Market-oriented interest rate, deposit insurance system and bank runs
A dynamic model perspective

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Abstract
Purpose – In the transitional process of promoting market-oriented interest rate, China is confronted with an important theoretical and practical issue: how to avoid bank runs and realize the smooth operation of the financial system. The purpose of this paper is to construct a bank-run dynamic model by taking into account a market environment with the transmission of multiple rounds of noise information, a comprehensive consideration of depositors’ expectation of return on assets (or earning rate/yields of assets), the efficiency of information processing and dissemination, and the different motives for premature withdrawal.

Design/methodology/approach – The authors discussed the dynamic process of bank runs, furnished the ratio and number of each round of bank run, and characterized the corresponding dynamic equilibrium as well. Furthermore, the authors expanded the benchmark model by incorporating the deposit insurance system (DIS) to discuss the action mechanism of DIS overruns.

Findings – The results show that DIS implementation has two opposite effects: stabilized expectation and moral hazard, by virtue of its influence over the two types of premature withdrawal motives of depositors; the implementation effect of DIS rests with the dual-effect comparison, which is endogenous to the institutional environment.

Originality/value – The policy implications are as follows: while implementing DIS, it is necessary to establish and improve the corresponding institutional construction and supporting measures, to consolidate market discipline and improve the supervisory role of the bank’s internal governance mechanism, so as to reduce the potential moral hazards. The financial system reform shall be furthered and the processing and dissemination efficiency of information be elevated to prompt depositors to form stable withdrawal expectations, thereby enhancing the stabilizing effect of DIS.

Keywords Moral hazard, Bank runs, Deposit insurance system, Market-oriented interest rate

Paper type Research paper
1. Introduction
Since the reform and opening up, the relative development lag of China’s financial industry has been regarded as one of the major issues confronting China’s economic transition. It is becoming more and more urgent to improve the capital allocation and utilization efficiency via initiating financial system reforms, wherein the interest rate marketization (market-oriented interest rate) reform constitutes an important initiative for financial system transformation (Yi, 2009). However, international experience shows that financial marketization, while improving the efficiency of financial resource allocation, may exacerbate the fragility of the financial system, leading to bank crises and even economic crises (Bekaert et al., 2005). International Monetary Fund (IMF) statistics show that since the prevalence of financial deregulation (liberalization) in the 1990s, more than 100 banking crises have occurred all over the world. Of the 181 member states of the IMF, 130 have experienced different levels of banking crises. How to avoid the banking crisis in the process of promoting interest rate marketization (liberalization) and realize the smooth operation of the financial system is a major theoretical and practical problem faced by the economies in transition.

The experience and research of various countries show that, considering the strong externalities of financial industry, while promoting institutional reforms such as interest rate liberalization (marketization), it is necessary to establish a series of supporting systems, such as deposit insurance systems (DISs), to promote the smooth implementation of financial system reform[1]. China’s central decision-making level has been exploring and promoting the introduction of DIS as a supporting condition for the steady implementation of interest rate liberalization[2]. In March 2015, China’s State Council promulgated the “Regulations on Deposit Insurance,” which was officially implemented on May 1 in the same year, marking the transition from implicit full guarantee (blanket guarantee) to explicit DIS in China. The existing research and practice show that the emergence of DIS mainly stems from the bank’s concern about the depositors’ bank runs; the potential information asymmetry between depositors will further lead to panic in the banking system and expand the chain reaction of the run, causing systemic banking crisis and extremely adverse effects on economic development. DIS can effectively restrain the behavior of depositors, reduce the occurrence of depositors’ run-off (bank run) behavior, and stabilize the financial system reform (Fama, 1980; Diamond and Dybvig, 1983). However, DIS may also cause serious moral hazard problems in banks, mainly in three aspects: First, DIS has not made it necessary for banks to pay higher interest rates to depositors for additional risks of investment. For higher returns, banks have incentives to invest in high-risk projects, thus increasing the instability of bank operations. Second, deposit insurance (DI) institution provides repayment guarantees for banks facing liquidity crisis; hence, DIS weakens the banks’ internal governance mechanism, reduces the risk management ability of banks and increases the probability of bank failures. Third, depositors expect DI institutions to provide withdrawal guarantees; hence, their incentives to supervise bank investment behaviors wane, leading to weak market discipline and prompting banks to opt for high-risk investment projects (Demirgüç-Kunt and Kane, 2002, 2004). Based on this, while implementing DIS, all countries face a trade-off between the stabilizing effect of preventing bank runs and the potential moral hazard. Furthermore, whether DIS can be effectively implemented is often closely related to the institutional environment. In an economy with a relatively sound financial system, the government intervenes little and provides less guarantees in banking business; thus, the run-off behavior poses a great threat to banks. Therefore, DIS implementation can protect the interests of depositors and the stability of the banking system. However, in the financial markets of transition economies surfeited with government intervention, the government usually acts as a hidden guarantor to exert influence on banking business. Therefore, DIS implementation may lead to serious moral hazard problems. With the gradual advancement of the financial system reform, the government’s implicit guarantee mechanism shall be flaw ridden and no longer applicable. On the contrary, the explicit DIS has become the world-wide institutional protocol.
Then, in the transition period of the financial system, how to establish and effectively implement the explicit DIS is an issue worthy of further study.

Existing research shows that explicit DIS tends to increase the likelihood of banking crises, the more so when the design characteristics of DI is taken into account (Demirgüç-Kunt and Kane, 2002). A natural question is why, in the process of marketization of interest rates in the economies with imperfect financial system, DIS implementation is often shadowed by bank runs. It is necessary to discuss the relationship between DIS and bank run in the context of interest rate liberalization. As far as the facts are concerned, DIS first appeared in developed countries in Europe and America, where the economic environment and mechanism of demand faced by DIS are significantly different from developing countries in the transition period (Fan and Cao, 2006). The United States can be taken as an example. When the United States established the explicit DIS, it faced the financial crisis of 1929–1933. Its mechanism of demand was “financial crisis→to establish DIS→to maintain financial stability.”

In transitional economies such as China, while implementing DIS, it faces a financial system reform whose mechanism of demand is “to reform financial system→to establish an explicit DIS→to establish a market-based risk management mechanism.” Among them, one of the major measures for financial system reform is market-oriented interest rate reform. Different mechanisms of demand are bound to face different mechanisms of interaction.

Therefore, when we discuss the relationship between DIS and bank run, it is necessary to closely follow this institutional background.

In view of this, this paper constructs a general dynamic model for bank run to analyze the basic logic of depositors’ runs in the process of interest rate marketization and further incorporates DIS into the benchmark model to examine the impact of DIS on bank runs. The process is in two steps: First, to clearly characterize the process of bank runs, in the dynamic model, we break down the depositor run process into multiple rounds and the run-off behavior into two types of motivations. Given an expected decline of long-term return on assets (ROA/assets earning rate), in round of the run, the marginal replacement rate of depositors in the two periods of consumption changes, causing them to withdraw money in advance, thereby releasing a signal upon which depositors will process and form new judgments, and decide whether to run in the next round. This process lasts for several rounds, and as a result, banks face the risk of running out of liquidity. It should be noted that the first round of the run is often different from the subsequent run: in the first round of the run, the depositor realizes the improvement of personal welfare by taking out the long-term deposit for consumption in period 1, defined as the first category of motivation; in other rounds of runs, depositors take long-term assets for future consumption, which is a panic behavior, defined as the second category of motivation. Furthermore, the error caused by information asymmetry has a lasting influence, which is endowed by the depositor’s choice of time weight while processing information, and it is often related to the institutional environment such as the degree of government intervention and the level of interest rate marketization (Demirgüç-Kunt et al., 2008). Then, DIS is introduced into the benchmark model to analyze its impact on the behavior of banks and depositors, respectively, and thus the impact mechanism of DIS on bank-run process and the dynamic equilibrium.

The rest of the paper is arranged as follows: Section 2 reviews the literature; Section 3 constructs a general theoretical model for bank-run benchmark and solves and analyzes dynamic equilibrium; Section 4 introduces DIS to expand the benchmark model and explore the action mechanism of DIS on bank runs; Section 5 concludes and gives policy implications; the proofs of the lemma and propositions in the text are shown in the Appendix.

2. Literature review

In the wake of the 2008 world financial crisis, the bank-run problem has once again become a hot topic in academic research. The relevant literature focuses mainly on two aspects.
First, the decline of the banks' real ROA gives rise to bank runs, which was an emphasis of the actual impact first proposed by Bryant (1980). Second, the D&D Model was first proposed by Diamond and Dybvig, who held that the bank run is based on the herd behavior caused by “Sunspot” (Diamond and Dybvig, 1983), that is, uncertainty of individual behavior in an imperfect market. The rule of “First-come, First-served” stipulated by the deposit contract triggers bank runs, and the restriction of “sequential service” for depositors leads to negative payoff externality effects. Postlewaite and Vives (1987) described bank runs as an equilibrium of a static game with incomplete information; Wallace (1988) pointed out that demand deposits and the “first come, first served” sequential rule are the two essential factors that lead to bank runs; Jacklin and Bhattacharya (1988) compared two types of bank run models caused by panic and real return, and proved that the critical point of bank run depends on the variance of long-term ROA by using square root utility function and the model specification with invariant consumer expectation of long-term ROA; Schotter and Yorulmazer (2009) established a dynamic bank-run model for information transmission, to examine the impact of information externalities and the herd effect in bank runs on the equilibrium outcome. At the same time, a series of studies have been conducted to discuss the relationship between DIS and the risk of bank runs. Diamond and Dybvig (1983) argued that the risk of bank runs brought about by demand deposits would lead to bank failures and real economic losses, whereas the implementation of a DIS would facilitate an equilibrium; Chen (1999) posed a DIS design idea that could eliminate the bank run panic and guide depositors to focus on accurate information. DIS may create moral hazard problems while preventing bank runs, whereas liquidity support policies can prevent bank runs and curtail moral hazard issues. Starr and Yilmaz (2007) studied the bank run in Turkey in 2001 and pointed out that the use of DIS in emerging economies can reduce the withdrawal behavior of depositors who lack information when bank liquidity is uncertain, and thus help prevent the occurrence of bank runs. It can be seen that the existing research has made instructive discussions on bank runs and the impact of DIS on bank runs. However, most of these models are carried out in the context of static information transmission, without considering the dynamic process of information transmission in reality, and especially the potential bank runs in the process of financial system reform in transitional economies. Below we will build a bank-run model based on dynamic perspectives considering the different premature withdrawal motives of depositors and the process of processing and dissemination of expectation bias, to analyze the process of bank runs in depth, which constitutes a helpful complement to the existing research.

With the active promotion of market-oriented interest rate reform and DIS in recent years, many Chinese scholars have embarked upon preliminary research on issues such as interest rate liberalization and the introduction and implementation of DIS. Xie et al. (2001) introduced the relevant theoretical research and international comparison of DIS. Huang (2001) divided the risks brought by interest rate liberalization into transitional risk and everlasting risk, pointing out that after interest rate liberalization, deposit interest rates will rise significantly in the short term. Qian (2004) held that the prerequisites for establishing an effective DIS include improving bank governance structure, prudent bank supervision and popularized DI concepts, etc., and proposed effective institutional arrangement measures for DI design. Su (2005) took into account the practical problems existing in China's financial development and made propositions on the nature of DI institutions, sources of funds and payment methods. Wang (2006) studied and summarized the experience of the US Federal Deposit Insurance Corporation (FDIC), and put forward suggestions on the establishment and supervision functions of China's DI institutions. Tang (2008) investigated, from the perspective of DIS in various countries, how to improve the corporate governance mechanism to overcome the potential moral hazard of introducing a DIS. Yao and Xia (2012) put forward the design idea of DIS with minimum cost and incentive compatibility based on the empirical analysis of DIS in Germany, Brazil and Russia. By employing the principal-agent model, Yao et al. (2013) discussed the pros and cons of the implicit...
and explicit DIS under systemic risk from the perspective of the internal governance level of commercial banks. It can be seen that the existing Chinese research mainly focuses on drawing on the international experience. It discusses the necessity of introducing DIS and proposes the corresponding design ideas, but rarely involves the relationship between DIS and bank run against the backdrop of interest rate marketization, which is the focus of this paper.

In summary, the existing research mainly considers the economic environment faced by developed economies, and considers less the demand regime (mechanism) for DIS in transitional economies, and the economic environment for introducing DIS. In addition, most of the existing researches use the research framework of the classic D&D model, that is, an idea to explore DIS implementation with the goal of maximizing social utility or the utility of depositors, but lacking consideration of the bank’s own behavior. The more realistic situation is that banks, as independent decision-making bodies, have behavioral characteristics that pursue their own utility maximization. Furthermore, in terms of depositor behavioral limits, the D&D model assumes that depositors do not know whether they are spending in the first or second period at the beginning of the period, but in reality, depositors will form the optimal consumption plan through continuous collection and processing of information.

In view of this, this paper will expand the existing model from two aspects. First, the participants’ behavioral assumptions are expanded. Assuming that the bank’s goal is to maximize its expected utility, a liquidity management strategy is attached and the depositor’s consumption decisions are internalized, depending on their expectations for deposit rates in different periods. Second, the existing bank-run model is expanded by the following method. First, the dynamic bank-run model of multi-round noise information transmission is established; the depositor’s premature withdrawal motives are divided into two categories, and the impact of long-term ROA distribution, information processing and effect of spreading efficiency on equilibrium is analyzed. Second, the ordering rule in the dynamic model is defined, and the concept of strong equilibrium is proposed to characterize the influence of the uncertainty of the signal obtained by the depositor in each round of the run on the equilibrium result, thereby boosting the reality emulation of bank-run equilibrium. Third, DIS is introduced into the model and the impact mechanism of the system implementation on the dynamic run is analyzed. The results of this paper show that DIS has two opposite effects of forming a stable expectation and causing moral hazard by affecting the two motives of depositors’ premature withdrawal, and the strength of the two effects depends on the institutional environment. Whether the introduction and implementation of DIS can improve the efficiency of social financing, reduce the probability of bank runs and promote the smooth transformation of the financial system relies on the establishment and improvement of market discipline, the internal governance mechanism of banks, the efficiency of market information processing and dissemination, and other supporting systems related to the financial marketization reform.

3. The dynamic bank-run model

This section expands the existing model to establish a bank-run dynamic model with a wider coverage. The specific analysis idea is to describe the economic environment, as well as the decision-making sequence of depositors and banks, to characterize the behavioral assumptions of banks and depositors, to analyze the specific dynamic process of bank runs, and to define and discuss the dynamic equilibrium.

3.1 Economic environment

Consider an economic environment with three periods (0, 1, 2), where there is a bank and a continuum of depositors in the [0, 1] range. The bank has proprietary technology for converting period 0 assets into period 1 and period 2 consumer goods, and investing the acquired deposits in two types of assets: short-term security assets and long-term risk

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assets, of which security assets can be considered as a storage technology that converts a unit of assets of period 0 into a certain unit of assets of period 1. Long-term risk assets are determined by a specific and uncertain technology, and they can generate $r$ units of consumer goods in period 2, here, $r$ is a random variable.

At the beginning, the depositor owns all the assets and the assets are not classified and converted. In period 0, the bank provides two types of contracts: the first type is a demand deposit. If the bank has a sufficient liquidity, in period 1, a unit asset will receive $R_1$ units of consumer goods; if the bank has insufficient liquidity, the bank’s liquid assets will be evenly distributed among depositors. The second type of contract is a fixed time deposit. If the long-term assets held by the bank have a higher ROA, a unit asset will receive $R_2(> R_1)$ units of consumer goods in period 2, or $R_1$ units in period 1; if the bank’s liquidity in period 1 is insufficient, it will be evenly distributed among all the withdrawals of period 1. Considering the reality of China’s dual-track interest rate system, the ROA $r$ of long-term risk assets is a random variable, which can be regarded as the equilibrium interest rate in the market of loanable funds. It is the embodiment of interest rate marketization, and the corresponding probability density function is $q(r)$. Taking into account the realities of interest rate control, the bank’s deposit interest rates of $R_1$ and $R_2$ in the two periods change relatively little, and they are subject to the constraints of the ceiling on deposit rates as well as the floor on lending rates (Yi, 2009; He and Wang, 2011). With the gradual advance of interest rate marketization, $R_1$ and $R_2$ will be more flexible. The decision sequence of the bank and depositor is shown in Figure 1.

3.2 Participant’s behavior setting
3.2.1 Bank. In period 0, the bank provides two types of contracts to absorb deposits and invests the acquired deposits in $L$ units of short-term (current) asset and $X$ units of long-term (risk) assets, respectively. Assuming that the bank is risk-neutral, the goal is to maximize the expected utility by asset allocation of the absorbed deposits. Suppose that in period 0, the information set faced by the bank is $I_0$. Given the ROA $r$ of long-term assets, the bank’s problem of maximizing the expected utility is:

$$\max_{(L,X)} E \left[ L - D_1 R_1 + \sigma (Xr - D_2 R_2) \right] | I_0$$

s.t. $L + X \leq D$,

where $\sigma$ is the bank’s intertemporal risk adjustment factor, which measures the expected return and risk preference (appetite) of the bank for long-term investment, and satisfies $\sigma \geq (1/r)$, that is, the adjustment of yield of long-term risk assets at least realizes the present discounted value. The optimization problems faced by banks can be further clarified as the following:

$$\max_{(L,X)} E \left[ (1 - \sigma) L + \sigma Dr - D_1 R_1 - \sigma D_2 R_2 \right].$$
Since $1 - \sigma r \leq 0$ and $r$ is independent of other variables, it is directly inferred that banks will prefer to invest in long-term risk assets, that is, there is a “maturity mismatch” problem. To ensure the safety of the operation, it is assumed that the bank implements a liquidity management strategy $\rho(E_1R_1 + E_2R_2)$, where the parameter $\rho$ indicates the bank’s pre-judgment of the probability of run-off behavior, and it can also be used to indirectly characterize the depositor’s belief in the long-term ROA. Therefore, short-term liquid assets can be expressed as $L = \max \{\rho(D_1R_1 + D_2R_2), D_1R_1\}$, corresponding to the capital required by the government’s banking regulatory policies. According to the Basel Accords, the relevant provisions on bank capital shall satisfy $\rho D_2R_2 > (1-\rho)D_1R_1$, so $L = \rho(D_1R_1 + D_2R_2)$ will be obtained. In this way, the bank’s investment decision becomes the problem of maximizing the expected utility of the additional liquidity management strategy.

3.2.2 Depositor (consumer). According to the participant’s decision-making sequence in Figure 1, in the face of the two types of deposit contracts provided by the bank, the depositor will select $D_1$ units of demand deposits and $D_2$ units of time deposits. $D_1$ and $D_2$ are all functions of $R_1$ and $R_2$, and $D$ is the sum of the total deposits. The two asset allocations have certain substitutability $D = D_1(R_1, R_2) + D_2(R_1, R_2)$. It is assumed that the representative depositor has a utility function in Gorman’s Form: $u(c_1) + \beta u(c_2)$, where $u(\cdot)$ satisfies the Inada Condition, $\beta$ is a depositor’s intertemporal discount factor. In period 0, the depositor and the bank have the same information set $I_0$, based on which the problem of maximizing the expected utility faced by the representative depositor can be expressed as:

$$
\max_{[D_1, D_2]} E[u(c_1, c_2) | I_0] = [u(D_1R_1) + \beta u(D_2R_2)]
$$

s.t. $D_1 + D_2 \leq D$.

Solving the above maximization problem, the Euler equation is obtained, $u'(c_1) = \beta u'(c_2)$, namely, $u'(D_1R_1)R_1 = \beta R_2u'(D_2R_2)$. It shows that the depositor’s marginal utilities in the two periods of consumption are equal, that is, the marginal utilities of the depositor’s short-term or long-term capital allocation are equal; the expected utility is maximized.

3.2.3 The dynamic process of bank runs. Realities and researches have shown that the emergence of bank runs often involves two aspects: the decline of long-term ROA and wrong contractual arrangements. First, two types of premature withdrawal motives are defined: in the face of declining expected long-term ROA, depositors take out long-term deposits for consumption in period 1, which is defined as the Type 1 motivation. As depositors are worried about the safety of bank deposits, they will choose to withdraw money in period 1 for consumption in period 2 in the future, which is defined as the Type 2 motivation. It can be seen that although the depositors are taking out deposits in advance because of the expected reduction in the yield of long-term assets, the Type 1 motivation will increase the depositor’s consumption in period 1 and improve their own welfare, while the Type 2 motivation may cause a bank run panic and lead to economic disturbances. The following two stages will examine the bank-run process caused by the depositor’s two types of premature withdrawal motives.

The following scenario can be considered: depositors have incomplete information about the rate of return on long-term assets $r$ held by banks. Referring to the ideas of Goldstein and Pauzner (2005), the signals obtained by representative agent (depositor) $i$ are expressed as $
abla_i = r + \epsilon_i$, where $\epsilon_i$ denotes the potential error of depositor’s interpretation of the information, which has a probability density function $q_1(\epsilon_i)$ defined within the support $[-\epsilon_1, \epsilon_1]$, and accordingly, the probability density function $q_1(\nabla_i - r)$ of $\nabla_i$ is defined within the interval $[r - \epsilon_1, r + \epsilon_1]$, and $q_1(\cdot)$ satisfies the condition $\int_{r - \epsilon_1}^{r + \epsilon_1} q_1(\nabla_i - r)d\nabla_i = 1$. Lemma 1 gives
the nature of the conditional probability density function of long-term asset return that the depositor judges based on information:

Lemma 1. For any depositor $i$ and $j$, if the probability density functions $q_1(\mathbf{e}_1)$ and $q_2(\mathbf{e}_2)$ of the inference error are independent of each other, then there is a positive number $\delta$ such that $f'_2(\hat{r}_j \pm \delta) = f'_2(\hat{r}_j)$, that is, for depositor $i$ and $j$, the inferred information of the long-term asset return is symmetrical.

In period 1, the short-term and long-term deposits of the asset allocation of depositor $i$ are expressed as follows: $D'_1 + D'_2 = D$. If the depositor $i$ observes or predicts that the long-term ROA is reducing, based on the Type 1 motivation, part of the long-term deposit, $\Delta'_1 > 0$, will be withdrawn in advance in period 1, for the consumption of period 1, and the Euler equation for the maximum expected utility of the depositor derives:

$$u' \left( (D'_1 + \Delta'_1)R_1 \right) R_1 = E \left[ \beta R_2 u' \left( (D'_2 - \Delta'_1)R_2 \right) \right].$$

(1)

Here, the right side of the equation can be expressed as:

$$\int_{(r-e_1)}^{(r+e_1)} \beta u' \left( (D'_2 - \Delta'_1)R_2 \right) R_2 q_1(\hat{r}_1-r) d\hat{r}_1,$$

where $R_2 = (X\hat{r}_1^*(1-\rho)D'_2) - (D'_1/D'_2)R_1$.

For any long-term ROA, it is inferred from the above formula (1) that $\Delta'_1 = g(r, e_1, D'_1, R'_1)$. Only the Type 1 motivation is considered to cause the depositor to withdraw in advance, and therefore only $\Delta'_1$ appears in the expected utility function of the depositor in period 1, and the amount of premature withdrawal $\Delta_1$ is uniquely determined by $\Delta_1 = \int_0^1 \Delta'_1(\cdot) di$. Specifically, in the case that depositor $i$ regards the signal $r'_1$ as the ROA distribution center, $r'_1$ represents the threshold value of the long-term ROA upon which the bank run will (not) just happen, determined by $D_2 R_2 = \rho(D_1 R_1 + D_2 R_2) - D_1 R_1 + Xr'_1$. There are three scenarios to discuss the number of bank runs that occur:

1. If $r \geq r'_1 + 2e_1$, then $\Delta_1 = 0$, namely, when the long-term ROA is high enough, no bank run will occur;

2. If $r'_1 \leq r < r'_1 + 2e_1$, then $\Delta_1 = \lim_{D'_1 \rightarrow 0} \int_{r'_1-e_1}^{r'_1+e_1} \Delta'_1 q'_1(\hat{r}'_1-r) d\hat{r}'_1 = \lim_{D'_1 \rightarrow 0} \int_{r'_1-e_1}^{r'_1+e_1} g(r, e_1, D'_1, R'_1) q'_1(\hat{r}'_1-r) d\hat{r}'_1$;

3. If $r < r'_1$, then $\Delta_1 = \lim_{D'_1 \rightarrow 0} \int_{r-e_1}^{r+e_1} g(r, e_1, D'_1, R'_1) q'_1(\hat{r}'_1-r) d\hat{r}'_1$.

In the case of a given deposit contract, we further have $\Delta_1 = h(r, e_1)$, and $(\partial \Delta_1/\partial r) < 0$, $(\partial \Delta_1/\partial e_1) > 0$, indicating that the number of runs moves inversely to the long-term ROA and in the same direction with depositors’ interpretation error of the ROA information.

In the second period, depositor $i$ receives a run signal with noise, and the judgment on the long-term ROA is updated as $r'_1$. The depositors process the information in the same way as described above, but their expectations for the signal change. At this time, depositor $i$ expects the long-term ROA as $r'_1 = r + e'_2$, where $e'_2$ indicates the error of the inference depositor $i$ makes based on the information of the previous round. It has a probability density function $q_2(e'_2)$ defined within the interval $[-e_2, e_2]$, and then the probability density function of $r'_2$ is $q_2(r'_2-r)$, defined within $[r-e_2, r+e_2]$. In order to clearly characterize the depositor’s information updating process, we consider using adaptive expectation equation to weigh the
information of the two adjacent periods, namely, \( r_2^i = t_1 r_1^i + (1-t_1) r_0^i = r + \tilde{u}_2^i \), to obtain new inferences, where \( \tilde{u}_2^i = e_2 + (1-t_1) e_0^i \) is aggregated error and \( t_1 \) is the weight the depositor assigned to the first round of information during processing.

The bank-run behavior of depositors under the Type 2 motivation will be considered below. Assuming the number of bank runs to be \( \Delta_2 \), the depositor’s consumption decision in period 2 is given by \( E[\mu_2((D_2-\Delta_2-\Delta_4)R_2 + \Delta_2 R_1)] \). \( r_2^* \) denotes the threshold value of the long-term ROA upon which the bank run will (not) just happen, thereby satisfying the following condition:

\[
\int_{r_2^* - u}^{r_2^* + u} \left[ X \hat{r}_1^i + \rho(D_1 R_1 + D_2 R_2) - E_1 R_1 - \Delta_1 R_1 \right] f_2(r_2^i \mid r_2^*) d\hat{r}_1^i = (E_2 - \Delta_1) R_1. \tag{2}
\]

Here, \( f_2(r_2^i \mid r_2^*) \) indicates the conditional probability density function of the depositor \( i \)'s expected long-term ROA in the case of given information \( r_2^i \) and satisfies the normalization condition \( \int_{r_2^* - u}^{r_2^* + u} f_2(r_2^i \mid r_2^*) d\hat{r}_1^i = 1 \).

Consider the homogeneity of information transmission, that is, each depositor has the same judgment on the long-term ROA held by the bank. The above formula (2) derives \( r_2^* \), that is, if the long-term ROA is lower than \( r_2^* \), it will trigger a bank run. According to Lemma 1, the ratio of runs can be expressed as a truncated cumulative distribution function, and the following proposition is compiled:

**P1.** Given the required threshold value of the long-term ROA, upon which the bank run will (not) just happen, to be \( r_2^* \), the depositor’s bank-run ratio is \( \lambda_1(r_2^*, u) = \int_{r_2^* - u}^{r_2^* + u} f_2(r_2^i \mid r_2^*) d\hat{r}_1^i \), and the corresponding number of runs is \( \Delta_2 = (D_2 - \Delta_1) \lambda_1 \).

Obtained directly from P1, the balance of the depositor’s term deposit in period 2 is \( D_2 - \Delta_1 - \Delta_2 = (1 - \lambda_1) (D_2 - \Delta_1) \). Similarly, based on \( \Delta_2 \), the depositors can further update their pre-judgment information regarding the \( r \) in period 3.

Considering a more generalized case, in the \( t \geq 2 \) round, based on historical information, the depositor expects the long-term ROA information to be \( \hat{r}_t^i = r + e_t^i \), where \( e_t^i \) has a probability density function \( q_t(e_t^i) \) defined within the interval \([-e_n, e_n]\) and the following defining and sorting rules will characterize the depositors’ information structure in different rounds:

**Definition 1.** (Sorting rule): If the depositor \( i \) gets the information \( r_t^i = r + e_t^i \) in the round \( t \), and the signal received in the \( t + 1 \) round is \( r_{t+1}^i = r + e_{t+1}^i \), then they represent a sorting rule \( \Omega \), where \( \Omega \) denotes the set of ordering rules.

In particular, if in the 1st round, the depositor receives the worst signal, \( e_1^i = -e_1 \), and the worst signals in each subsequent round, \( e_2^i = -e_2, e_3^i = -e_3 \ldots e_n^i = -e_n \), then they represent a sorting rule of the worst signal denoted as \( \omega \). According to the sorting rule, the depositor uses the weighted method of the adaptive expectation equation to estimate the long-term ROA, i.e.:

\[
r_3^i = t_2 r_2^i + (1-t_2)(r + e_2^i); \quad r_4^i = t_3 r_3^i + (1-t_3)(r + e_3^i); \quad r_t^i = t_{t-1} r_{t-1}^i + (1-t_{t-1})(r + e_t^i).
\]

For any \( n \) round, there is \( r_n^i = t_{n-1} r_{n-1}^i + (1-t_{n-1})(r + e_n^i) = r + u_n \), where \( u_n \) is the comprehensive error of the depositor’s prediction of long-term ROA in the round \( n \), i.e.:

\[
u_n = \prod_{i=1}^{n} t_i e_1 + \prod_{i=2}^{n} t_i(1-t_1) e_2 + \prod_{i=3}^{n} t_i(1-t_2) e_3 + \ldots + (1-t_{n-1}) t_n e_n + (1-t_n) e_{n+1}.
\]
We define $\Phi = \{e_1, e_2, e_3, \ldots\}$ as the noise sequence signifying the inference error of the depositor on the long-term ROA in different rounds; at the same time, $T = \{t_1, t_2, t_3, \ldots\}$ is defined as the weight sequence, indicating the depositor's weight selection while processing the information from different rounds. Both are the basic elements composing $\Omega$, that is, depositors' expectations of long-term ROA are often affected by $\Omega$, i.e., the combined influence of noise sequences $\Phi$ and weighting sequences $T$.

Similar to the above analysis, the nature of the conditional probability density function of any round's long-term ROA is discussed and organized into the following Lemma:

**Lemma 2.** For any round $n, n \geq 2$, and any depositor $i$ and $j$, if the probability density functions $q_1(\hat{e}_1), q_2(\hat{e}_2), \ldots, q_n(\hat{e}_n)$ of the inference error are independent of each other, then there is a positive number $\delta$, such that $f_{n}(\hat{r}_j \pm \delta) = f_{n}(\hat{r}_j)$, i.e., for any two depositors $i$ and $j$, their inferred information of long-term ROA is symmetrical.

Define $r_{n+1}^*$ as the threshold value of the long-term ROA upon which the bank run will (not) just happen at round $n$, thereby satisfying the following conditions:

$$
\int_{r-u}^{r+u} \left[ X^{n+1}_{n} + \rho(D_{n+1}R_{n+1} - D_{n}R_{n} + \sum_{j=1}^{n} \Delta_{n}R_{j}) f_{n+1}(\hat{r}_{n}^{i} | r_{n+1}^{i} = r_{n+1}^{*}) \right] d\hat{r}_{n}^{i} = (D_{n} - \sum_{j=1}^{n} \Delta_{j}) R_{1}, \quad (3)
$$

where $f_{n+1}(\hat{r}_{n}^{i} | r_{n+1}^{i} = r_{n+1}^{*})$ represents the conditional probability density function of the depositor $i$'s inference error on the long-term ROA in round $n$.

Consider a more general situation. For any round $n, n \geq 2$, the above formula (3) denotes the threshold value $r_{n+1}^*$ of the long-term ROA upon which the bank run will (not) just happen, that is, when the long-term ROA is lower than $r_{n+1}^*$, it will trigger a bank run. Combined with Lemma 2, the ratio of the depositors' bank runs is expressed as a truncated cumulative distribution function, which is organized as follows:

**P2.** For any round $n, n \geq 2$, given the threshold value $r_{n+1}^*$ of the long-term ROA triggering a bank run, the ratio of the depositors' bank runs is $\lambda_n(r_{n+1}^*, u) = \int_{r-u}^{r+u} f_{n+1}(\hat{r}_{n}^{i} | r_{n+1}^{i} = r_{n+1}^{*}) d\hat{r}_{n}^{i}$; the corresponding amount of runs is $\Delta_n = (D_{n} - \sum_{j=1}^{n-1} \Delta_{j}) \lambda_n$.

**P2** is a more general characterization of **P1**. Considering the homogeneity of information transmission, each depositor has the same inference for long-term ROA. The depositor's balance in round $n$ can be directly obtained by **P2**:

$$
E_2 - \sum_{i=1}^{n+1} \Delta_i = \prod_{i=1}^{n} (1-\lambda_{i})(E_2 - \Delta_1).
$$

Combining **P1** and **P2**, the cumulative number of bank runs in $n \geq 2$ rounds is:

$$
\Delta = \sum_{i=1}^{n} \Delta_i = \Delta_1 + (D_2 - \Delta_1) \prod_{j=2}^{n} (1-\lambda_{j-2}) \lambda_{j-1},
$$

where $\lambda_0 = 0$.

It should be noted that in analyzing the dynamic process of bank runs, special attention should be paid to the depositor's inference error (noise) sequence $\phi$ of long-term ROA and
the weighting sequence $T$ of information processing. Both are determined by the information processing capacity of the depositor and the efficiency of information dissemination of the economy itself. They are often unrelated to the ROA $r$ and can be regarded as variables that characterize the financial institutional environment. This paper mainly deals with interest rate marketization (market-oriented interest rate). At different marketization levels, depositors and banks have different interpretations and dissemination efficiency of information, and varied processing capabilities as well, that is, different noise sequences $\Phi$ and weight sequences $T$ are used for characterization.

3.2.4 Dynamic equilibrium. We first define the dynamic equilibrium of the bank run and then propose a way to find dynamic equilibrium:

**Definition 2.** (Dynamic Equilibrium): There exists a round $t^*$, such that at the round $t \geq t^*$, any depositor $i$ will no longer participate in a bank run, i.e., $D_i^{t+1} = 0$. Specifically, if the distribution information of this round's signal is given, and the following conditions are satisfied for any depositor $i$:

$$
\int_{r+1}^{r+u} \left[ X\bar{r}_n + \rho(D_1R_1 + D_2R_2) - D_1R_1 - \sum_{i=1}^{n} \Delta_i R_i \right] f_{n+1}(\bar{r}_{n+1}^{*}) d\bar{r}_{n+1}^{*} \geq \left( D_2 - \sum_{i=1}^{n} \Delta_i \right) R_i. \tag{3}
$$

This state is called dynamic equalization.

However, equation (3) may not guarantee that the depositor who got the lower ROA information in the previous period can obtain a higher ROA information in this round. Therefore, it is necessary to consider the problems caused by this uncertainty. In particular, when the signal received by the depositor is invariably $e_i^t = -e_i$ in any round $t$, according to the worst-sorting rule $\omega^*$, there is a certain round after which any depositor will stop the run. We thus call it the strong equilibrium state, which is summarized as follows:

**Definition 3.** (Strong equilibrium): If the equilibrium state is independent of the sorting rules, it is called a strong equilibrium.

It should be noted that the strong equilibrium state can only occur when the long-term asset yield $r$ is large enough and the probability density function of the inference error $e_i$ has a sufficiently high concentration on the right side of the integration interval, that is, the depositor has a higher probability of acquiring higher ROA signal. It can be seen as a constraint on the sorting rules. For a particular depositor, a higher expected assets earning rate will be received:

$P3$. Given a noise sequence $\Phi$, a weight sequence $T$ and a probability density function $q_t(e_t)$ of the error, there is a dynamic equilibrium and a strong equilibrium if and only if the sorting rule is the worst-sorting rule.

$P3$ provides a way to find (strong) equilibrium, for which we only need to verify whether the expected bias of the depositor’s long-term ROA satisfies the worst-sorting rule. Given the similarity of other variables, the probability density function $q_t(e_t)$ of inference error affects the expected value of the left part of the threshold value in equation (3). It can be seen that the form of probability density function of the depositor acquired signal also affects the equilibrium result.

According to the above analysis, the factors affecting the bank runs mainly involve three aspects: first, changing the weight sequence $T$ of depositor information processing; second, changing the depositor’s interpretation of long-term ROA information and dissemination error sequence $\Phi$. Both are endogenous in the financial system environment. In the environment with high degree of marketization, the market participants can accurately interpret and disseminate information, and thus the error is small. At the same time, the processing of information tends to be clear and rational, especially if depositor endows more weight to the next period of information. If the market information transmission efficiency is low, the
occurrence of the run still cannot be prevented. Third, changing the probability density function \( q_t(e_t) \) of the depositor's pre-judgment information. Furthermore, the sorting rules are often determined endogenously by the depositor's attributes and cannot be easily and accurately identified, and measured by banks and policy designers. Therefore, such uncertainties may also lead to instability in the banking system.

4. Model extension via the incorporation of DIS

This section introduces a DIS to expand the benchmark model and examine the potential impact of DIS implementation on the dynamic equilibrium of bank runs. Suppose that the DI quantity of the bank (depositor) provided by DI regulatory agencies is \( F = \gamma D, 0 \leq \gamma \leq 1 \), that is, when a bank run occurs, the bank (depositor) can obtain a financial aid with a total amount of \( F \), where \( \gamma \) indicates the coverage (ratio) of DIS and is thus one of the key elements in the design of DIS. Assuming that whether the government implements DIS and the specific DI contract are common information shared by all participants such as depositors and banks, the implementation of explicit DIS will affect the bank's managerial decision-making via its effects on the bank's investment decisions, the depositors' expectations and behaviors commensurate with changes in long-term ROA, and the mechanism of market discipline, etc., thereby affecting the dynamic process and equilibrium of bank runs.

4.1 Changes in the behavior of participants

Under the background of the financial system reform of interest rate marketization, DIS implementation will cause changes in the behavior of depositors and banks and other participants, which, in turn, will affect the bank-run process caused by depositors' expected liquidity shocks of long-term assets held by the bank. This is embodied in Figure 1, which characterizes the sequence of participants' decision-making: at \( t = 0 \), the government begins to implement a DIS, which will influence the bank’s investment decisions at \( t = 0.5 \) and will have an impact on the depositors' withdrawal behavior at \( t = 1 \).

4.1.1 Changes in depositor behavior. When DIS is implemented, it mainly affects the behavior of depositors in two aspects: first, as deposits are guaranteed, depositors may weaken their incentives to supervise the bank's investment decisions, leading to the waning of market discipline mechanism; second, expectations of depositors on the impact of changes on long-term ROA have changed, \( q(r,F) \). With the advancement and improvement of market-oriented interest rate, the depositor's ability to process information will be elevated, and the efficiency of information interpretation and dissemination will also escalate, thereby reducing the depositor's inference error sequence \( \Phi \) for long-term ROA. Meanwhile, the weighting sequence \( T \) used in the information processing process also becomes clearer and more rational, which will enhance the accuracy of the depositors' interpretation and processing of information, making them more patient to fluctuations of long-term ROA in banks, and more likely to hold stable the withdrawal expectations and decisions.

4.1.2 Changes in banking behavior. The impact of DIS implementation on bank investment decisions is mainly reflected in the following two aspects. First, it affects the allocation of investment assets of banks, prompting banks to reduce the number of holdings of current assets and beef up the investment amount of risk assets. With the advancement of interest rate marketization, the short-term and time deposit interest rates will be more flexible. Considering the increasing competition in the banking industry, the interest rate on deposits will increase, whereas the interest rate on loans will fall, resulting in narrowing interest rate spreads and more incentives for banks to invest in high-yield and high-risk projects. Second, the guarantee of DIS on behalf of the repayment of deposits may render the banks with loose self-discipline, especially for the economies in transition. The internal governance mechanism of banks is far from perfect; with the implementation of the explicit
DIS, banks will be more motivated to invest in high-risk projects. It can be seen that the above two aspects have invariably boosted the moral hazard behavior of banks.

At this point, bank's investment decision-making behavior that maximizes the expected utility changes. When the amount of DI is $F$, the bank's problem of maximizing the expected utility becomes:

$$\max_{(L,X)} \int \left[ L - D_1 R_1 + \sigma (X \bar{\tau} - D_2 R_2) \right] q(\bar{\tau}, F) d\bar{\tau}$$

s.t. $L + X \leq D$,

where $q(\bar{\tau}, F)$ denotes the probability density function of long-term ROA while introducing DIS.

Further, the issue of maximizing the expected utility of banks is re-expressed as:

$$\max_{(L,X)} \int ((1 - \rho) L + \sigma D \bar{\tau} - D_1 R_1 - \sigma D_2 R_2) q(\bar{\tau}, F) d\bar{\tau}.$$ 

As with the benchmark model, since $1 - \rho < 0$, the bank's asset allocation to maximize expected utility is $L = 0$, that is, the bank prefers to engage in high-risk investment projects. Therefore, the government usually requires banks to implement a liquidity management discipline through regulatory policies, i.e., $L = \rho (D_1 R_1 + D_2 R_2)$. The difference is that DIS implementation will weaken this discipline. The specific manifestation is that the lowering of the parameter $\rho$ can be explained by the bank's lowering of the pre-judged probability of the occurrence of the run-off behavior or the weakening of its internal governance mechanism, which will lead to its reduced holding of liquid assets, while adding up the amount of investment on risk assets. Coupled with the narrowing of potential interest margin gains in the process of interest rate liberalization, the banks' incentives for high-risk businesses to pursue high returns have led to serious moral hazard problems. Moreover, when there is insufficient information processing capability, the bank will have an overoptimistic estimate of the stability of DIS, which will lead them to relax their self-discipline and invest in high-risk projects. It can be seen that limited information processing capabilities may also lead to moral hazard issues.

In summary, DIS implementation will have an impact on the bank's investment decisions and depositors' behavior, and it will bring about the opposite dual effects of preventing run-off and eliciting moral hazard. The payment guarantee brought by DIS will decelerate the process of depositors' expectations on bank failures, thus having a restraining effect on bank runs. DIS implementation directly changes the bank's asset allocation decisions while weakening the supervision over banks and depositors, both of which are confronted with moral hazard problems while banks investing more in high-risk assets, and these effects are substantially sustainable. The following section will analyze the impact of DIS implementation on the bank-run process and dynamic equilibrium.

### 4.2 Bank run and dynamic equilibrium

Continuing the previous analysis, the following section will examine the impact of DIS implementation on the two types of premature withdrawal motives of depositors, which, in turn, affects the process and dynamic equilibrium of bank runs.

#### 4.2.1 Bank-run process

First of all, this paper analyzes the process of bank run in the new economic environment of DIS, which is divided into two periods.

In period 1, the depositor has incomplete information on the earnings rate of long-term assets $r$ and error $e_1$ in the interpretation of information, and there is a probability density function $q_1(e_1, F)$ defined within the interval $[-e_1, e_1]$. Define the signal obtained by depositor $i$ as $r_i = r + e'_1$, then the probability density function of $r'_i$ is $q_1(r'_i - r, F)$. In the new economic environment, the threshold value of the long-term ROA triggering a bank run is...
subject to changes. The new threshold is defined as \( r^*_d \), and it is determined by 
\[ D_2 R_2 = \rho (D_1 R_1 + D_2 R_2) - D_1 R_1 + X r^*_d + F. \]

When depositor \( i \) observes or expects a decrease in the long-term ROA, based on the Type 1 motivation, she/he will take out some of the long-term deposits in advance in period 1 to maximize the expected utility. As per the analysis of the benchmark model, for any long-term asset yield \( r \), given the deposit arrangement of a depositor, we can invariably get \( \Delta^i_1 = g(r, e_1, L; F) \). It should be noted that both \( e_1 \) and \( L \) will be affected by the DI amount \( F \) (a detailed analysis will be made below). Considering the following three scenarios, the number of bank runs that occurred due to Type 1 motivation of the round 1 depositors is discussed:

1. when \( r \geq r^*_d + 2e_1 \), then \( \Delta^i_1 = 0; \)
2. when \( r^*_d \leq r < r^*_d + 2e_1 \), then \( \Delta^i_1 = \lim_{D_1 \to 0} \int_{r_d}^{r^*_d + e_1} g(r_i, e_1, L) q_1(\hat{r} - r, F) d\hat{r} \); and
3. when \( r < r^*_d \), then \( \Delta^i_1 = \lim_{D_1 \to 0} \int_{r - e_1}^{r^*_d} g(r_i, e_1, L) q_1(\hat{r} - r, F) d\hat{r}. \)

It can be seen that the depositor’s premature withdrawal amount \( \Delta^i_1 \) only appears in the expected utility function of period 1 and is uniquely determined by the equation \( \Delta^i_1 = \int_{0}^{1} \Delta^i_1(\cdot) d\hat{r}. \)

The following analysis of the impact mechanism of DIS implementation on the first type of premature withdrawal motives mainly involves two aspects. First, DIS implementation will affect the threshold value of the long-term ROA triggering a bank run. As DIS provides guarantee for repayment of deposits, the depositor’s tolerance for the decline in long-term asset yields increases, causing the threshold value of long-term asset yields to fall. It can be further inferred that with the increase in the ratio of DI coverage \( r \), the decrease rate of threshold value will increase. Since \( r^*_d \) is the upper limit to trigger (when it is lower than \( r^*_d \)) the depositor’s bank run, the decline of \( r^*_d \) can reduce the probability of bank run. Second, the bank’s asset allocation structural changes can cause moral hazard problems. The bank expects that depositors’ attitudes toward fluctuations in long-term ROA will change, coupled with the narrowing interest rate differentials brought about by interest rate liberalization. They will be more preferred to engage in high-yield and high-risk businesses, which will both help the banks to reduce liquidity assets and to augment investment incentives for risky assets. According to the above analysis, the two opposite effects are expressed as the following:

\[
\frac{\partial \Delta^i_1}{\partial F} = \frac{\partial \Delta^i_1}{\partial r^*_d} \frac{\partial r^*_d}{\partial F} + \frac{\partial \Delta^i_1}{\partial L} \frac{\partial L}{\partial F} ,
\]

where the first part (A) on the right side of Equation (4) indicates that DIS implementation affects the threshold value of the long-term ROA triggering a bank run, and thus affecting the premature withdrawal amount of deposits. Given \( (\partial \Delta^i_1 / \partial r^*_d) > 0 \) and \( (\partial r^*_d / \partial F) < 0 \), then \( A < 0 \), i.e., DIS implementation helps to curb the occurrence of bank runs, which is called the stabilizing effect. The second part (B) indicates that DIS implementation affects the probability of bank runs via its impact on the bank’s asset allocation changes. Given \( (\partial L / \partial F) < 0 \) and \( (\partial \Delta^i_1 / \partial L) < 0 \), we have \( B > 0 \), that is, DIS implementation will encourage incentives for banks to engage in high-risk business, leading to the negative effects of moral hazard.

In the second period, the depositor \( i \) receives a run signal with noise, and the judgment on the long-term ROA is updated to \( r'_1 = r + e'_2 \), where \( e'_2 \) represents the depositor \( i \)’s inference error based on the previous round of information, with a probability density function \( q_2(e'_2, F) \) defined within the interval \([-e_2, e_2]\). Accordingly, the probability density function of long-term ROA is \( q_2(r'_1 - r, F) \) defined within \([r-e_2, r+e_2]\). As in the analysis process of
In the third section, the depositor uses the adaptive expectation model to weigh the information of the two adjacent periods to form a new inference. Considering a more general case, we define \( r_{d,n+1}^* \) as the threshold value of the long-term ROA triggering a bank run in round \( n \), which satisfies the following conditions:

\[
\int_{r_{d,n+1}^*}^{r_{d,n+1}} \left[ X \tilde{r}_n + \rho (D_1 R_1 + D_2 R_2) - D_2 R_1 \sum_{i=1}^{n} \Delta \tilde{r}_i \right] f_{n+1}^\prime (\tilde{r}_n | r_{d,n+1}^*) d\tilde{r}_1 = \left( D_2 - \sum_{i=1}^{n} \Delta \right) R_1 + F,
\]

where \( f_{n+1}^\prime (\tilde{r}_n | r_{d,n+1}^*, F) \) represents the conditional probability density function of the depositor \( i \)'s inference error on the long-term ROA in round \( n \geq 2 \), and \( \tilde{r}_n = r + u_n \).

Given the deposit composition of a depositor, it can be inferred from the monotonicity of the left side of the above equation that \( r_{d,n+1}^* < r_{d,n+1}^* \), that is, DIS implementation will cause a decline of the threshold value of the long-term ROA upon which the bank run will (not) just happen, thus reducing the probability of a bank run. In turn, the ratio and number of bank runs of depositors at any round are obtained, which is expressed as the following proposition:

**P4.** In the case of DIS implementation, for any round \( n, n \geq 2 \), given the threshold value of the long-term ROA triggering a bank run, \( r_{d,n+1}^* \), the ratio of bank runs is thus:

\[
\lambda_u (r_{d,n+1}^*, u_n, F) = \int_{r_{d,n+1}^*}^{r_{d,n+1}} f_{n+1}^\prime (\tilde{r}_n | r_{d,n+1}^*, F) d\tilde{r}_1.
\]

The corresponding number of bank runs is \( \Delta_n = \left( D_2 - \sum_{i=1}^{n-1} \Delta_i \right) \lambda_u (r_{d,n+1}^*, u_n, F) \).

In the context of interest rate liberalization, the impact of DIS on the second type of premature withdrawal motives of depositors involves three aspects. First, DIS implementation will affect the threshold value of the long-term ROA triggering a bank run, which is similar to the impact of the Type 1 motivation. Second, the inference error of the depositor on the long-term ROA will be reduced, and the weighting process of the information becomes more rational and clear, that is, taking into account the small probability for large deviations, it is given a lighter weight. Conversely, a heavier weight is given to the smaller deviations, which leads to stable expectations and decision-making of withdrawal. Third, the bank may adjust the asset allocation structure and reduce liquid assets, increase venture capital, which triggers moral hazard issues that are similar to the impact of the Type 1 motivation. Specifically, in any round, the impact of DIS implementation on the bank-run ratio can be broken down into three parts:

\[
\frac{\partial \lambda_u (r_{d,n+1}^*, u_n, F)}{\partial F} = \frac{A}{\partial r_{d,n+1}^*} + \frac{C}{\partial u_n} + \frac{B}{\partial L} + \frac{C}{\partial F},
\]

where the first part (\( A \)) indicates that DIS implementation directly affects the threshold value of the long-term ROA triggering a bank run, which, in turn, affects the ratio of depositors' bank runs. From \( \frac{\partial \lambda_u (\cdot)}{\partial r_{d,n+1}^*} > 0 \) and \( \frac{\partial r_{d,n+1}^*}{\partial F} < 0 \), we derive \( A < 0 \); the second part (\( C \)) analyzes the impact of DIS implementation on the depositor pre-judgment bias sequence and weight sequence as a channel to influence the bank-run ratio. From \( \frac{\partial \lambda_u (\cdot)}{\partial u_n} > 0 \) and \( \frac{\partial u_n}{\partial F} < 0 \), we derive \( C < 0 \). It can be seen that both results show that DIS implementation has a restraining effect on bank runs, namely, \( D = A + C < 0 \). Thus, they jointly form a stabilizing effect. The third part (\( B \)) indicates the transmission path of moral hazards in banks after DIS implementation when the banks are more inclined to invest in high-risk assets because they expect the DI institutions to provide repayment guarantees for potential crises. At the same
time, the narrowing of bank interest spreads brought about by the advancement of interest rate marketization will strengthen this effect. *Ceteris paribus*, it is bound to increase the probability and ratio of bank runs, namely, \( B > 0 \).

Based on the analysis of the above two periods and multiple rounds of bank-run process, and the results of Equations (4) and (5), DIS implementation has two opposite effects of stabilizing withdrawal expectation and inducing moral hazard, which are mainly reflected in the following three aspects. First, DIS implementation can reduce the threshold value of bank runs when depositors expect changes in long-term ROA, and it can cut down the probability and ratio of bank runs, which are reflected in Equation (4) of Type 1 motivation and the first part of Equation (5) of Type 2 motive. Second, with the advancement of the interest rate marketization, DIS implementation will reduce the depositor’s inference error sequence for the long-term ROA, and at the same time, the weighted sequence of weights used in information processing also becomes more rational and clear, thereby reducing the population deviation of the depositor’s pre-judgment and reducing the probability and ratio of bank runs, which are reflected in the second part of Equation (5) of Type 2 motive. Third, DIS implementation encourages banks to have greater incentives to engage in high-risk businesses to reap greater yields, and it uplifts depositors’ expectations of DIS providing all or part of the deposit payment security, thus weakening their oversight of the banks and eliciting more moral hazard behaviors in the latter. These are reflected in the second part of Equation (4) of Type 1 motivation and third part of Equation (5) of Type 2 motivation. The first and second aspects above show that DIS implementation has a stabilizing effect on bank runs, whereas the third aspect will lead to moral hazard problems in banks. The total effect depends on the strength comparison between the two.

As defined by dynamic equilibrium, we need to determine a threshold round \( t^* \); after \( t \geq t^* \) rounds, any depositors no longer initiate a bank run. From the above analysis, it can be seen that DIS implementation can affect the threshold round triggering a bank run. Specifically, the stabilizing effect achieved by DIS implementation can reduce the threshold rounds, so that the bank run ends early. Conversely, the moral hazard effect can augment the probability and ratio of bank runs, and eventually the runs end late. Therefore, policy makers need to integrate and balance the stabilizing effects and moral hazards while introducing and implementing DIS.

### 5. Conclusions and policy implications

As one of the three pillars of the financial safety net, DIS, together with prudential supervision, and the central bank as lender of last resort, promotes the healthy development of the financial system, helps prevent and resolve risks in a timely manner, and maintains the stability of the financial system. In May 2015, the “Regulations on Deposit Insurance” officially came into force, marking the formal entry into the implementation level of DIS that the government and academia have discussed for many years. Then, in the context of interest rate marketization, how to effectively put DIS “in place” and achieve the expected goal is an urgent problem to be studied.

In view of the interest rate marketization, we comprehensively considered the economic environment for banks’ investment decision-making and depositors’ consumption decision-making. Based on the dynamic perspective, this paper constructed a bank-run model of multi-round noise information transmission, discussed the dynamic process of bank run in stages, and separately considered the two types of premature withdrawal motives of the depositors. The ratio and quantity of bank runs are derived from the fluctuations of long-term ROA held by the bank, the sorting rule of the depositor’s pre-judgment error, the information processing and disseminate efficiency under information asymmetry and other factors, and the corresponding dynamic equilibrium is characterized. We further incorporated the DIS into our benchmark model, described the impact of DIS implementation on bank and depositor behavioral decision-making, and then studied the influence mechanism of DIS on bank-run process and the corresponding dynamic equilibrium by analyzing its impact on depositors’ two types of
premature withdrawal motives. The research results show that DIS implementation has two opposite effects of stabilizing expectation and eliciting moral hazards, which are mainly reflected in the following three aspects. First, DIS implementation can reduce the threshold value of bank runs when depositors expect changes in long-term ROA, and it can cut down the probability and ratio of bank runs. Second, with the advancement of the interest rate marketization, DIS implementation will reduce the depositor’s inference error sequence for the long-term ROA, and at the same time, the weighted sequence of weights used in information processing also becomes more rational and clear, thereby reducing the population deviation of the depositor’s pre-judgment and reducing the probability and ratio of bank runs. Third, DIS implementation encourages banks to have greater incentives to engage in high-risk businesses to reap greater yields, and it uplifts depositors’ expectations of DIS providing all or part of the deposit payment security, thus weakening their original oversight of the banks and eliciting more moral hazard behaviors in banks. The effect of implementing a DIS depends on the comparison of stabilizing effects and moral hazards, which rests with the institutional environment, thus requiring policy makers to comprehensively consider the trade-off between stabilizing effects and moral hazards while introducing and implementing DIS.

DIS is an important groundwork system for national finance. It provides institutional guarantee for the reform of the financial system, especially the promotion of interest rate marketization, and it improves the operational efficiency of the financial market. However, in the implementation process of DIS, it is necessary to take into account the moral hazards caused by imperfect supporting policies and measures. This requires proper handling of the dynamic relationship between interest rate marketization, DIS and financial transformation and development. The study has the following policy implications. While implementing DIS, on the one hand, efforts should be made to reinforce the construction of financial regulatory mechanisms, strengthen market discipline, improve the role of the bank’s internal governance mechanisms, and reduce the negative impact of potential moral hazards. On the other hand, efforts should be made to further deepen the reform of the financial system, improve the efficiency of processing and dissemination of information, reduce the deviation of depositors’ pre-judgment of bank yield information, and encourage depositors to form stable withdrawal expectations in the face of changes in long-term asset yields of banks, thereby enhancing the stabilizing effect of DIS.

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Notes
1. Given the strong externalities of the financial industry, the financial industry is the most regulated industry, even in mature developed market economies.
2. For the demonstration process of DIS implementation, refer to Su (2005).

References


Appendix

(1) Proof of PI: Proof: by Equation (1), we have:

\[ B_i = \int_{(r_i^2 + u)}^{(r_i^2 + u)} \tilde{r}_i f_2(\tilde{r}_i | r_i^2 = r_i^2) d\tilde{r}_i. \]

By Lemma 1, for depositors \( i \) and \( j \), we have:

\[ f_2(\tilde{r}_j - \delta) = f_2(\tilde{r}_j). \]

Here:

\[ \delta = t_1 e_1 + (1-t_1)e_2 - t_1 e_1' - (1-t_1)e_2' > 0, \]

and thus:

\[ B_i = \int_{r_i + t_1 e_1 + (1-t_1)e_2 - u}^{r_i + t_1 e_1 + (1-t_1)e_2 + u} \tilde{r}_i f_2(\tilde{r}_i) d\tilde{r}_i, \]

\[ B_j = \int_{r_i + t_1 e_1 + (1-t_1)e_2 - u}^{r_i + t_1 e_1 + (1-t_1)e_2 + u} \tilde{r}_j f_2(\tilde{r}_j) d\tilde{r}_j. \]

Let \( \hat{r}_i = \tilde{r}_i + \delta \), and by placing it in equation \( B_i \), we have:

\[ B_i = \int_{r_i + t_1 e_1 + (1-t_1)e_2 - u}^{r_i + t_1 e_1 + (1-t_1)e_2 + u} (\hat{r}_i - \delta) f_2(\hat{r}_i - \delta) d\hat{r}_i = \int_{r_i + t_1 e_1 + (1-t_1)e_2 - u}^{r_i + t_1 e_1 + (1-t_1)e_2 + u} (\hat{r}_i - \delta) f_2(\hat{r}_i) d\hat{r}_i = B_i - \delta < B_i. \]

Therefore, for depositors with signal \( r_i^{2} < r_i^2 \),

\[ \int_{r_i + t_1 e_1 + (1-t_1)e_2 - u}^{r_i + t_1 e_1 + (1-t_1)e_2 + u} \tilde{r}_i f_2(\tilde{r}_j) d\tilde{r}_i = B_i < B_i. \]

Here:

\[ B_i = \int_{r_i^2 - u}^{r_i^2 + u} \tilde{r}_i f_2(\tilde{r}_i) d\tilde{r}_i, \]

where \( r_i^2 \) is the threshold value for a run.

Also,

\[ f_2(\tilde{r}) = \int_{(r_i^2 - u)(r - c_2)}^{(r_i^2 - u)(r + c_2)} q_1 \left( \frac{r - y}{r_i^2 - u} \right) q_2 \left( \frac{r - (1-t_1)r}{1-t_1} \right) dy. \]

Thus, the ratio of depositors who conduct bank runs is \( \lambda_1 = \int_{r_i^2 - u}^{r_i^2 + u} f_2(\tilde{r}) d\tilde{r}. \]

(2) Proof of P2:

Proof: Consider \( n = 2 \), let:

\[ C_i = \int_{r_i^2 - u}^{r_i^2 + u} \tilde{r}_i f_3(\tilde{r}_i | r_i^2 = r_i^2) d\tilde{r}_i. \]

Here, \( u = t_1 e_1 + (1-t_1)e_2 + (1-t_2)e_3. \)
As per Lemma 2, let:

\[ \delta = (1-t_2)e_3^l + t_2[t_1 e_1^l + (1-t_1)e_2^l] - (1-t_2)e_3^l - t_2[t_1 e_1^l + (1-t_1)e_2^l] > 0. \]

We have:

\[ C_i = \int_{r + (1-t_2)e_3^l + t_2[t_1 e_1^l + (1-t_1)e_2^l]}^{r^*_3 + (1-t_2)e_3^l + t_2[t_1 e_1^l + (1-t_1)e_2^l]} \hat{r}_j f_3^l(\hat{r}_i) d\hat{r}_i, \]

\[ C_j = \int_{r + (1-t_2)e_3^l + t_2[t_1 e_1^l + (1-t_1)e_2^l]}^{r^*_3 + (1-t_2)e_3^l + t_2[t_1 e_1^l + (1-t_1)e_2^l]} \hat{r}_j f_3^l(\hat{r}_j) d\hat{r}_j. \]

Replacing \( \hat{r}_i = \hat{r}_j + \delta \) into equation \( C_j \) yields:

\[ C_j = \int_{r + (1-t_2)e_3^l + t_2[t_1 e_1^l + (1-t_1)e_2^l]}^{r^*_3 + (1-t_2)e_3^l + t_2[t_1 e_1^l + (1-t_1)e_2^l]} (\hat{r}_j - \delta) f_3^l(\hat{r}_i - \delta) d\hat{r}_i \\
= \int_{r + (1-t_2)e_3^l + t_2[t_1 e_1^l + (1-t_1)e_2^l]}^{r^*_3 + (1-t_2)e_3^l + t_2[t_1 e_1^l + (1-t_1)e_2^l]} (\hat{r}_j - \delta) f_3^l(\hat{r}_j) d\hat{r}_j = C_i - \delta < C_i. \]

Thus, for depositors with signal \( r_3^* < r_f^* \), it derives:

\[ \int_{r + (1-t_2)e_3^l + t_2[t_1 e_1^l + (1-t_1)e_2^l]}^{r^*_3 + (1-t_2)e_3^l + t_2[t_1 e_1^l + (1-t_1)e_2^l]} \hat{r}_j f_3^l(\hat{r}_j) d\hat{r}_j = C_i < C^*, \]

where:

\[ C^* = \int_{r^*_3 - u}^{r^*_3 + u} \hat{r}_j f_3^l(\hat{r}_j) d\hat{r}_j. \]

\( r_f^* \) is the threshold of the run and satisfies the following:

\[ f_3(\hat{r}_j) = \int_{(1-t_2)(r + e_2)}^{(1-t_2)(r + e_2)} f_2 \left( \frac{\hat{r}_j - y}{t_2} \right) q_3 \left( \frac{y - (1-t_2)r}{1-t_2} \right) dy. \]

Therefore, the ratio of depositors participating in bank runs is \( \lambda_2 = \int_{r^*_3 - u}^{r^*_3 + u} f_3(\hat{r}_j) d\hat{r}_j \). When \( u > 2 \), by the above similar analysis process, we obtain:

\[ \lambda_n(r^*_{n+1}, u) = \int_{r^*_n - u}^{r^*_n + u} f_{n+1}(\hat{r}_n | r^*_{n+1} = r^*_{n+1}) d\hat{r}_n : \text{Q.e.d.} \]

(3) Proof of \( P3:\)

Proof: \((\Leftarrow)\) Obviously holds. \((\Rightarrow)\) For depositors with a signal satisfying \( e_1 = -e_1, e_2 = -e_2, e_3 = -e_3, \ldots \), it can be inferred that no one can get a worse signal. Since all depositors process information according to the same method, it can be seen from the proof process of Lemma 2 and \( P2 \) that the expected value of the long-term ROA of the depositor corresponding to the worst-sorting method is the lowest. If the depositor does not run, no other depositors will, and it is thus a strong equilibrium.

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