On the Ratifiability of Efficient Cartel Mechanisms in First-Price Auctions with Participation Costs and Information Leakage

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Abstract

This paper investigates whether the efficient all-inclusive cartel mechanism studied by McAfee and McMillan (1992) can still preserve its efficiency when bidders can update their information through the cartel's collusive mechanism and there is a cost to participate in the seller's auction in an independent, private values setting. It is shown that, when the seller uses the first-price sealed-bid auction, the usual efficient cartel mechanisms will no longer be ratifiable in the presence of both participation costs and potential information leakage. The bidder whose value is higher than a cutoff in the cartel will have an incentive to betray the cartel, sending a credible signal of his high value and thus, discouraging other bidders from participating in the seller's auction. However, the cartel mechanism is still efficient if either participation cost or information leakage is absent.

Keywords: Ratifiability, efficient cartel mechanism, first-price auction, information leakage, participation cost.

JEL Classification: D42, D62, D82

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1 Introduction

An auction is an effective way to extract private information by improving the competitiveness of potential buyers and thus increasing allocation efficiency. Efficiency is diminished when buyer' collusion occurs. However, the stability of the collusion is affected when bidders have private information. As Osborne (1976) indicates, when an efficient cartel mechanism is not freely implemented, a cartel may face both external and internal problems.[1] A ratifiable collusive mechanism has features that try to solve these issues.[2] In this paper, we study the ratifiability of an efficient collusive mechanism with participation costs and potential information leakage where bidders may update their information through pre-auction knockouts.

The synthesis of standard mechanism design literature in McAfee and McMillan (1992) states that, bidders' collusion is possible if they use a strong cartel mechanism that results the efficiency by implementing a prior auction before the seller's legitimate auction. However, there are two forces that may destabilize the cartel mechanism. One is the information leakage problem where bidders may update their information from others' participation decisions in the prior auction. Cramton and Palfrey (1995) show that in a two-stage game, bidders' participation decisions in the prior auction would disintegrate the optimal cartel mechanism. Bidders would update other bidders' values from their choices of whether or not to veto the collusive mechanism in the prior auction. This may reveal that the vetoer bidders are likely to have higher valuations than other bidders', which disadvantages other bidders in the seller's auction. The other is that bidders may incur some participation costs when they participate in an auction.[3] The existence of participation costs will further reduce the incentive for bidders with low valuations to enter the seller's auction.

Tan and Yilankaya (2007) apply the ratifiability introduced by Cramton and Palfrey (1995) to investigate whether efficient collusive mechanisms are affected by potential information leakage and participation costs when the seller uses a second-price auction. They find that the standard efficient cartel mechanisms are not ratified by cartel members.

Similar problems occur in first-price auctions. However, they have received little attention in the literature despite first-price auctions being more common in practice. Tanno (2008) in-

[1]The internal problems include designing the rule, and dividing the profit, detecting and deterring cheating. A cartel also has to anticipate and prevent outside production in order to avoid an external threat.

[2]Much of the literature on collusion then tries to overcome a cartel's problems. For related literature, see Cooper (1977), Roberts (1985), Graham and Marshall (1987), Cramton and Palfrey (1990), Graham, Marshall, and Richard (1990), Mailath and Zemsky (1991), etc.

[3]See Cao and Tian (2010) for related literature.

vestigated the ratifiability of efficient cartels in first price auctions with the assumptions of two potentially bidders and uniform distributions on values, without bidder participation costs. The extension to first-price auction setting is not only non-trivial but also important. Costly participation typically induces asymmetry among bidders after entry occurs and bidding strategies involving asymmetric bidders are much more complicated than strategies in second-price auctions that result in truth-telling; this increases the technical difficulty of the analysis. Besides, the analysis of such an extension is arguably more important as empirical studies show that evidence of collusion is found typically in a first-price sealed-bid auction, as mentioned by Tan and Yilankaya (2007, page 392). So this paper studies the ratifiability of the efficient collusive mechanism when the seller uses a first-price sealed-bid auction.

We consider a two-stage ratification game in a first-price sealed-bid auction format, following Cramton and Palfrey (1995) and Tan and Yilankaya (2007), allowing the presence of both participation costs and information leakage in the strong cartel mechanism studied in McAfee and McMillan (1992). We determine the veto set, such that if a bidder's value belongs to the set, he will choose to betray the cartel. Other bidders can then update their beliefs on the vetoer bidder's value distribution. A bidder vetoes the cartel only when the expected revenue from the post first-price auction with these updated beliefs is larger than that of staying in the cartel.

We show that, when the seller uses a first-price auction, the usual efficient cartel mechanism will no longer be ratifiable in the sense that a bidder vetoes a pre-auction in the presence of both participation costs and potential information leakage. The bidder with a value greater than a critical point in the cartel will have an incentive to veto. By vetoing the mechanism, a bidder sends a credible signal that he has a relatively high value, which discourages other bidders from joining the seller's auction when there are positive participation costs. However, even if the information leakage problem exists, such that bidders can update their information through a cartel's collusive mechanism, the usual efficient cartel mechanism is still ratifiable provided there is no participation cost since the vetoer's betraying signal is seen as an incredible threat. One implication of our results is that, in practice, the seller can charge some entry fee to the auction to decrease the possibility that bidders may form a cartel and thus increase the competition of bidders. This in turn increases the expected revenue of the seller.

The remainder of the paper is organized as follows. Section 2 describes the economic environment. Section 3 considers the benchmark case where no information leakage is allowed. Section 4 allows the presence of information leakage and investigates the ratifiability of efficient cartel mechanisms. Section 5 concludes. All proofs are included in the Appendix.

2 Economic Environment

Consider a standard independent private values economic environment with one seller and n ($n \geq 2$) risk-neutral potential bidders. The seller values her object at 0. [4] Bidder i' value for the object being auctioned is v_i, which represents i's willingness to pay for the object in the auction, and $v = (v_1, ..., v_n)$ is the vector of n bidders' profiles. Variable v_i is private information which is a random draw from the same cumulative distribution function $F(\cdot)$ with continuous and strictly positive density function $f(\cdot)$ supported on $[0, 1]$.

In order to submit a bid, each bidder needs to pay a non-refundable participation cost[5] common to all bidders and denoted by $c \in [0, 1)$. Bidders do not know the participation decisions of others when they make their own decisions. When a bidder is indifferent between participating and not participating in the seller's auction, we assume that he participates for illustration convenience. When a bidder submits a bid, he knows the participation decisions of all other bidders. Only the winner in the seller's auction must pay his bid, and all participants in the seller's auction have to pay the participation costs.

Bidders may form a cartel. The seller is assumed to be passive; i.e., she does not know whether she faces a cartel.

3 Auction with No Information Leakage

In this section, we assume there is no information leakage problem and restrict our attention to the equilibria with passive beliefs, that is, that after the cartel's prior auction, bidders have no updated information on other bidders' valuation distributions. This is the benchmark for our investigation in the Section 3.1. To know whether bidders prefer to form a cartel, we compare bidders' choices between the non-collusive and collusive games without information leakage.

3.1 Non-Collusive First-Price Auction

The auction format for the seller is the first-price sealed-bid auction. Let v^* be the cutoff point for participation, which is determined by $c = v^* F(v^*)^{n-1}$. It is obvious that $v^* > c$. Following

[4] A positive reserve price only complicates our derivation without getting additional insights on our main result. Although a higher reserve price may affect the seller's expected revenue, we do not investigate the seller's optimal reserve price strategy since our main concern is the ratifiability of the efficient cartel mechanism.

[5] Participation costs can be entry fees (charged by the seller) or sunk costs. Even though entry fees may be correlated with the seller's revenue, given that the major concern in our paper is the ratifiability of the efficient cartel mechanism, i.e., the behavior of the bidders, we do not investigate the optimal entry fee strategy but rather take it as exogenously given.

Cao and Tian (2010), when there are $k \geq 2$ bidders participates in the seller's auction, there exists a unique (up to changes for a measure zero set of values) symmetric (Bayesian-Nash) equilibrium where each bidder's bidding function $\lambda(v_i)$ is monotonically increasing when bidder i with value v_i participates in the seller's auction. This is given by

$$\lambda(v_i, v^*) = v_i - \frac{\int_{v^*}^{v_i} [F(y) - F(v^*)]^{k-1} dy}{[F(v_i) - F(v^*)]^{k-1}}$$

whre $v_i \geq v^*$. If $v_i < v^*$, then bidder i does not participate in the auction. When $k = 1$, the bidder who participates in the auction just bids zero. When $v_i \geq v^*$, the non-collusive expected profit $\pi_i^s(v_i)$ for bidder i is

$$\pi_i^s(v_i) = v_i F(v^*)^{n-1} + \sum_{k=2}^{n} C_{n-1}^{k-1} F(v^*)^{n-k} [v_i - \lambda(v_i, v^*)][F(v_i) - F(v^*)]^{k-1} - c,$$

where the first term on the right side is the expected revenue when all others do not participate in the sellers auction, the second term gives the expected revenue when $k \geq 2$. With some simplifications, bidder i's expected profit from the non-collusive first price auction is given as

$$\pi_i^s(v_i) = \begin{cases} 0 & v_i < v^* \\ \int_{v^*}^{v_i} F(y)^{n-1} dy & v_i \geq v^*. \end{cases}$$

3.2 Efficient All-Inclusive Cartel Mechanism

Now suppose transfers among bidders are possible. Bidders may form a cartel and design an all-inclusive ex post efficient cartel mechanism outside of the sellers auction, which maximizes the sum of bidders' expected profits with transfer payments.[6] This efficient cartel mechanism exists for $n \geq 2$.[7] Because of the ex post efficiency, the highest value bidder earns the object if his value is higher than the participation cost. It does not matter whether there is a participation cost or a reserve price in the seller's auction provided that their magnitudes are the same.

After dropping the bidder's indices to simplify the notation, it follows from McAfee and McMillan (1992) that there is an all-inclusive ex post efficient (symmetric) cartel mechanism.[8]

[6] This efficient cartel mechanism is called a strong cartel, which is shown to be ratifiable in McAfee and McMillan (1992).

[7] Che and Kim (2006) show that agents' collusion imposes no cost in a broad class of circumstances with more than two agents ($n \geqslant 3$) for correlated types and for more than one agent ($n \geqslant 2$) for uncorrelated types. As for two-agent nonlinear pricing environments with correlated types, Meng and Tian (2013) show that collusive behavior may not be prevented freely.

[8] As in the assumption in McAfee and McMillan (1992), some punishment is available to the cartel, so to dissuade cartel members from breaking the ring when the ring dictates that he bids to lose. This is because we focus on the constraints of the cartel that result from the privacy of the cartel members' information. The cartel should ensure obedience to the cartel's orders when the cartel is ratified.

The expected profit of a cartel member with value v can be written as

$$\pi^m(v) = \begin{cases} \pi^m(0) & if \quad v < c \\ \pi^m(0) + \int_c^v G(u)du & if \quad v \geq c, \end{cases}$$

where $G(u) = F(u)^{n-1}$ and

$$\pi^m(0) \quad = \quad \int_c^1 [y - \frac{1-F(y)}{f(y)} - c]G(y)dF(y),$$

which is the transfer payment that each loser receives in the cartel.

Since $v^* > c$ and $\pi^m(0) > 0$, it follows that

$$\pi^m(v) > \pi^s(v), \tag{1}$$

which means that the efficient cartel mechanism satisfies bidders' individual rationality and incentive compatibility conditions. Bidders in the cartel earn more profit, whether they win the object or not.

As compared with the non-collusive auction, all bidders prefer to form a cartel. This is true for all $0 \leq c < 1$. That is, it does not matter whether there is a participation cost in the seller's auction or not. We then have the following proposition.

Proposition 1 *Suppose there is no information leakage. Then the strong cartel mechanism is efficient and interim individually rational with respect to symmetric equilibrium payoffs in the seller's auction, no matter whether there is a participation cost or not.*

4 Ratifiability of Efficient Cartel Mechanisms

The assumption that there is no information leakage is unrealistic. As long as bidders are rational, they will update their information, if any, from the cartel's prior auction before they participate in the seller's auction. In this section, we investigate the ratifiability of efficient cartel mechanisms with both participation costs and the possibility that bidders may update their beliefs through bidders' participation decisions. In this case, the bidding strategy of a bidder in the seller's auction usually depends on the bidder's beliefs about others' values, which in turn may be affected by the ratifiability of the cartel.

Following Cramton and Palfrey (1995) and Tan and Yilankaya (2007), we consider a two-stage ratification game to analyze the stability of the efficient cartel. In the first stage, bidders simultaneously vote for or against the efficient cartel mechanism. In the second stage, if the cartel mechanism is unanimously accepted, then it is implemented; otherwise, bidders participate in the first-price auction knowing who vetoed the cartel mechanism, and thus having updated their beliefs about vetoers' values.

In order to show that bidders may have an incentive to exit the cartel, we define a veto set A_i. If bidder i vetoes the cartel,[9] then his valuation is updated to be in the set A_i by other bidders. Insofar as vetoing the cartel brings more profit to the vetoer than he would gain in the collusive case and so the cartel mechanism may not be supported. For illustrative simplicity, we assume that if a bidder is indifferent between staying in and vetoing the cartel, he chooses to stay in the cartel. Let $\pi_i^v(v_i, b^*)$ denote the vetoer's expected payoff at equilibrium b^* in the post-veto auction with updated beliefs on the vetoer's valuation and $\pi_i^m(v_i)$ be the payoff when he stays in the cartel. Formally, we have the following definitions.

Definition 1 *A set A_i with $\emptyset \neq A_i \subseteq [0, 1]$ for bidder i is said to be a credible veto set if there exists an equilibrium b^* in the post-veto auction with updated beliefs on the vetoer's valuation such that $\pi_i^v(v_i, b^*) > \pi_i^m(v_i) \Leftrightarrow v_i \in A_i$.*

Definition 2 *The cartel mechanism is ratifiable, if there is no credible veto set for all $i \in N$.*

Intuitively, low-value bidders have relatively more to gain by participating in the cartel than high-value bidders. By vetoing the cartel mechanism, a high-value bidder sends a signal that he has high value, which would deter other bidders from entering the seller's auction. On the contrary, a low-value bidder does not have much to gain from vetoing. This makes the higher value bidder's veto credible. Thus a bidder, if any, only vetoes the collusive mechanism when his value exceeds a cutoff value.

Suppose that when bidder i vetoes the cartel, others believe that his value is in $(v_N, 1]$, where N for saying no to the cartel mechanism (the vetoer), and v_N is an upper bound at which the bidder is indifferent between vetoing and staying in the cartel. We will show that there is an asymmetric equilibrium of the auction with these updated beliefs, such that the vetoer's payoff at equilibrium is larger than his payoff in the cartel if his value is larger than v_N. As such, the vetoer has an incentive to betray the cartel and $A_i = (v_N, 1]$ is then a credible veto set for bidder i.

When bidder i vetoes the cartel, his value is updated to be distributed on $(v_N, 1]$ according to $F_N(v) \equiv \frac{F(v)-F(v_N)}{1-F(v_N)}$, which is derived from $F(.)$ using the Bayesian rule. For all other bidders, the expected profit of participating in the seller's auction is a non-decreasing function of their

[9] Since we are looking for the possibility that all bidders unanimously ratify the cartel mechanism, it is sufficient to only think the unilateral deviation.

true values. Thus, with participation costs, a bidder uses a cutoff strategy in which he submits a bid if and only if his value is greater than or equal to a cutoff.

Now suppose that all ratifier bidders use the same strategy, including the participation decision and bidding function, i.e., the cutoff points used by all collusive bidders to decide whether or not to participate in the seller's auction are the same and denoted by v_Y, where Y denotes saying yes (ratifiers).[10] If any ratifier bidder participates in the auction, then his value is updated to be distributed on $(v_Y, 1]$ according to $F_Y(v) \equiv \frac{F(v)-F(v_Y)}{1-F(v_Y)}$. Thus, when both the vetoer and any ratifier bidder participate in the seller's auction, we have an asymmetric first-price auction in which bidders' valuation distributions are on different supports.

Remark 1 Only when a bidder vetoes the cartel can we infer that his value is among $(v_N, 1]$. This does not mean any bidder with a value in $(v_N, 1]$ will veto the cartel. It is possible that a bidder with a value among $(v_N, 1]$ decides to remain in the cartel mechanism. Thus, for a ratifier bidder, we can only infer from their participation behavior in the seller's auction that his value is above v_Y or not. If he participates, his value is updated to be distributed on $(v_Y, 1]$. Otherwise, we have no information to judge whether his value is less than v_N or not. [11]

Since bidders can observe who else participates in the auction when they submit bids, the bidding functions hinge on the number of other bidders. Thus we can specify the strategy as a function of the number of ratifier bidders participating in the seller's auction, k, as follows.

For the vetoer,

$$b_i^k(v_i) = \lambda_i^k(v_i) \quad \forall v_i \in (v_N, 1].$$

For a typical ratifier bidder j $(j \neq i)$,

$$b_j^k(v_j) = b_{-i}^*(v_j) = \begin{cases} N_0 & v_j < v_Y \\ \lambda_j^k(v_j) = \lambda^k(v_j) & v_j \geq v_Y \end{cases} \quad \forall j \neq i,$$

where N_0 denotes non-participation and k is the number of ratifiers participating in the auction. When there is no ratifier bidder participating, the vetoer bidder just bids zero, i.e., when $k = 0$, $b_i^k(v_i) = 0$.

[10] It should be noted that each collusive bidder may use a different cutoff value as studied in Cao and Tian (2010), which may complicate our analysis considerably. However, if $F(\cdot)$ is inelastic, i.e., $F(v) \geq vf(v)$ for all $v \in [0, 1]$, then there is a unique equilibrium (see Cao and Tian, 2010).

[11] In a special case with only two bidders, Maskin and Riley (2000, p. 425) show that the payoff of the vetoer is lower than what he would get if both bidders were weak, which in turn is lower than what he would get in the cartel mechanism. No type in $[v_N, 1]$ would like to veto the mechanism if a veto signals that the vetoer's value is in $[v_N, 1]$.

Note from Cao and Tian (2010) that $v_Y \geq v_N$, since otherwise a collusive bidder with value v_Y has no chance to win in the seller's auction but still incurs the participation cost c. This makes a v_Y type collusive bidder's participation irrational. Without loss of generality, we assume that a bidder with zero probability of winning will bid his true value when he participates. Let $v_i^k(b)$ and $v_j^k(b)$ be the optimal inverse bidding functions when both types of bidders have a positive probability of winning. Then the maximization problem for typical ratifier bidder j is

$$\max_{b_j^k}(v_j - b_j^k)(\frac{F(v_j^k(b_j^k(v_j))) - F(v_Y)}{1 - F(v_Y)})^{k-1}\frac{F(v_i(b_j^k(v_j))) - F(v_N)}{1 - F(v_N)}.$$

Similarly, for vetoer bidder i,

$$\max_{b_i^k}(v_i - b_i^k)[\frac{F(v_j(b_i^k(v_i))) - F(v_Y)}{1 - F(v_Y)}]^k.$$

Then, by Cao and Tian (2010), there is a unique optimal bidding strategy where bidders with the same value distribution use the same bidding function, which is characterized in the following lemma.[12]

Lemma 1 *Suppose there are $k \geq 1$ bidders whose values are distributed on the interval $[v_Y, 1]$ with cumulative distribution function $F_Y(v) = \frac{F(v)-F(v_Y)}{1-F(v_Y)}$, and there is one bidder whose value is distributed on the interval $[v_N, 1]$ with cumulative distribution function $F_N(v) = \frac{F(v)-F(v_N)}{1-F(v_N)}$ who participates in the first-price auction, where $v_N \leq v_Y$. Let $\underline{b}_k = \arg\max_b(F(b) - F(v_N))(F(b) - F(v_Y))^{k-1}(v_Y - b)$. The optimal inverse bidding functions $v_i^k(b)$ and $v_j^k(b)$ are uniquely determined by*

(1) $v_i^k(b) = b$ for $v_N \leq b \leq \underline{b}_k$ and

(2) for $\underline{b}_k < b \leq \bar{b}_k$,

$$\begin{cases} \frac{kf(v_i(b))v_i^{k\prime}(b)}{F(v_i^k(b))-F(v_N)} + \frac{(k-1)f(v_j^k(b))v_j^{k\prime}(b)}{F(v_j^k(b))-F(v_Y)} = \frac{1}{v_j^k(b)-b} \\ \frac{kf(v_j^k(b))v_j^{k\prime}(b)}{F(v_j^k(b))-F(v_Y)} = \frac{1}{v_i^k(b)-b}. \end{cases}$$

with boundary conditions $v_j(\underline{b}_k) = v_Y$, $v_i(\underline{b}_k) = \underline{b}_k$ and $v_i(\bar{b}_k) = v_j(\bar{b}_k) = 1$.

Remark 2 Note from Lemma 1 that when $k = 1$, for any ratifier bidder in the seller's auction, his only rival is the vetoer bidder and thus $\underline{b}_1 = \arg\max_b(F(b) - F(v_N))(v_Y - b)$. When $k > 1$, for any ratifier bidder, his rivals include the vetoer and other ratifier bidders, in this case, $\underline{b}_k = v_Y$.

[12] The proof of Lemma 1 can be followed from that of Cao and Tian (2010) directly, and is thus omitted here.

Remark 3 When $v_N < v_i < \underline{b}_1$, the vetoer bidder can win the auction when he is the only bidder in the seller's auction and thus he can do no better than bidding zero. When $\underline{b}_1 < v_i < v_Y$, if there is more than one ratifier bidder participating in the seller's auction, then the vetoer cannot win the auction.

Since all ratifier bidders use the same bidding strategy, if there are more than two ratifier bidders in the seller's auction, then the probability for the v_Y ratifier bidder to win is zero. He has positive revenue only when he is the only ratifier bidder in the seller's auction and his bid $\underline{b}$ is higher than that of the vetoers. Let $\tilde{v}_Y$ be the solution to

$$c = (\tilde{v}_Y - \underline{b}) F(\tilde{v}_Y)^{n-2} \frac{F(\underline{b}) - F(v_N)}{1 - F(v_N)},$$

that is, the payoff of a $\tilde{v}_Y$ bidder is equal to his participation cost, whenever $\tilde{v}_Y \leq 1$. We then have $v_Y = \min\{1, \tilde{v}_Y\}$. Notice that v_Y is the cutoff point where ratifier bidders are indifferent to participating in the seller's auction. Since $v_Y > v_N$, an increase in v_N leads to a higher v_Y. Thus we have that v_Y is a strictly increasing function of v_N until it reaches 1 for some value of v_N and stays there for a greater value of v_N.

The expected revenue for the vetoer bidder with a value in $(v_N, 1]$ depends on the number of ratifier bidders participating in the seller's auction. Let $H_k(v_i)$ be the probability that all other bidders' bids are less than vetoer i's bid when there are k ($k \geq 0$) ratifier bidders participating. The expected payoff of vetoer i (given the ratifier's belief that the vetoer's value is in $(v_N, 1]$ and the equilibrium in the asymmetric first price auctions, i.e., b^*) is

$$\pi_i^v(v_i, b^*) = \sum_{k=0}^{n-1} C_{n-1}^k [1 - F(v_Y)]^k F(v_Y)^{n-k-1} [v_i - \lambda_i^k(v_i)] H_k(v_i) - c,$$

where $C_{n-1}^k [1 - F(v_Y)]^k F(v_Y)^{n-k-1}$ is the probability that there are just k ratifier bidders participating in the seller's auction.

More specifically, when $v_N < v_i < \underline{b}_1$, the bidder can win the auction only when no others participate in the auction, which happens with probability $F(v_Y)^{n-1}$. In this case the vetoer bidder just bids zero, thus

$$\pi_i^v(v_i, b^*) = v_i F(v_Y)^{n-1} - c.$$

When $\underline{b}_1 < v_i < v_Y$, the vetoer bidder can win the auction when there are less than two ratifier bidders participating in the seller's auction. With the participation of more than two ratifier bidders, who would submit bids higher than or equal to v_Y, the vetoer bidder has no chance of winning. Then

$$\begin{aligned}\pi_i^v(v_i, b^*) &= v_i F(v_Y)^{n-1} + C_{n-1}^1[1-F(v_Y)]F(v_Y)^{n-2}(v_i - b_i^1)\frac{F(v_j(b(v_i))) - F(v_Y)}{1-F(v_Y)} - c \\ &= v_i F(v_Y)^{n-1} + C_{n-1}^1 F(v_Y)^{n-2}(v_i - b_i^1)(F(v_j(b(v_i))) - F(v_Y)) - c,\end{aligned}$$

where the first term is the expected revenue when no others participate. The second term is the expected revenue when only one other bidder participates, which happens with probability $C_{n-1}^1[1-F(v_Y)]F(v_Y)^{n-2}$.

Accordingly when $v_i \geq v_Y$,

$$\begin{aligned}\pi_i^v(v_i, b^*) &= v_i G(v_Y) + c_{n-1}^1[1-F(v_Y)]F(v_Y)^{n-2}(v_i - b_i^1)\frac{F(v_j(b(v_i))) - F(v_Y)}{1-F(v_Y)} \\ &+ \sum_{k=2}^{n} C_{n-1}^k[1-F(v_Y)]^k F(v_Y)^{n-1-k}[v_i - b^k(v_i)](\frac{F(v_j(b^k(v_i))) - F(v_Y)}{1-F(v_Y)})^k - c \\ &= v_i F(v_Y)^{n-1} + C_{n-1}^1 F(v_Y)^{n-2}(v_i - b_i^1)(F(v_j(b(v_i))) - F(v_Y)) \\ &+ \sum_{k=2}^{n} C_{n-1}^k F(v_Y)^{n-1-k}[v_i - b^k(v_i)](F(v_j(b^k(v_i))) - F(v_Y))^k - c,\end{aligned}$$

where the first two terms are interpreted as before, and the third term is the summation of the expected revenue for each of k, $k \geq 2$, which happens with probability $C_{n-1}^k[1-F(v_Y)]^k F(v_Y)^{n-1-k}$. Note that if $n = 2$, the third term does not exist.

We will show that there is a unique asymmetric equilibrium of the auction such that the vetoer's expected profit at equilibrium is larger than that obtained from staying in the cartel if his value is larger than v_N. We first consider the case where $c = 0$.

Proposition 2 *The efficient cartel mechanism is ratifiable when $c = 0$ and the information leakage problem exists.*

The intuition is that when the participation cost is zero, if some bidder votes the cartel, we have $v_Y = v_N = 0$, i.e., all bidders enter the seller's auction symmetrically. Then in this case, the game becomes the basic non-collusive model in McAfee and McMillan (1992). It is obvious that $\pi_i^m(v_i) \geq \pi_i^v(v_i, b^*) = \pi_i^s(v_i)$. Therefore, in this case, the bidder with value $v_i \in A_i$ does not veto the cartel. This is because the vetoer's betraying signal becomes an incredible threat when there are no participation costs since now the losers in the cartel's auction can participate in seller's auction without a participation cost.

Remark 4 Tanno (2008) reached the same conclusion with a numerical example in government procurement auctions without participation costs, focusing on the two bidders and uniform distributions on values assumptions. Our result is more general in the sense that it allows for any number of potential bidders in the cartel and general distributions on values.

When participation costs are positive, the following proposition applies.

Proposition 3 *In a first-price sealed-bid auction, suppose $c > 0$ and there is information leakage. Then the strong efficient cartel mechanism is no longer ratifiable.*

The intuition is based on the following two effects. The first is the *participation effect*. Having updated their beliefs that the vetoer's value belongs to A_i, other bidders with low values would not participate in the seller's auction because they would have to pay the non-refundable participation costs, which in turn leads to a higher expected profit for the vetoer due to weakened competition. The second is termed the *bidding effect* and is induced by a veto. Given $v_N < v_Y$, once a ratifier bidder participates, the vetoer, known as the weak bidder, will bid more aggressively in the first-price auction (Maskin and Riley, 2003), which makes the ratifier bidder less chance of winning.[13] Note that the bidding effect works in the same direction as the participation effect, making a veto more desirable for someone with a value higher than v_N. As a result, the cartel mechanism is not ratifiable.

Remark 5 As indicated by the proof of Proposition 2, if a bidder with value v_i finds that it is more beneficial to betray than to stay in the cartel, it must be the same case for $v_i' > v_i$. Given that v_N is uniquely determined, the veto set is unique.

By combining the results of Propositions 2 and 3, one can see that there is a discontinuity at $c = 0$. The efficient cartel mechanism cannot be ratifiable when both the information leakage problem and positive participation costs exist. When participation costs exist without an information leakage problem, as in Proposition 1, the strong cartel mechanism is still efficient. This efficient cartel mechanism is designed to maximize bidders' ex post profit. Without the information leakage problem, bidders cannot update their beliefs through the cartel's auction, so no one has an incentive to exit the cartel. On the other hand, when the information leakage problem exists without participation costs, the cartel mechanism is still ratifiable. Since bidders can submit a bid in the seller's auction for free, the vetoer cannot earn extra profit from betraying the cartel. Thus, the efficient cartel mechanism is still possible even if there is a participation cost or an information leakage problem, but not when both are present.

To illustrate the main conclusion of this section, we give the following example.

[13] The bidding effect does not exist for the case considered in Tan and Yilankaya (2007) where the seller uses second-price auctions.

Example 1 Suppose v is uniformly distributed on $[0,1]$ and $n = 2$. v_N is determined by the indifference between staying in and vetoing the cartel, thus, we have the first condition

$$\pi^m(0) + \int_c^{v_N} F(u)du = v_N F(v_Y) - c,$$

where the left side is the expected payoff for staying in the cartel while the right side is the expected payoff from participating in the seller's auction (when $v_i = v_N$, he can win the auction only if he is the sole bidder because if there is any other bidder, it must be $v_j \geq v_Y > v_N$ and thus a v_N vetoer has no chance of winning). With $F(v) = v$, the above equation is equivalent to

$$v_N v_Y - c = \frac{1}{2}v_N^2 - \frac{c}{2} - \frac{1}{6}c^3 + \frac{1}{6}$$

where v_Y is determined by the zero net payoff for a ratifier bidder who participates in the seller's auction. For the ratifier bidder j with $v_j = v_Y$, his expected payoff from participating in the auction is given by (assuming $v_Y \leq 1$ since otherwise the ratifier bidder never participates)

$$\max_b \frac{F(b) - F(v_N)}{1 - F(v_N)}(v_Y - b) - c = 0.$$

This maximization problem gives $b = \frac{v_Y + v_N}{2}$ and the zero net payoff gives us the second condition

$$v_Y - v_N = \sqrt{4c(1 - v_N)}.$$

Now we have two conditions on v_Y and v_N. Our numerical examples show that, when c is less than 0.1755, v_Y is less than 1 and the ratifiers with values higher than v_Y have the chance to win in the seller's auction. When $c \geq 0.1755$, the ratifier bidder never participates in the seller's auction. Thus, when $c \geq 0.1755$, $v_Y = 1$, and v_N is determined by $v_N - c = \frac{1}{2}v_N^2 - \frac{c}{2} - \frac{1}{6}c^3 + \frac{1}{6}$. For example, when $c = 0.1755$, $v_N = 0.2980$; when $c = 2/3$, $v_N = 0.686$.

Now consider an extreme case where c approaches zero. From the first condition, we have $v_Y v_N = \frac{1}{2}v_N^2 + \frac{1}{6}$, and from the second condition, we get $v_Y - v_N = 0$. These two conditions give $v_N = 0.577$, which implies that, even when $c = 0$, the cartel mechanism is not ratifiable. However, the problem here is that when $c = 0$, the vetoer bidder needs to worry not only about the ratifier bidder with a value in $[v_Y, 1]$, but also about the ratifier bidder with a value in $[0, v_Y)$.[14] As such, bidders do not have any incentive to deviate. To see this, consider the first-price auction in which the vetoer bidder's value is distributed on $[v_N, 1]$ with $F_{v_N}(v) = \frac{v - v_N}{1 - v_N}$ and the ratifier bidder's value is distributed on [0,1] with $F(v) = v$. For the vetoer with value v_N, he maximizes $(v_N - b)F(b)$ with respect to b, which gives an expected payoff of $\frac{1}{4}v_N^2$. The payoff for staying in the cartel is $\frac{1}{2}v_N^2 + \frac{1}{6}$. Then, in this case, it is impossible to find a $v_N \in [0,1]$

[14] A positive participation cost will eliminate the ratifier bidder of this type.

such that $\frac{1}{4}v_N^2 = \frac{1}{2}v_N^2 + \frac{1}{6}$. So, when $c = 0$, no bidder has any incentive to veto the cartel. Thus there is a discontinuity between $c = 0$ and $c > 0$.

5 Conclusion

This paper studies the ratification of the efficient collusive mechanism in the setting where the seller uses a first-price sealed-bid auction. We find that when the seller uses a first-price auction, the usual efficient cartel mechanism will no longer be ratifiable in the sense that a bidder vetoes a pre-auction in the presence of both participation costs and potential information leakage. We also find that when either a participation cost or the information leakage problem is absent, the efficient cartel mechanism is still ratifiable. The implication of our results is that, in practice, the seller can charge some entry fee to decrease the possibility that bidders may form a cartel and thus, increase the competition of bidders. This in turn increases the expected revenue of the seller.[15]

There are several empirical studies that show evidence of collusion and these studies generally involve first-price sealed-bid auctions.[16] Our theoretical results shed some light on why the cartel becomes more difficult to form; it may be because of the presence of both participation costs and potential information leakage. In the real world, there may also be other factors that affect the formation of a cartel. For instance, in the real world, members in a successful cartel play repeated games where vindictive strategies are a method usually used to enforce collusion. Moreover, an external threat is possible in the real world. In order to raise the expected profit and avoid non-ring members' damages, bidders may have strong incentive to form a ring.

Finally, it should be noted that in our model, all bidders are assumed to be initially in an efficient cartel. However, it is possible that in reality, not all bidders are in the cartel and there may exist some outside bidders. With participation costs and an outside threatener, the cartel mechanism is not as the same as the efficient cartel mechanism without an outside threatener. Marshall and Marx (2007) investigate similar cases in the absence of participation costs and show that bidders in the cartel cannot satisfy the incentive compatible constraint. Besides, the bidders' expected profit should be affected by both the outsiders, and members' bids and value

[15]This involves the optimal entry fee from the perspective of the seller. It would be a important issue for us to discuss how the seller should charge an optimal entry fee to deter collusion in a first price auction or implement the optimal collusive proof allocation mechanism as in Laffont and Martimort (1997). However, since our paper mainly focuses on the optimal behavior of the bidders, i.e., whether or not to deviate from the efficient cartel mechanism, but not on the optimal behavior of the seller, we leave these interesting questions for future research.

[16]Examples can be seen in Porter and Zona (1993), Baldwin et al. (1997), etc.

distributions. Whether the not-all-inclusive cartel is ratifiable and how the cartel operates, if it does, when the seller uses a first-price auction with participation cost, are potentially interesting questions for future research.

Appendix: Proofs

Proof of Proposition 3

To prove the proposition, we need to show there is a $v_N \in (c, 1)$ such that $\pi_i^v(v_i, b^*) > \pi_i^m(v_i)$, if and only if $v_i > v_N$.

When vetoer i vetoes the cartel, his expected payoff can be one of the following four cases. The first case is that $v_i G(v_Y) < c$ with $G(y) \equiv F(y)^{n-1}$; i.e., his expected payoff is less than his participation cost. No matter whether or not other bidders participate in the seller's auction, the vetoer will not join the auction. The second case occurs when $c \leq v_i G(v_Y) \leq \underline{b}_1 G(v_Y)$, with $\underline{b} = \arg\max_b [F(b) - F(v_N)](v_Y - b)$. In this case, the vetoer chooses to participate in the seller's auction because his expected payoff is larger than c. But in this case, the vetoer can win the auction only when no ratifier bidder participates. The third case occurs when $\underline{b}_1 \leq v_i \leq v_Y$. In this case, the vetoer can win the auction if there is only one ratifier bidder participating in the auction.[17] The fourth case is when $v_i > v_Y$. Based on the derivations in the text,

$$\pi_i^v(v_i, b^*) = \begin{cases} 0 & v_i < \frac{c}{G(v_Y)} \\ v_i G(v_Y) - c & \frac{c}{G(v_Y)} \leq v_i \leq \underline{b}_1 \\ v_i G(v_Y) + c_{n-1}^1 [1 - F(v_Y)] F(v_Y)^{n-2} [v_i - b(v_i)][F(v_j(b(v_i))) - F(v_Y)]) - c & \underline{b}_1 < v_i \leq v_Y \\ v_i G(v_Y) + c_{n-1}^1 [1 - F(v_Y)] F(v_Y)^{n-2} [v_i - b(v_i)][F(v_j(b(v_i))) - F(v_Y)] & \\ \qquad + \sum_{k=2}^n c_{n-1}^k F(v_Y)^{n-1-k} [v_i - b^k(v_i)][F(v_j(b^k(v_i))) - F(v_Y)]^k - c & v_i > v_Y. \end{cases}$$

$b_i^k(v_i)$ represents the bidding function of the vetoer bidder when there are k ratifiers in the auction. When $n = 2$, the third case and fourth case will be combined into one case.

The expected profit in the cartel is:

$$\pi_i^m(v_i) = \begin{cases} \pi_i^m(0) & v_i < c. \\ \pi_i^m(0) + \int_c^{v_i} G(y) dy & v_i \geq c, \end{cases}$$

where $\pi_i^m(0) = \int_c^1 [y - c - \frac{1-F(y)}{f(y)}] G(y) dF(y)$. We want to show:

$$\pi_i^v(v_i, b^*) > \pi_i^m(v_i) \quad \forall v_i > v_N.$$

We first find a value v_N for which $\pi_i^v(v_N, b) = \pi_i^m(v_N)$, and then check the inequality.

Step 1: Show that there exists $v_N \in (c, 1)$ such that $\pi_i^v(v_N, b) = \pi_i^m(v_N)$. Since $\frac{c}{G(v_Y)} \geq c$, when $v_N \leq c$, we have $\pi_i^m(v_N) > \pi_i^v(v_N, b*) = 0$, which is impossible. When $\underline{b}_1 \geq v_N > c$,

[17] If there are more than two ratifiers participating in the auction, then their minimal bids would be at least higher than v_Y, which is higher than v_i.

we have $\pi_i^v(v_N, b^*) = v_N G(v_Y(v_N)) - c$, and $\pi_i^m(v_N) = \pi_i^m(0) + \int_c^{v_N} G(y)dy$; i.e., we need $v_N G(v_Y(v_N)) - c - \pi_i^m(0) - \int_c^{v_N} G(y)dy = 0$. Let

$$\phi(v_i) = v_i G(v_Y(v_i)) - \int_c^{v_i} G(y)dy - c - \pi_i^m(0),$$

and

$$\begin{aligned}\phi'(v_i) &= G(v_Y(v_i)) + v_i G'(v_Y(v_i))v_Y'(v_i) - G(v_i) \\ &= [G(v_Y(v_i)) - G(v_i)] + v_i G'(v_Y(v_i))v_Y'(v_i).\end{aligned}$$

Since $G(\cdot)$ is an increasing function and $v_Y(v_i) \geq v_i$, we have $G(v_Y(v_i)) - G(v_i) > 0$ and $v_Y'(v_i) > 0$. Therefore, $\phi'(v_i) > 0$ and

$$\phi(c) = cG(v_Y(c)) - c - \pi_i^m(0) = c[G(v_Y(c)) - 1] - \pi_i^m(0) < 0,$$

so that

$$\begin{aligned}\phi(1) &= G(v_Y(1)) - \int_c^1 G(y)dy - c - \pi_i^m(0) \\ &= 1 - c - \int_c^1 G(y)dy - \pi_i^m(0).\end{aligned}$$

Now we need to show $\phi(1) > 0$. Note that

$$\begin{aligned}1 - c - \int_c^1 G(y)dy - \pi_i^m(0) &= 1 - c - \int_c^1 G(y)dy - \int_c^1 [y - \frac{1-F(y)}{f(y)} - c]G(y)dF(y) \\ &= \int_c^1 (1 - F(y)^n)dy - \frac{1}{n}[1 - \int_c^1 F(y)^n dy] + \frac{c}{n} \\ &= \frac{n-1}{n}[1 - c - \int_c^1 F(y)^n dy].\end{aligned}$$

Since $F(y)^n \leq 1$, $\int_c^1 F(y)^n dy \leq \int_c^1 1dy = 1 - c$. We have $\phi(1) \geq 0$.

For $v_i < 1$ and $v_Y(v_i) \geq v_i$, since $\phi(v_i)$ is continuous, $\phi(c) < 0$ and $\phi(1) \geq 0$. A unique solution to $\phi(v_i) = 0$ exists and is our candidate for v_N.

Step 2: We want to show $\pi_i^v(v_i, b^*) > \pi_i^m(v_i) \;\; \forall v_i > v_N$. Given c, v_N and v_Y are fixed, then we have $c \leq \frac{c}{G(v_Y)} \leq v_N \leq v_Y$. The payoff difference $\pi_i^v(v_i, b^*) - \pi_i^m(v_i)$ is continuous, and can be given by:

I: $\pi_i^v(v_i, b^*) - \pi_i^m(v_i) < 0, \quad when \;\; v_i \leq \frac{c}{G(v_Y)}$.

II: When $\frac{c}{G(v_Y)} < v_i \leq \underline{b}$, let $\varphi(v_i) = v_i G(v_Y) - c - \int_c^{v_i} G(y)dy - \pi_i^m(0)$, as $\varphi(v_N) = \phi(v_N) = 0$, we have $\varphi'(v_i) = G(v_Y) - G(v_i) > 0$ when $v_i < \underline{b}$, and $\varphi'(v_Y) = 0$. Therefore,

$$\varphi(v_i) \begin{cases} < 0 & v_i < v_N \\ = 0 & v_i = v_N \\ > 0 & v_i > v_N. \end{cases}$$

Thus, $\varphi(v_Y)$ is strictly positive.

III: We need to consider a special case where $n=2$ first, then we prove the general case for $n \geq 3$ in IV. When $n=2$, the payoff $\pi_i^v(v_i, b^*)$ is given by:

$$\pi_i^v(v_i, b^*) = \begin{cases} 0 & v_i < \frac{c}{G(v_Y)} \\ v_i F(v_Y) - c & \frac{c}{G(v_Y)} \leq v_i \leq \underline{b} \\ v_i F(v_Y) + [v_i - b(v_i)][F(v_j(b(v_i))) - F(v_Y)] - c & v_i > \underline{b}. \end{cases}$$

Note that when $v_i > \underline{b}$, we have a different $\pi_i^v(v_i, b^*)$ from the case when $n \geq 3$, because there is only one ratifier competing with the vetoer in the auction. The minimal bid for the ratifier is not v_Y. The proof of I and II are the same as in the general case. We need to show $\pi_i^v(v_i, b^*) - \pi_i^m(v_i) \geq 0$ when $v_i > \underline{b}$. Then

$$\begin{aligned} \frac{d}{dv_i}[\pi_i^v(v_i, b^*) - \pi_i^m(v_i)] &= \frac{d}{dv_i}[v_i F(v_Y) + [v_i - b(v_i)][F(v_j(b(v_i))) - F(v_Y)] - c - \int_c^{v_i} G(y)dy - \pi_i^m(0)] \\ &= \frac{d}{dv_i}[v_i F(v_j(b(v_i))) - v_i F(v_Y) - b(v_i) F(v_j(b(v_i))) \\ &+ b(v_i) F(v_Y) - c - \int_c^{v_i} G(y)dy - \pi_i^m(0)] \\ &= F(v_j(b(v_i))) + F(v_Y) - F(v_Y) + v_i f(v_j(b(v_i))) v_j'(b) v'(v_i) - b'(v_i) F(v_j(b(v_i))) \\ &- b(v_i) f(v_j(b(v_i))) v_j'(b) b'(v_i) + b'(v_i) F(v_Y) - G(v_i) \\ &= [(v_i - b(v_i)) f(v_j(b(v_i))) v_j'(b) b'(v_i)] + [F(v_j(b(v_i))) - F(v_i)] \\ &- b'(v_i)[F(v_j(b(v_i))) - F(v_Y)]. \end{aligned}$$

Substitute $v_i(b) = b + \frac{F(v_j(b)) - F(v_Y)}{f(v_j(b)) v_j'(b)}$ in the above equation. We have

$$\begin{aligned} [\frac{F(v_j(b)) - F(v_Y)}{f(v_j(b)) v_j'(b)}] f(v_j(b(v_i))) v_j'(b) b'(v_i) &+ [F(v_j(b(v_i))) - F(v_i)] - b'(v_i)[F(v_j(b(v_i))) - F(v_Y)] \\ &= F(v_j(b(v_i))) - F(v_i), \end{aligned}$$

where $v_j(b(v_i))) - v_i \geq 0$. So $[\pi_i^v(v_i, b^*) - \pi^m(v_i)]$ is an increasing function of v_i whenever $v_i > \underline{b}$. Thus, $\pi_i^v(v_i, b^*) > \pi^m(v_i)$ whenever $v_i > v_N$.

IV: Now we prove the general case for $n \geq 3$. First we show that when $\underline{b} < v_i \leq v_Y$, $\frac{d}{dv_i}[\pi_i^v(v_i, b^*) - \pi_i^m(v_i)] \geq 0$. So

$$\begin{aligned} \frac{d}{dv_i}[\pi_i^v(v_i, b^*) - \pi_i^m(v_i)] &= \frac{d}{dv_i}\{v_i F(v_Y)^{n-1} - \int_c^{v_i} F(y)^{n-1}dy - \pi_i^m(0) \\ &+ c_{n-1}^1 F(v_Y)^{n-2}[v_i - b(v_i)][F(v_j(b(v_i))) - F(v_Y)]\} \end{aligned}$$

Note that

$$\frac{d}{dv_i}[v_i F(v_Y)^{n-1} - \int_c^{v_i} F(y)^{n-1}dy - \pi_i^m(0)] = F(v_Y)^{n-1} - F(v_i)^{n-1} \quad \text{(A.1)}$$

$$
\begin{aligned}
\frac{d}{dv_i}c_{n-1}^1 F(v_Y)^{n-2}[v_i &- b(v_i)][F(v_j(b(v_i))) - F(v_Y)] \\
&= (n-1)F(v_Y)^{n-2}\{f(v_j(b(v_i)))v_j'(b(v_i))b'(v_i)[v_i - b(v_i)] \\
&+ (1-b'(v_i))[F(v_j(b(v_i))) - F(v_Y)]\} \\
&= (n-1)F(v_Y)^{n-2}(F(v_j(b(v_i))) - F(v_Y)) \qquad \text{(A.2)}
\end{aligned}
$$

by noting that $(v_i(b) - b)f(v_j(b(v_i)))v_j'(b) = F(v_j(b)) - F(v_Y)$ from the first-order condition of the vetoer bidder when there is only one ratifier bidder participating in the seller's auction. Since $v_j(b(v_i))) - v_i \geq 0$ and $v_Y \geq v_i$, we have $\frac{d}{dv_i}[\pi_i^v(v_i, b^*) - \pi_i^m(v_i)] \geq 0$ when $\underline{b}_1 < v_i \leq v_Y$.

Next, we continue to prove $\frac{d}{dv_i}[\pi_i^v(v_i, b^*) - \pi_i^m(v_i)] \geq 0$ when $v_i > v_Y$, such that

$$
\begin{aligned}
\pi_i^v(v_i, b^*) &- \pi_i^m(v_i) = v_i F(v_Y)^{n-1} + c_{n-1}^1[1 - F(v_Y)^{n-2}[v_i - b(v_i)][F(v_j(b(v_i))) - F(v_Y)] \\
&+ \sum_{k=2}^{n} c_{n-1}^k F(v_Y)^{n-1-k}[v_i - b^k(v_i)][F(v_j(b^k(v_i))) - F(v_Y)]^k - c \\
&- \int_c^{v_i} F(y)^n dy - \pi_i^m(0).
\end{aligned}
$$

Now, we have

$$
\begin{aligned}
\frac{d}{dv_i}\sum_{k=2}^{n} c_{n-1}^k[1 &- F(v_Y)]^k F(v_Y)^{n-1-k}[v_i - b^k(v_i)][F(v_j(b^k(v_i))) - F(v_Y)/(1-F(v_Y))]^k \\
&= \frac{d}{dv_i}\sum_{k=2}^{n} c_{n-1}^k F(v_Y)^{n-1-k}[v_i - b^k(v_i)][F(v_j(b^k(v_i))) - F(v_Y)]^k \\
&= \sum_{k=2}^{n} c_{n-1}^k F(v_Y)^{n-1-k}[F(v_j(b^k(v_i))) - F(v_Y)]^{k-1}\{k[v_i - b^k(v_i)]f(v_j(b^k(v_i)))v_j'(b^k(v_i))b^{k'}(v_i) \\
&+ [F(v_j(b^k(v_i))) - F(v_Y)](1 - b^{k'}(v_i))\} \\
&= \sum_{k=2}^{n} c_{n-1}^k F(v_Y)^{n-1-k}[F(v_j(b^k(v_i))) - F(v_Y)]^{k-1}[F(v_j(b^k(v_i))) - F(v_Y)] \\
&= \sum_{k=2}^{n} c_{n-1}^k F(v_Y)^{n-1-k}[F(v_j(b^k(v_i))) - F(v_Y)]^k,
\end{aligned}
$$

by noting that $k(v_i^k(b) - b)f(v_j^k(b(v_i)))v_j^{k'}(b) = F(v_j^k(b)) - F(v_Y)$ from the first-order condition of the vetoer bidder when there are exactly k ratifier bidders participating in the seller's auction. To finish the proof, we make the following claim:

Claim 1 *When $k \geq 2$, $v_j(b^k(v_i)) = v_i$.*

Proof of Claim 1: Note that when $k \geq 2$, the minimal bid of the ratifier bidder is v_K and thus, in this case, the vetoer bidder with a value less than v_Y will not submit a bid. For the ratifier bidder, if they observe that the vetoer bidder submits a bid, then it must be the case that $v_i \geq v_Y$. Then, the vetoer's value can be further updated to be distributed on $[v_Y, 1]$ with $F_Y(.)$, which is the same as the ratifier bidders. In this case, bidders are involved in a symmetric first-price auction and the equilibrium is symmetric.

With Claim 1,

$$
\begin{aligned}
\frac{d}{dv_i}\sum_{k=2}^{n} c_{n-1}^{k} \quad & F(v_Y)^{n-1-k}[v_i - b^k(v_i)][F(v_j(b^k(v_i))) - F(v_Y)]^k \\
= \quad & \sum_{k=2}^{n} c_{n-1}^{k} F(v_Y)^{n-1-k}[F(v_i) - F(v_Y)]^k \\
= \quad & F(v_i)^{n-1} - F(v_Y)^{n-1} - c_{n-1}^{1} F(v_Y)^{n-2}[F(v_i) - F(v_Y)]. \qquad \text{(A.3)}
\end{aligned}
$$

From Equations (A.1), (A.2) and (A.3), we have

$$
\begin{aligned}
\frac{d}{dv_i}(\pi_i^v(v_i, b^*) \quad - \quad & \pi_i^m(v_i)) \\
= \quad & F(v_Y)^{n-1} - F(v_i)^{n-1} + (n-1)F(v_Y)^{n-2}(F(v_j(b(v_i))) - F(v_Y)) \\
+ \quad & F(v_i)^{n-1} - F(v_Y)^{n-1} - c_{n-1}^{1} F(v_Y)^{n-2}[F(v_i) - F(v_Y)] \\
= \quad & (n-1)F(v_Y)^{n-2}(F(v_j(b(v_i))) - F(v_i)) > 0.
\end{aligned}
$$

Q.E.D.

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