Characterizing Higher-Order Ross More Risk Aversion by Comparison of Risk Compensation

Guoqiang Tian [a*] Yougong Tian [b,c]

[a] Department of Economics, Texas A&M University, College Station, TX77843, USA

[b] School of Economics, Shanghai University of Finance and Economics, Shanghai, 200433, China

[c] Department of Mathematics, Lanzhou University of Finance and Economics, Lanzhou, 730020, China

---

Abstract

This paper provides a characterization of higher-order Ross more risk aversion based on risk compensation instead of risk premium, extending the results involving WTA in Denuit and Eeckhoudt (2013) to more general risk changes. Furthermore, we provide a characterization of comparative precautionary saving strength based on precautionary compensation, complementing the analysis of Liu (2014) based on precautionary premium.

Keywords: Risk premium; Risk compensation; Higher-order Ross more risk aversion; Precautionary premium; Precautionary compensation

---

1 Introduction

In order to measure the degree of risk aversion, Pratt (1964) introduces the concepts of risk premium and certainty equivalent. When the risk is zero mean, the risk premium is equivalent to " willingness to pay " (WTP), which is the maximal amount an agent is willing to pay to avoid some risk, while the certainty equivalent is equivalent to " willingness to accept " (WTA), which is the minimal amount an agent is willing to accept to sell some risk.[1]

* Corresponding author. Financial support from the National Natural Science Foundation of China (NSFC-71371117) is gratefully acknowledged. Email: gtian@tamu.edu.

[1] Note that La Vallée (1968) uses the terms 'ask prices' and 'bid prices', which are equivalent to WTP and WTA, respectively.

Specifically, the risk premium is defined by Pratt (1964) as π such that an agent u with the utility function $u(\cdot)$ is indifferent between receiving a zero mean risk $\tilde{\epsilon}$ and receiving a certain cost π, i.e., $u(x-\pi) = Eu(x+\tilde{\epsilon})$, where x is the non-random initial wealth. However, Ross (1981) observes that the initial wealth is not always non-random in the real world, and then provides a new description of the risk premium, i.e., the risk premium π is determined by $Eu(\tilde{x}-\pi) = Eu(\tilde{x}+\tilde{\epsilon})$, where $\tilde{x}$ is the random initial wealth. Afterwards, Modica and Scarsini (2005) propose the risk premium in a specific risk setting, i.e., the risk premium π is determined by $Eu(\tilde{x}-\pi) = Eu(\tilde{y})$, where $\tilde{y}$ has more downside risk than $\tilde{x}$.[2] Recently, Li (2009) and Denuit and Eeckhoudt (2010) present a more general version of the risk premium for an increase in nth degree risk, i.e., the risk premium π is determined by $Eu(\tilde{x}-\pi) = Eu(\tilde{y})$, where $\tilde{y}$ has more nth degree risk than $\tilde{x}$.

In contrast, the literature pays little attention to WTA. Indeed as indicated by Denuit and Eeckhoudt (2013): " Ross's (1981) contribution and subsequent ones limited themselves to WTP. This restriction is unsatisfactory because quite often decision-makers have to monetarize risk deteriorations." As a result, except for WTP, Denuit and Eeckhoudt also use WTA to characterize the degree of nth degree Ross more risk aversion for risk changes of nth degree risk increase and of nth degree mean-preserving stochastic dominance (MPSD). For risk changes above from $\tilde{x}$ to $\tilde{y}$, the risk compensation (WTA) m is defined with the equation $Eu(\tilde{x}) = Eu(\tilde{y}+m)$, where the risk compensation (WTA) is explained as the minimal amount an agent is willing to accept for bearing some risk increase. More recently, Liu (2014) only extends the results related to WTP in Denuit and Eeckhoudt (2013) to more general risk changes, i.e., $\tilde{x}$ dominates $\tilde{y}$ in the nth degree first l $(1 \leq l \leq n-1)$ moments-preserving stochastic dominance (l-MPSD).

This paper is the first to extend the results involving WTA in Denuit and Eeckhoudt (2013) to more general risk changes, thus we show that u is kth degree Ross more risk averse than v for $k = l+1, \ldots, n$, if and only if the risk compensation (WTA) to u is not less than the risk compensation (WTA) to v. Compared with the risk premium, however, the risk compensation in particular has a great advantage in the empirical setting. More specifically, Ebert and Wiesen (2014) use the risk compensation to measure the intensity of risk aversion, prudence, and temperance, and find evidence for these risk attitudes in a laboratory experiment.[3] The present paper theoretically

[2] Menezes, Geiss and Tressler (1980) introduce the concept of downside risk aversion and show that an agent is downside risk averse if and only if $u''' > 0$, which is equivalent to prudence coined by Kimball (1990).

[3] Ebert and Wiesen (2014) explain the reason why they choose the risk compensation to measure the intensity of risk aversion, prudence, and temperance in their footnote 4. More importantly, they think that the risk compensation is most convenient for experimentation.

establishes the validity of risk compensation (WTA) as a measure of (higher-order) risk aversion.

Furthermore, we examine the classical precautionary saving model. Suppose that the undesirable changes in the distribution of future income incur a deterioration in the sense of nth degree l-MPSD. Following Kimball (1990) and Eeckhoudt and Schlesinger (2008), for every nth degree l-MPSD deterioration in the future income (the income in period 1), our result indicates that the precautionary compensation (defined below in Section 4) to the consumer u with the utility function $u_1(x)$ in period 1, is not less than the precautionary compensation to the consumer v with the utility function $v_1(x)$ in period 1, if and only if $-u_1'(x)$ is kth degree Ross more risk averse than $-v_1'(x)$ for $k = l+1, \ldots, n$. Our result accordingly complements the analysis on the precautionary saving motive based on the precautionary premium in Liu (2014).

The remainder of the paper is organized as follows. Section 2 reviews notation, definitions and the existing results related to our paper. In Section 3, we characterize higher-order Ross more risk aversion by comparison of risk compensation. Section 4 defines precautionary compensation for a general deterioration in future income in the sense of nth degree l-MPSD, and characterizes comparative precautionary saving motive by higher-order Ross more risk aversion. Section 5 concludes this paper.

2 Definitions and Preliminaries

Let F and G be the cumulative distribution functions of two random variables $\tilde{x}$ and $\tilde{y}$ with support contained in $[a, b]$, respectively. As such, $F(a) = G(a) = 0$ and $F(b) = G(b) = 1$. Rewriting $F^1(z) = F(z)$ and $G^1(z) = G(z)$, for the sake of convenience, we define recursively

$$F^k(z) = \int_a^z F^{k-1}(x)dx, \quad \text{for} \quad k = 2, 3, \ldots, n,$$

and

$$G^k(z) = \int_a^z G^{k-1}(x)dx, \quad \text{for} \quad k = 2, 3, \ldots, n.$$

Ekern (1980) extends the concept of mean-preserving spread developed by Rothschild and Stiglitz (1970) to the cases of higher-order increases in risk.

Definition 2.1 (Ekern, 1980) $\tilde{y}$ has *more nth degree risk* than $\tilde{x}$, if

(i) $G^k(b) = F^k(b)$, for $k = 1, 2, \ldots, n$;

(ii) $G^n(z) \geq F^n(z)$, for all $z \in (a, b)$ with strict inequality holding for some z.

DEFINITION 2.2 (JEAN, 1980) $\tilde{y}$ is dominated by $\tilde{x}$ in the *nth degree stochastic dominance*, denoted by $\tilde{x} \succcurlyeq_n \tilde{y}$, if

(i) $G^k(b) \geq F^k(b), \quad$ for $k = 1, 2, \ldots, n$;

(ii) $G^n(z) \geq F^n(z), \quad$ for all $z \in (a, b)$ with strict inequality holding for some z.

Note that $G^k(b) = F^k(b)$ for $k = 1, 2, \ldots, n$, this implies that $\tilde{x}$ and $\tilde{y}$ have the same first $n-1$ moments. Thus $\tilde{y}$ has more nth degree risk than $\tilde{x}$ if and only if $\tilde{y}$ is dominated by $\tilde{x}$ in the nth degree stochastic dominance and the two random variables have the same first $n-1$ moments.

Ekern (1980) builds the linkage between the nth degree risk aversion and the sign of the derivative of the utility function.

THEOREM 2.1 (EKERN, 1980) *$\tilde{y}$ has more nth degree risk than $\tilde{x}$ if and only if $\tilde{x}$ is preferred to $\tilde{y}$ by all utility functions that are nth degree risk averse, or equivalently,*

$$(-1)^n u^{(n)}(x) < 0, \quad \textit{for all } x \in (a, b),$$

where $u^{(n)}$ denotes the nth derivative of $u(x)$.

Jindapon and Neilson (2007) present a further generalization of nth degree Ross more risk aversion.

DEFINITION 2.3 (JINDAPON AND NEILSON, 2007) For two increasing utility functions $u(x)$ and $v(x)$ that are both nth degree risk averse, $u(x)$ is nth degree Ross more risk averse than $v(x)$ if there exists $\lambda > 0$ such that

$$\frac{u^{(n)}(x)}{v^{(n)}(x)} \geq \lambda \geq \frac{u'(y)}{v'(y)}$$

for all $x, y \in [a, b]$.

Note that, when $n = 2$, nth degree Ross more risk aversion reduces to Ross more risk aversion developed by Ross (1981); when $n = 3$, it corresponds to more downside risk aversion proposed by Modica and Scarsini (2005).

Recently, Denuit and Eeckhoudt (2013) propose a new notion of nth degree MPSD.

DEFINITION 2.4 (DENUIT AND EECKHOUDT, 2013) $\tilde{y}$ is dominated by $\tilde{x}$ in the *nth degree MPSD (mean-preserving stochastic dominance)*, if $\tilde{x} \succcurlyeq_n \tilde{y}$ and $E(\tilde{x}) = E(\tilde{y})$.

The concept of the nth degree MPSD is further generalized by Liu (2014) to the more general cases, i.e., the nth degree l-MPSD, for $l \in \{1, 2, \ldots, n-1\}, n \geq 2$.

DEFINITION 2.5 (LIU, 2014) For any integer l, $1 \leq l \leq n-1$, $\tilde{x}$ dominates $\tilde{y}$ in the *nth degree l-MPSD (first l moments-preserving stochastic dominance)* if and only if $\tilde{x} \succcurlyeq_n \tilde{y}$ and $E(\tilde{x}^k) = E(\tilde{y}^k)$ for $k = 1, \ldots, l$.

It is easy to see that the two risk changes—an increase in nth degree risk introduced by Ekern (1980) and an increase in nth degree MPSD developed by Denuit and Eeckhoudt (2013)—are the two extreme cases of the nth degree l-MPSD. Specifically, the nth degree risk increase corresponds to the nth degree $(n-1)$-MPSD, while the nth degree MPSD is the nth degree 1-MPSD.

Similar to Ekern (1980), Liu (2014) establishes the connection between the nth degree l-MPSD and the signs of different order derivatives of utility function.

THEOREM 2.2 (LIU, 2014) *Suppose that $n \geq 2$ and $1 \leq l \leq n-1$. $\tilde{x}$ dominates $\tilde{y}$ in the nth degree l-MPSD if and only if $\tilde{x}$ is preferred to $\tilde{y}$ by all utility functions $u(x)$ that are kth degree risk averse for $k = l+1, \ldots, n$, or equivalently,*

$$(-1)^k u^{(k)}(x) < 0, \quad \textit{for all } x \in (a, b) \textit{ and } k = l+1, \ldots, n.$$

3 Characterization of higher-order Ross more risk aversion based on risk compensation

It is worth to mention that Denuit and Eeckhoudt (2013) first use the risk compensation (WTA) to characterize the degree of higher-order Ross more risk aversion for two extreme cases of risk changes, i.e., the nth degree risk increase and the nth degree MPSD. These results have greatly enriched the approach to quantify the degree of higher-order Ross more risk aversion. In the section, we provide a comparative characterization of higher-order Ross more risk aversion based on risk compensation (WTA) for more general risk changes.

PROPOSITION 3.1 *Suppose that $n \geq 2$ and $1 \leq l \leq n-1$. For any two increasing utility functions $u(x)$ and $v(x)$ that are both kth degree risk averse for $k = l+1, \ldots, n$, the following statements are equivalent:*

(i) $u(x)$ is kth degree Ross more risk averse than $v(x)$ for $k = l+1, \ldots, n$;

(ii) There exist $\lambda > 0$ and $\phi(x)$ such that $u(x) = \lambda v(x) + \phi(x)$, where $\phi : R \to R$ with $\phi'(x) \leq 0$ and $(-1)^{k+1}\phi^{(k)}(x) \geq 0$ for all $x \in [a, b]$ and for $k = l+1, \ldots, n$;

(iii) $m_u \geq m_v$ *for all* $\tilde{x}$ *and* $\tilde{y}$ *such that* $\tilde{x}$ *dominates* $\tilde{y}$ *in the* n*th degree* l*-MPSD, where* $Eu(\tilde{x}) = Eu(\tilde{y} + m_u)$ *and* $Ev(\tilde{x}) = Ev(\tilde{y} + m_v)$*.*

PROOF. $(i) \Rightarrow (ii)$. According to the definition that $u(x)$ is kth degree Ross more risk averse than $v(x)$, we know that there exists $\lambda_k > 0$ such that

$$\frac{u^{(k)}(x)}{v^{(k)}(x)} \geq \lambda_k \geq \frac{u'(y)}{v'(y)}$$

for $k = l+1, \ldots, n$, and for all $x, y \in [a, b]$. Let $\lambda = \min_{l+1 \leq k \leq n} \{\lambda_k\}$. Then we have

$$\frac{u^{(k)}(x)}{v^{(k)}(x)} \geq \lambda \geq \frac{u'(y)}{v'(y)} \tag{1}$$

for $k = l+1, \ldots, n$, and for all $x, y \in [a, b]$.

Define $\phi(x) = u(x) - \lambda v(x)$, thus we have $\phi'(x) = u'(x) - \lambda v'(x)$ and $\phi^{(k)}(x) = u^{(k)}(x) - \lambda v^{(k)}(x)$ due to differentiability of $u(x)$ and $v(x)$. Then according to the inequality (1), we have $\phi'(x) \leq 0$, $(-1)^{(k+1)}\phi^{(k)}(x) \geq 0$ for all $x \in [a, b]$ and $k = l+1, \ldots, n$.

$(ii) \Rightarrow (iii)$. Since m_u and m_v satisfy the equation $Eu(\tilde{x}) = Eu(\tilde{y} + m_u)$ and $Ev(\tilde{x}) = Ev(\tilde{y} + m_v)$ respectively, we have

$$\begin{aligned}
\int_a^b u(z + m_u)dG(z) &= \int_a^b u(z)dF(z) \\
&= \int_a^b [\lambda v(z) + \phi(z)]dF(z) \\
&\geq \lambda \int_a^b v(z)dF(z) + \int_a^b \phi(z)dG(z) \\
&\geq \lambda \int_a^b v(z + m_v)dG(z) + \int_a^b \phi(z + m_v)dG(z) \\
&= \int_a^b u(z + m_v)dG(z),
\end{aligned}$$

where the first inequality holds because $\phi(x)$ is kth degree risk averse for $k = l+1, \ldots, n$, and the second inequality holds because $\phi'(x) < 0$. Thus $m_u \geq m_v$.

$(iii) \Rightarrow (i)$. Suppose that $m_u \geq m_v$ for all $\tilde{x}$ and $\tilde{y}$ such that $\tilde{x}$ dominates $\tilde{y}$ in the nth degree l-MPSD. Note that this implies that $m_u \geq m_v$ for all $\tilde{x}$ and $\tilde{y}$ such that $\tilde{y}$ is a kth degree risk increase from $\tilde{x}$ for all $k = l+1, \ldots, n$. Naturally, $m_u \geq m_v$ for all $\tilde{x}$ and $\tilde{y}$ such that $\tilde{y}$ is an sth degree risk increase from $\tilde{x}$ (or G is an sth degree risk increase from F), $l+1 \leq s \leq n$.

Now we use proof by contradiction. Suppose that $u(x)$ is not kth degree Ross more risk averse than $v(x)$ for $k = l+1, \ldots, n$, then there at least exists some s such that $u(x)$ is not sth degree

Ross more risk averse than $v(x)$, $l+1 \leq s \leq n$, or equivalently,

$$\frac{u^{(s)}(x)}{v^{(s)}(x)} < \frac{u'(y)}{v'(y)} \qquad \text{for } x, y \in [c,d] \subseteq [a,b], \tag{2}$$

under the assumption that $u(x)$ and $v(x)$ are all sth degree risk averse, then the inequality (2) implies

$$\frac{(-1)^{s-1}u^{(s)}(x)}{(-1)^{s-1}v^{(s)}(x)} < \frac{u'(y)}{v'(y)} \Longleftrightarrow \frac{u'(y)}{(-1)^{s-1}u^{(s)}(x)} > \frac{v'(y)}{(-1)^{s-1}v^{(s)}(x)}$$

for all $x, y \in [c,d] \subseteq [a,b]$.

Let $Q(\cdot)$ be a distribution function, which is positive in interval $[c,d]$ and zero elsewhere. We then have

$$\frac{\int_a^b u'(z)dQ(z)}{(-1)^{s-1}u^{(s)}(x)} > \frac{\int_a^b v'(z)dQ(z)}{(-1)^{s-1}v^{(s)}(x)}.$$

Assume that $T(\cdot)$ is a nonnegative function, which is positive in interval $[c,d]$ and zero elsewhere, we have

$$\frac{\int_a^b (-1)^{s-1}u^{(s)}(z)T(z)dz}{\int_a^b u'(z)dQ(z)} < \frac{\int_a^b (-1)^{s-1}v^{(s)}(z)T(z)dz}{\int_a^b v'(z)dQ(z)}. \tag{3}$$

Define

$$H(z,t) = (1-t)G(z) + tF(z),\ t \in (0,1).$$

$H(z,t)$ is also a distribution function that has more sth degree risk than F and in turn less sth degree risk than G.

Accordingly, let $m_u(t)$ denote the risk compensation to agent u for a change from $H(z,t)$ to $G(z)$, we have

$$\begin{aligned}
\int_a^b u[z+m_u(t)]dG(z) &= \int_a^b u(z)dH(z,t) \\
&= \int_a^b u(z)d[(1-t)G(z)+tF(z)] \\
&= \int_a^b u(z)dG(z) - t\int_a^b u(z)d[G(z)-F(z)] \\
&= \int_a^b u(z)dG(z) - t\int_a^b (-1)^s u^{(s)}(z)[G^s(z)-F^s(z)]dz \\
&= \int_a^b u(z)dG(z) + t\int_a^b (-1)^{s-1} u^{(s)}(z)[G^s(z)-F^s(z)]dz.
\end{aligned}$$

Differentiating with respect to t yields

$$\frac{d\, m_u(t)}{dt} = \frac{\int_a^b (-1)^{s-1}u^{(s)}(z)[G^s(z)-F^s(z)]dz}{\int_a^b u'[z+m_u(t)]dG(z)},$$

and by $m_u(t)|_{t=0} = 0$, we have

$$\left.\frac{d\, m_u(t)}{dt}\right|_{t=0} = \frac{\int_a^b (-1)^{s-1} u^{(s)}(z)[G^s(z) - F^s(z)]dz}{\int_a^b u'(z)dG(z)}.$$

Similarly, let $m_v(t)$ denote the risk compensation to agent v for a change from $H(z,t)$ to $G(z)$, we have

$$\frac{d\, m_v(t)}{dt} = \frac{\int_a^b (-1)^{s-1} v^{(s)}(z)[G^s(z) - F^s(z)]dz}{\int_a^b v'[z + m_v(t)]dG(z)},$$

and by $m_v(t)|_{t=0} = 0$, we have

$$\left.\frac{d\, m_v(t)}{dt}\right|_{t=0} = \frac{\int_a^b (-1)^{s-1} v^{(s)}(z)[G^s(z) - F^s(z)]dz}{\int_a^b v'(z)dG(z)}.$$

Now $m_u(t)$ and $m_v(t)$ can be written as the forms of Taylor expansion:

$$m_u(t) = \frac{\int_a^b (-1)^{s-1} u^{(s)}(z)[G^s(z) - F^s(z)]dz}{\int_a^b u'(z)dG(z)} t + o(t),$$

and

$$m_v(t) = \frac{\int_a^b (-1)^{s-1} v^{(s)}(z)[G^s(z) - F^s(z)]dz}{\int_a^b v'(z)dG(z)} t + o(t).$$

Thus let us take $T(x) = G^s(x) - F^s(x)$ and $Q(x) = G(x)$ in the inequality (3) above, we know that, as $t \to 0^+$, we have $m_u(t) < m_v(t)$ for $F(x)$ and $G(x)$ such that $Q(x)$ and $T(x)$ are positive, for all $x \in [c, d]$ and zero elsewhere, which is a contradiction. ■

Specifically, Proposition 3.1 shows that u is kth degree Ross more risk averse than v for $k = l+1, \ldots, n$, if and only if the risk compensation (WTA) to u is not less than the risk compensation (WTA) to v, for all $\tilde{x}$ and $\tilde{y}$ such that $\tilde{x}$ dominates $\tilde{y}$ in the nth degree l-MPSD. In other words, any risk increase in nth degree l-MPSD that is undesirable for v is inevitably undesirable for u.

It needs to highlight that Proposition3.1 generalizes the results involving WTA in Denuit and Eeckhoudt (2013) to more general risk changes. More specifically, when $l = 1$, Proposition 3.1 corresponds to Proposition 2 in Denuit and Eeckhoudt (2013); when $l = n - 1$, Proposition 3.1 corresponds to Proposition 1 in Denuit and Eeckhoudt (2013). More importantly, Proposition 3.1 complements the result obtained by Liu (2014), who only extends the result related to WTP in Denuit and Eeckhoudt (2013) to more general risk changes.

4 Precautionary compensation for higher-order Ross more risk aversion

In this section, we consider a consumer who lives for two periods. In a two-period framework, assume that the consumer u chooses the level of saving s to maximize his total intertemporal utility, i.e.,

$$u_0(x_0 - s) + Eu_1[\tilde{x}_1 + s(1+r)],$$

where u_0 and u_1 are the utility function in period 0 and period 1 respectively. We assume that utility is increasing in wealth in each period ($u_0'(x) > 0, u_1'(x) > 0, \forall x$) and the consumer is risk averse in both periods ($u_0''(x) < 0, u_1''(x) < 0, \forall x$). x_0 is the certain income in period 0; $\tilde{x}_1$ is the random income in period 1; r is the certain rate of return on saving and the time preference rate is equal to zero. The optimal level of saving is determined by the first order condition

$$-u_0'(x_0 - s) + (1+r)Eu_1'[\tilde{x}_1 + s(1+r)] = 0.$$

Following Kimball (1990), Eeckhoudt and Schlesinger (2008) and Liu (2014), we make the same assumption that the first order condition has an interior solution.[4] In doing so, the concavity of $u_i(\cdot)$ ensures that the solution to the first order condition, denoted by s^*, is both unique and the expected utility maximizing level of saving. In the following, we assume that the future income $\tilde{x}_1$ in period 1 is faced with a risk deterioration due to some uncertainty, i.e., the future income changes from $\tilde{x}_1$ to $\tilde{y}_1$. More generally, $\tilde{y}_1$ is an nth degree l-MPSD deterioration from $\tilde{x}_1$.

To quantify the degree of the precautionary saving motive,[5] Kimball (1990) introduces the precautionary premium, which is defined as the fixed reduction in the future income that has the same effect on saving as the addition of a mean zero risk to the future income. Similarly, we here make a technical assumption, i.e., the optimal level of saving remains unchanged when the future income incurs a risk deterioration from $\tilde{x}_1$ to $\tilde{y}_1$.[6] To this end, it needs to compensate an amount

[4] Eeckhoudt and Schlesinger (2008) provide a general characterization of the precautionary saving motive against nth degree deteriorations in future income and in interest rate risk.

[5] Leland (1968) and Sandmo (1970) show that decision makers with a precautionary saving motive are characterized by a convex marginal utility function.

[6] Note that the technique here is quite different from the method used in the existing literature, for example, Kimball (1990), Eeckhoudt and Schlesinger (2008), Liu (2014) and so on, which looks on the optimal level of saving as benchmark when the future income is $\tilde{y}_1$, while this paper views the optimal level of saving as benchmark when the future income is $\tilde{x}_1$.

of money to the consumer, which characterizes precautionary saving strength. Correspondingly, the amount of monetary compensation is referred to as the precautionary compensation.

Specifically, the solution to

$$-u_0'(x_0 - s^*) + (1+r)Eu_1'[\tilde{x}_1 + s^*(1+r)] = 0$$

and the solution to

$$-u_0'(x_0 - s^*) + (1+r)Eu_1'[\tilde{y}_1 + m_u + s^*(1+r)] = 0$$

are identical. Namely, the precautionary compensation m_u is determined by

$$Eu_1'[\tilde{x}_1 + s^*(1+r)] = Eu_1'[\tilde{y}_1 + m_u + s^*(1+r)].$$

Similarly, for another consumer v, the precautionary compensation m_v is determined by

$$Ev_1'[\tilde{x}_1 + s^*(1+r)] = Ev_1'[\tilde{y}_1 + m_v + s^*(1+r)].$$

In order to measure the degree of the precautionary saving motive, we obtain the following result, which complements the analysis on precautionary saving motive against any deteriorations in future income in Liu (2014).

PROPOSITION 4.1 *Suppose that $n \geq 2$, $1 \leq l \leq n-1$, and $u_1(x)$ and $v_1(x)$ are two increasing and concave utility functions that are both $(k+1)$th degree risk averse for $k = l+1, \ldots, n$. Then the following statements are equivalent:*

(i) *$-u_1'(x)$ is kth degree Ross more risk averse than $-v_1'(x)$ for $k = l+1, \ldots, n$, or equivalently, there exists $\lambda_k > 0$ such that*

$$\frac{u_1^{(k+1)}(x)}{v_1^{(k+1)}(x)} \geq \lambda_k \geq \frac{u_1^{(2)}(y)}{v_1^{(2)}(y)}$$

for $k = l+1, \ldots, n$, and for all x and y;

(ii) *$m_u \geq m_v$ for all nth degree l-MPSD deteriorations in future income.*

PROOF. Since $u_1(x)$ and $v_1(x)$ are both increasing and concave utility functions, thus $-u_1'(x)$ and $-v_1'(x)$ are both increasing and kth degree risk averse for $k = l+1, \ldots, n$. Moreover, we know that $\tilde{x}_1$ dominates $\tilde{y}_1$ in the nth degree l-MPSD. From Proposition 3.1, we conclude that (i) and (ii) are equivalent. ■

As a special case, an increase in nth degree risk in future income corresponds to the nth degree $(n-1)$-MPSD deteriorations in future income. Intuitively, to hold the optimal level of saving fixed, the greater the nth degree Ross more risk aversion, the larger the precautionary compensation. More specifically, $-u_1'(x)$ is nth degree Ross more risk averse than $-v_1'(x)$, if and only if the precautionary compensation to u is not less than the precautionary compensation to v. As such, we have the following corollary.

COROLLARY 4.1 *Suppose that $n \geq 2$ and that $u_1(x)$ and $v_1(x)$ are two increasing and concave utility functions that are both $(n+1)$th degree risk averse. Then the following statements are equivalent:*

(i) $-u_1'(x)$ *is nth degree Ross more risk averse than* $-v_1'(x)$*, or equivalently, there exists $\lambda > 0$ such that*

$$\frac{u_1^{(n+1)}(x)}{v_1^{(n+1)}(x)} \geq \lambda \geq \frac{u_1^{(2)}(y)}{v_1^{(2)}(y)}$$

for all x and y;

(ii) $m_u \geq m_v$ *for all nth degree risk increases in future income.*

5 Conclusion

In this paper, we characterize comparative higher-order Ross more risk aversion based on risk compensation (a notion of willingness to accept), instead of the usual risk premium (a notion of willingness to pay). Besides, we characterize comparative precautionary saving strength based on the precautionary compensation instead of the precautionary premium that has been the focus of all other studies on this topic since Kimball (1990).

References

[1] Denuit, M. and Eeckhoudt, L. (2010). Stronger measures of higher-order risk attitudes, Journal of Economic Theory 145, 2027-2036.

[2] Denuit, M. and Eeckhoudt, L. (2013). Risk attitudes and the value of risk transformations, International Journal of Economic Theory 9, 245-254.

[3] Ebert, S. and Wiesen, D. (2014). Joint measurement of risk aversion, prudence, and temperance, Journal of Risk and Uncertainty 48, 231-252.

[4] Eeckhoudt, L. and Schlesinger, H. (2008). Changes in risk and the demand for saving, Journal of Monetary Economics 55, 1329-1336.

[5] Ekern, S. (1980). Increasing Nth degree risk, Economic Letters 6, 329-333.

[6] Jean, W. (1980). The geometric mean and stochastic dominance, Journal of Finance 35, 151-158.

[7] Jindapon, P. and Neilson, W. (2007). Higher-order generalizations of Arrow-Pratt and Ross risk aversion: A comparative statics approach, Journal of Economic Theory 136, 719-728.

[8] Kimball, M. (1990). Precautionary saving in the small and in the large, Econometrica 58, 53-73.

[9] La Vallée, I. (1968). On cash equivalence and information evaluation in decisions under uncertainty, part I: Basic theory, Journal of the American Statistical Association 63, 252-276.

[10] Leland, H. (1968). Saving and uncertainty: the precautionary demand for saving, Quarterly Journal of Economics 82, 465-473.

[11] Li, J. (2009). Comparative higher-degree Ross risk aversion, Insurance: Mathematics and Economics 45, 333-336.

[12] Liu, L. (2014). Precautionary saving in the large: nth degree deteriorations in future income, Journal of Mathematical Economics 52, 169-172.

[13] Menezes, C., Geiss, C. and Tressler, J. (1980). Increasing downside risk, American Economic Review 70, 921-932.

[14] Modica, S. and Scarsini, M. (2005). A note on comparative downside risk aversion, Journal of Economic Theory 122, 267-271.

[15] Pratt, J. (1964). Risk aversion in the small and in the large, Econometrica 32, 122-136.

[16] Ross, S. (1981). Some stronger measures of risk aversion in the small and in the large with applications, Econometrica 49, 621-663.

[17] Rothschild, M. and Stiglitz, J. (1970). Increasing risk I: A definition, Journal of Economic Theory 2, 225-243.

[18] Sandmo, A. (1970). The effect of uncertainty on saving decisions, Review of Economic Studies 37, 353-360.