## ECON 323

## Microeconomic Theory <br> Guoqiang Tian

Lecture Notes of Prof. Guoqiang Tian

## I. Math Review and Economics Review

## 1. Math Review

1.1 Equation for straight line $y=a x+b$
where $a=s l o p e$
$b=$ intercept of $y$


1.2 Solve two equations with two unknowns.
example
i) $y=60-3 x$, where slope $=-3$
intercept ${ }^{\circ f y}=60$


$$
\begin{aligned}
& x=0 \Rightarrow y=60 \\
& y=0 \Rightarrow x=20
\end{aligned}
$$

ii) $y=5+2 x$, where slope $=2$ intercept of $\begin{array}{r}\text { of } \\ =\end{array}$


$$
\begin{aligned}
& x=0 \Rightarrow y=5 \\
& y=0 \Rightarrow x=-\frac{5}{2}
\end{aligned}
$$

iii) Find the intersection of these two lines
graphically:


$$
\begin{aligned}
& \text { algebraically: } \\
& y=60-3 x \\
& y=5+2 x \\
& \Rightarrow \quad 60-3 x=5+2 x \\
& 55=5 x \\
& x *=11 \\
& \Rightarrow \quad y *=60-3 x * \quad \text { or } \quad y *=5+2 x * \\
& =60-33 \\
& =5+22 \\
& =27 \\
& =27
\end{aligned}
$$

This is all the math you need for this course.

Economics - the study of how limited resources are allocated among competing uses.

Resources - Land - all natural resources egg., soil, forest, minerals, water.

Labor - includes the mental and physical skills provided by workers, egg. teachers, managers, dancers, steel workers.

Capital - includes man-made aids to production. e.g. machinery, buildings, tools.

Problem - Resources are limited but society has unlimited wants. Thus we have scarcity.

Because of scarcity, economic decisions are necessary, such as
i) what goods should be produced and in what quantities.
ii) how should goods be produced ( which feasible technology)
iii) for whom are the goods produced
iv) Who. will make decisions and by what process?

Solutions depend upon the type of economic system.

Centralized system

- all production and consumption decisions are - consumption and made by a central planning production decisions are board
- no unemployment, no inflation, no free enterprise

Decentralized system
(price or market system) made with markets

- consumers and producers
are motivated by selfinterest
We will study the price system to learn how markets work. Microeconomics - study of individual consumers, firms or markets, as well as how markets are organized.
Economic agents - consumers - who generate a demand for goods and services via utility maximization.
- producers (firms) - who supply quantities of goods and services via profit maximization.

Relative Price and Absolute Price
Absolute price or nominal price is the price without considering the changing value of money.

Relative price or real price is the price with considering the changing value of money. Eg. The price of the first class ticket for Titanic in 1912 was $\$ 7,500$ which is the equivalent of roughly $\$ 80,000$ in 1997 dollars.

## Three Basic Assumption:

1 Self-interest behavior. Individuals are self-interested. Every one pursues his/her personal goal.
2. Rationality Decision. Market participants are engage in ration behavior. Every one makes rational decisions.
3. Scarcity of Resources. Market participants confront scarce resources.

Opportunity Cost: The cost of a unit of a good measured in terms of other goods that must be forgone to obtain it. Whenever you pursue one desire, you limit the extent to which your other desire can be satisfied with your scare resources. The sum of the explicit and implicit costs associated with suing some resources in a particular way is defined "opportunity cost" or the resources's economic cost.

Explicit Costs: The payments made for resources which are purchased or hired from outside sources._Eg. wages, interest paid on borrowed money, rent for land owned by outside party.

Implicit Costs: The costs of resources which are used but neither purchased or hired from outside sources.

Production Possibility Frontier (PPF): A production possibility frontier shows all the different combinations of goods that a rational individual with certain personal desire can attain with a fixed amounts of resources.


## II. Microeconomics and Market Analysis

## 1. Positive and Normative Statements

Positive Statements $=$ tell what is, was, or will be. Any disputes can be settled by looking at facts.
e.g. "the sun will rise in the east tomorrow" , "decreasing unemployment will result in higher inflation"

Normative Statements - opinions or value judgments; tell us what should or ought to be. Disputes cannot be settled by looking at facts.
e.g. "It would be better to have low unemployment than low inflation."
2. Demand, Supply and Price Determination
2.1 Demand

Demand $[D(p)]: A$ schedule which shows the maximum amounts of a good or service which the consumer is willing and able to purchase at specific price, ceteris paribus.
e.g. schedule.

| price of <br> recordsbs $(p)$ | quantity demand of of <br> recordersper year |
| :---: | :---: |
| $\$ 20$ | 2 |
| 15 | 5 |
| 10 | 10 |
| 5 | 15 |
| 1 | 25 |

- willingness to purchase reflects tastes (preferences)
- ability to purchase depends upon income
- ceteris paribus - all other things remaining constant (ie, preferences, income, prices of other goods, environmental conditions, expectation, size of markets).

Demand curve - graphical representation of demand schedule.


Law of Demand - inverse relationship between price and quantity demanded. That is, the lower the price, the larger will be the quantity demanded and vice versa.

Demand Function (linear) looks like


Traditionally, economists reverse the axes when graphing:


Example: $\quad D(p)=60-4 p$
slope $=-\frac{1}{4}$


Above, we have made the assumption that all other influences on quantity demanded are held constant as the price changes.

Price changes are the only cause of a change in quantity demanded (movement along demand curve).

Market demand is the sum of single individual demands.

## Changes in Quantity Demanded versus Changes in Demand

## Change in Quantity demanded

i) caused by a change in price;
ii) represented by a movement along the demand curve.


Quantity Demanded


Quantity Demanded

## Change in Demand

i) caused by a change in something other than the price;
ii) represented by a shift in the demand curve.

"increase in demand"


Factors Causing a change in Demand
(ie, factorswhich shift the demand curve)
a) Size of market
as city grows
eg. Better marketing $\Rightarrow$ increase in \# of consumers $\uparrow \quad \Rightarrow$ increase in demand
b) Income
normal goods
As income rises, demand rises
eg. most goods are "normal" goods
inferior goods
As income rises, demand falls
eg. potatoes, bread (poverty goods)

c) Prices of Related Goods substitute goods ("competing goods") eg. butter and margarine


Price of margarine rises $\Rightarrow>$ will substitute butter

$$
\Rightarrow \text { demand for butter rises }
$$

Complementary goods - goods which "go together"
eg. hamburger and buns
The price of hamburgers falls $\Rightarrow>$ quantity demanded rises

d) Tastes (Preferences)
eg. saccharin causes cancer
$\Rightarrow$ demand for saccharin falls
e) Expectations
eg. paper towels go on sale next week
buy
$\Rightarrow$ people by them next week
$\Rightarrow$ demand this week falls
f) Environmental Conditions

eg. weather conditions affect demand for air conditioners;
ice cream; winter coats

### 2.2 Supply

Supply $[S(p)]:$ a schedule which shows the maximum quantities of a good or service that potential sellers are willing and able to sell at specific price, ceteris paribus.


Law of Supply: A direct relationship between price and quantity supplied. That is, the higher the price, the larger will be the quantity supplied, and vice versa.

Linear supply function:
$S(p)=a p+b$
direct relationship $\Rightarrow>0$
Example $S(p)=-10+6 p, \quad$ slope $=\frac{1}{6}$


Why a direct relationship?

## Substitution of Expansion in Production

As the price of a good rises a producer will shift resources into the production of this relatively high priced good and away from production of relatively low priced goods. Alternatively the producer has an incentive to hire extra resources.

Market supply is the sum of single firm supplies.

## Changes in Quantity supplied

i) caused by a change in price;
ii) represented by a movement along the supply curve.


Change in Supply
i) caused by a change in something other than the price;
ii) represented by a shift in the supply curve.


Increase in supply


Decrease in supply

## Factors Causing a Change in Supply

(factors which shift the supply curve)
a) number of firms
b) prices of related goods
c) technology
d) expectations
e) environmental conditions

Examples of change in supply
i) price of resources
increase in wage $\Rightarrow$ increase in costs of production $\Rightarrow$ decrease in supply.
ii) advancement in technology $\Rightarrow$ decrease in costs of production
$\Rightarrow$ increase in supply.
2.3 Determination of Price and Quantity

Notation $=\quad \mathrm{q}^{\mathrm{d}} \equiv$ quantity demanded
$q^{S} \equiv$ quantity supplied
$\mathrm{p} \equiv$ price
Equilibrium Price ( ${ }^{e}$ ) - is established at the price where quantity supplied equals quantity demanded.


Equilibrium price: $p^{e}=\$ 10$
We say that "pe clears the market"
Equilibrium quantity: $q^{e}=3,000$
$\left(q^{e}=q^{s}=q^{d}\right)$


## Example

$$
\begin{gathered}
D(p)=80-4 p \\
S(p)=-10+6 p
\end{gathered}
$$

Find the market equilibrium price, $p^{e}$, and equilibrium quantity, $q^{e}$.
$p=0$ implies $D(0)=80$ and $S(0)=-10$
$q_{d}=0$ implies $p \overline{\bar{p}} 20$, and $q_{s}=$ implies $=10 / 6=5 / 3$.


At equilibrium,

$$
D\left(p^{e}\right)=S\left(p^{e}\right)
$$

so that

$$
80-4 p^{e}=-10+6 p^{e},
$$

and thus

$$
90=10 p^{e}
$$

which gives us $p^{e}=9$.
Substituting $p^{e}=9$ into either the demand or supply equation, we have

$$
q^{e}=D\left(p^{e}\right)=S\left(p^{e}\right)=80-4 x 9=44
$$

$$
\begin{aligned}
& 2.4 \text { Market Adjustment } \\
& \text { Suppose } p>p^{e} \\
& \text { Then } q^{s}>q^{d} \Rightarrow \text { surplus } \\
& \text { Producers compete to rid of the surplus by price cutting. } \\
& \text { Price falls } \Rightarrow q^{s} \text { falls, } q^{d} \text { rises } \\
& \text { Eventually } q^{s}=q^{d} \text { at } p^{e} \\
& \\
& \text { Suppose } p<p^{e} \\
& \\
& \text { Then } q d>q^{s} \Rightarrow>\text { shortage } \\
& \text { Consumer compete and force the price up } \\
& \\
& \text { As price rises } \Rightarrow q^{d} \text { falls, } q \text { rises. } \\
& \text { Eventually, } q=q^{s} \text { at } p^{e} \text {. }
\end{aligned}
$$

Changes in Supply and Demand: Effect on Equilibrium
Examples



Results Increase in $S \Rightarrow p^{e}$ falls, $q^{e}$ rises
3. Supply and demand rise

Results Increase in $S$ and $D$


Simultaneous Changes in Demand 'Supply ? their Effects on $p^{e}$ and $q^{e}$ -


## Government Intervention: Price Controls

Markets can be thought of as self-adjustment mechanism; they automatically adjust to any change affecting the behavior of buyers and sellers in the market. But for this mechanism to operate, the price must be free to move in response to the interplay of supply and demand. When the government steps in to regulate prices, the market does not function in the same way.

There are two types of price controls: price ceiling and price floor.
Price Ceiling: A price above which buying or selling is illegal. It is aimed to help consumers.


Allocation Methods:
i) First come, first served;
ii) rationing (using coupons)

Effects of Price Ceiling:
a. in general it results in shortage;
b. there is a tendency to form a black market;
c. bad service and bad quality of goods;
d. production is reduced;
e. provide wrong information about production and consumption.
f. it hurts producers who provide goods, consumers sometimes are also worse off.

Price Floor (Price Support): A price below which buying or selling is prohibited. It is aimed to help producers. Examples include setting prices of agricultural products and minimum wage rate.

a. in general it results in surplus;
b. provide unnecessary service;
c. over investment;
d. provide wrong information about production and consumption.

Methods for maintaining the price support:
i) the government purchases surplus, the total revenue of the producer $=p_{f} q_{s}$.
ii) output is restricted at $q^{d}$, the total revenue of the producer $=$ $p_{f} q_{d}$.

## Example

$$
\begin{aligned}
& D(p)=90-20 p \\
& S(p)=-15+10 p
\end{aligned}
$$

a) Find the market equilibrium price and quantity.

Setting $D(p)=S(p)$, we have $90-20 \mathrm{p}=-15+10 \mathrm{p}$ which gives us $p^{e}=$ $105 / 30=3.5$ and $q^{e}=20$.
b) Suppose a price support is set at $\$ 4$. What is the surplus?

Since

$$
\begin{gathered}
D(4)=90-20 \times 4=10 \\
S(4)=-15+10 \times 4=25
\end{gathered}
$$

so the surplus is given by

$$
S(4)-D(4)=25-10=15
$$

## III. Theory of Consumer Choice

### 3.1 The Budget Line

The Budget Line - a straight line representing all possible combinations of goods that a consumer can obtain at given prices by spending a given income.

Eg. two goods $x \& y$

$$
p_{x}=10, \quad p_{y}=5
$$

Income, $I=\$ 100$


Combination $\mathrm{P}_{\mathrm{x}}$ - Units of $\mathrm{x}+\mathrm{p}_{\mathrm{y}}$ - Units of $\mathrm{y} \quad$ = Income


The budget line can be written as

$$
\begin{aligned}
& p_{x} x+p_{y} y=I \\
& \Rightarrow y=\frac{I}{p_{y}}-\frac{p_{x}}{p_{y}} \cdot x \\
& \Rightarrow \text { slope }=-\frac{p_{x}}{p_{y}} \quad \text { intercept }=\frac{I}{p_{y}}
\end{aligned}
$$

In the above example,

$$
\text { slope }=-\frac{\mathrm{P}_{\mathrm{x}}}{\mathrm{P}_{\mathrm{y}}}=-\frac{10}{5}=-2
$$

What if prices and income increase at the same rate?
eg. both double

$$
\left(2 p_{x}\right) x+\left(2 p_{y}\right) y=\not 2 I
$$

nothing changes.
Changes in Budget Line
What if only one price changes?
If the $p_{y}$ is changed to $p_{y}^{*}=10, P_{x} \underset{y}{y}{ }_{20} \hat{i}^{\text {and }} I$ are the same as before, then new budget line
$10 x+10 y=100$
slope $=-\frac{p_{x}}{p_{y}}=-1$
What if only income changes?


If the income changes from $\$ 100$ to $\$ 150$, then new budget line is
$10 x+5 y=150$


### 3.2 Preferences of the Consumer

The consumer is assumed to have preferences over bundles of goods.
Suppose there are 2 goods available: $x, y$ and bundles of these goods: A, B where each bundle contains a given amount of $x$ and $y$.


We make the following assumptions on the consumer's preferences.
i) between any 2 bundles, the consumer can only make one of the following statements
$A$ is preferred to $B \quad(A 母 B)$
$B$ is preferred to $A$ ( $A(D)$
$A$ is indifferent to $B \quad(A \Theta B)$
ii) transitivity of preferences

If $\mathrm{A} \mathcal{O} \mathrm{B}$ and $\mathrm{B} \boldsymbol{\theta} \mathrm{C}$, then $\mathrm{A}(\mathcal{C}$
iii) more is preferred to less

$$
\text { If } A=\left(x^{A}, y^{A}\right) \quad B=\left(x^{A}, y^{A}+c\right) \quad c>0, \quad \text { then } B \mathscr{P}
$$

Indifference Curve - a curve on which the consumer is indifferent.
Thus, a consumer is indifferent between any two bundles that lie on the same indifferensecurve.

To find an indifference curve, start at bundle A. Subtracting one unit of good $y$, puts us at a point $A^{1}$, less preferred to $A$ by assumption iii), $A^{1} \Theta A$. However, if we add more $x$ to $A^{1}$, we know that $A^{1}$ will be less preferred to this new point. If we add "enough" $x$ to $A^{1}$, then we find a point $B$ with the property $A \Theta B$. Etc.


Similarly we can trace out a family of indifference curves, and indifference map.

where $A \Theta B, \bar{A} \Theta \bar{B}, \bar{A} \Theta \bar{B}$, but any point on $U^{1}$ is preferred to any point on $\mathrm{U}^{\mathrm{o}}$, and less preferred to any point on $\mathrm{U}^{2}$ by assumptions ii) \& iii).

### 3.3 Properties of Indifference Curves

i) indifference curves slope downward by assumption ii)
ii) indifference curves do not intersect.

Suppose they did intersect,

$A \Theta B$ and $C \theta B$
$c(\theta A B C O B$ but $C O B$
a contradiction.
iii) indifference curves are convex to the origin.
$\Rightarrow$ Diminishing marginal rate of substitution.


As $A \longrightarrow B$, we gain one unit of $x$ and sacrifice enough $y$ to remain indifferent.
3.4 Marginal Rate of Substitution of $x$ for $y$ along $U$.
$\left(\mathrm{MRS}_{x y}\right)=\begin{gathered}\text { maximum } \\ \text { numbers }\end{gathered}$ of units of $y$ that must be given up for one extra unit of $x$ if the consumer is to remain indifferent. MRS $_{x y}=\left|\frac{\Delta y}{\Delta x}\right|=\quad$ absolute value of the slope of the indifference curve where " $\Delta$ " are very small in change.

Diminishing MRS - The amount of good $y$ the consumer is willing to give up for one additional unit of $x$ decreases as units of $x$ obtained increases.

That is, MRS decreases as we move from left to right along $U$.
In the above graph, as at $A, y$ is abundant, $x$ is scarce consumer is willing to give up a relatively large amount of the plentiful good to obtain the scarce.

At $E, y$ is scarce, $x$ is abundant; consumer is willing to give up amounts 8 for another unit of $x$.


## Limiting Shapes of Indifference Curves

i) Perfect substitutes (Linear Indifference Curves)

Since the slope of a straight is constant, MRS is constant.

ii) Perfect Complements

iii)

iv)


### 3.5 Utility Functions

Indifference curves, which represent consumer preferences, enable us to rank commodity bundles.


That is, $A \otimes B$
B 8 C
Thus $\quad \# 1=C$
$\# 2=B$
\#3 $=\mathrm{A}$

Sometimes it is convenient to summarize these rankings with a numerical index.

After a consumer reveals preferences, a utility function can be derived which is consistent with the consumer's indifference map.
$\underline{U(x, y)}$ : assigns a number to commodity bundle ( $x, y$ ) such that whenever
(a) $U(x, y)>U(\bar{x}, \bar{y})$ then bundle $(x, y)$ is preferred to bundles ( $\bar{x}, \bar{y}$ ).
(b) $U(x, y)=U(\bar{x}, \bar{y})$ then the consumer is indifferent between $(\mathrm{x}, \mathrm{y})$ and $(\overline{\mathrm{x}}, \overline{\mathrm{y}})$.

Thus $U(x, y)$ can be used to rank commodity bundles. Such a function is called an ordinal utility function.

To find an indifferent curve from a utility function:
example 1.
Let $U(x, y)=x+2 y$
Along an indifferent curve the consumer is indifferent between bundles. That is, along an indifference curve $U(x, y)=\bar{U}$ is constant.

Suppose $\bar{U}_{1}=10$. To trace an indifference curve we find all combinations of $(x, y)$ which satisfy $U(x, y)=\bar{U}_{1}=10 \Rightarrow x+2 y=10$.

Suppose $\bar{U}_{2}=15 \Rightarrow x+2 y=15$


Since $U(x, y)=10<U(x, y)=15$, the latter indifference curve represents bundles preferred to the original set. That is consistent with the indifference curve being further from origin.

## Example 2

Suppose $\mathrm{U}(\mathrm{x}, \mathrm{y})=\mathrm{xy}$


If we know a particular utility function, we can compute MRS of the indifference: Since slope of utility function is $\frac{\Delta y}{\Delta x}=-\frac{M U}{M U}$, we have

$$
\mathrm{MRS}=\frac{\mathrm{MU}_{\mathrm{x}}}{\mathrm{MU}_{\mathrm{y}}}
$$

Example 1
$U(x, y)=x+2 y$
$\mathrm{MU}_{\mathrm{x}}=1 \quad \mathrm{MU}_{\mathrm{y}}=2$
$\Rightarrow \quad$ MRS $=\frac{1}{2}$

$$
\begin{array}{lll}
\text { Example 2 } & U(x, y)=x y \\
\text { Let } \bar{U}=x y & \Rightarrow & y \Delta x+x \Delta y=0 \\
\Rightarrow & \Delta y \\
\Delta x & =-\frac{y}{x} \quad \Rightarrow & M R S=\frac{y}{x}
\end{array}
$$

## Summary

Consumers want to choose commodity bundles which maximize preferences, or as we have seen, maximize utility. Thus, when given a choice the consumer picks a bundle on the furthest indifference curve from the origin.

### 3.6 Consumer's Optimization Problem (the Consumer's Choice)

The consumer maximizes preferences subject to his budget line by choosing an ( $\mathrm{x}, \mathrm{y}$ ) . (Inters Solution)

Case 1 Indifference curves are strictly convex and do not cross the axes


The consumer chooses a bundle on the furthest indifference curve permitted by his budget.

C - too expensive
$B \Rightarrow A$
$B$ is preferences maximizing bundle.

Notice that preferences maximization occurs at the point where the indifference curve is tangent to (has the same slope as) the budget line.

Thus, at the optimal bundle, we have
(1) slope of the indifference curve equals slope of budget line. Since slope of indifference curve $=-M R S$ and slope of the budge line $=$ $-\frac{p_{x}}{p_{y}}$
$\Rightarrow \mathrm{P}^{\mathrm{y}} \mathrm{MRS}=\frac{\mathrm{p}_{\mathrm{x}}}{\mathrm{p}_{\mathrm{y}}}$ at B
(2) $p_{x} x+p_{y} y=I$

In other words, if $\left(x^{*}, y^{*}\right)$ is the preference maximizing bundle, then $\left(x^{*}, y^{*}\right)$ must satisfy (1) and (2).

Example $U(x, y)=x y \quad P_{x}=2 \quad P_{y}=1 \quad I=100$
Since $\operatorname{MRS}=\frac{y}{x}$, from $M R S=\frac{P_{x}}{P_{y}} \Rightarrow$
(1) $\frac{y}{x}=\frac{p_{x}}{p_{y}}=\frac{2}{1}$
$\Rightarrow \quad y=2 x$
(2) $p_{x} x+p_{y} y=100 \Rightarrow 2 x+y=100$

Since $y=2 x \Rightarrow 2 x+2 x=100$

$$
\Rightarrow \quad 4 x=\dot{1} 00
$$

$x=25$ and $y=50$
Case 2. Corner Solution (Linear Indifference Curve)
Suppose $U=x+2 y . \quad$ Then $\operatorname{MRS}=\frac{1}{2}$
The optimal bundle depends on the slopes of the budget line and the indifference curve.
i) Let $\mathrm{P}_{\mathrm{x}}=1, \mathrm{p}_{\mathrm{y}}=1, \mathrm{I}=20$

The budget line $x+y=20$ slope $=-1 \neq$ MRS


So $y=\frac{I}{p_{y}}=20$
$\mathrm{x}=0 \quad \mathrm{y}=20$
ii) Let $\mathrm{p}_{\mathrm{x}}=1$


So $x:=\frac{I}{P_{x}}=20$
$y=0$
iii) $p_{x}=1, \quad p_{y}=2$


The slopes of the budget line and the indifferences are the same, any bundle $\left(\mathrm{x}^{*}, \mathrm{y}^{*}\right)$ satisfying $\mathrm{x}+2 \mathrm{y}=20$ is optimal bundles.

### 3.7 The Composite - Good Convention

So far, we developed our analysis only for a two-good world, but the general principles can apply to a world of many goods. Unfortunately, many goods, cannot be shown on a two-dimensional graph. Still, it is possible to deal with a multitude of goods in two dimensions by treating a number goods as group.

Suppose there are many goods $x, y, \ldots, z$. We can continue to measure consumption for $x$ by treating other goods ( $y, \ldots, z$ ) as composite good. Consumption of the composite good is gauged by total outlays on it, in other words, total outlays on all goods other than $x$.

Thus, all analysis and conclusions for a two-good world also hold for a world of many goods.

Notice that price of the composite is equal to one. Then, slope of budget line $=-\frac{P_{x}}{1}$
$\operatorname{MRS}=\frac{\mathrm{P}_{\mathrm{x}}}{1}=\mathrm{P}_{\mathrm{x}}$

IV. Individual and Market Demand

### 4.1 Income Change

Income Consumption Curve (ICC) - traces response of
demand: to change in income.
To find ICC - change I, leaving price fixed.


Above, both $x$ and $y$ rise when income rises.
A good is said to be a normal good if, when income rises with $p_{x}$, $p_{y}$ constant,$~-~ d e m a n d$ of it increases.

Most goods are normal goods. A good is said to be a inferior good if, when income rises with $p_{x}, p_{y}$ constant demand decreases.

$I^{\prime}>I, x^{*}>x^{*}$ and $y^{\prime}<y^{*} . \quad x$ is normal good and $y$ is inferior good. If there are only two goods in the economy they cannot be both inferior goods at the same time.

## Food Stamp Program

Under the federal food stamp program, eligible low-income families receive free food stamps, which can be used only to buy food. Consumer theory can be used to evaluate this program.

We show this by considering a specific example in which a consumer receives $\$ 50$ worth of food stamps each week. We assume that the consumer has a weekly income of $\$ 100$ and the price of food is $p_{f}=5 /$ per unit.


The presubsidy budget line is $A Z$. The food stamp subsidy shifts the budget line to $A A^{\prime} Z^{\prime}$. Over the $A A^{\prime}$ range, the budget line is horizontal since the $\$ 50$ in free good stamps permits the recipient to purchase up to 10 units of good while leaving the consumer his or her entire income of $\$ 100$ to be spent on other goods. Over 10 units of food, the consumer has to pay for it by $\$ 5$ per unit. Thus, the $A^{\prime} Z^{\prime}$ portion of the budget line has a slope of -5 . Note that this new budget line is not straight line, but is a kinked at $A^{\prime}$.

The food stamp subsidy will affect the recipient in one of two ways. The above Fig. shows a possibility. If the consumer spends more than $\$ 50$ on food, the equilibrium, $W^{\prime}$, occurs on the $A^{\prime} Z^{\prime}$ portion of the budget line. The consequences of the food stamp subsidy are exactly the same as when the consumer receives a cash grant of $\$ 50$ in which case the budget line is $A^{\prime \prime} Z^{\prime}$.

The following diagram shows another possible outcome of the food stamp subsidy. With a direct cash grant of $\$ 50$, the consumer prefers to consume at point $W^{\prime}$ which is prohibited by the food stamp subsidy. The consumer has to choose among the options shown by the $A A^{\prime} Z^{\prime}$ budget line, and the best choice under the food stamp subsidy case is the kinked point $A^{\prime}$. Thus, the consumer would be better off if the subsidy is given as cash instead of as the food stamps.

Thus, in the first case, the consumer is equally well off under either giving case or food stamps, and in the second case, the consumer would be better off if the subsidy is given as cash. There is no case, however, where the consumer is better off with a food stamp subsidy.

4.2 Derivation of the Demand Curve from preferences maximization.
i) Graphical: To find demand for $x$ : change $P_{x}$, hold $P_{y}$ and $I$ constant.


Demand for x is a function of $\mathrm{P}_{\mathrm{x}}, \mathrm{P}_{\mathrm{y}}$ and I .
ii) Algebraic Derivation
(1) $\quad$ MRS $=\frac{\mathrm{p}_{\mathrm{x}}}{\mathrm{p}_{\mathrm{y}}}$
(2) $p_{x} x+p_{y} y=I$

We want to find $x$ as function of $p_{x}, p_{y}$ and $I$.
Example Suppose preferences are such that

$$
\operatorname{MRS}=\frac{y}{2 x} \quad 0 \quad\left(u n \quad(x, y)=x^{\overline{3}} y^{\frac{\overline{3}}{3}}\right)
$$

(1) $\Rightarrow \frac{y}{2 x}=\frac{p_{x}}{p_{y}} \Rightarrow y=\frac{2 x p_{x}}{p_{y}}$
(2) $p_{x} x+p_{y}\left(\frac{2 x p_{x}}{p_{y}}\right)=I$
$\Rightarrow \quad 3 p_{x} x=I \quad \Rightarrow \quad x=\frac{I}{3 P_{X}}=D_{x}$
Let $I=50, P_{y}=2 . \quad$ Then $x=\frac{50}{3 p_{x}}$

To derive the demand curve:

$$
\begin{aligned}
& P_{x}=1 \quad \Rightarrow \quad x=\frac{50}{3}=16.6 \\
& P_{x}=5 \quad \Rightarrow \quad x=\frac{50}{15}=\frac{10}{5}=3.3 \\
& P_{x}=10 \Rightarrow x_{i}=\frac{50}{30}=\frac{5}{3}=1.6 \\
& P
\end{aligned}
$$

4.3 Income and Substitution Effects of a Price Change



Initial optimum ar A with $x \%$, when $p_{x}, p_{y}$, forgiven After $p_{x}$ increases to $p_{x}^{\prime}$. New optimum at $C$ with $x^{\prime \prime}$, when $p_{x}^{\prime}, p_{y}$, I. The decrease in $x^{*}$ is the combined result of two effects of the price increase.

1) Income effect: $p_{x}$ increases $\Rightarrow \frac{1}{p_{x}}$ decreases

The amount of $x$ that can be purchased with the same income has decreased [A loss in purchasing power, or real income].
2) Substitution Effect - When $p_{x}$ increases $p_{y}$ fixed. $y$ has become relatively cheaper and the consumer will substitute $y$ for $x$.

Let's isolate these effects on the graph above.

To find the pure substitution effect, we must compensate the consumer with enough income so that the consumer does not suffer a utility loss due to the price change. That is, at the new price ratio $\mathrm{p}_{\mathrm{x}} / / \mathrm{p}_{\mathrm{y}}$, the consumer must be given enough (imaginary) income to reach U again.
$-p_{x}^{\prime} / p_{y}=s l o p e$ of new budget line. Any parallel line also has the same slope $=-p_{x} \prime / p_{y}$. Thus, we only need to find a parallel line which is tangent to $\mathrm{U}_{0}$


Thus $A$ to $B$ represents the effect of the increase in $p_{x}$ on $x$ while the consumer maintains the same level of utility. This is called the substitution effect. $\left(=x^{\stackrel{\rightharpoonup}{*}}-x^{\prime}\right)$.

The remaining change, from $B$ to $C$ is the income effect. The size of the shift in the budget line represents the amount of loss income it would take to fall from $U_{0}$ to $U_{1} . \quad\left(=x^{\prime}-x *^{\prime}\right)$

Total Effect $=$ substitution effect + income effect
$=\left(x^{*}-x^{\prime}\right)+\left(x^{\prime}-x^{* \prime}\right)$
$=x^{*}-x^{\prime}+x^{\prime}-x^{\prime \prime}$
$=x^{*}-x^{*}$
4.4 From Individual to Market Demand

We have derived demand curves for individual consumer. To obtain a market demand curve, you must add all individual demands at given prices.
eg. $P_{x} \quad D_{x}$ for Mr. A $D_{x}$ for Mr. B Market $D_{x}$

| $\$ 1$ | 10 | 12 | 22 |
| :--- | :--- | :--- | :--- |

$\begin{array}{llll}\$ 2 & 7 & 5 & 12\end{array}$
$\begin{array}{llll}\$ 3 & 3 & 1\end{array}$


### 4.5 An Example of Violation of Law of Demand

So far, the demand curves we derived are all downward-sloping. that is, demand for $x$ will decrease if its price increases. However, it is possible for a consumer to have indifference curves so that the law of demand does not hold for some good.

Eg. A lower price leading to less consumption.


The consumer purchases less of good $x$ when it's price falls. Note that the indifference curves that produce this result are downward sloping, nonintersecting, and convex; that is, they do not contradict any of our basic assumptions about preferences.

## Giffen Good

A good is said to be a giffen good if when price of good falls quantity demanded also falls. What kind of goods can be giffen goods? We know

Total effect $=$ substitution effect + income effect
In the example, when $p_{x}$ decreases to $p_{x}$ '

$$
\left(x^{\prime}-x^{x^{\prime}}\right)+\left(x^{*}-x^{\prime}\right)=x^{*}-x^{\prime \prime}>0
$$

Thus when a good is inferior and its income effect is larger than its substitute effect (in absolute value), the demand curve of the good will have a positive slope, ie., the good is giffen good.

So a giffen good must be an inferior good, but an inferior good may not be a giffen good.

### 4.6 Elasticity of Demand

Elasticity - measure responsiveness of quantity demanded (supplied) to a change in a given variable (such as own price, prices of the other goods, income).

Can we use slope of the curve to measure the elasticity? No, because a change in scale can make a curve look flatter or steeper without altering absolute responsiveness.

Thus, changing units (say from $\$$ to $\$$ ) will alter the slope. For this reason, economistsuse a unitless measure.

Price Elasticity of Demand ("own" Price Elasticity) - a measure of responsiveness of quantity demanded to changes its price.

| $E^{p}$ | $=\frac{\text { percentage change in quantity demanded }}{\text { percentage change in its price }}$ |
| ---: | :--- |
|  | $=\frac{\Delta q(p) / q(p)}{\Delta p / p}$ |
| where $" \Delta "=$ "change". |  |

Note: Since demand curves in general, slope downward, $E_{d}^{P}$ will be negative. Thus we ignore the sign.

Range of Value for $E_{d}^{p}$
(a) Inelastic Demand : $E_{d}^{\mathrm{P}}<1$
$\frac{\Delta q_{x} / q_{x}}{\Delta p_{x} / p_{x}}<1 \Rightarrow \Delta q_{x} / q_{x}<\Delta p_{x} / p_{x}$
So $q_{x}$ is relatively unresponsive to change in price.
(b) Elastic Demand $=\mathrm{E}_{\mathrm{d}}^{\mathrm{P}}>1$

That is $\Delta q_{x} / q_{x}>\Delta P_{x} / P_{x}$
So $q_{x}$ is relatively responsive to change in idsice.
(c) Unit Elasticity of Demand $=E_{d}^{P}=1$

That is $\Delta q_{x} / q_{x}=\Delta P_{x} / P_{x}$
(d) Perfectly Inelastic Demand

That is $\Delta q_{x} / q_{X}=0$ for all changes in price - totally unresponsive.
(e) Perfectly Elastic Demand $=E_{d}^{p}=\infty$

A very small percentage change in price leads to a huge percentage change in quantity demanded.




Example

| $\mathbf{P}_{\mathrm{x}}$ | $\mathrm{q}_{\mathrm{x}}$ <br> $\$ 1$ <br> $\$ 2$ |
| :---: | ---: |
|  | 10 |
| 5 |  |

$$
\begin{aligned}
& \Delta \mathrm{q}_{\mathrm{x}} / \mathrm{q}_{\mathrm{x}}=50 \% \\
& \Delta \mathrm{p}_{\mathrm{x}} / \mathrm{p}_{\mathrm{x}}=100 \% \\
& \mathrm{E}_{\mathrm{d}}^{\mathrm{p}}=\frac{50}{100}=\frac{1}{2} \\
& \Delta \mathrm{q}_{\mathrm{x}} / \mathrm{q}_{\mathrm{x}}=100 \% \\
& \Delta \mathrm{p}_{\mathrm{x}} / \mathrm{p}_{\mathrm{x}}=\frac{2-1}{2}=50 \% \\
& \text { So } \mathrm{E}_{\mathrm{d}}^{\mathrm{p}}=\frac{1000}{500}=2
\end{aligned}
$$

| $\mathrm{P}_{\mathrm{x}}$ | $\mathrm{q}_{\mathrm{x}}$ |
| :---: | :---: |
| $\$ 2$ | 5 <br> $\$ 1$ |

This demonstrates how $E_{d}^{P}$ varies according to which $P-q$ combination is used as a reference point. To avoid this problem we use the:

Midpoint Formula: $E_{d}^{p}$ between points ( $\mathrm{P}_{\mathrm{x}}, \mathrm{q}_{\mathrm{x}}$ ) and ( $\mathrm{p}_{\mathrm{x}}^{\prime}, \mathrm{q}_{\mathrm{x}}^{\prime}$ )

$$
=\frac{\Delta q_{x} / \frac{1}{2}\left(q_{x}+q_{x}^{\prime}\right)}{\Delta p_{x} / \frac{1}{2}\left(p_{x}+p_{x}^{\prime}\right)}
$$

This just uses the midpoints as the reference points.

$$
\begin{aligned}
& \text { Example (same as above) } \\
& \left.\frac{\mathrm{p}_{\mathrm{x}}}{\$ 2} \right\rvert\, \frac{\mathrm{q}_{\mathrm{x}}}{5} \\
& \$ 1 \\
& \Delta \mathrm{q}_{\mathrm{x}} / \frac{1}{2}\left(\mathrm{q}_{\mathrm{x}}+\mathrm{q}_{\mathrm{x}}^{\prime}\right)=\frac{5}{1 / 2 \cdot 15}=\frac{2}{3} \\
& \Delta \mathrm{p}_{\mathrm{x}} / \frac{1}{2}\left(\mathrm{p}_{\mathrm{x}}+\mathrm{p}_{\mathrm{x}}^{\prime}\right)=\frac{1}{1 / 2 \cdot 3}=\frac{2}{3} \\
& \text { So } \mathrm{E}_{\mathrm{d}}^{\mathrm{p}}=\frac{2 / 3}{2 / 3}=1
\end{aligned}
$$

Only very special demand curves have the same elasticity between any two points. Straight line demand curves do not have constant elasticity throughout.

$$
\begin{aligned}
\mathrm{E}_{\mathrm{d}}^{\mathrm{p}} & =\frac{\Delta \mathrm{q}_{\mathrm{x}} / \mathrm{q}_{\mathrm{x}}}{\Delta \mathrm{p}_{\mathrm{x}} / \mathrm{p}_{\mathrm{x}}}=\frac{\Delta \mathrm{q}_{\mathrm{x}}}{\Delta \mathrm{p}_{\mathrm{x}}} \cdot \frac{\mathrm{p}_{\mathrm{x}}}{\mathrm{q}_{\mathrm{x}}} \\
& =\frac{1}{\frac{\Delta \mathrm{p}_{\mathrm{x}}}{\Delta \mathrm{q}_{\mathrm{x}}}} \cdot \frac{\mathrm{p}_{\mathrm{x}}}{\mathrm{q}_{\mathrm{x}}}
\end{aligned}
$$

$$
=\frac{1}{k} \frac{p_{x}}{q_{x}} \neq \text { constant because } \frac{p_{x}}{q_{x}} \text { is not constant even though }
$$

slope of straight line demand curve is constant.

## Elasticity Along Straight Line Demand Curve

Since slope of straight line is constant (ie. $\frac{\Delta q_{x}}{\Delta p_{x}}$ is constant),

$$
\frac{\Delta \mathrm{q}_{\mathrm{x}}}{\Delta \mathrm{p}_{\mathrm{x}}}=\frac{\mathrm{LB}}{\mathrm{AL}} . \text { So }
$$

$$
E^{p}=\frac{\Delta q_{x}}{\Delta p_{x}} \cdot \frac{p}{q_{x}}=\frac{L B}{A L} \cdot \frac{O L}{L B}=\frac{0 L}{A L}
$$

Thus if $B$ is the midpoint of the demand curve, $E_{d}^{P}=1$. If $B$ is a point above the midpoint, $A L<O L, \Rightarrow E_{d}^{P}>1$. If $B$ is a point below the midpoint, $\mathrm{AL}>\mathrm{OL}, \mathrm{E}_{\mathrm{d}}^{\mathrm{p}}<1$.

### 4.7 Elasticity and Total Revenue

Total Revenue $(T R)=$ Total Expenditure $=p_{x} \cdot q_{x}$
Along the demand curve we have: if $p_{x}$ rises, then $q_{x}$ falls. We are now interested in what happens to $\left(p_{x} \cdot q_{x}\right)$, ie, total revenue. Since $p_{x}$ and $q_{x}$ are inversely related by the law of downward sloping demand, we need information about the magnitude of the changes in $p_{x}$ and $q_{x}$ to determine the direction of the effect on $\left(p_{x} \cdot q_{x}\right)$. To do this, we use the price elasticity of demand. They have the following relationships:

$\left.$|  |
| :---: |
| $P_{\mathrm{x}}$ rises |$\left|\frac{\mathrm{E}_{\mathrm{d}}^{\mathrm{P}}>1}{\text { TR falls }}\right| \frac{\mathrm{E}_{\mathrm{d}}^{\mathrm{P}}<1}{\text { TR rises }} \right\rvert\, \frac{E_{d}^{\mathrm{P}}=1}{\text { TR are constant }}$

In other words, when demand is elastic, price and total expenditure move in opposite directions. When demand is inelastic, price and total expenditure move in the same direction. When demand is unit elastic, total expenditure main constant when the price varies.




Algebraically, we can show these relationship holds.
Eg. Suppose $\mathrm{E}_{\mathrm{d}}^{\mathrm{P}}>1$ and $\mathrm{P}_{\mathrm{y}}^{\prime}>\mathrm{P}_{\mathrm{X}}$.

$$
\frac{\Delta \mathrm{q}_{\mathrm{x}} / 1 / 2\left(\mathrm{q}_{\mathrm{x}}+\mathrm{q}_{\mathrm{x}}^{\prime}\right)}{\Delta \mathrm{p}_{\mathrm{x}} / 1 / 2\left(\mathrm{p}_{\mathrm{x}}+\mathrm{p}_{\mathrm{x}}^{\prime}\right)}>1
$$

$\Rightarrow \frac{\Delta q_{x}}{q_{x}+q_{x}^{\prime}}>\frac{\Delta p_{x}}{p_{x}+p_{x}}$,
$\Rightarrow\left(q_{x}-q_{x}^{\prime}\right)\left(p_{x}+p_{x}{ }^{\prime}\right)>\left(p_{x}^{\prime}-p_{x}\right)\left(q_{x}+q_{x}{ }^{\prime}\right)$
$\Rightarrow p_{x} q_{x}-p_{x}^{\prime} q_{x}^{\prime}>p_{x} q_{x}+p_{x}^{\prime} q_{x}^{\prime}$
$\Rightarrow 2 p_{x} q_{x}>2 p_{x}{ }^{\prime} q_{x}^{\prime}$
$\Rightarrow \quad p_{x} q_{x}>p_{x}{ }^{\prime} q_{x}{ }^{\prime}$
This shows if $\mathrm{P}_{\mathrm{x}}$ rises TR falls.
Other Elasticity
Income Elasticity $\left(\mathrm{E}_{\mathrm{d}}^{\mathrm{I}}\right)$ :

$$
\mathrm{E}_{\mathrm{d}}^{\mathrm{I}}=\frac{\Delta \mathrm{D}_{\mathrm{x}} / \mathrm{D}_{\mathrm{x}}}{\Delta \bar{I} / \mathrm{I}}
$$

## Cross-Price Elasticity of Demand

Let $P_{y}$ be the price of good $y$. The cross-price elasticity of demand for $x$ with respect to the price is defined as

$$
\mathrm{E}_{\mathrm{d}}^{\mathrm{p}}{ }_{\mathrm{x}}=\frac{\Delta \mathrm{D}_{\mathrm{x}} / \mathrm{D}_{\mathrm{x}}}{\Delta \mathrm{p}_{\mathrm{y}} / \mathrm{p}_{\mathrm{y}}}
$$

## Consumer Surplus

Consumers purchase goods and services because they are better off after the purchase than they were before, otherwise, the purchase would not take place. The term consumer surplus refers to the net benefit, or gain.

To obtain the measure of consumer surplus associated with purchasing a certain amounts of a good or service, we may ask the question: What is the maximum amount you would be willing to pay, that is, what is the maximum total benefit, and what is the total cost? After we have the answers, the consumer surplus is defined by

## Consumer Surplus $=$ Total Benefit - Total Cost.

That is, the consumer surplus is the difference between that the consumer would be willing to pay !or the consumption of a good and what the person actually has to pay for it.


To show this, let us consider a specific example. Suppose a person buy 6 cups of espresso at the price, $p=3$. The total cost $=3 \times 6=18$. Total benefit from purchasing six units at a price 3 is the sum of marginal benefit (the sum of six units shaded rectangles of the above diagram), i.e.,
total benefit $=\$ 8+\$ 7+\$ 6+\$ 5+\$ 4+\$ 3=33$
Thus, the consumer surplus $=$ total benefit - total cost $=33-18=15$.

In general, with a smooth demand curve indicated in the following diagram, consumer surplus equals the area $T E P$.


The concept of consumer surplus can also be used to identify the net benefit of a change in the price of a commodity. In the following diagram, at a price of 25 cents per unit, consumer surplus is $T A P$. At a price of 15 cents per unit, consumer surplus is $T E P^{\prime}$. The increase in consumer surplus from the price reduction is thus the shaded area $P A E P^{\prime}$, which is a measure of the benefit to consumers of a reduction in the price from 25 to 15 cents.


```
V. Exchange, Efficiency, and Prices
(Welfare Economics for Exchange Economy)
```


#### Abstract

In this chapter a model of pure exchange is described and used to analyze the nature and consequences of voluntary exchange and to introduce the concept of economic efficiency. A new tool of analysis is introduced as well - the Edgeworth exchange box diagram.

\subsection*{5.1. Two-Person Exchange}

Voluntary exchange is mutually beneficial; that is, people will not trade voluntarily unless they believe they will benefit from the trade. Pure exchange can be studied using the Edgeworth exchange box diagram.


Pure Exchange Economies - deal with pricing and allocation in economies in which $\underline{n}$ individuals exchange and consume fixed quantities of $\underline{m}$ commodities.

Each individual is endowed with one or more of the commodities and is free to buy and sell at the prevailing market prices.

Let $m=2$, commodities $x$ and $y$ $n=2$, consumers $A$ and $B$

Endowment of $x$ and $y$ for $A$ : $\left(x_{0}^{A}, y_{0}^{A}\right)$
Endowment of $x$ and $y$ for $B$ : $\quad\left(x_{0}^{B}, y_{0}^{B}\right)$.

The Edgeworth Exchange Box Diagram (The Edgeworth Box) - a geometric picture of a 2 -person, 2-commodity pure exchange economy and can be used to examine the allocation of fixed total quantities of two goods between two consumers.

Total economy endowments determine the dimension of the box:

$$
\begin{aligned}
& \text { length }=x_{0}^{A}+x_{0}^{B} \\
& \text { width }=y_{0}^{A}+y_{0}^{B} .
\end{aligned}
$$

That is, the horizontal and vertical dimensions of the box indicate the total quantities of the two goods.


How is the Edgeworth box derived?
A's preference map is pictured in Figure a.

B's preference map is pictured in Figure b.
Now Figure b is rotated 180 degrees with origin $O^{B}$ we obtain Figure $c$. Putting Figure a and Figure $c$ together we come up with the Edgeworth box.



### 5.2 Efficiency of Allocations (Pareto Optimality)

When the marginal rates of substitution differ mutually beneficial trade between the parties is possible. Differing MRS's imply intersecting indifference curves and a corresponding lens-shaped area of potential mutual gains in the Edgeworth box. Thus, there should be a tendency for trade until there is no intersection of two consumers's indifference curves, that is, until a Tangency is reached at point A. The point $A$ is efficient.

An allocation (distribution) is efficient (Pareto Optimal) for a fixed total quantities of goods if it is impossible to make one person better off without making someone else worse off.


All tangent points are efficient.
Contract Curve. A line through all the efficient allocation is called the contract curve.

Contract curve connects the points of tangency between indifference curves. Thus, efficient distributions are characterized by an equality between marginal rates of substitution, i.e.

$$
\mathrm{MRS}^{\mathrm{A}}=\mathrm{MRS}^{B}
$$

The contract curve defines the set of all efficient way to decide the total endowments between the consumers. In contrast, all points off the contract curve are inefficient allocations.

Inefficiency. An inefficient allocation of goods is one in which it is possible, through a change in the distribution, to benefit one person without harming the other.

Thus inefficient allocations are shown as points where the indifference curves of the two parties intersect, that is, where

$$
\operatorname{MRS}^{A} \neq \operatorname{MRS}^{B}
$$

We know that if an allocation is efficient then

$$
\begin{equation*}
\mathrm{MRS}^{\mathrm{A}}=\mathrm{MRS}^{\mathrm{B}} \tag{1}
\end{equation*}
$$

Is an allocation which satisfies (1) necessarily efficient? Answer is no by considering the following figure.


With given prices and preferences:
A: wants to buy $y$ and to sell $x$
B: wants to buy $y$ and to sell $x$

Both want to buy $y$, sell $x$ but at existing prices this arrangement is impossible. So this allocatiots is not efficient (because indifferent curves of the two consumers are not tangent). However, $\operatorname{MRS}^{A}=M R S^{B}$ still holds.

Under what conditions is an allocation satisfying $\operatorname{MRS}^{A}=\operatorname{MRS}^{B}$ efficient?

If an allocation satisfies
(1) $\quad$ MRS $^{A}=\operatorname{MRS}^{B}$
(2) $x^{A}+x^{B}=x_{0}^{A}+x_{0}^{B}$ and $y^{A}+y^{B}=y_{0}^{A}+y_{0}^{B}$.
the allocation is efficient. Condition (2) is call balance condition.

### 5.3 Competitive (Market) Equilibrium and Efficiency

In this section we show that a competitive market equilibrium is efficient

Competitive Markets - a market is competitive if every person (buyer or seller) takes the prices as given, i.e., each individual cannot determine prices of goods.

A Competitive Equilibrium in the pure exchange economy consists of prices $\left(P_{x}, P_{y}\right)$ and final holdings $\left(\left(x^{* A}, y^{* A}\right),\left(x^{* B}, y^{* B}\right)\right)$ such that a) $\left(x^{* A}, y^{* A}\right)$ is an optimal choice under his budget constraint.
b) $\left(x^{* B}, y^{* B}\right)$ is an optimal choice under his budget constraint.
c) Market equilibrium (balance condition) holds:

$$
\begin{aligned}
& x^{* A}+x^{* B}=x_{0}^{A}+x_{0}^{B} \\
& y^{* A}+y^{* B}=y_{0}^{A}+y_{0}^{B}
\end{aligned}
$$

Proposition 1. Competitive equilibrium allocation is efficient.
Recall from Consumer Theory that if a bundle is optimal for a consumer then

$$
\begin{aligned}
\text { MRS }^{A} & =\frac{P_{x}}{P_{y}} \\
\text { MRS }^{B} & =\frac{P_{x}}{P_{y}}
\end{aligned}
$$

Since the prices are the same for both consumers, we have

$$
\operatorname{MRS}^{\mathrm{A}}=\mathrm{MRS}^{B}
$$

Also market equilibrium condition c) holds for a competitive equilibrium allocation. So we have shown that a competitive equilibrium allocation is efficient.


The above figure shows although each consumer acts independently in choosing a market bundle, the result is an efficient distribution of the goods.

Proposition 2: Any efficient allocation can be achieved as competitive equilibrium allocation given an appropriate redistribution of endowment.


Point $E$ is efficient and is competitive equilibrium allocation under new endowment $\omega$.

## VI. Theory of Production

### 6.1 Relating Output to Inputs

Inputs (Factors of production) - are the ingredients used by a firm to produce a good or a service.
(egg. labor., land or capital)

Firm - any organization that engages in production.

Two important aspects of production process

1) a set of input requirements
2) a production technique or production function.

Production Function - is a relationship between inputs and outputs. It identifies the maximum output which the firm can produce with a particular combination of inputs per time period.

We can present a production function in tabular, graphical, or mathematical form
egg. mathematical form: $Q=F(k, L)$
where $Q$ is number of units of output, $k$ number of units of capital input, $L$ number of units of labor input, and $F$ is production function where $F(k, L)=A k^{\alpha} L^{\beta}$ and $\alpha=\beta=\frac{1}{2}, A=1$

$$
Q=k^{1 / 2} L^{1 / 2} \quad(\operatorname{cosb} \text { - Douglas induction fan.) }
$$

Thus for a particular combination of inputs, say, $k=16, L=36$, the maximum output obtainable with this technology is $Q=(16)^{1 / 2}(36)^{1 / 2}=$ $4 \times 6=24$.

A production is technologically efficient if the maximum quantity of a commodity can be produced by each specific combination of inputs.

Thus, production specified by production function is technologically efficient. If a firm is rational it always operates in a technologically efficient way to produce outputs.

### 6.2 Production Isoquants

Production Isoquants - is a curve that shows all the combinations of inputs that, when used in a technologically efficient way, will produce a certain level of output.

e.g. Suppose $Q=K^{1 / 2} L^{1 / 2}$. The isoquant for $Q=2$ must include the following pair of inputs.
$K=1, \quad L=4$
$\mathrm{K}=2, \mathrm{~L}=2$

# $K=3, \quad L=\frac{4}{3}$ <br> $K=4, L=1$ 

### 6.3 Short-Run and Long-Run Production Responses

Short Run is a period of time in which changing the employment levels of some inputs is impractical.

Fixed Inputs - are resources that a firm cannot feasibly vary over the time period involved.

Long Run is a period of time in which the firm can vary all its inputs.
e.g. Factory - short run is long enough to hire an extra worker but not long enough to build an extra production line.

### 6.4. Production in Short Run

Assume there are 2 factors of production. Labor (L) and Capital (K).

In the short run we usually assume $K$ is the fixed resources and $L$ is the variable resources.

Given fixed $K$, the firm can vary L to produce different amounts of output.
(a) Total Product (TP): total output produced
(b) Average Product (AP): total product per unit of variable factor.
$A P=$ total product/variable factor.
e.g. average product of labor:

$$
A P_{L}=\frac{T R}{L}
$$

where $L=$ quantity of labor
(c) Marginal Product (MP) - the change in total output that results from a one-unit change in the amount of the input, holding the quantities of other inputs constant.
e.g. marginal product of labor

$$
M P_{L}=\Delta T P / \Delta L
$$

## Example

| Fixed amount of Land | Amount of Labor (L) | $\mathrm{TP}_{\mathrm{L}}$ | ${ }^{A P}{ }_{L}$ | ${ }^{M P}{ }_{L}$ |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 0 | 0 | -- | -- |
| 5 | 1 | 50 | 50 | 50 |
| 5 | 2 | 150 | 75 | 100 |
| 5 | 3 | 300 | 100 | 150 |
| 5 | 4 | 400 | 100 | 100 |
| 5 | 5 | 480 | 96 | 80 |
| 5 | 6 | 540 | 90 | 60 |
| 5 | 7 | 580 | 83 | 40 |
| 5 | 8 | 610 | 76 | 30 |
| 5 | 9 | 610 | 68 | 0 |
| 5 | 10 | 580 | 58 | -30 |

Relationship between marginal and averages.
If $M P_{L}>A P_{L}$, then $A P_{L}$ must rise.
If $M P_{L}<A P_{L}$, then $A P_{L}$ must fall.


### 6.5. The Law of Diminishing Marginal Returns (DMR)

Law of Diminishing Marginal Returns: states that as more and more a variable inputs are used together with a fixed amounts of other inputs and fixed technology, a point is reached beyond which the marginal product of the variable input begins to fall.
e.g. farm adds fertilizer (variable) to an acre of land (fixed).
e.g.

| $\mathrm{K}, \mathrm{L}$ | Q | $\mathrm{MP}_{\mathrm{L}}$ |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| 1,0 | 0 | - |  |
| 1,1 | 10 | 10 |  |
| 1,2 | 30 | 20 |  |
| 1,3 | 40 | 10 |  |
| 1,4 | 45 | 5 | Diminishing Returns |
| 1,5 | 48 | 3 |  |

The Geometry of Production Curves
Average Product $\quad A P_{L}=\frac{T P}{L}=\frac{Q}{L}$
Note that the slope of any line from the origin to a point on the TP curve has slope $=\frac{\Delta Q}{\Delta L}=\frac{Q-0}{L-0}=\frac{Q}{L}$. That is, AP $L$ is the slope of the line which connects the origin to the TP curve.


At point $C . A P_{L}$ reaches a maximum since the ray $O C$ is the stepest ray from the origin that still touches the $T P$ curve.
Marginal Product: Recall $M P_{L}=\frac{\Delta T P}{\Delta L}=\frac{\Delta Q}{\Delta L}$. That is, $M P$ is the slope of the TP curve. The Law of Diminishing Marginal Product (DMP) tell us about the shape of the $T P$ curve. $M P_{I}$ increases to a point but then decreases by the Law of DMP. This is, slop of TP is increasing and then eventually starts decreasing. When $M P_{L}=0$, the $T P$ reaches its maximum.

Note - ${ }^{M P} P_{L}=A P_{L}$ at the maximum of $A P_{L}$.

### 6.6. Production in Long Run

Let us return to the isoquant curves. Isoquants are very similar to indifference curves in their characteristics. By analogous reasoning, we can explain several of the characteristics of isoquants
i) isoquant slopes downward. If we increase quantity of one input employed and wish to keep output unchanged, we must reduce the amount the amount of the other inputs.
ii) two isoquants can never intersect.
iii) isoquants lying further to the northeast identify greater levels of outputs.
iv) isoquants will generally be convex to the origin.

The slope of an isoquant measures the marginal rate of technical substitution between the inputs.

Marginal Rate of Technical Substitution (MRTS) - is a rate at which one input can be substituted for another without loss of output.

$$
\operatorname{MRTS}_{L K}=-\frac{\Delta K}{\Delta \mathrm{~L}}
$$

Also

$$
\mathrm{MRTS}_{\mathrm{LK}}=\frac{\mathrm{MP}}{\mathrm{~L}} \mathrm{MP}_{\mathrm{K}}
$$

Proof.

$$
\Delta \mathrm{Q}=\Delta \mathrm{L} \cdot \mathrm{MP}_{\mathrm{L}}+\Delta \mathrm{K} \cdot \mathrm{MP}_{\mathrm{K}}
$$

Along, an isoquant, $\Delta Q=0,0=\Delta L \cdot M P_{L}+\Delta K M P_{K}$. Thus, we have

$$
-\frac{\Delta \mathrm{K}}{\Delta \mathrm{~L}}=\frac{\mathrm{MP}}{\mathrm{~L}} \mathrm{MP}
$$

So, $\quad \mathrm{MRTS}=\frac{\mathrm{MP}_{\mathrm{L}}}{\mathrm{MP}_{\mathrm{K}}}$.
Isoquants are convex means the marginal rate of technical substitution diminishes. It is called Law of Diminishing MRTS.

Extreme Cases:
i) Perfect Substitutions $\quad$ MRTS $=$ Constant

ii) Perfect Complement. (Fixed Proportions Production function).


Fixed Proportions Production function can be written as

$$
\mathrm{Q}=\mathrm{F}(\mathrm{~K}, \mathrm{~L})=\min \{\mathrm{K}, \mathrm{~L}\}
$$

### 6.7. Returns to Scale

Return to Scale: the effect on output of equal proportionate change in
all inputs.
i.e. double inputs $\Rightarrow$ what happens to outputs?
if output doubles: called Constant Returns to Scale (CRS)
if output more than doubles: Increasing Returns to Scale (IRS)
if less than doubles: Decreasing Returns to Scale (DRS)

The Production function can be checked in the following way.

$$
F(\lambda K, \quad \lambda L)=\lambda^{t} F(K, L), \quad \lambda>0
$$

if $t<1, F(K, L)$ displays DRS
if $t=1, F(K, L)$ displays CRS
if $t>1, F(K, L)$ dispiays IRS
e.g. Suppose $F(K, L)=5 K^{1 / 3} L^{2 / 3}$
then $F(\lambda K, \lambda L)=5(\lambda K)^{1 / 3}(\lambda L)^{2 / 3}$

$$
\begin{aligned}
& =5 \lambda^{1 / 3+2 / 3} \mathrm{~K}^{1 / 3} \mathrm{~L}^{2 / 3} \\
& =5 \lambda \mathrm{~K}^{1 / 3} \mathrm{~L}^{2 / 3} \\
& =\lambda F(\mathrm{~K}, \mathrm{~L})
\end{aligned}
$$

Since $t=1$, it is CRS.

$$
\text { e.g. } \begin{aligned}
F(K, L) & =7 K+6 L \\
\text { then } F(\lambda K, \lambda L) & =7(\lambda K)+6(\lambda L) \\
& =\lambda(7 K+6 L) \\
& =\lambda F(K, L)
\end{aligned}
$$

Since $t=1$, it is CRS
e.g. $\quad F(K, L)=K L$

$$
\begin{aligned}
\mathrm{F}(\lambda \mathrm{~K}, \quad \lambda \mathrm{~L}) & =(\lambda \mathrm{K})(\lambda \mathrm{L}) \\
& =\lambda^{2} \mathrm{KL} \\
& =\lambda^{2} \mathrm{~F}(\mathrm{~K}, \mathrm{~L})
\end{aligned}
$$

```
Since t = 2, it is IRS
```

Graphically, returns to scale are illustrated below:


Note: As a general proposition, increasing returns to scale are likely to be the case when the scale of operations is small, perhaps followed by an intermediate range when constant returns prevail, with decreasing returns to scale becoming important for large-scale operations. In other words, a production function can embody increasing, constant, decreasing returns to scale at different levels of output.

## VII. Cost of Production

We will discuss types of production costs and analyze the relationship between cost of production and output of production.

### 7.1 The Nature of Costs

In economic analysis a firm's costs of production are the sum of explicit and implicit costs.

Explicit Costs - The payments made for resources which the firm purchases or hires from outside sources.
e.g. wages, interest paid on borrowed money, rent for land owned by outside party.

Implicit Costs - the costs of resources which the firm uses but neither buys nor hires from outside sources.

- Provided these resources have an alternative use there is a cost involved although no explicit monetary payment is made. To the firm


# these implicit costs are the monetary payments which resources could earn in their best alternative use. 

e.g. If you own your own building implicit costs of running a small store include the rent that could have been earned if the building was leased to another firm.
e.g. Salary that could be earned by owner if employed in another business.
e.g. interest that could be earned by lending money to someone else.

The sum of explicit and implicit costs may be regarded as opportunity costs.

Opportunity Cost - the cost of a unit of a good measured in terms of other goods that must be forgone to obtain it.
-follows from idea that resources are scarce and if they are used to produce one good, they are not available to produce other goods.
e.g. cost of washing machine might be \# of refrigerators.

### 7.2. Short-Run Costs of Production

(a) Total Fixed Cost (TFC): costs of fixed factors of production in the short-run. These costs do not change as level of output changes.
e.g. cost of factory - incurred even when output is zero.
(a') Average Fixed Cost (AFC)
$A F C=\frac{T F C}{Q}$ where $Q=$ output level.
(b) Total Variable Cost (TVC): costs incurred by the firm that depend on how much output it produces. These costs are associated with the variable inputs.
e.g. labor cost depends on the number of workers hired.
(b') Average Variable Cost (AVC)

$$
\mathrm{AVC}=\frac{\mathrm{TVC}}{\mathrm{Q}}
$$

(c) Total Cost (TC)
$\mathrm{TC}=\mathrm{TFC}+\mathrm{TVC}$
(c') Average Total Cost (ATC)
$A T C=\frac{T C}{Q}$
Since $\mathrm{TC}=\mathrm{TFC}+\mathrm{TVC}$, then
$A T C=\frac{T F C+T V C}{Q}$
$=\frac{T F C}{Q}+\frac{T V C}{Q}$
So. $\quad A T C=A F C+A V C$
(d) Marginal Cost (MC) - is the change in total cost that results from a one-unit change in output.

$$
M C=\frac{\Delta \mathrm{TC}}{\Delta \mathrm{Q}}=\frac{\Delta \mathrm{TFC}+\Delta \mathrm{TVC}}{\Delta \mathrm{Q}}
$$

$\Delta T F C=0$, since only $T V C$ varies with $Q$.
So $\quad M C=\frac{\Delta T V C}{\Delta Q}$
Also $T V C=$ sum of $M C$

## Examp1e:

Short Run Cost Schedule For
an Individual Firm


Note: $\quad Q=$ total product

$$
\begin{aligned}
& \mathrm{TC}=\mathrm{TFC}+\mathrm{TVC} \\
& \mathrm{MC}=\frac{\Delta \mathrm{TC}}{\Delta \mathrm{Q}}=\frac{\Delta \mathrm{TVC}}{\Delta \mathrm{Q}} \\
& \mathrm{AFC}=\frac{\mathrm{TFC}}{\mathrm{Q}} \\
& \mathrm{AVC}=\frac{\mathrm{TVC}}{\mathrm{Q}} \\
& \mathrm{ATC}=\frac{\mathrm{TC}}{\mathrm{Q}}
\end{aligned}
$$



Note that the vertical distance between TC and

7.3. The Short-Run Cost Curves

Now let us examine what shapes the short-run cost curves have. We will show that all short-run cost curves have the same shapes as the shapes implied by the data in the previous example.

Short Run Cost Schedule for Individual Firm:


Note: MC cuts AVC and ATC at their minimum. This is because:
IF MC < AVC, the AVC must be falling.
IF MC $>$ AVC, the AVC must be rising.
IF $M C=A V C$, the AVC must be at minimum.

The marginal cost curve is U-shaped, with the cost of additional units of output first falling, reaching a minimum, and then rising. The shape of the marginal cost curve is attributable to the law of diminishing marginal returns. To see why, recall the MC is defined as

$$
M C=\frac{\Delta T V C}{\Delta Q}
$$

We know TVC = wL where $w$ is the wage rate and $L$ is the amount of the variable input (labor). Thus $\triangle T V C=w \Delta L$, therefore we have

$$
M C=\frac{\Delta T V C}{\Delta Q}=\frac{W \Delta L}{\Delta Q}=w / \frac{\Delta Q}{\Delta L}=w / M P_{L} .
$$

Thus MC and $M P_{L}$ have a reverse relationship. Because of the law of diminishing marginal returns. $\quad \mathrm{MP}_{\mathrm{L}}$ varies with the amount of output and therefore, so must $M C$. At low levels of output $M P_{L}$ is rising, so, correspondingly, $M C\left(=w / M P_{L}\right)$ must be falling. When $M P_{L}$ reaches a maximum, then Mc must be at a minimum. After that $M P_{L} f a l l s$, then MC must rises. That is, $M P_{L}$ rises and then falls, the $M C$ will first fall and then rise.

The average variable curve (AVC) must be also U-shaped. Recall AVC is defined as

$$
A V C=\frac{T V C}{Q}=\frac{W L}{Q}=W / \frac{Q}{L}=W / A P_{L}
$$

In the previous chapter we saw that the law of diminishing marginal returns leads to an $A P_{L}$ shaped like an inverted $U$. That is, $A P_{L}$ rises, reaches a maximum, and then falls. As a result, $W / M P_{L}$ (MC) mus U-shaped, i.e., MC will fall, reach a minimum, and then rise.

### 7.4. Long-Run Costs of Production

In the long run:
(a) Firm can enter/leave the industry (discussed later).
(b) A firm can vary all resources thus

- the law of diminishing marginal returns does not apply.
- all costs are variable so there is no distinction between fixed and variable costs. The only types of cost are TC, ATC, MC.

Isocost Line- shows all combinations of inputs the firm can purchase with a fixed amount of total cost. (Compare with the budget line)

Isocost equation:

$$
C=w L+r K
$$

where $C=$ total cost, $w=$ wage/hour of labor, $r=$ rent/unit of $K$, slope of isocost line $=-\frac{w}{r}$



Suppose the firm wants to produce a given level ${ }^{\circ} \mathrm{F}$ (utput, $\mathrm{Q}=\overline{\mathrm{Q}}$. We can use the isoquant to determine the possible input combinations which make $\bar{Q}$ feasible. Which will be chosen?

The firm would choose whichever combination of inputs that

1) yield $\bar{Q}$ units of output
2) costs less than any other input combination which also produces $\overline{\mathrm{Q}}$.

## Cost Minimization Problem:

The firm wishes to find the least cost input combination which can be used to produce a given level of output. This implies the firm must find the point on the isoquant $Q=\bar{Q}$ which is tangent to an isocost line.


Both D and E produce $\overline{\mathrm{Q}}$ but D cost more.
At the cost minimizing bundle.
Slope of isocost $=$ slope of isoquant

$$
-\frac{w}{r}=-M R T S
$$

Recall MRS $=-\frac{M P_{L}}{M P_{K}}$ We can write the previous expression as

$$
\frac{M P_{L}}{M P_{K}}=\frac{w}{r}
$$

Rearranging terms, we obtain

$$
\frac{{ }^{M P_{L}}}{w}=\frac{{ }^{M P_{K}}}{r}
$$

The last equality means a firm should employ inputs in such a way the marginal product cost per dollar's worth of all inputs is equal.

Expansion Path: shows the input combinations that represent cost minimizaiton, it is formed by connecting all tangency points when the level of output varies.



### 7.5. Long-Run Cost Curve

The long-run total cost shows the minimum cost at which each rate of output may be produced just as the expansion path does. The long-run marginal cost and average cost curves are derived from the total cost curve in the same way the short-run per-unit curves are derived from the short-run total cost curves.


We have drawn the TC curve to imply a U-shaped long-run $A C$ and MC curves. Why would the $A C$ and $M C$ have this shape? In long-run all inputs are variable, the law of diminishing marginal returns is not responsible for their U-shapes.

However, the shape of the long-run $A C$ and MC curves reflect the return to scale which characterize the technology. As we explained before, increasing returns to scale are likely to be common at low rates of output, while decreasing returns to scale are likely to prevail at high output levels. Therefore, the long-run $A V$ and $M C$ must have a U-shape.

## Increasing Returns to Scale

Double input cost $\Rightarrow$ more than double output.


$M C=s l o p e$ of $T C$ falls and $A C=$ slope of connecting line also falls.

## Decreasing Returns to Scale

Double input cost $\Rightarrow$ less than double output



MC increases, $A C$ increases.

## Constant Returns to Scale

Double input $\Rightarrow$ double outputs. Thus, input cost per unit as constant, i.e. $T C=a Q$ where $a$ is cost of producing one unitoutput.


Note: MC crosses AC at its minimum. When $A C$ is falling, MC is below it.
When $A C$ is rising, $A C$ is above it.

Relationship Between Short-Run and Long-Run Average Cost Curves Short-Run AC Curve:

Consider five different plant capacities


Plant capacities 1 \& 2: small firms
Plant capacities 3: medium firms
Plant capacities $4 \& 5$ : large firms
Observe that the short run ATC's decline from small to medium sized firms and then increase as firms become large.

## Long-Run AC Curve

The long run $A C$ curve shows the lowest per unit cost at which any output can be produced, given that the firm has sufficient time to vary all resources, including plant capacity


The long run $A C$ curve consists of segments of the short run $A C$ curves. Given an unlimited number of possible plant sizes the long run $A C$ curve is made up of points of tangency with the unlimited number of short run $A C$ curves.

Thus the long run $A C$ curve shows the most efficient way to produce a given level of output.

### 7.6. Input Price Changes and Cost Curves

A change in inputs prices will cause the entire cost curves to shift.


Initially, the firm is producing $Q_{1}$, by employing $K$ and $L$ at point $E$. With a lower wage rate the cost of producing each level of output falls. Input combination $E^{\prime}$ becomes the least costly way to produce the same $Q_{1}$ after the wage reduction. Thus the change in $w$ shifts the $A C$ and $M C$ curves downward to $A C^{\prime}$ and $M C^{\prime}$.

## Using Cost Curves: Controlling Pollution

Many problems can be clarified by posing them in terms of marginal cost. Here is an example of using cost curves to controlling pollution in a cheapest way. There are two firms which release pollutants into the air in the process of production. The government steps in and restricts the total pollution to a certain level, say 200 units.

In the following diagram, the amount of pollution generated by each firm is measured from right to left. For example, before the government restricts its activity, firm A discharges $O P_{1}$ (300 units), and firm B discharges $O P_{2}$ ( 250 units). Measuring pollution from right to left is the same as measuring pollution abatement-the number of units by which pollution is reduced from its initial level-from left to right. For example, if firm $B$ cuts back its pollution from 250 to 100 units, it has produced 150 units of pollution abatement, the distance $p_{2} X$.


One way to reduce pollution to 200 is to ask each firm reduces to 100 , this way may not be the cheapest way to do it. At 100 units of pollution, the marginal cost of reducing pollution to firm A is $\$ 4,000$, but firm B's cost only $\$ 2,000$. Thus, if firm B reduces one more unit of pollution, it would add only $\$ 2,000$. But if we let firm A increase one more unit of pollution, its cost would fall by $\$ 4,000$. As a result, we can reduce the two firms' combined cost by $\$ 2,000$.

In fact, as long as the marginal costs differ, the total cost of pollution abatement can be reduced by increasing abatement where its marginal cost is less and reducing abatement where its marginal cost is higher. Thus, to minimize the cost of pollution control, firm should be producing at a point where their marginal costs are equal. For the above example, to reduce pollution to 200 units in the cheapest way, firm A should discharges 150 units and firm B, 50 units.

## VIII. Profit Maximization and Competitive Firm

## Characteristics of Competitive Firm

(a) The firm is one of many which produce identical products
(b) The firm is price taken in inputs and outputs markets, ide., no firm supplies a sufficiently large amount of the product to have any effect of price. This means the demand curve faced by the firm is horizontal.

### 8.1. Demand Curve Under Competition

Industry Market
Firm


Price taking is actually a result of (a) "Many" firm implies each firm is infinitesimilly small and thus has no market power. "Identical Products" implies each firm's output is a perfect substitute for all other firms' output. Recall "the more substitutes, the more elastic the demand curve".

Assume: the firm's objective is to maximize profit ( $\pi$ ) where
Profit $=$ Total Revenue - Total Cost

$$
\pi=\mathrm{TR}-\mathrm{TC}
$$

TR, $A R$, MR in a Competitive Firm
Total Revenue (TR): $T R=P \cdot q$
Average Revenue $(A R): A R=\frac{T R}{q}=\frac{P \cdot q}{q}=P$
Marginal Revenue $(M R): M R=\frac{\Delta T R}{\Delta q}=\frac{P \Delta q}{\Delta q}=P$

Conclusion: $P=A R=M R$.

### 8.2. Short Run Profit Maximization

The competitive firm has no control over prices so a profit maximizing policy must be related to the quantity produced.

The following figure shows how we identify the most profitable level of output by using the typical $T R$ and $T C$ curves. $T C$


Notice that $\pi$ is distance between $T R$ and $T C$, and is greatest at $q^{*}$, and that at $q *$ the slopes of the $T R$ and $T C$ curves are equal.

## Should the firm produce any output?

(i) Yes, if there is a level of output which can earn a (positive) profit, ide. if $T R>T C(o r A R>A C)$.
(ii) Yes, if it cannot make a profit but it can make a loss smaller than fixed cost.
if $\mathrm{q}=0, \mathrm{TC}=\mathrm{TFC} ; \mathrm{TR}=0$
so profit $=T R-T C=-T F C$
$\Rightarrow$ Loss $=$ PFC

```
if q > 0 and loss occurs then
    Loss = TC - TR
    =(TFC + TVC) - TR
    = TFC + (TVC - TR)
```

    So loss < TFC if TVC < TR (AVC < AR)
    In (ii) above it is important to recall that in the short run a firm must pay its FC no matter the level of output. By producing output ( $q>0$ ) the firm may be able to reduce its loses from the no production case ( $q=0$ ).

Summary The firm should produces some output if $T R>T V C$ (or $A R>A V C$ ).
Since $P=A R=M R, P>A V C$.

## What Quantity Should the Firm Produce? (Suppose AR > AVC)

The firm should produce the quantity which maximizes profits or minimizes losses.

It is worthwhile producing units for which $M R>M C$.
It is not worthwhile continuing to produce when MR $<~ M C$. (Recall MC must rise eventually).

Thus a firm will produce up to the prime where

$$
\begin{aligned}
M C & =M R \\
\text { or } \quad M C & =P \text { since } P=M R
\end{aligned}
$$

Let us put this together graphically:
Summary: Profit Maximization in Short Run


When the price is $\mathrm{P}^{1}$, the profit-maximizing output is $\mathrm{q}^{1}$. The total profits are the rectangle $A B C D$. When the price is $p^{2}$ the most profitable output is $q^{2}$. The profits, however, are zero since the price just equals average cost of production. When the price is $\mathrm{p}^{3}$, the firm can just cover its variable cost by producing $q^{3}$, where AVC equal the price. At this point the firm would be operating at a short-run loss; the loss is exactly equal to its total fixed cost. At a price below $P^{3}$ the firm is unable to cover its variable cost and short down.

## Perfect Competition: Profit Maximization



Note: $T C=A T C \cdot q, T R=p \cdot q$
$M R>M C$ for $q=1,2, \ldots, 9$. $M R<M C$ for $q=10 . q=9$ for profit maximization Since $A R>A T C$ at $q=9$ a profit is made.


Example B:
Perfect Competition: Loss Minimization

$M R=M C$ at $q=6$
At $q=6$ Loss $=\$ 64$ so Loss $<T F C=\$ 100$
This is because $A R<A T C$ implies loss minimization. However, at $q=6 A R>$

$A t(C) M R=M C \Rightarrow G^{x}=6$

$$
\begin{aligned}
T R & =A R \times q^{x} \\
& =O f C q^{*} \\
T C & =O a b q^{*} \\
L o s s & =T C-T R \\
& =a b c f
\end{aligned}
$$

Example C
Perfect Competition: Shutdown Production

$M R=M C$ at $q=5$ which looks optimal. However, $A R<A V C$ at $q=5$ which implies Loss $>$ PFC $(115>100)$. Thus loss is minimized when $q=0$, ie.


At (d) $M R=M C \Rightarrow q^{*}=5$
At $q=5, T R=$ oed $q^{*}$
$T C=0 a b q^{*}$
$\Rightarrow$ Loss $=$ abd
At $q=0, T R=0$
$T C=T F C=A F C \times q$

$$
=a b c f
$$

Thus we have from the above discussion:

## Conditions for Short-Run Profit Maximization

If output $q^{*}$ satisfies the following two conditions then it is the unique profit-maximizing level of output.
(A) at $q^{*}, \quad P=M R\left(q^{*}\right)=M C\left(q^{*}\right)$
(B) at $q^{*}, \quad P \geq \operatorname{AVC}\left(q^{*}\right)$

Otherwise, $q^{*}=0$.
Remarks
(1) If (B) were not satisfied, then the firm is best off producing q* $=0$ units so that $T R=0, T C=F C+0=F C$ and thus $\pi=-T C$ (lossing FC is better than lossing $\mathrm{FC}+$ some VC).
(2) When $P=M R\left(q^{*}\right)=A V C\left(q^{*}\right)$ and $(A)$ is satisfied, then $q^{*}$ and $q=0$ both return the same $\pi$ (i.e., $\pi=-F C$ ).

Example:
Suppose $T C(q)=200+9 q+5 q^{2}$
and $\quad M C(q)=9+10 q$

$$
P=16
$$

(a) Find the AVC.
$A V C=\frac{V C}{q}$ where $V C$ is the portion of $T C$ which involves terms with "q".

$$
V(q)=9 q+5 q^{2}
$$

Thus $A V C=\frac{9 q+5 q^{2}}{q}=9+5 q$
b) Find the profit maximizing level of output.
(A) $\quad P=M C\left(q^{*}\right)$
$\Rightarrow \quad 16=9+10 q^{-}$
$\Rightarrow \quad q^{*}=\frac{7}{10}$
(B) $\quad P \geq$ AVC (q*) ?

$$
\begin{aligned}
\operatorname{AVC}\left(q^{*}\right) & =9+5 q^{*} \\
& =9+5 \frac{7}{10}=9+\frac{7}{2}=12.5
\end{aligned}
$$

Thus $16 \geq 12.5$ and $q^{*}=\frac{7}{10}$ is profit-maximizing level of output.
If we change $P$, we find a new $q$. Repeating this we can derive a supply curve.

### 8.3. Output Response to a Change in Prices

Changes in Output Price
$M C=P$ shows that a competitive firm will produce more at higher price because increased production becomes profitable at higher prices.

## Changes in Input Prices

A Change in the price of an input, with an unchanged product price, changes the profit-maximization output. If. an input price falls, MC shift to $M C^{\prime}$, and output increases, where $M C^{\prime}$ equals the unchanged price.


### 8.3. Long-Run Profit Maximization

The same principle we used for the short-run setting can apply to long-run profit maximization, but now we employ long-run cost curves. The firm maximizes profits in the long run by producing where $P=M C$.

with a price $p^{*}$, the most profitable output will be $q^{1}$ in the long run, the profit is rectangled $A B C D$. In the short run the most profitable output will be $q^{2}$, rectangle CEFG is the profitable output which is less than the profit in the long run.

## IX. The Competitive Industry

In this chapter the emphasis shifts from the individual firm to the competitive industry.

## Assumptions of Perfect Competition

1. Large numbers of buyers and sellers, which will normally guarantee that the firms and consumers behave as price takers.
2. Unrestricted mobility of resources
3. Homogeneous product
4. Possesion of all relevant information

### 9.1. The Short-Run Industry Supply Curve

In the short run a competitive firm will produce at point where the marginal cost equals the price, as long as the price is above the minimum point on its average variable cost.

How is the short run industry supply curve determined:

The horizontal sum of MC curves for each firm (above AVC) gives the industry supply curve. That is, the short urn industry supply curve is derived by simply adding the quantities produced by each firm.


Note that the short run supply curve, SS, slope upward. Remember that each firm's marginal cost curve slopes upward because it reflects the law of diminishing marginal returns to variable inputs. Thus, the law of diminishing marginal returns is the basis determinant of the shape of the industry's short-run supply curve.

## Price and Output Determination in the Short Run

The interaction of supply and demand in the market determines the market price and output. In previous figure the intersection of the demand curve $D$ with the supply curve identifies the price where total quantity demanded equals total quantity supplied. Thus $P$ is the equilibirum price and total industry output is $Q$, where $Q=q_{1}+q_{2}+$ $q_{3}$.

In the short run an increase in the market demand leads to a higher price and higher output. When demand increases to $D^{\prime}$ the equilibrium price $P^{\prime}$ and total industry output is $Q^{\prime}$.

### 9.2. Long-Run Competitive Equilibrium

In the long-run competitive equilibrium the independent plans of firms and consumers mesh perfectly. Each firm has adjusted its scale of operations in light of the prevailing price and is able to sell as mush as it chooses. Consumers are able to purchase as much as they want at the prevailing price. There are no incentives for any firm to alter its scale of operation or to leave the industry and no incentive
for outsiders to enter the market. Unless market conditions change, the price and rate of output will remain stable.



## The Conditions for Long-Run Competitive Equilibrium

(a) At the prevailing market price each firm must be producing the output the maximized its profits, that is, $P=L M C$.
(b) Firms must making zero economic profit. Therefore, there are no incentive for firms to enter or leave the industry.
(c) The combined quantity of output of all firms at the prevailing price must just equal the total quantity consumers wish to purchase at that price.

Entry or Exit of Firms
Entry of Firms Eliminates Profits

(i) The representative firm is in $L R$ equilibrium. $p^{e}=$ minimum of ATC.

Normal Profits earned (EP $=0$ ).
(ii) Suppose demand increases from $D_{1}$ to $D_{2}$. Price rises and exceeds minimum ATC.

Economic profits are earned
(iii) New Firms enter industry. Supply increases from $S_{1}$. Price starts to fall.
(iv) Supply continues to increase while economic profits are being made. Hence supply stops at $s^{2}$, where $p^{e}=\min$ ATC
(v) In the LR, a larger quantity is supplied at the same price.

## Exit of Firms Eliminates Losses




Industry
(i) The representative firm is in $L R$ equilibrium. $p^{e}=\min A T C$.

Normal profits are earned (EP $=0$ )
(ii) Suppose demand decreases form $D_{1}$ to $D_{3}$. Price falls below min ATC. Economics profits are negative, i.e., firm's suffer a loss.
(iii) Firms leave the industry Supply falls from $S_{1}$, price rises.
(iv) Supply continues to fall while economic loss is experienced.

Supply stops at $S_{3}$ where $\mathbf{p}^{\mathbf{e}}=\min A T C$.
(v) In LR, smaller quantity at same price.

### 9.3. Price Elasticity of Supply

The Price Elasticity of Supply - a measure of the responsiveness of quantity supplied to a change in price. It is defined as the percentage change in quantity supplied divided by the percentage change in price.

$$
\begin{aligned}
E_{S}^{P} & =\frac{\Delta S(P) / S(P)}{\Delta P / P} \\
& =\frac{P}{S(P)} \cdot \frac{1}{\Delta P / \Delta S(P)} \\
& =\frac{P}{S(P)} \cdot \frac{1}{\text { slope of } S(P)}
\end{aligned}
$$

Note: $E_{S}^{P}>0$ when supply slopes upwards.
If $\left|E_{S}^{P}\right|>1$ then supply is elastic.
If $\left|E_{S}^{P}\right|<1$ then supply is inelastic.
If $\left|E_{S}^{P}\right|=1$ then supply is unit elastic.
If $\left|E_{S}^{P}\right|=0$ then supply is perfectly inelastic.

If $\left|E_{S}^{P}\right|=\infty$ then supply is perfectly elastic.



### 9.4. The Long-Run Supply Curve

Economists distinguish among three different types of competitive industries, constant-cost, increasing-cost, and decreasing-cost. The distinction depends on how a change in industry output affects the prices of inputs.

## Constant Cost Industry: LR Supply Curve

When the entry and exit of firm do not affect costs, price of output remains constant for all levels of quantity. Hence, LR supply is perfectly elastic.



Increasing Cost Industry: LR Supply
As firms enter the industry, they compete for scarce resources. Consequently resource prices rise as do production costs of firms. So
cost curves shift up, (ATC, MC). Hence Industry supply curve is upward sloping.



## Decreasing Cost Industry $=$ LR Supply Curve

A decreasing-cost industry is one that has a downward-sloping longrun supply curve. This means the expansions of output by the industry in some way lowers the cost curves of the individual firms. The entry of firms may result in lower per unit cost.


## X. Monopoly

While perfect competition is characterized by many firms selling in the same market, monopoly is characterized by only one firm selling in a given market. In this chapter, we explain how a monopoly determines price and output, and we compare the results with those of the competitive industry.
10.1 The Nature of Monopoly

Monopoly - form of market structure in which there is only one producer of some product that has no close substitutes.

As a result, the monopoly is the industry since it is the only producer in the market.

Monopoly is the opposite of perfect competition since there is no competition. The monopoly need not be concerned with the possibility that other firms may undercut its price.
10.2 Sources of Monopoly Power

How does a monopoly come about?
(1) Exclusive Ownership of a unique resource. e.g. Debeers Co. of South Africa owns most of the world's diamond mines.
(2) Economies of Scale.

If large economies of scale exist then a firm's LRATC curve will fall over a sitable range as output is increased.


The first firm to enter this industry has a competitive advantage because it can take advantage of low per unit costs at higher levels of output, whereas a new firm would have higher per unit costs producing at low levels of output. Thus the existing firm could charge a price lower than new firms could afford. Therefore, rivals cannot enter the market and monopoly power is maintained.

Such a monopoly is called a nature monopoly.
e.g., public utilities, telephone services.

## (3) Government-Granted Monopoly

The government can grant the exclusive right to produce and sell a product or service. The government can grant such monopoly power through patents, licenses, copyrights, or exclusive franchises.

```
patents - give inventor a monopoly position for life-time of
                                    patent--17 years in the U.S.A.
                                    e.g. IBM, Xerox.
```


#### Abstract

copyrights - give writers and composers exclusive legal control over production and reproduction of their work for a period of time.

Licenses - limit the number of producers but rarely give monopoly power. e.g. need license to practice medicine, law, cut hair or sell liquor. public utilities - Competition is impractical so industries are given exclusive franchise by the government. If firms share the market none would be able to take advantage of the large economies of scale. If only one firm supplied the market it could take advantage of the lower per unit costs. In return for this the government granted monopoly position the utility must agree to let the government regulate the price of the product.


### 10.3 The Monopoly's Demand and Marginal Revenue Curves.

In perfect competition each firm is a price taker and thus faces a horizontal demand curve.

By contract, the monopolist comprises the entire industry. Thus, the firm's (monopolist's) demand curve is the industry demand curve. Then a monopolist's demand curve is downward sloping-i.e., the firm is a price "maker".

TR: $\quad$ Total Revenue $=p \cdot q$
MR: $\quad$ Marginal Revenue $=\frac{\Delta T R}{\Delta q}$
AR: Average Revenue $=\frac{(p \cdot q)}{q}=p$
Demand and Revenue for a Monopolist

| q | $\mathrm{p}=\mathrm{AR}$ | TR | MR |
| :--- | :---: | :--- | :--- |
|  |  |  |  |
| 0 | $\$ 11$ | 0 | - |
| 1 | 10 | 10 | 10 |
| 2 | 9 | 18 | 8 |
| 3 | 8 | 24 | 6 |
| 4 | 7 | 28 | 4 |
| 5 | 6 | 30 | 2 |
| 6 | 5 | 30 | 0 |
| 7 | 4 | 28 | -2 |
| 8 | 3 | 24 | -4 |
| 9 | 2 | 18 | -6 |
| 10 | 1 | 10 | -8 |

For a monopolist, the demand curve is the AR curve, and the marginal revenue is always less than price when the demand curve slopes downward except for the first unit sold. Why?
$M R=\frac{\Delta T R}{\Delta q}$
$=\frac{\mathrm{p} \Delta \mathrm{q}-\mathrm{q} \Delta \mathrm{p}}{\Delta \mathrm{q}}$
$=p-q \frac{\Delta p}{\Delta q}$
since $q>0 \& \frac{\Delta p}{\Delta q}>0$,
so

$$
\mathbf{M R}<\mathbf{p}
$$



If the demand function is linear, i.e. $p=a+b q$
then $\quad T R=p . \quad q=a q+b q^{2}$
$\Rightarrow \quad M R=\frac{\Delta T R}{\Delta q}=a+2 b q$.

Thus the slope of the $M R$ is exactly twice the slope of the demand function.


What are relationships among price, elasticity of demand, and marginal revenue?

$$
\begin{aligned}
M R & =p-q \frac{\Delta p}{\Delta q} \\
& =p\left(1-\frac{q}{p} \cdot \frac{\Delta p}{\Delta q}\right) \\
& =p\left(1-\frac{1}{E_{d}^{p}}\right)
\end{aligned}
$$

Note $E_{d}^{P}$ is defined by

$$
\mathrm{E}_{\mathrm{d}}^{\mathrm{p}}=\frac{\Delta \mathrm{q} / \mathrm{q}}{\Delta \mathrm{p} / \mathrm{p}}
$$

1. When the elasticity of demand is infinity (a horizontal demand curve), $M R=p$ :

$$
M R=p\left(1-\frac{1}{\infty}\right)=p
$$

2. When demand is unit elastic $\left(E_{d}^{p}=1\right), M R=0$ :

$$
\mathrm{MR}=\mathrm{p}\left(1-\frac{1}{1}\right)=0
$$

3. When demand is elastic $\left(\mathrm{E}_{\mathrm{d}}^{\mathrm{p}}>1\right), \mathrm{MR}>0$ :
e.g. Let $E_{d}^{p}=2$

$$
M R=p\left(1-\frac{1}{2}\right)=\frac{1}{2} p>0
$$

4. When demand is inelastic $\left(\mathrm{E}_{\mathrm{d}}^{\mathrm{p}}<1\right), \mathrm{MR}<0$ :
e.g. $\operatorname{Let} E_{d}^{P}=\frac{1}{2}$

$$
\mathrm{MR}=\mathrm{p}\left(1-\frac{1}{1 / 2}\right)=\mathrm{p}(1-2)=-\mathrm{p}
$$

### 10.4 Monopolist's Profit Maximizing Rule

The monopolist can affect both price and quantity. It has both pricing and output policies which are not independent.

Suppose AR $>$ AVC. Then the monopolist will produce units for which $M R>M C$ until $M R=M C$.

Suppose $A R<A V C$ for all level of output. Then the profit maximizing output is at $q=0$.

$M C=M R \Rightarrow q^{*}$ is profit maximizing level of output.

$$
\left(q^{*} \Rightarrow p^{*}\right)
$$

Profit Maximization

$$
\mathrm{AR}>\mathrm{ATC}>\mathrm{AVC}
$$



1. Use $M R=M C$ to get $q *$
2. Use $q^{*}$ and $p=A R$ (-demand) to get $p^{*}$
3. $T R=A R \times q^{*}=p^{*} x q^{*}$
4. Use $q^{*}$ and ATC to get TC

$$
T C=A T C \times q^{*}
$$

5. Profit $=T R-T C$

Loss Minimization

$$
\mathrm{ATC}>\mathrm{AR}>\mathrm{AVC}
$$


Shutdown Production
$\mathrm{ATC}>\mathrm{AVC}>\mathrm{AR}$

$q^{*}=0$ is loss minimizing level of output

## Normal Profit



$$
\begin{aligned}
\mathrm{TR}=\mathrm{TC} & \Rightarrow \text { Economic Profit }=0 \\
& \Rightarrow \text { a normal profit is earned. }
\end{aligned}
$$

Long Run: A monopolist may earn positive economic profits in the long run since barriers to competition prevent new firms from entering the industry. (In perfect competition this
involved a shift in the supply curve which reduced the equilibrium price and eventually reduced profits to zero.)

### 10.5 Further implications of Monopoly Analysis

## MC curve supply curve for Monopolist

In perfect competition we noticed that at any given price there is one and only one quantity of output that the firm is willing to supply.

Conversely, at any given output level there is one and only one price that makes the firm willing to supply that output level.

This relationship does not exist for the monopolist.
Consider 2 possible demand curves below:

$M R_{1}$ and $M R_{2}$ intersect $M C$ at same point.
If $D_{1}$, then price is $p_{1}$
If $\mathrm{D}_{2}$, then price is $\mathrm{P}_{2}$
Hence given output level $q^{*}$ corresponds to two possible prices.
Thus, a monopoly has no supply curve. However, the absence of a monopoly supply curve does not mean that we are unable to analyze the output choice of a monopoly since we have been doing just that.
10.6. Monopoly Versus Perfect Competition


Perfectly Competitive Industry $S=\Sigma M C$
$\mathrm{S}=\mathrm{D} \Rightarrow \mathrm{p}^{\mathrm{c}}, \mathrm{q}^{\mathrm{c}}$


Therefore, when an industry is a monopoly consumers pay a higher price and receive less than would be the case under perfect competition.

## Monopoly and the Distribution of Income

If a competitive industry becomes a monopoly, there will be a change in the distribution of real income among members of society. The monopolist will gain. Consumers will lose because of higher price they pay. The higher price reduces the real purchasing power, or real income, of the consumers. Thus, by chafing a price above average cost, the monopolist gains at the expense of the consumers-a redistribution of income from consumers to the owners of the monopoly.

The Welfare Cost of Monopoly.
Monopoly has another effect that involves a net loss in welfare because it leads to an inefficient allocation. Economists refer to this net loss as a welfare cost of monopoly.


The monopolist first sells $q_{m}$ units at $p_{m}$, then sells more output at some other price. The consumer is better off since they are freely purchasing the extra unit of output, and the monopolist is better off since he can sell the extra unit at a price that exceeds the cost of its production. These extra monopoly profits can be distributed so as to make everyone better off. Here we are allowing the monopolist to be discriminapt in his pricing.

## Monopoly and Price Discrimination

Price discrimination: selling the same good or service at different prices to different buyers.

Monopolists use price discrimination to realize greater profits. They have ability to do this since they are the only supplier of a good.
e.g. movie theaters charge different prices for children and adults.
e.g. utility companies charge different rates for businesses and residences.
e.g. senior citizens pay $10 ¢$ for bus ride; we pay $60 ¢$.

Conditions which aid price discrimination:

- resale not possible
- must be able to segment market by classifying buyers in separate, identifiable groups.
- monopoly control
- different demand elasticities
e.g. Mr. A's demand is inelastic

Mr. B's demand is elastic
monopolists can increase TR by increasing price for MR. A decreasing price for MR. B

Assume below that the $M C=A T C$ which is constant.


If same price is charged for all goods then profit is maximized at $q_{m}$, sold at $p_{m}$. Profit $=$ area abcd.
If monopolist uses price discrimination to charge the maximum price buyers are willing to pay for each additional unit, then the demand curve is the same as the $M R$ curve. Thus profits are maximized where $D=M C$; i.e., at $q_{d}$. Profit is now given by area aef.
Clearly, area aef $>$ area abcd.

## XI. Monopolistic Competition and Oligopoly

Competition and monopoly lie at opposite ends of the market spectrum. Competition is characterized by many firms, unrestricted entry, and homogeneous product, while a monopoly is the sole producer of a product with no close substitutes. Falling between competition and monopoly are two other types of market structures, monopolistic competition and oligopoly, which describe the major remaining market firms. In substance, monopolistic competition is closer to competition; it has many firms and unrestricted entry, like the competitive model, but its product is differentiated. Oligopoly, on the other hand, is more like monopoly; it is characterized by a small number of large firms producing either a homogeneous products like steel, or a differentiated product like automobiles. In the following, we will examine some of these models, noting the similarities as well as the differences between these models.

### 11.1 Monopolistic Competition

Characteristics
a) many firms
b) each firm's product is slightly different from other firms in the industry $\Rightarrow$ demand curve is downward sloping
c) freedom of entry and exit
d) firms engage in nonprice competition--advertising is important example - Chinese restaurants in NYC.

Short Run Equilibrium: same as monopolist.
e.g. Profit Maximization


## Long Run Equilibrium:

Recall that a monopolist can earn positive economic profits in the long run. The above equilibrium could represent a monopolist's LR equilibrium.

This does not hold for a monopolistically competitive firm as there are no barriers to entry.

## Entry Eliminates Profits

Thus in the long run firms will enter the industry. Industry demand must be divided between more firms $\Rightarrow$ demand for each firm is reduced. (D and MR shift to the left until profits are normal.)


Long Run Equilibrium: ( $\mathrm{p}_{2}, \mathrm{q}_{2}^{*}$ )

## Exit Eliminates Loss

As firms exit, the market demand is spread over fewer firms $\Rightarrow$ demand for each remaining firm's output increases $\Rightarrow D \& M R$ shift to the right.

11.2 Oligopoly

Oligopoly is an industry characterized by a few large firms producing most or all of the output of some product.

## Characteristics

a) Economies of scale -


It only takes a few firms of size $q$ to supply the whole market.
b) Mutual interdependence among firms--since there are only a few
firms in the market each firm must react to other firms' actions.
c) Nonprice competition and price rigidity

Price war is last alternative
(fear of lowering profits)
Competition relies on advertising and product differentiation.
d) temptation for firms to collude in setting prices

Firms may want to maximize collective profits. This is illegal
in the U.S.
e) Incentive for firms to merge

There is "perfect collusion" when the industry becomes a monopoly.
f) Substantial barriers to entry--such as
(i) economies of scale


D: demand faced by the first firm with no competitors.
$D_{1}$ : demand faced by the first firm with one competitor.
$\mathrm{D}_{2}$ : demand faced by competitor ( $\mathrm{D}=\mathrm{D}_{1}+\mathrm{D}_{2}$ )
ATC: same for both firms, with economies of scale.
Strategy for firm 1:
Charge a price below $P_{2}$ and above $P_{1}$. The potential loss of firm 2 will keep it from joining the market.
note: $A T C^{\prime} \Rightarrow$ 1st firm cannot use price to keep 2nd firm out of the market.


Assume:

- firm 2 steals half the market
- costs for new firm are higher. ( $\mathrm{ATC}_{2}>\mathrm{ATC}_{1}$ )

Strategy:
The first firm can keep 2nd firm out by choosing a price between $P$ and $P_{L} . \quad P_{L}$ is referred to as the limit price because any price greater than this will cause entry.

In the following we will examine several models of oligopolistic behavior. As we discuss the models, keep in mind how different assumptions about rival behavior change the outcome.

### 11.3 The Cournot Model

The model shows how uncoordinated output decisions between rival firms could interact to produce an outcome that lies between the competitive and monopolistic equilibria.

Assumptions:

1. produce identical products.
2. marginal costs are zero.
3. the market demand curve is known to both sellers and is linear.
4. each firm believes that its rival is insensitive to its own output.


Initially, firm $A$ views $D_{T}$ as its demand curve and produce $Q_{1}=\frac{T}{2}$ where $M C$ equals $M R_{T}$. Firm B enters the market and assumes firm $A$ will continue to produce $Q_{1}=\frac{T}{2}$ units so it sees $D_{B}$ as its demand curve and produce $Q_{2}=\frac{T-Q_{1}}{2}$. Firm $A$ then readjusts its output to $Q 3=\frac{T-Q_{2}}{2}$. The adjustment process continues until an equilibrium is reached with each firm producing $1 / 3$ of $T$.

Note that $D_{B}$ is a residual demand curve which is obtained by subtracting the amount sold by firm $A$ at all prices from the market $Q_{2}=\frac{T-Q_{1}}{2}$, firm $A$ will readjust its output and will view $D_{A}$ as its new
demand curve. $D_{A}$ is a residual demand curve and is obtained subtracting firm $B^{\prime}$ s output from $D_{T}$.

In the Cournot model, uncoordinated rival behavior produces a determinate equilibrium that is more than the monopoly output $Q_{1}=\frac{T}{2}$, and two-thirds the competitive equilibrium output.

The Cournot model can be written in the formula:

Suppose the market demand function is $P\left(q_{1}+q_{2}\right)$. Firm 1 wishes to find $q_{1}$ which maximizes profits by taking firm 2's level of output as fixed.
$\max _{\mathrm{q}} \mathrm{p}\left(\mathrm{q}_{1}+\bar{q}_{2}\right) \mathrm{q}_{1}-\mathrm{c}_{1}\left(\mathrm{q}_{1}\right)$
Simifarly, firm $2^{\prime}$ s problem is to find $q_{2}^{*}$.
$\max _{2} p\left(\bar{q}_{1}+q_{2}\right) q_{2}-C_{2}\left(q_{2}\right)$
Under what circumstances will the actions of two firms be consistent? They will be consistent when the choices each firm makes are compatible with the other firm's expectations. That is $\bar{q}_{1}=q_{1}^{*}, \bar{q}_{2}=q_{2}^{*}$.

### 11.4 The Kinked Demand Curve Mode1

Each firm in an oligopolistic industry is aware that any price change it makes will affect the sales of other firms in the industry.

Thus, $D$ and MR curves must take into account reactions of rivals.


D: firm's demand curve when rivals do not react to price changes.
$\mathrm{D}_{\mathrm{r}}$ : firm's demand curve when rivals match any price change.
Along $D$, as $P$ decreases, $Q$ increases due to :
i) substitutability of this product for similar products in other industries; and
ii) purchases by customers who switch away from other firms in this industry.

Along $D_{r}$, as $P$ decreases, $Q$ increases but not by as much as along $D$.
This is because ii) above will no longer be true.
no rival reactions
$P_{1}$ decreases to $P_{2} \Rightarrow a \longrightarrow b$
$P_{1}$ decreases to $P_{3} \Rightarrow a \longrightarrow c$
w/ rival reactions
$P_{1}$ decreases to $P_{2} \Rightarrow a \rightarrow b^{*}\left(b^{\prime}<b\right)$
$P_{1}$ decreases to $P_{3} \Rightarrow a \longrightarrow c^{\prime}\left(c^{\prime}<c\right)$

If $P_{1}$ increases to $P_{0}$ no rival will increase $P$. Original firm loses customers to other firms.

Oligopolists assume rivals will match price cuts but not price hikes.
$\Rightarrow$ Kinked demand curve


Recall: MR is twice as steep as demand curve. To find MR here:
i) identify $q_{K}$
ii) draw $M R$ for $D$ until $q_{K}$ is reached
iii) extend $D_{r}$ to vertical axis and draw $M R$ beginning at $q_{K}$ In the vertical segment $b$ to $b$ the profit maximizing rule $M R=M C$ yields the same ( $p, q$ ) for any $M C . \quad\left(M C_{2} \geq M C \geq M C_{0}\right)$ => price rigidity

### 11.5 Game Theory

Game theory is a mathematical technique that provides us with another way to examine the nature of interdependence among rival firms and to understand the role of uncertainty in their pricing and output decisions.

The game is described by its payoffs, the rules of the game, the number of players, and the information available to the players.

One game theory model that has implications for oligopolist behavior is called the prisoners' dilemma. The prisoners' dilemma demonstrates how rivals could act to their mutual disadvantage.
e.g. Suppose two suspects $A \& B$, are apprehended and questioned separately about their involvement in crime. Without a confession, the district attorney has insufficient evidence for a conviction. The prisoners are unable to communicate with each other, and they are interrogated separately. During interrogation each is separately told the following outcomes about years in jail given in the "payoff matrix".

Person B

| confesses |
| :--- | :--- | :--- |
| confesses |
| Person $A$ |
| doesn't |
| confess |$(5,5) \quad(1,10)$

Note that each "player" must individually choose a strategy (confess or not confess), but that the outcome of that choice depends on what the other player does.

In this setting, it is quite likely that players $A$ and $B$ will confess when one doesn't know what the other will do. Confessing makes sense since each prisoner is attempting to make the "best" of the "worst" outcomes.

To see the relevance of the prisoners' dilemma to oligopoly theory, let's suppose that player $A$ and player $B$ are the firms in the same industry. The following matrix identifies the payoffs about profits from an agreement to fix prices and share the market or from cheating on the collusive agreement.
player B
cheats doesn't cheat

| cheats <br> player A <br> doesn't <br> cheat | $(10,10)$ | $(20,5)$ |
| :---: | :---: | :---: |
| $(5,20)$ | $(15,15)$ |  |

By the same reasoning as before, both are likely to cheat despite the fact that both will be worse off by cheating. Advertising. Another way to illustrate the usefulness of the game theory approach is to examine the interdependence of advertising decisions. Suppose two firms are considering their advertising budgets. They have two strategies, a large budget or small budget. The payoff matrix is the profits they get at different strategies.

B

| Small Budget |  | Large Budget |
| :---: | :---: | :---: |
| Small budget |  |  |
| A |  |  |
| Large budget |  |  |$|$| (10, 10) |
| :---: |
| $(18,5)$ |

### 11.6 Dominant Firm Price Leadership Model

Price Leadership Model is another way to resolve the uncertainty of rivals' reactions to price changes. If one firm in the industry initiates a price change, and the rest of the firms traditionally follow the leader, there is no uncertainty about rival behavior.

Price leadership by the dominant firms occurs when the dominant firm in the industry sets a price that maximizes its profit and lets its smaller rivals sell as much as they want at the set price.

Suppose the industry demand curve is DD. Since the other smaller firms in the industry will follow any price change initiated by the dominant firm, they become price takers, and adjust output until price equals their marginal cost. The residual demand curve confronting the dominant firm is obtained by subtracting the quantity the other smaller firms will produce shown by $S_{D}$ from market demand, yielding $P_{1} A D$. The dominant firm produces $G_{D}$, where $M C_{D}=M R_{D}$, and charges $P_{3}$. Smaller firms produce $\mathrm{q}_{0}$.


## Cartels and Collusion

In all models discussed so far, the individual firms were assumed to behave independently. Each firm makes a specific conjecture regarding how other firms will response to its action without any concern for how this affects the profits of the other firms. That is, no cooperation among firms.

The most important cooperative model of oligopoly is the cartel model. A cartel model is an explicit agreement among independent producers to coordinate their decision so each of them will earn monopoly profits.

## Cartelization of a Competitive Industry

Let us see how a group of firms in a competitive market can earn monopoly profit by coordinating their activities. We assume that the industry is initially in long-run equilibrium, and then will identify the short-run adjustments (with existing plants) that the industry's firms can make to reap monopoly profits for themselves. The following figure show this possibility.

(a)

(b)

Under competitive conditions, industry output is $Q$ and price is $P$. If the firms in the industry form a cartel, output is restricted to $Q_{1}$ in order to charge price $P_{1}$, the monopoly outcome. Each firm produces $q_{1}$ and makes a profit at price $p_{1}$.

Firms can always make a larger profit by colluding rather than by competing. Acting along, competitive firms are unable to raise price by restricting output, but when they act jointly to limit the amount supplied, price will increase.

## Why Cartels Fail

If cartels are profitable for the members, why are not there many more? One reason is that in the United States they are illegal. But they were rare and were short-lived even before there were such laws. Three important factors appear to contribute to cartel instability.

1. Each firm has strong incentive to cheat on the cartel agreement. (Say, the above figure, if one firm enlarge its output with price $p_{1}$, it can earn much more profit.) Yet, if many firms do so, industry output will increase significantly, and price will fall below the monopoly level. It is in each firm's interest to have other firms restrict their output while it increases its own output.
2. Members of the cartels will disagree over appreciate cartel policy regarding prices, output, market shares, and profit sharing. This is true when cost, technology, size of firms are different.
3. Profit of the cartel members will encourage entry into the industry. If the cartel achieves economic profits by raising the price, new firms have an incentive to enter the market. If the cartel cannot block entry of new firms, price will be driven back down to the competitive level as production from the "outsiders" reaches the market.

The OPEC (Organization Petroleum Exporting Countries) which is formed in 1960 is a good example of a cartel.
XII. Employment and Pricing of Inputs


#### Abstract

The emphasis now shifts from product markets to input markets. We will begin to look more closely at factors that determine the level of employment and prices of inputs used to produce the products. Firms are suppliers in product markets, but they are demanders in input markets. Households and individuals are the demanders in product markets and the suppliers in input markets. In this chapter, we will discuss the basic principles common to all input markets analysis, whether the input is labor, capital, or raw materials.


### 12.1 The Input Demand Curve of A Competitive Firm The Firm's Demand Curve: One Variable Input

Suppose that only one input (labor) is allowed to vary and the others are fixed. This is a short-run setting in which labor is the only variable input. By the law of diminishing marginal returns, the labor's marginal production ( $M P^{L}$ ) curve slopes downward beyond some point. Convert labor's marginal product curve $M P$ into the marginal value product curve $\mathrm{MVP}_{\mathrm{L}}$ by multiplying the marginal product of labor by the price of the commodity produced. The MVP ${ }_{L}$ curve is the competitive firm's demand curve for labor when all other inputs are fixed.


Suppose that the daily wage rate is $\$ 30$ per worker. What is the optimal number of workers? The firm will hire up to the point where the input's marginal value product is just equal to its marginal cost:

$$
\mathrm{W}=\mathrm{MVP}_{\mathrm{L}}
$$

In our case, $L=20$.
Note that $M V P_{L}=M P_{L} \cdot P_{X}$, thus we have

$$
\frac{W}{M P_{L}}=P_{x}
$$

Recall that the ratio $W / \mathrm{MP}_{\mathrm{L}}=\mathrm{MC}$. Then the above equation is equivalent to the condition of profit maximization (MC $=P$ ).

## The Firm's Curve: All Inputs Variable

In general, a change in the price of an input will lead the firm to alter other inputs. What is the demand curve when all inputs are variable?

Suppose that at initial equilibrium the daily wage rate $=30 \$$ and $L$ $=20$. The firm is operating at point $A$ on $M V P_{L}$ where $(K=10)$. Now suppose wage rate decreases to $W=20 \$$. If $k$ keep constant $(k=10)$, the firm will increase employment of labor to $\mathrm{L}=25$. If capital increases to $k=12$ (for more workers need more "tools"), the entire MVP ${ }_{L}$ curve shifts to upward. This adjustment leads to a further increase in labor to point C. Connecting points $A$ and $C$, we get the firm's demand curve d.


The Firm's Demand Curve: An Alternative Approach
By looking at the new expansion path curve, we can also get the firm's demand curve, but it gives more explicit attention to the output market demand for other inputs.


A lower wage rate for labor causes the firm to employ more labor as it substitutes labor for capital-the movement from $E$ to $E$, (see Fig. a). At a lower wage rate output will expand from $x_{1}$ to $x_{2}$, as the lower wage rate causes the marginal cost curve to shift downward. This output effect further increases the employment of labor--the movement from $E_{1}$ to $E_{2}$ in Fig. a.
Substitution Effect: the increase in the quantity employed when output is held constant and labor substituted for capital.

Output Effect: the increase in the quantity employed when output is increased.

Since both the substitution and the output effects imply greater employment at a lower input price, and lower employment at a higher input price, the firm's demand curve for an input must slope downward.

### 12.2 The Input Demand Curve of a Competitive Industry

The total quantity of an input hired by an industry is the sum of the quantities employed by the firms in the industry. To derive the industry demand curve, we must therefore aggregate the demand curves of the firms. However, when we derived the firm's demand curve, we assume that the price of the product remained unchanged. This cannot be true for an industry. When all firms simultaneously increase output, they can sell more output only at a lower price. So we must take into account this fact.


12.3 The Supply of Inputs

The supply side of input markets deals with the quantities of inputs available at alternative prices.

The supply curve of inputs to all industries in the economy almost vertical. For example, the total amount of labor can increase only if workers decide to work longer hours or if more people enter the labor force. Such responses to a higher wage rate may be so small.

Although the supply of input to all industries taken together may be vertical, this fact does not mean that the supply curve confronting any particular industry is vertical. The amount employed one particular industry is subject to great variation. For example, if the wage rate paid to workers in the shoe industry should increase, workers in other industries would leave their jobs to go to work making shoes.



Because the shoe industry is only a small part of the entire labor market, its labor supply curve will be more elastic than the supply curve of labor for the economy. In fact, the supply curves of most inputs to most industries are likely to be either perfectly horizontal or gently upward sloping, as in the above Figure.

### 12.4 Industry Determination of Price and Employment of Inputs

As usual, the market equilibrium of an input for a particular industry will be established when the quantity demand equals the quantity supplied Graphically, the equilibrium is shown by the intersection of the industry demand and supply curve. The position of a firm in equilibrium can be similarly determined.


Note that each firm is in the position employing the quantity of the input at which the marginal value product equals the price.

## Process of Input Price Equalization Across Industry

When several industries employ the same input, the input tends to be allocated among industries so that its price is the same in every industry. If this were not true--if workers were receiving $\$ 40$ in industry $A$ and $\$ 30$ in industry $B$--input owner would have incentive to shift inputs to industries where pay is higher, and this process tends to equalize input prices.


12.5 Input Demand and Employment by Monopoly

A monopoly is defined as a firm that is the sole seller of some product, but a firm that has monopoly power in its output market does not necessarily have market power in its input markets.

Like a competitive firm, a monopoly bases its decisions about input use on the way its marginal cost equals marginal revenues:

Suppose input is labor. $M C_{L}=M R_{X}$ at equilibrium.

Since $M C_{L}=\frac{W}{M P_{L}}$, we have $W=M P_{L} \cdot M R_{X}$.

The $M P_{L}$. $M R_{x}$ is called marginal revenue product ( $M R P_{L}$ ). The $M R P_{L}$ curve is the demand curve for the input. As we know, $\mathrm{MR}_{\mathrm{x}}$ of a monopoly must be lower than the price of output $x, i . e ., M_{x}<P$ at each level of output and at each level of employment of labor, so the $\mathrm{MRP}_{\mathrm{L}}$ curves lie below the $M V R_{L}$ curve which is the competitive demand curve.


Note that the employment of labor, or any other input, is lower under monopolistic condition than under competitive condition. This result
is consistent with the result we got before: A monopoly produces less output than does a competitive industry.

### 12.6 Monopsony

Monopsony means "single buyer". A monopsony is a single firm that is the sole purchaser of some type of input. It faces the market supply curve of the input, a curve that is frequently upward sloping. An upward-sloping supply curve for labor means that the firm must pay a higher wage rate to increase the number of workers it employs. Thus, the marginal cost of hiring another worker is not equal to the wage rate it must pay to all workers $\left(M C_{L}>W\right)$, and therefore $M C_{L}$ lies above S.

The profit-maximizing level of employment of a monopsony is

$$
M V P_{L}=M C_{L}>W, \quad \text { or } M R P_{L}=M C_{L}
$$

The intersection of $M C_{L}$ and the demand curve determines employment. (The demand curve will be the generalized MVP ${ }_{L}$ curve if the firm is a competitor in its output market; it will be $M R P_{L}$ curve if the firm is a monopoly in its output market.)


In comparison with competitive input conditions, employment is lower under monopsony and so is the wage rate paid. A similarity between monopsony and monopoly is apparent from this conclusion. A monopoly restricts output in order to obtain a higher price; a monopsony restricts output in order to pay a low wage. A monopoly is able to charge a high price because it faces a downward-sloping demand curve; a monopsony is able to pay a lower wage because it faces an upwardsloping supply curve.
XIII. Wage, Rent, Interest, and Profit

In this chapter we will extend the general analysis to specific input markets to see how wage, rent, interest, and profits are determined.

### 13.1 The Income-Leisure Choice of the Worker

In our discussion of consumer's choice, we assumed the consumer's income to be fixed. However, most people's income is not fixed but depends instead on, among other things, the decision about how much time the person will work. To investigate, we assume only income is labor income, the wage rate is fixed.

Let $I=$ the weekly income;
$\mathrm{L}=$ the weekly leisure time;
$\mathrm{H}=$ the weekly working hours
$\mathrm{W}=10 \$ /$ per hour
Then the income the consumer earns is
$I=10 H$
Since a week has $24 \times 7=168$ hours, $H+L=168$.
Thus $I=10(168-L)$, or
$I+10 L=1680$
This is the consumer's budget constraint. The consumer's problem is choose a combination of ( $I, L$ ) such that he has highest utility.

Suppose the consumer's indifference curves are strictly convex over (I,L). We can find the optimal bundle by graphical or mathematical approach.


As usual, the equilibrium is the point of tangency between the budget line and an indifference curve.

### 13.2 The Supply of Hours of Work

The above assumed that the wage rate was fixed. What happens if the wage rate changes? Will workers work longer hours at a higher wage rate? The answer depends on the consumer's preference.
i) The substitution effect dominates the income effect:

Suppose initial wage rate $\mathrm{W}_{0}=\$ 5$, the optimal bundel is E . If the wage increases to $W_{1}=\$ 8$, the new optimal bundel is E'. The substitution effect $=L_{3}-L_{1}<0$. The substitution effect of a higher wage rate means to encourage a worker to have less leisure time, or to supply more hours of labor. The income effect $=L_{2}-$ $L_{3}>0$, which means the higher wage rate encourages the consumer to have more leisure time, or to supply less hours of labor since income and leisure are both normal good. The total effect of the
higher wage rate is the sum of the income and substitution effects. Although these effects operate in opposite directions, in this case the substitution effect is large, so the total effect is an increase in hours of work from $\mathrm{NL}_{1}$ to $\mathrm{NL}_{2}$.

ii) The income effect dominates the substitution effect.

When the income effect is larger than the substitution effect, the conclusion is different from the above. The higher wage rate leads to a decrease in hours worked. Suppose $W$ increases


Total effect $=\left(L^{2}-L^{3}\right)+\left(L^{3}-L^{1}\right)=L^{2}-L^{1}>0$, which means the consumer increases the leisure time.

## Backward-bending Labor Supply Curve

In general, for low values of $w$, the substitution effect dominates. Beyond w* the income effect dominates.



Labor Supply Curve can be a verticle line.

## Example

$U=L^{\alpha} I^{\beta} \quad \Rightarrow \quad \operatorname{MRS}=\frac{\alpha I}{\beta L}$, where $\alpha>0, \beta>0 \quad \alpha+\beta=1$.
Determination of the consumer's choice of $I$ and $L$.
(1) $\quad \mathrm{MRS}=\mathrm{W} \quad \Rightarrow \quad \frac{\alpha \mathrm{I}}{\beta \mathrm{L}}=\mathrm{W} \quad \Rightarrow \quad \mathrm{I}=\beta \mathrm{WL} / \alpha$
(2) budget constraint:

$$
\begin{array}{r}
I+W L=168 \mathrm{~W} \\
\Rightarrow \quad \frac{B W L}{\alpha}+W L=168 \mathrm{~W} \\
\left(\frac{\beta}{\alpha}+1\right) W L=168 \mathrm{~W} \\
L^{*}=\alpha 168
\end{array}
$$

The consumer works $24 \alpha$ hour a day.

This is true for all wage rate $W$. That is, $L^{s}$ is independent of $W$.


## The Market Supply Curve

To go from an individual's supply curve of hours of work to the market supply curve, we need only add (horizontally sum) the responses of all workers competing in a given labor market. Thus, the market supply curve can also slope upward, bend backward, or be a vertical line.

### 13.3 The General Level of Wage Rate

We still use supply and demand analysis to investigate the level of wage rate. The supply curve of labor for this problem should indicate the total quantity of labor that will be supplied by all persons at various wage levels. The appropriate supply curve is the aggregate supply curve of hours of work discussed in the previous section.

The aggregate demand curve for labor reflects the marginal productivity of labor to the economy as a whole. The aggregate demand curve for labor interacts with the aggregate supply curve to determine the general level of wage rates. Over time, normally both supply and
demand increase. If demand increases faster than supply, wage rates tend to rise over time.


The productivity of labor is a main factor influencing the level of wages. This explains why real wage rates are so much higher in the U.S.A. than the less developed countries. Marginal productivity is higher because of the factors that determine the position of the demand curve: capital, technology, and skill.

### 13.4 Why Wages Differ

We know that there is a tendency for wage rates among firms or industries to equalize under the assumptions that workers were identical, and they evaluated the desirability of the jobs only in terms of the money wage rates. Dropping these assumptions leads to the conclusion that wage rates can differ among jobs and among people employed in the same line of work. Why is the wage rate for engineers higher than the wage rate for clerks? These differences are in full equilibrium with no tendency for the wage rates to equalize.


## Factors Resulting Equilibrium Wage Differences

a) Equalizing Wage Differentials

Workers currently employed as clerks may prefer their current jobs despite the financial difference. Monetary consideration is not the only factor, and sometimes not the most important factor, that influences the job choices of individuals. The differences of wage generated by preference on job choices are called equalizing wage differentials because the less attractive jobs must pay more to equalize the real advantages of employment among the jobs.
b) Differences in Human Capital Investment

Acquiring the skills to become an engineer may have a significant cost. The wage for engineers may not be sufficiently high to compensate clerks for the training costs they would have to bear to become engineers.
c) Differences in Ability

Even if there were no training costs, clerks may not have the aptitude for science and mathematics necessary to work as engineers.

### 13.5 Economic Rent

Rent -- the payments made to lease the services of land, apartments, equipment, or some other durable asset.

Economic Rent -- portion of the payment to the supplier of an input that is in excess of the minimum amount necessary to retain the input in its present use.

## Economic Rent with Vertical Supply Curve

The vertical curve (of land) intercuts with the demand curve for the services (of land) to determine its price. The price and quantity are specified per month to indicate that we are not concerned with the sale price of the land but with the price for the services yielded by the use of land. When the supply curve of an input is vertical, the entire renumeration of the input represents economic rent since the same quantity would be available even at a zero price.


## Economic Rent with an Upward-sloping Supply Curve

Consider the supply of college professors. With an upward-sloping input supply curve, part of the payment to input owners represents rent. In this case individuals $A, B, C$, and $D$ receive rents equal to areas $A_{2}, B_{2}, C_{2}$, and $D_{2}$, respectively.


Whenever the supply curve slopes upward, part of the payments to inputs will be rent. The more inelastic the supply curve, the large rents are as a fraction of total payments. Note that rents are the net benefits received by owners of inputs from their current employment. They measure the gains from voluntary exchange.

### 13.6 Borrowing, Lending, and the Interest Rate

Interest Rate -- the price paid by borrowers for the use of funds, or the rate of return earned by capital as an input. That is, interest rate is a return on loaned funds or invested capital.

Suppose there is no investment demand for funds. The BorrowingLending Equilibrium can be determined by the demand for consumption loans and the supply of saving.


### 13.7 Investment and the Marginal Productivity of Capital

Now let's expand the analysis to account for the fact that saving also provides funds used to finance investments.

Let $C$ be the initial cost, $g$ the rate of return of productivity, and $R$ the resulting addition to output (capital's gross marginal value product).

For one time period, we have the following formula

$$
C=R /(1+g)
$$

For more than one period (T periods),

$$
C=\frac{R_{1}}{(1+g)}+\frac{R_{2}}{(1+g)^{2}}+\ldots+\frac{R_{T}}{(1+g)^{T}}
$$

where $R^{t}$ is the resulting addition to output at period $t$ $(\mathrm{t}=1,2, \ldots, \mathrm{~T})$.

Given the initial cost of the equipment and the contribution to output in each period, we can solve the expression for $g$

Eg. $\quad C=100 \quad R=120$
$100=\frac{120}{(1+g)} \quad \Rightarrow \quad g=0.2$ or 208
The Investment Demand Curve -- a curve indicating the rate of return generated by investment at different levels, and it shows the amount


An expansion in investment in capital will occur as long as the rate of return is greater than the cost of borrowed funds, and such an expansion causes the rate of return to fall. Equilibrium results when investments yield a return just sufficient to cover the interest rate on borrowed funds. Thus, the rate of return on investment in capital tends to become equal the interest rate for borrowed funds.

### 13.8 Saving, Investment, and the Interest Rate

So far we have discussed the demand for consumer loans, $D_{c}$ and the investment demand curve, $D_{1}$ separately. The total demand for funds supplies by savers, $D_{T}$ is the sum of these two demand curves. the
intersection of $D_{T}$ and $S_{S}$ determines the interest rate. Investment is $\mathrm{OI}_{1}$, saving is OT , and consumption loans are $\mathrm{OL}_{1}$. $+\frac{1}{2}=1$


## Equalization of Rates of Return

We pointed out previously a number of reasons why wages differ because of differences in the productivities and preferences of people. However, there is a tendency for capital to be allocated among firms and individuals so that the wage rates are equal. A person investing funds usually doesn't care whether the funds are used to finance a computer in the aerospace industry or a truck in the construction industry; all that matters is the rate of return earned on the saving.

### 13.9 Why Interest Rates Differ

Even though interest rates tend to equal in general, differences can exist in specific interest rates equilibrium. The most important reasons for these differences are

1. Differences in risk. The greater the risk that the borrower will default on the loan, the higher is the interest a lender will charge.
2. Differences in the duration of loan. Borrowers will generally pay more for a loan that doesn't need to be repaid for a long time since that gives them greater flexibility. Lender also must receive a higher interest rate to part with funds for extended periods.
3. Cost of administering loans. A small loan usually involves greater bookkeeping and servicing costs per dollar of the loan than a long loan.
4. Differences in tax treatment.

## Real Versus Nominal Interest Rates

Our analysis of interest rate has focused entirely on the real rate of interest. The nominal, or money, rates of interest are commonly used. The difference between the two measures depends on what is happening to the price level.

Eg. Borrow $\$ 100$ this year and agree to pay back $\$ 110$ next year, the normal rate of interest is $10 \%$. If price rise by $8 \%$ over the year, the real rate of interest paid is $2 \%$.

In general, real interest rate $=$ nominal interest rate minus the rate of change in the price level.
i.e.

$$
i_{t}^{r}=i_{t}^{n}-\frac{P_{t+1}}{P_{t}}
$$

