$

(20)

Let $u_i$ be the usual residual in the linear working hours equation based on true variables. The model in (2) becomes:

$$
h_i = \begin{cases} 
\alpha w_{ij} + \beta \tilde{y}_{ij} + u_i & \text{ if } S_{ij} = 1 \\
H^*_{ij} + u_i & \text{ if } K_{ij} = 1
\end{cases}
$$

(21)

Rewrite (21) into (22) with observed variables.

$$
h_i = \begin{cases} 
\alpha w_{ij} + \beta \tilde{y}_{ij} + u_i + \beta \epsilon_i m_{ij} & \text{ if } S_{ij} = 1 \\
H_{ij} + u_i - \epsilon_i/w_i & \text{ if } K_{ij} = 1
\end{cases}
$$

(22)
Again, following Hausman (1981), the observations that \( h_i = 0 \) are obtained from two sources: those whose optimal hours are zero, and those that the error \( u_i \) are negative enough such that \( h_i \leq 0 \). The decision rules that \( h_i = 0 \) are given in (23):

\[
\begin{align*}
    h_i = 0 & \quad \text{if} \quad \begin{cases} 
        \alpha w_{i1} + \beta \tilde{y}_{i1} + \beta \epsilon_{i1} m_{i1} \leq 0 \\
        S_{ij} = 0 & \quad \text{and} \quad u_i \leq - (\alpha w_{ij} + \beta \tilde{y}_{ij} + \beta \epsilon_{ij} m_{ij}) \\
        K_{ij} = 0 & \quad \text{and} \quad u_i \leq -H_{ij}
    \end{cases}
\end{align*}
\]

Next, we derive the pdf of observed hours. For notational convenience, define:

\[
\begin{align*}
    s_{ij-1}(\lambda) &= \frac{H_{ij-1} - \alpha w_{ij} - \beta \tilde{y}_{ij}}{(1 - \lambda)\tilde{x}_{ij} + \lambda(1/w_i + \beta m_{ij})} \\
    s_{ij}(\lambda) &= \frac{H_{ij} - \alpha w_{ij} - \beta \tilde{y}_{ij}}{(1 - \lambda)\tilde{x}_{ij} + \lambda(1/w_i + \beta m_{ij})} \\
    s_{ij+1}(\lambda) &= \frac{H_{ij} - \alpha w_{ij+1} - \beta \tilde{y}_{ij+1}}{(1 - \lambda)\tilde{x}_{ij+1} + \lambda(1/w_i + \beta m_{ij+1})} \\
    s_{i0}(\lambda) &= \frac{-\alpha w_i - \beta \tilde{y}_i}{(1 - \lambda)\tilde{x}_i + \lambda(1/w_i + \beta m_i)} \\
    s_{iN_i}(\lambda) &= \frac{H_{\text{imax}} - \alpha w_{iN_i} - \beta \tilde{y}_{iN_i}}{(1 - \lambda)\tilde{x}_{iN_i} + \lambda(1/w_i + \beta m_{iN_i})}
\end{align*}
\]

When we set \( \lambda = 1 \), we have:

\[
\begin{align*}
    s_{ij-1}(1) &= \frac{H_{ij-1} - \alpha w_{ij} - \beta \tilde{y}_{ij}}{1/w_i + \beta m_{ij}} \\
    s_{ij}(1) &= \frac{H_{ij} - \alpha w_{ij} - \beta \tilde{y}_{ij}}{1/w_i + \beta m_{ij}} \\
    s_{ij+1}(1) &= \frac{H_{ij} - \alpha w_{ij+1} - \beta \tilde{y}_{ij+1}}{1/w_i + \beta m_{ij+1}} \\
    s_{i0}(1) &= \frac{-\alpha w_i - \beta \tilde{y}_i}{1/w_i + \beta m_i} \\
    s_{iN_i}(1) &= \frac{H_{\text{imax}} - \alpha w_{iN_i} - \beta \tilde{y}_{iN_i}}{1/w_i + \beta m_{iN_i}}
\end{align*}
\]

When \( h_i > 0 \), we have:

\[
f(h_i) = \sum_{j=1}^{N_i} \int_{s_{ij-1}(1)}^{s_{ij}(1)} f_u(h_i - \alpha w_{ij} - \beta \tilde{y}_{ij} - \epsilon_i \beta m_j) dF(\epsilon_i)
\]
When \( h_i = 0 \), we have:

\[
Pr(h_i = 0) = 1 - F_\epsilon \left( -\frac{\alpha w_{i1} + \beta \tilde{y}_{i1}}{\beta m_{i1}} \right)
\]

\[
+ \sum_{j=1}^{N_i-1} \int_{s_{ij}(1)}^{s_{ij+1}(1)} f_u(h_i - H_{ij} + \epsilon_i/w_i)dF(\epsilon_i)
\]

\[
+ \left[ F_\epsilon \left( \frac{H_{ij} - \alpha w_{ij} - \beta \tilde{y}_{ij} + 1}{1/w_i + \beta m_{ij}} \right) - F_\epsilon \left( \frac{H_{ij} - \alpha w_{ij} - \beta \tilde{y}_{ij}}{1/w_i + \beta m_{ij}} \right) \right] F_u(-H_{ij})
\]

\[
+ \left[ 1 - F_\epsilon \left( \frac{H_{imax} - \alpha w_{iN_i} - \beta \tilde{y}_{iN_i}}{1/w_i + \beta m_{iN_i}} \right) \right] F_u(-H_{imax})
\]

The log likelihood function is simply \( \sum_i 1_{h_i>0} \log f(h_i) + 1_{h_i=0} \log Pr(h_i = 0) \). We henceforth refer equations (24) and (25) as the MENLI model, for “Measurement Error in Non-Labor Income.”

The MENLI model has the same advantage of the Triest Additive model: it does not require the econometrician to have precise assignment of each individual’s budget segment or kink point. In this regard, it resolves the Hausman-MacCurdy concern. More importantly, the measurement error in non-labor income leads to the uncertainty for the econometrician about each individual’s budget constraint. This is exactly in-line with Heckman’s concern.

One analogy to the densities in (16) and in (24) is the familiar regime switching model of Hamilton (1989): \( \epsilon_i \) is a random variable which determines which segment or kink point that individual \( i \) will settle, and \( u_i \) is the density when segment \( j \) or kink point \( j \) is given.

It is possible to nest the Dual Additive Model and the MENLI models with one encompassing model. The density when \( h_i > 0 \) is in (26):

\[
f(h_i) = \sum_{j=1}^{N_i} \int_{s_{ij-1}(\lambda)}^{s_{ij}(\lambda)} f_u(h_i - \alpha w_{ij} - \beta y_{ij} - \epsilon_i(\beta m_j \lambda + (1 - \lambda)x_{ij}))dF(\epsilon_i)
\]
\[ + \sum_{j=1}^{N} \int_{s_{ij} - 1}^{s_{ij} + 1(\lambda)} f_u(h_i - H_{ij} + \lambda \epsilon_i/w_i) dF(\epsilon_i) \] (26)
\[ + \int_{-\infty}^{s_0(\lambda)} f_u(h_i + \lambda \epsilon_i/w_i) dF(\epsilon_i) \]
\[ + \int_{s_{im}(\lambda)}^{\infty} f_u(H_{imax} - h_i - \lambda \epsilon/w_i) dF(\epsilon) \]

The probability when \( h_i = 0 \) is in (27):
\[
Pr(h_i = 0) = F(\alpha \bar{w}_{i1} + \beta \bar{y}_{i1} - \frac{\alpha \bar{w}_{i1} + \beta \bar{y}_{i1}}{1 - \lambda} \bar{x}_{i1} - \lambda \beta m_{i1})
\]
\[ + \sum_{j=1}^{N} \int_{s_{ij} - 1(\lambda)}^{s_{ij}(\lambda)} F_u(-\alpha \bar{w}_{ij} - \beta \bar{y}_{ij} - \epsilon_i(\lambda \beta m_{ij} + (1 - \lambda)\bar{x}_{ij})) dF(\epsilon_i) \] (27)
\[ + \sum_{j=1}^{N-1} \int_{s_{ij} - 1(\lambda)}^{s_{ij} + 1(\lambda)} F_u(-H_{ij}) dF(\epsilon_i) \]
\[ + \int_{s_{N1}(\lambda)}^{\infty} F_u(-H_{imax}) dF(\epsilon_i) \]

When \( \lambda = 0 \), this encompassing model becomes the Dual Additive model. When \( \lambda = 1 \), it becomes MENLI model. By virtue of this nesting, likelihood ratio tests can be performed.

3 Estimations

In this example, we apply the various models discussed in previous section to two different data sets: the Panel Study of Income Dynamics (PSID) of 1984 and a recent data set of Current Population Survey (CPS) of March, 2001. Typically, the errors \( \epsilon_i \) and \( u_i \) in the Hausman framework are assumed to be joined normal.

3.1 Estimation based on the PSID (1984) data set

The first data set is extracted from the Panel Study of Income Dynamics (PSID) in the year of 1984. This is the data set used in Triest (1990). Table 2 lists the tax rates and income brackets
for calculating each person’s budget constraints. We assume all individuals take the standard deduction and file jointly.

Table 3 lists the results for three different models. The first column corresponds to Triest Dual Additive model in (15) and (16) with $\tilde{x}_{ij} = 1$. The second column is the MENLI model, with density given by (24) and (25). The third column corresponds to the encompassing model that nests the two models. When the nesting parameter $\lambda = 1$, the encompassing model becomes the MENLI model. When $\lambda = 0$, the model becomes Triest Dual Additive model.

All parameters in various models are estimated accurately, with signs as expected. The parameter of the wage variable $\alpha$ in the dual additive model is .023 (.0091), and the parameter of the virtual income variable $\beta$ is -.026 (.0022). In the MENLI model, the estimate $\alpha$ is .0055 (.061), which is not statically significant. The estimate for $\beta$ is -.008 (.00044), which is statistically significant from zero but is much smaller in magnitude than the estimate based on the dual additive model. The likelihood value is much higher for the MENLI model (-723.5) than the dual additive model (-1242.6), indicating our model is a better specified model. In the nesting model, the nesting parameter is .973, very close to 1. As expected, the parameter estimates are closer to our model than to the dual additive error model.

Given the large difference in the key parameters, it is not surprising to see the large difference in elasticities. Our model says that the uncompensated elasticity is about 0.037% and the compensated elasticity is .096, both are very small. However, based on the dual additive error model, the uncompensated elasticity is .158, and the compensated elasticity is .345.

3.2 Estimation using March CPS, 2001

The second data set is extracted from CPS in March, 2001. We adopt the similar data extraction criterion. We consider married women, between age 25 and 55. Table 2 lists the tax rates and income brackets for calculating each person’s budget constraints. We assume all individuals take the standard deduction and file jointly.
The summary statistics of the variables used in the estimation is listed in the last column (Column 3) of Table 4. We estimate two models using this data set. In the first model, we assume that the measurement error in non-labor income has zero mean. The estimation result from this model is reported in the first column in Table 4. Similar to our estimates using PSID data in 1984, the wage coefficient is very small, 0.00080(.00061), not statistically different from zero. The coefficient of non-labor income is -.0017 (.00013), small in magnitude but significantly different from zero. An increase in non-labor income of $50,000 will reduce the working hours by about 85 hours.

In the previous estimations, we assumed that all households chose standard deductions because of the data availability. In reality, individual households choose the one with higher deduction between the standard deduction and the itemized deduction. Applying only the standard deduction underestimates the actual amount of deductions. A higher deduction implies a higher non-labor income. To illustrate this point, consider the second panel in Table 2. At present, the standard deduction is $7,600 for this household. Some households may choose itemized deduction if higher deductions can be achieved. If the itemized deduction is $7,600 + \Delta. The after-tax-income (non-labor) is higher by \Delta. This can be considered as an increase in non-labor income in \( Y_{ni}^{*n} \).

Therefore, we let the mean of the measurement error in non-labor income \( \epsilon \) vary. From Equation (17), \( Y_{ni}^{*n} \) is the "true" non-labor income that the person \( i \) uses to obtain her budget set. If \( E(\epsilon) > 0 \), \( E(Y_{ni}^{*n}) > E(Y_{ni}^n) \). The estimated model with non-zero mean \( \epsilon \) is listed in Column (2) in Table 4. This new model yields very similar parameter estimates with the model in Column (1) with a higher likelihood. On the other hand, the mean of the \( E(\epsilon) = $614 \). On average, taking itemized deduction is equivalent to an increase of non-labor income of $614.

4 Conclusion

The Hausman framework to estimate labor supply on piecewise-linear budget constraints has many advantages over the reduced-form approach. The simple version of this framework implies bunching
of individual observations at kink points. This becomes the major concern of this framework since we do not empirically observe the bunching at the kink points.

In this paper, we consider a new version within the Hausman framework. We introduce measurement error in non-labor. In particular, we assume that individuals know what their incomes are. But econometricians do not have perfect knowledge of individual’s non-labor income. An uncertain non-labor income leads to an uncertain budget set for each individual, which seems to be precisely in-line with comments in Heckman (1983). Our empirical estimates show that our model performs better statistically and yields much smaller uncompensated and compensated elasticities.

Appendix:

A The Data Set of PSID (1984)

We extract data from the PSID following the procedure in Triest (1990). Some difference appears between our data and the Triest data. Our data set has 1,136 observations while the Triest data set has only 978 observations. The summary statistics for our data set and the Triest data are very close. One possible explanation is that the new version of the PSID has fewer missing values.

We use Heckman’s two stage least square to estimate the wage rates of those who not working.

References


Table 1: Summary of budget segments

<table>
<thead>
<tr>
<th>Budget Segment 1</th>
<th>Budget Segment $j &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>function for after-tax income $y^a$</td>
<td>$y^a = y^n + w(1 - t_1)h$</td>
</tr>
<tr>
<td>kink points for income $y^a$</td>
<td>$y_0^a = y^n$</td>
</tr>
<tr>
<td>kink points for working hours $h$</td>
<td>$H_0 = 0$</td>
</tr>
<tr>
<td>virtual income $\tilde{y}$</td>
<td>$\tilde{y}_1 = y^n$</td>
</tr>
</tbody>
</table>

This table is reproduced from Fullerton and Gan (2001).

We define $t_1$ as the first bracket applied to labor income of this person (after taxation of non-labor income). Using the person’s non-labor income, $t_j$ and $Y_j$ are also individual-specific, but can be found from the tax table.
Table 2: Tax tables

<table>
<thead>
<tr>
<th>Income</th>
<th>Rates</th>
<th>Income</th>
<th>Rates</th>
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<tbody>
<tr>
<td>0 – $3,400</td>
<td>0</td>
<td>0 – $7,600</td>
<td>0</td>
</tr>
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<td>$3,400 – $5,500</td>
<td>.11</td>
<td>$7,600 – $51,450</td>
<td>.15</td>
</tr>
<tr>
<td>$5,500 – $7,600</td>
<td>.13</td>
<td>$51,450 – $113,550</td>
<td>.28</td>
</tr>
<tr>
<td>$7,600 – $11,900</td>
<td>.15</td>
<td>$113,550 – $169,050</td>
<td>.31</td>
</tr>
<tr>
<td>$11,900 – $16,000</td>
<td>.17</td>
<td>$169,050 – $295,950</td>
<td>.36</td>
</tr>
<tr>
<td>$16,000 – $20,200</td>
<td>.19</td>
<td>$295,950 +</td>
<td>.396</td>
</tr>
<tr>
<td>$20,200 – $24,600</td>
<td>.23</td>
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</tr>
<tr>
<td>$24,600 – $29,900</td>
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<td>$29,900 – $35,200</td>
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<td>$35,200 – $45,800</td>
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<td>$45,800 – $60,000</td>
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<tr>
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<td>$109,000 +</td>
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Table 3: Estimation Results using PSID (1983)

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<td>(.025)</td>
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<td></td>
<td>(.0022)</td>
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<td>(.0044)</td>
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<tr>
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<td>-.063</td>
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<tr>
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<tr>
<td></td>
<td>(.18)</td>
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Table 4: Estimation Results using CPS (March, 2001)

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<td>non-labor income (in $1000)</td>
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<td>(.00013)</td>
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<td>(.0080)</td>
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<tr>
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<td>(.0047)</td>
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<tr>
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<td>(.0037)</td>
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<td>education (in years)</td>
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<td></td>
<td>(.0021)</td>
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<td>$\mu_\epsilon$</td>
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