State Aid in Government Procurement

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Abstract

We revisit the topic of national favoritism in government procurement in the context of international trade, focusing on the impact of a participation/transition cost difference between domestic and foreign firms. Our public tender model generates two kinds of equilibrium outcomes, where a higher participation cost foreign firm is more aggressive in participation in one kind, but less aggressive in another kind. However, the latter equilibrium dissipates when the difference in the transition costs becomes sufficiently large. Thus a favoritism policy would probably result in unintended consequences in a undesired equilibrium. Some simulation results are provided to quantify the restrictions on the model parameters that eliminate one non-intuitive equilibrium.

Journal of Economic Literature Classification Number: F12, F13, L10

Key Words: public tender, state aid, government procurement, international trade

1 Introduction

We revisit the topic of national favoritism in government procurement in the context of international trade. Government adopting preferential policies towards domestic firms is a common empirical phenomenon in many countries, as pointed out by the prior literature, e.g. Bronco (1994). In the US, for example, there have been two federal statutes (Buy American Act of 1933 and Buy America Act of 1982) so far, and various buy-America clauses embedded in other legislation. The issue has been extensively addressed in global free trade agreement negotiations.

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GATT Article III requires that WTO member countries provide what is called "national treatment" to all other members in that it stipulates that members must not apply internal taxes or other internal charges, laws, regulations and requirements affecting imported or domestic products so as to afford protection to domestic production. Nevertheless favoritism still persists in subtle ways in some countries.

What is less documented in the literature however is that favoritism can also go the other way – a host country sometimes also curries favor to foreign companies. For example, in China in the 1990s and early 2000s, foreign multinational companies routinely received preferential treatment in taxes, investment incentives and other discounted public utility services that put them in an above-national-treatment status. It is also quite common to observe some states in the U.S. to put forth billion-dollar incentive packages to entice foreign direct investment (FDI).[1] The phenomenon has been traditionally justified on the grounds of attracting FDI and creating jobs. In this paper, we provide possibly an alternative explanation underlying such favoritism. In our model, the asymmetry of the transition cost structures between domestic and foreign firms can lead to equilibrium outcomes of favoritism to either type of firms in a public tender.

That foreign companies may suffer a cost disadvantage, ceteris paribus, is self-evident and well documented in the international trade literature. These additional trade-related transition costs can be policy-induced, for things like tariffs, quotas and other nontariff barriers, or environment-determined, for things like transportation, hazard insurance and time cost. Anderson and Wincoop (2004) provided a survey of all trade costs in 121 countries, using the UNCTAD's TRAINS database, and they found that trade costs are indeed large when broadly defined to include all transition costs involved in getting a good from country A to country B. From a theoretical perspective, a foreign company's cost disadvantage could manifest in both transition costs and Samuelson (1954) type of iceberg costs. In line with the former, it is a customary practice in recent trade models to assume a transition cost incurred for exports, for example, in Melitz (2003). In line with the latter, a percentage markup over the production cost is usually assumed to measure what Eaton and Kortum (2002) call the geographic barriers to trade.

Our paper focuses on the impact of participation costs in international trade on government procurement policies. In our model, foreign firms incur a higher transition/participation cost than domestic firms do. This fixed cost differential would prescribe a different set of firms to participate in public tenders respectively for the two camps, even though, a priori, their

[1] For example, the Trump administration announced a \$3 billion incentive package to lure Foxconn to the state of Wisconsin for building a large factory in December 2017.

production costs could belong to the same distribution function. We show multiple equilibrium outcomes would arise that impart opposing consequences of state aid policies. If favoritism is indeed a government policy objective, that is, if the government wants to encourage a particular type of firms to participate in a public tender, then the intended consequence could be ambiguous and go either way in a multiple-equilibria environment, one of which is possible to entirely contradict the original policy objective.

Our public tender model developed in this paper generates two kinds of type-symmetric equilibrium outcomes where a higher participation cost foreign firm is more aggressive in participation in one kind, but less aggressive in another kind. However, the latter equilibrium outcome dissipates when the difference in the participation costs becomes sufficiently large. Thus from a policy perspective, subsidizing a domestic firm in the first equilibrium, which we call the intuitive equilibrium, would result in encouraging only those most competitive foreign firms to participate in a public tender, but at the cost of including fewer foreign participants. On the other hand, subsidizing a higher transition cost foreign firm would provide incentive for the firm to participate, while at the same time increasing the likelihood of a domestic firm winning. In the other equilibrium, which we call the non-intuitive equilibrium, the results are the exact opposite.

The theoretical literature with respect to favoritism in government procurement is initiated by McAfee and McMillan (1989), who first demonstrate that price-preference policies is justified on an efficiency argument in that optimal discrimination can lead to procurement cost minimization. Kim (1994) compares the tariff policies with price-preference policies, and shows their equivalence in effect both in terms of the government's procurement costs and the domestic and foreign firm's expected profits. Bronco (1994) shows that discrimination in favor of domestic firms can be motivated by distributional concerns, because the government's welfare objective function would only contain domestic firms' but not foreign firms' welfare. Naegelen and Mougeot (1998) show, in a more generalized model in which both efficiency and distributional concerns could arise, that the optimal policy can be implemented by a modified Vickery auction or by a complex modified first price auction. We contribute to this strand of literature by investigating the impact of participation cost differences between domestic and foreign firms on government procurement policies, and show that favoritism has an ambiguous effect, depending on the government's policy objective.

Our theoretical results are based on an auction model with participation costs. Krasnokutskaya and Seim (2011) first develop the insight that the impact of preference policies hinges very much on firms' participation decisions. Tan and Yialankaya (2006) and Cao and Tian (2013) provide formal analysis of second price auction models with participation cost, and they found multiple equilibria would result under a convex value distribution. Cao and Tian (2010) extended their result to a first price auction setting. To explain the results obtained in the paper, we build up a theoretical model that extends the existing literature to heterogeneous participation costs with multiple firms on both sides to generate multiple equilibria in the context of government procurement in an international trade environment. Our model also generates policy implications with respect to government preference decisions in public tenders that involve foreign players.

The rest of the paper is arranged as follows. Section 2 shows the basic model setup and presents our formal analysis as well as the main theoretical results. Section 3 provides a simulation analysis to gauge the effect of the transition cost difference on equilibrium outcomes, and discusses the resulting consequence of state aid. And Section 4 concludes with discussions of policy implications.

2 The Model

Suppose a government offers a contract for the delivery of a certain project that he values as 1. Both producers of domestic and foreign origins are invited to submit bids for the contract, whereby the firm who makes the lowest offer below 1 is granted the contract.

There are n_1 domestic firms and n_2 foreign firms who compete for this government contract and all firms are risk neutral. Domestic firms incur a lower transition cost, k_1, and foreign firms incur a higher transition cost, k_2, which are both sunk costs after participating in the bidding. Regardless of domestic or foreign firms, firm i's production cost, c_i, is assumed to be known to the firm itself. The other firms as well as the government perceive this cost to be independently and identically drawn from a probability distribution $G(.)$. Assume $G(c_i)$ is continuously differentiable, with derivative $g(c_i)$ fully supported on $[0, 1]$ [2]. Here we effectively assume the only difference between domestic and foreign firms lies in the participation costs, which are assumed to be common knowledge, with $k_i \in (0, 1]$ for all i.

We assume the public tender is in the form of a second price auction for modeling convenience.[3] Then the individual rational action for any firm includes whether to participate in the

[2] Here "0" denotes the production cost is zero whereas "1" is a normalization of the highest possible cost among all bidders.

[3] Our analysis in this paper applies to descending-price auctions. In this scenario, firms who participate will stay in the auction until the price reaches their costs.

government procurement and how much to bid if he chooses to participate. Firm i incurs the cost if and only if he chooses to participate.

If a firm finds participating in this government procurement is in its optimal interest, he cannot do better than bidding his true production cost.[4] A firm participates in the procurement whenever its expected revenue is no less than its participation cost. Given the behavior of other firms, the expected revenue of a firm from participating in the procurement is a decreasing function of its participation cost. Thus, firms use cutoff strategies, i.e., for firm i, he participates whenever $c_i \leq c_i^*$.[5] If $c_i^* = 0$, then firm i will never participate.

For the setup described above, each firm's action is to choose a cutoff and decide how to bid when he participates. Further, once the cutoffs are determined, the game is reduced to the standard second price auction and each firm bids his true production cost. Thus it is sufficient to focus exclusively on cutoffs, since they are sufficient to describe an equilibrium. We assume firms with the same participation cost use the same cutoff, i.e., we focus on the symmetric cutoff equilibrium of domestic and foreign firms in public tenders in the context of international trade.[6]

Definition 1 A symmetric equilibrium of domestic and foreign firms in a public tender in international trade is a cutoff vector $(c_1^*, c_2^*) \in \mathbb{R}_+^2$, where c_1^* (*resp.* c_2^*) is the cutoff for domestic firms (*resp. foreign firms*), such that each type i's action is optimal, given the other type's cutoff strategies.

Intuitively, firms with higher transition costs are less likely to enter the public tender. However, as we will see later, it is possible for firms with higher transition costs to enter with a larger probability. This is because it is possible that once a firm finds it is in its interest to participate, it would act more aggressively given the fact that the participation cost is already sunk. To account for these two scenarios, we distinguish two types of equilibria: intuitive equilibria and non-intuitive equilibria which are defined formally below.

Definition 2 A symmetric equilibrium of domestic and foreign firms in public tenders for government procurement $(c_1^*, c_2^*) \in \mathbb{R}_+^2$ is called an *intuitive equilibrium* (*resp. non-intuitive equilibrium*) if a firm has a lower (higher) participation cost implies that it would have a higher

[4] There may exist an equilibrium in which bidders do not bid their true production cost when they participate.

[5] See Lu and Sun (2007) for a detailed analysis.

[6] Formally, if we let $b_i(c_i, k_1, k_2)$ denote firm i's strategy. The bidding decision function of each firm can be characterized as

$$b_i(c_i, k_1, k_2) = \begin{cases} c_i & \text{if } c_i \leq c_i^*(k_1, k_2) \\ \text{No parcitipating} & \text{otherwise.} \end{cases}$$

cutoff, i.e., for any two firms i and j, $k_i < k_j$ implies $c_i^* > c_j^*$ (resp. $k_i < k_j$ implies $c_i^* \leq c_j^*$) and $k_i = k_j$ implies $c_i^* = c_j^*$.

In other words, an intuitive equilibrium is where a higher participation cost foreign firm is less likely to participate in a public tender than a domestic firm, whereas in a non-intuitive equilibrium, this foreign firm would be more likely to participate.

Our first result is the following proposition, of which its analysis and the proof are relegated to the Appendix.

Proposition 1 *There exists a symmetric intuitive equilibrium of domestic and foreign firms in public tenders for government procurement, where participating foreign firms are more likely to have lower production costs than domestic firms.*

Proposition 1 indicates that in an intuitive equilibrium, higher participation cost foreign firms will use a more conservative cutoff strategy, but, ceteris paribus, be more competitive upon participation. That means those participating foreign firms, which usually incur a higher participation cost in a host country, are more likely to be lower-cost firms and more likely to win vis-a-vis domestic firms.

The intuition behind Proposition 1 is actually quite easy to understand. When a higher transition foreign firm considers participating, it regards its participation cost as a sunk cost and therefore there would be more incentives to win conditional upon participation. That means more aggressive bidding driven by a more competitive cost structure. And that is the reason why we call it an intuitive equilibrium.

The intuitive equilibrium is not the only equilibrium arising from our setup, however. It is possible that the opposite scenario - a higher participation cost foreign firm adopting a higher cutoff value - can still constitute an equilibrium, which we call the non-intuitive equilibrium. Indeed, we have the following proposition, of which the analysis and proof are relegated to the Appendix.

Proposition 2 *There exists a symmetric non-intuitive equilibrium of domestic and foreign firms in public tenders for government procurement where participating foreign firms are more likely to have higher production costs and are less competitive than domestic firms, when the distribution of the production cost is strictly concave and the foreign-domestic participation cost difference, i.e.,* $k_2 - k_1$*, is sufficiently small. However, this equilibrium does not exist under the convexity assumption.*

Proposition 2 depicts an alternative scenario where a higher participation cost foreign firm would be more eager to participate in a public tender than a domestic firm. At surface it may appear surprising, but the rationale behind it hinges critically upon the concavity assumption on the part of the production cost distribution. Concavity means there is a larger probability mass on smaller production costs and therefore firms are more competitive. For justifying the foreign firm's participation burdened with a higher transition cost, it has to have a slightly more profitable outlook during the bidding stage, which means it is more likely to be competitive on production cost. That is where the concavity assumption kicks in. However, the equilibrium of this what we call the non-intuitive equilibrium cannot be sustained with a large participation cost differential. Once the participation cost gap between the domestic camp and the foreign camp widens, this equilibrium dissipates. This equilibrium also does not exist under convexity with respect to the production cost distribution, meaning that all firms are likely to incur higher production costs.

3 Simulations and Policy Implications

In this section, we provide a simulation exercise to show the existence of multiple equilibria and illustrate that the non-intuitive equilibria will disappear when the difference of the participation costs between the two types of firms is sufficiently large. We also show some dynamic comparatives on the equilibria and its properties. Technically, to find an equilibrium with $c_1^* < c_2^*$, we consider the following two equations:

$$
\begin{aligned}
k_2 &= (1-y)[1-G(y)]^{n_2-1}[1-G(x)]^{n_1} \\
k_1 &= [1-G(x)]^{n_1-1}[(1-y)[1-G(y)]^{n_2} + \int_x^y [1-G(c_2)]^{n_2} dc_2],
\end{aligned}
$$

with $x < y$. The first equation implicitly defines y as a decreasing function of x, denoted as $y(x)$, which has a fixed point c_2^s determined by $k_2 = (1-c_2^s)[1-G(c_2^s)]^{n_1+n_2-1}$. Then, when $x < c_2^s$, we have $y > c_2^s$. Insert $y(x)$ into the right side of the second equation and let

$$
\phi(x) = [1-G(x)]^{n_1-1}[(1-y)[1-G(y)]^{n_2} + \int_x^y [1-G(c_2)]^{n_2} dc_2]
$$

be a function of x defined on $[0, c_2^s]$. The simulation results from this formulation are plotted in the following figures.

In Figure 1, we fix $k_2 = 0.3$, $n_1 = n_2 = 1$ and plot $\phi(x)$ for the case of $G(c) = c^{\frac{1}{2}}$ (plotted in blue color) and $G(c) = c^2$ (plotted in red color). When $G(c) = c^2$, $\phi(x)$ is a monotonically decreasing function over $[0, c_2^s]$ with $\phi(c_2^s) = k_2 > k_1$, which indicates that there is no equilibrium with $c_1^* < c_2^*$. However, when we use $G(c) = c^{\frac{1}{2}}$, $\phi(x)$ is first a decreasing function and then an increasing function over $[0, c_2^s]$ with $\phi(c_2^s) = k_2 > k_1$. Thus if k_1 is sufficiently close to k_2, as in our case when $0.2846 \leq k_1 < 0.3$, there exists an $x \in [0, c_2^s]$ with $\phi(x) = k_1$, i.e., a non-intuitive equilibrium exists when $G(c)$ is strictly concave and $k_2 - k_1$ is sufficiently small.

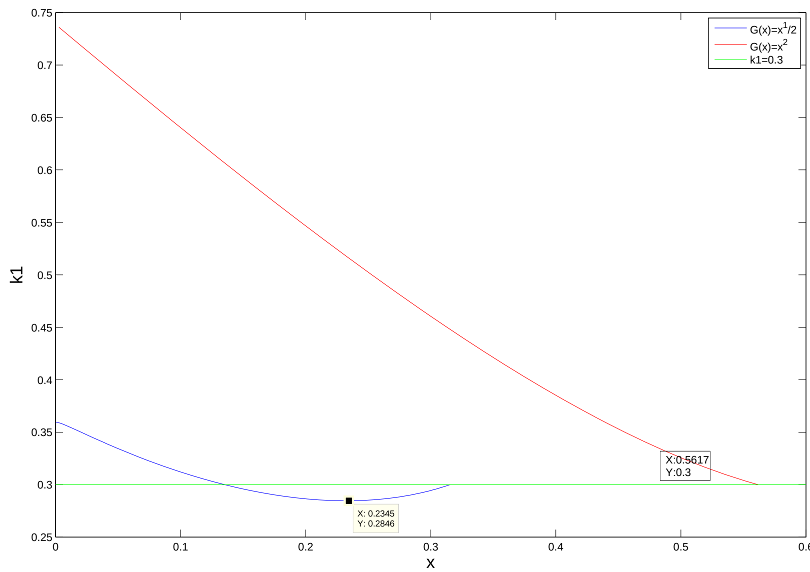

Figure 1: Existence and Non-existence of Non-intuitive Equilibria

In Figure 2, we explore the range of k_1 to support a non-intuitive equilibrium for different values of k_2, which is measured by $\frac{\min k_1}{k_2}$, for the distributions of $G(c) = c^{\frac{1}{2}}$ and $G(c) = c^{\frac{1}{3}}$, respectively. We find that as k_2 becomes smaller, $\frac{\min k_1}{k_2}$ will also become smaller. Indeed, when k_2 is smaller, so is k_1, and the advantage among the firms in terms of the participation cost will become relatively smaller. Thus, a smaller relative difference, which is measured by $\frac{\min k_1}{k_2}$, is required to result in a non-intuitive equilibria.

In Figure 3, we show how the non-intuitive equilibria will increase the number of foreign firms, holding the number of domestic firms fixed. As it can be seen from the graph, the cutoff for the non-intuitive equilibria will become smaller as the competition among firms intensifies, which is consistent with common sense.

The existence of two equilibria makes policy implications of state aid in public procurement more complex than previously believed. Conventional wisdom would suggest subsidies are usually directed at domestic firms and its intended purpose would be a simplistic interpretation to also help domestic firms. Neither is entirely true based on our analysis, and more importantly the implications of state aid might not be clear-cut under some circumstances.

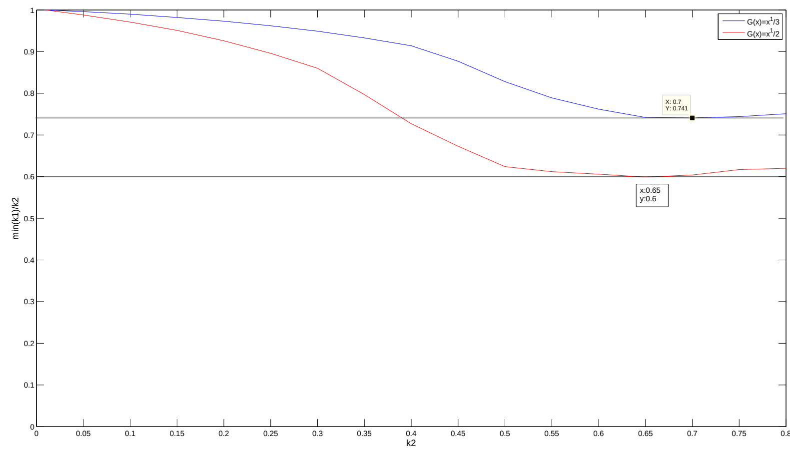

Figure 2: Relative Range of Fixed Cost for Non-intuitive Equilibria

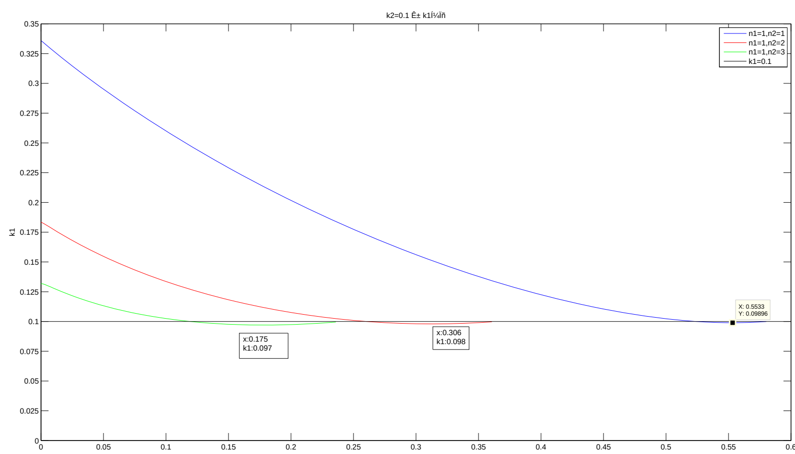

Figure 3: The Effects of Number of Firms on Cutoffs

Let us first consider the case of subsidies to domestic firms, which results in a decrease in k_1. In an intuitive equilibrium, that would lead to an increase in c_1^* but a decrease in c_2^*. That would indicate an increase in the number of domestic firms participating in the public tender, but a decrease of participating foreign firms. However these foreign firms that do participate are more competitive with lower production costs and more likely to win. On the other hand, a non-intuitive equilibrium, based on our previous analysis, shows that foreign firms are less enthusiastic about participating than domestic firms with the same production cost structure. However, a subsidy to domestic firms would essentially amplify their participation cost difference, thus further intensifying this abnormal effect but only to the point that this non-intuitive equilibrium is supported. With too large a participation cost difference, this equilibrium would just dissipate.

It is certainly possible that a subsidy would be showered on foreign firms as anecdotal evidences have indicated. Under such a circumstance, a decrease in k_2 would make the two respective two cutoff values of the two types of firms to move toward each other. That would mean an increase in the number of participating foreign firms and a decrease for domestic firms in an intuitive equilibrium. And expectedly, the result would be the exact opposite in a non-intuitive equilibrium, which again would dissipate if the subsidy is not large enough to make foreign firms' participation cost disadvantage greatly mitigated.

The overall message of our paper is that policy implications of state aid may not be as simple and straightforward as it appears. It depends on the contextual parameters of the competitive environment, and even when circumstances appear to be overwhelmingly tilted towards intending to favor domestic firms, multiple equilibria might emerge that indicates otherwise. However, the government does indeed have some leeway in influencing the type of equilibrium that eventually materializes, depending on its exact policy objective. The intuitive equilibrium is clearly preferred by the government if its objective is domestic favoritism. The non-intuitive equilibrium tends to cater to fostering more competition from abroad. Subsidies could be used to control the difference in the transition/participation costs between the two camps, and consequently the sustainability of the non-intuitive equilibrium.

Our non-intuitive equilibrium result also hinges critically on the concavity assumption on the product cost distribution. Under convexity, the government policy space is restricted to the intuitive equilibrium where it is much easier to implement domestic favoritism by providing direct subsidies to domestic firms. A subsidy to foreign firms would on the other hand encourage more foreign firms to participate, potentially increasing competition.

4 Conclusions

This paper investigates the implications of the transition cost asymmetry between domestic and foreign firms in public tenders for government procurement. We borrow from the auction literature with participation costs to build a simple theoretic model where these two sets of companies with different transition/participation costs use a cost value cutoff strategy to decide if participation is warranted. In line with the prior literature, we find multiple equilibrium outcomes would result, which interestingly yield exactly opposite behaviors. In one case, which we call the intuitive equilibrium, higher transition cost foreign firms would be less likely to participate than domestic firms unless they are highly competitive in production cost, whereas in the other case, which we call the non-intuitive equilibrium, the foreign firms incur a higher cutoff value and thus are more aggressive in participation under certain circumstances. However, this non-intuitive equilibrium dissipates, if the participation cost difference becomes too large.

The existence of these dual equilibria imparting opposite outcomes poses a somewhat perplexing dilemma for government policies, in that the intended consequence of a favoritism policy, no matter which party is the beneficiary notwithstanding, could go awry sometimes. In our intuitive equilibrium, a subsidy to domestic firms makes them more confident and thus more likely to participate, in relative terms, in public tenders vis-a-vis their foreign competitors, even though they may include less competitive domestic firms. But in a non-intuitive equilibrium, such a subsidy would only encourage those domestic firms with highly competitive production costs.

However, the government does have some leeway in achieving its preferred equilibrium by controlling the participation cost difference via state aid subsidies. When that difference is large, for example by heavily subsidizing domestic firms, the non-intuitive equilibrium dissipates. By subsidizing foreign firms on the other hand, the government is able to encourage them to participate in a public tender and foster more competition from abroad.

Which equilibrium dominates in the real world? We do not have a good empirical answer. And in our opinion, it is not even clear there is enough empirical evidence that the past favoritism policy intended to help domestic firms for instance in the US is indeed successful. Our theory does not touch upon the issue of predicting an equilibrium refinement, although we do point to the sun spot refinement concept developed in Campbell (1998) as a possible solution.

Appendix: Analysis and Proofs

To prove Propositions 1, we start by assuming, provisionally, that an equilibrium exists in which $c_1^* > c_2^*$. Then each domestic firm is indifferent between participating in the public tender and not participating when $c_1 = c_1^*$. The equilibrium zero expected payoff entails

$$k_1 = (1 - c_1^*)[1 - G(c_1^*)]^{n_1 - 1}[1 - G(c_2^*)]^{n_2}, \tag{1}$$

where $[1 - G(c_1^*)]^{n_1-1}$ is the probability that none of the other domestic firms participates and $[1 - G(c_2^*)]^{n_2}$ is the probability that none of the foreign firms participates. If $c_2^* = 0$ (i.e., foreign firms never participate), then $c_1^* = c_1'$, where c_1' is determined by $k_1 = (1 - v_1')[1 - G(c_1')]^{n_1-1}$.

Similarly, for a foreign firm with $c_2 = c_2^*$, we have

$$k_2 = (1 - c_2^*)[1 - G(c_1^*)]^{n_1}[1 - G(c_2^*)]^{n_2-1} + [1 - G(c_2^*)]^{n_2-1} \int_{c_2^*}^{c_1^*} (c_1 - c_2^*) d[1 - (1 - G(c_1))^{n_1}],$$

where the first part on the right side is the expected revenue when he is the only participating firm in the public tender. The second part is the expected revenue when he is the only participating foreign firm, and there is at least one domestic firm submitting a bid. $1 - (1 - G(c_1))^{n_1}$ is the distribution of the minimal production cost among the domestic firms, with $c_1 \in (c_2^*, c_1^*]$. Simplifying the above equation with integration by parts, we have

$$k_2 = [1 - G(c_2^*)]^{n_2-1}[(1 - c_1^*)[1 - G(c_1^*)]^{n_1} + \int_{c_2^*}^{c_1^*} [1 - G(c_1)]^{n_1} dc_1]. \tag{2}$$

By combining (1) and (2), we can prove Proposition 1 as follows.

Proof of Proposition 1: Consider the following cutoff reaction equations

$$\begin{aligned} k_1 &= (1 - x)[1 - G(x)]^{n_1-1}[1 - G(y)]^{n_2} && (3) \\ k_2 &= [1 - G(y)]^{n_2-1}[(1 - x)[1 - G(x)]^{n_1} + \int_y^x [1 - G(c_1)]^{n_1} dc_1] && (4) \end{aligned}$$

with $x > y$. From (3),

$$\frac{dx}{dy} = -\frac{n_2(1 - x)g(y)(1 - G(x))}{(1 - G(y))[1 - G(x) + (n_1 - 1)(1 - x)g(x)]} < 0,$$

which indicates that y is a decreasing function of x, denote by $y(x)$.

Let c_1' be determined by $k_1 = (1 - c_1')[1 - G(c_1')]^{n_1-1}$. Now if $(1 - c_1')[1 - G(c_1')]^{n_1} + \int_0^{c_1'} [1 - G(c_1)]^{n_1} dc_1 \leq k_2$, type 2 will never participate in the public tender and thus $c_1^* = c_1'$ and $c_2^* = 0$ constitute an equilibrium in which $c_1^* > c_2^*$. So we only need to consider the case that $(1 - c_1')[1 - G(c_1')]^{n_1} + \int_0^{c_1'} [1 - G(c_1)]^{n_1} dc_1 > k_2$.

From (3), let c_1^s be determined by $k_1 = (1-c_1^s)[1-F(c_1^s)]^{n_1-1}[1-F(c_1^s)]^{n_2}$, we have $x > c_1^s$ and $y < c_1^s$ by noting that $y = y(x)$ is a decreasing function with $y(c_1^s) = c_1^s$. By definition, we have $c_1' > c_1^s$.

Let $h(x) = (1-x)[1-G(x)]^{n_1}[1-G(y(x))]^{n_2-1} + [1-G(y(x))]^{n_2-1}\int_{y(x)}^{x}[1-G(c_1)]^{n_1}dc_1 - k_2$.

Since $h(c_1') = (1-c_1')[1-G(c_1')]^{n_1} + \int_0^{c_1'}[1-G(c_1)]^{n_1}dc_1 - k_2\, 0$ and $h(c_1^s) = k_1 - k_2 < 0$, there exists a $c_1^* \in (c_1^s, c_1']$ such that $h(c_1^*) = 0$. Thus, $c_1^* > c_1^s$ and $c_2^* = y(c_1^*) < c_1^s$ constitute an equilibrium. The proof is completed.

To prove Proposition 2, suppose there exists a type-symmetric equilibrium in which $c_1^* < c_2^*$. Similar to the previous analysis, the zero net-payoff condition requires that

$$
\begin{aligned}
k_2 &= (1-c_2^*)[1-G(c_2^*)]^{n_2-1}[1-G(c_1^*)]^{n_1}, &(5)\\
k_1 &= [1-G(c_1^*)]^{n_1-1}[(1-c_1^*)[1-G(c_2^*)]^{n_2} + \int_{c_1^*}^{c_2^*}(c_2-c_1^*)d[1-(1-G(c_2))^{n_2}]]. &(6)
\end{aligned}
$$

Integrating (6) by parts, we have

$$k_1 = [1-G(c_1^*)]^{n_1-1}[(1-c_2^*)[1-G(c_2^*)]^{n_2} + \int_{c_1^*}^{c_2^*}[1-G(c_2)]^{n_2}dc_2]. \tag{7}$$

By combining (5) and (7), we now prove Proposition 2 as follows.

Proof of Proposition 2: We first show when $G(.)$ is concave and $k_2 - k_1$ is sufficiently small, there exists a non-intuitive equilibrium with $c_2^* > c_1^*$. To see this, insert $y(x)$ into the right side of the second equation and let

$$\phi(x) = [1-G(x)]^{n_1-1}[(1-y)[1-G(y)]^{n_2} + \int_x^y [1-G(c_2)]^{n_2}dc_2]$$

be a function of x defined on $[0, c_2^s]$. Let k_m be the minimum of $\phi(x)$ on $[0, c_2^s]$. For notational convenience, define $F(x) = 1-G(x)$. Then $f(x) = F'(x) = -g(x) < 0$. Consider $\phi(x) = F(x)^{n_1-1}[(1-y(x))F(y(x))^{n_2} - \int_x^{y(x)} F(c_2)^{n_2}dc_2]$ with $x \in [0, c_2^s]$, where $y(x)$ is defined by $k_2 = (1-y)F(y)^{n_2-1}F(x)^{n_1}$. We have

$$y'(x) = \frac{n_1(1-y)f(x)F(y)}{F(x)[F(y)-(n_2-1)(1-y)f(y)]},$$

and

$$
\begin{aligned}
\phi'(x) &= (n_1-1)F(x)^{n_1-2}f(x)((1-y(x))F(y(x))^{n_2} + \int_x^{y(x)} F(c_2)^{n_2}dc_2)\\
&\quad + F(x)^{n_1-1}[-y'(x)F(y)^{n_2} + (1-y(x))n_2F(y)^{n_2-1}f(y)y'(x)\\
&\quad + F(y)^{n_2}y'(x) - F(x)^{n_2}],\\
&= F(x)^{n_1-2}\{(n_1-1)f(x)(1-y(x))F(y)^{n_2} + (n_1-1)f(x)\int_x^{y(x)} F(c_2)^{n_2}dc_2\\
&\quad + F(x)[-F(x)^{n_2} + n_2(1-y(x))F(y)^{n_2-1}f(y)y'(x)]\}.
\end{aligned}
$$

Inserting $y'(x)$ into $\phi'(x)$ and rearranging the terms, we have

$$\begin{aligned}\phi'(x) &= F(x)^{n_1-2}f(x)\{(n_1-1)(1-y(x))F(y)^{n_2}+(n_1-1)\int_x^{y(x)}F(c_2)^{n_2}dc_2\\ &\quad +F(x)[\frac{-F(x)^{n_2}}{f(x)}+(1-y(x))\frac{n_1n_2(1-y)F(y)^{n_2}f(y)}{F(x)[F(y)-(n_2-1)(1-y)f(y)]}]\}\\ &= F(x)^{n_1-2}f(x)\{(n_1-1)(1-y(x))F(y)^{n_2}+(n_1-1)\int_x^{y(x)}F(c_2)^{n_2}dc_2\\ &\quad +F(x)\frac{-F(x)^{n_2}}{f(x)}+(1-y(x))F(y)^{n_2}\frac{n_1n_2}{\frac{F(y)}{f(y)(1-y)}-(n_2-1)}]\}.\end{aligned}$$

When $x=c_2^s$, $y=c_2^s$, we have

$$\phi'(v_2^s)=F(c_2^s)^{n_1+n_2-2}f(c_2^s)\{(n_1-1)(1-c_2^s)-\frac{F(c_2^s)}{f(c_2^s)}+(1-c_2^s)\frac{n_1n_2}{\frac{F(c_2^s)}{f(c_2^s)(1-c_2^s)}-(n_2-1)}]\}.$$

Since $\frac{1-G(c_2^s)}{1-c_2^s}<g(c_2^s)=-f(c_2^s)$ by the strict concavity of $G(.)$, we have $-1<\frac{F(c_2^s)}{f(c_2^s)(1-c_2^s)}<0$, thus $\phi'(c_2^s)>0$, which indicates that $\phi(x)$ is increasing at $x=c_2^s$ with $\phi(c_2^s)=k_2$. Thus $\phi(x)$ has a minimum value $c_m<c_2$ in the interval $[0,c_2^s]$. Let $\phi(c_m)=k_m$.

When $k_1<k_m$, we have $\phi(x)>k_1$ for $x\in[0,c_2^s]$. However, to have an equilibrium with $c_2^*>c_1^*$, we need $\phi(x)\le k_1$. Therefore we do not have such an equilibrium.

When $k_1=k_m$, since $\phi(x_m)=k_m$, then (x,y) is the unique equilibrium with $c_2^*>c_1^*$, where $x=x_m$ and y is determined by $k_2=(1-y)F(y)^{n_2-1}F(x_m)^{n_1}$.

When $k_m<k_1<k_2$, we have at least two type-symmetric equilibria with $c_2^*>c_1^*$. Indeed, since $\phi(x_m)=k_m<k_1$ and $\phi(c_2^s)=k_2>k_1$, there is an $x_1\in[x_m,c_2^s]$ such that $\phi(x_1)=k_1$. On the other hand, when $\phi(0)<k_1$, we have an equilibrium with $c_2^*>c_1^*$ in which bidder 1 never participates. When $\phi(0)\ge k_1$, we can find $x_2\in(0,x_m)$ such that $\phi(x_2)=k_1$, since $\phi(0)\ge k_1$ and $\phi(x_m)=k_m<k_1$. Thus we can find at least two type-symmetric equilibria with $c_2^*>c_1^*$.

Next we show when $G(.)$ is strictly convex, there exists no non-intuitive equilibrium. To see this note that from (3) and (4), $k_2>k_1$ implies that

$$(1-c_2^*)[1-G(c_2^*)]^{n_2-1}[1-G(c_1^*)]^{n_1}>(1-c_1^*)[1-G(c_2^*)]^{n_2}[1-G(c_1^*)]^{n_1-1},$$

Simplifying the above equation, we have

$$\frac{1-G(c_1^*)}{1-c_1^*}>\frac{1-G(c_2^*)}{1-c_2^*},$$

which is not consistent with $c_1^*<c_2^*$ when $G(.)$ is convex. The proof is competed.

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