On Risk Compensation to Prudent Decision Maker

Guoqiang Tian[a*] Yougong Tian[b,c]

[a] Department of Economics, Texas A&M University, College Station, TX77843, USA

[b] School of Economics, Shanghai University of Finance and Economics, Shanghai, 200433, China

[c] Department of Mathematics, Lanzhou University of Finance and Economics, Lanzhou, 730020, China

---

Abstract

For a prudent decision maker, this note indicates that she prefers to receive risk compensation in the bad state when she is risk averse, while she prefers to receive risk compensation in the good state when she is risk loving. In particular, our results have a certain instructive significance to further capture experimental evidence supporting the observation of Eeckhoudt et al. (2009) and Crainich et al. (2013).

Keywords: Downside risk aversion; Prudence; risk compensation; Experiment design

---

1 Introduction

The risk preference of prudence is fundamental concept in the study of economic decision making. Within an expected utility (EU) framework, Menezes et al. (1980) introduce the concept of downside risk aversion and show that a decision maker (DM) with the utility function u is downside risk averse if and only if she has a convex marginal utility function, i.e., $u''' > 0$, this condition is also equivalent to prudence coined by Kimball (1990). Naturally, in order to characterize the strength of precautionary saving motive (Leland, 1968; Sandmo, 1970; Dréze and Modigliani, 1972), Kimball proposes a local index of absolute prudence $-u'''/u''$ in the spirit of Arrow (1971) and Pratt (1964). In contrast, Modica and Scarsini (2005) provide an alternative measure for the intensity

*Corresponding author. Financial support from the National Natural Science Foundation of China (NSFC-71371117) is gratefully acknowledged. Email: gtian@tamu.edu.

of local downside risk aversion u'''/u' in the spirit of Ross (1981) and present the corresponding comparative criterion. [1]

Surprisingly, Eeckhoudt and Schlesinger (2006) interpret the concept of downside risk aversion or prudence as a preference for disaggregating a zero mean risk and a sure reduction in wealth across two equally likely states of nature. In particular, Eeckhoudt et al. (2009) present an alternative explanation for viewing aversion to higher order risks as a type of lottery preference for combining relatively good outcomes with relatively bad outcomes; as opposed to the alternative of combining good with good and bad with bad. More surprisingly, Crainich et al.(2013) discover that risk lovers are also prudent so that the behavior of prudence is shared by risk averters and risk lovers. They argue that risk lovers consistently like to combine good with good and bad with bad.

By virtue of Eeckhoudt and Schlesinger's (2006) framework, Crainich and Eekhoudt (2008) use the concept of utility premium to further explain that the intensity of local downside risk aversion might be measured by u'''/u'. [2] Specifically, Crainich and Eeckhoudt demonstrate that the utility premium can be vanished by adding the risk compensation to the good state. As pointed out by Crainich and Eeckhoudt that many other forms of compensation are possible to explain u'''/u' as the measure of downside risk aversion or prudence. In fact, these risk compensations indeed capture the intensity of downside risk aversion. Recently, Ebert and Wiesen (2014) measure the intensity of risk aversion, prudence, and temperance based on the risk compensation in a laboratory experiment, where the risk compensation is simultaneously received in both states.

The motive of this note originates from such question: If a prudent DM has a chance to receive the risk compensation in the one of two states (the good and bad state), which state does the DM prefer to receive the risk compensation in ? Unlike Crainich and Eekhoudt (2008) and Ebert and Wiesen (2014), this note considers a different form of risk compensation, i.e., the risk compensation might be received in the bad state other than the good state or both states. Accordingly, our note also discusses the relations among these risk compensations received in the different states. More recently, outside EU, the growing number of experimental studies on higher order risk preferences has emerged (Deck and Schlesinger (2010, 2014), Ebert and Wiesen (2011, 2014), Noussair et al. (2014) and so on). Indeed the original intention of this note is to mainly elicit a clue to capturing experimental evidence supporting the important observation of Eeck-

[1] Jindapon and Neilson (2007) present a generalization of nth degree Ross risk aversion, while Li (2009) and Denuit and Eeckhoudt (2010) independently provide the comparative criterion for higher order Ross more risk aversion.

[2] Friedman and Savage (1948) originally introduce the concept of utility premium, which is defined as the loss in expected utility caused by an introduction of risk.

houdt et al. (2009) and Crainich et al. (2013). Undoubtedly, our result helps to experiment design of measuring the intensity of prudence.

This note is organized as follows. In the next section, within EU and outside EU, we separately analyze the risk preferences of the risk averters and the risk lovers to receiving the risk compensation. Section 3 presents the main result, i.e., the prudent DM prefers to receive the risk compensation in the bad state when she is risk averse, while she prefers to receive the risk compensation in the good state when she is risk loving. A brief conclusion is drawn in section 4.

2 Lottery Preference and Marginal Utility

Outside EU, let us briefly recall extremely attractive lottery defined by Eeckhoudt and Schlesinger (2006). For random variables X and Y, let $[X, Y]$ be a lottery that yields X or Y, each with equal probability. Consider two lotteries, $A = [x, x - k + \tilde{\varepsilon}]$ and $B = [x - k, x + \tilde{\varepsilon}]$, where k is a arbitrary positive constant and $\tilde{\varepsilon}$ is a zero mean risk. Eeckhoudt and Schlesinger define prudence as a preference for B over A. Thus, prudence displays a type of preference for disaggregation of a sure loss of size k and the addition of a zero mean random variable $\tilde{\varepsilon}$.

In particular, Eeckhoudt et al. (2009) further indicate that the risk preferences of a risk averse and prudent DM ($u''(x) < 0$ and $u'''(x) > 0$ for all x) can be characterized by preferences over 50-50 lotteries that display a preference for combining good with bad over combining good with good and bad with bad. Specifically, for lottery $A = [x, x-k+\tilde{\varepsilon}]$, x is relatively good state, while $x - k + \tilde{\varepsilon}$ is relatively bad state; For arbitrary $m > 0$, m is obviously better than 0, as a result, a risk averse and prudent DM prefers combining good with bad (x with 0, and m with $x - k + \tilde{\varepsilon}$) over combining good with good (x with m) and bad with bad (0 with $x-k+\tilde{\varepsilon}$). Consequently, the DM is risk averse and prudent if she prefers $A'' = [x, x - k + m + \tilde{\varepsilon}]$ to $A' = [x + m, x - k + \tilde{\varepsilon}]$.

Similarly, Crainich et al. (2013) show that the risk preference of a risk loving and prudent DM ($u''(x) > 0$ and $u'''(x) > 0$ for all x) can be characterized by preferences over 50-50 lotteries that display a preference for mixing good with good and bad with bad over mixing good with bad and bad with good. Specifically, for lottery $A = [x, x - k + \tilde{\varepsilon}]$ and lottery $[0, m]$, a risk loving and prudent DM prefers mixing good with good (x with m) and bad with bad ($x - k + \tilde{\varepsilon}$ with 0) over mixing good with bad (x with 0 and $x - k + \tilde{\varepsilon}$ with m). Consequently, the DM is risk loving and prudent if she prefers $A' = [x + m, x - k + \tilde{\varepsilon}]$ to $A'' = [x, x - k + m + \tilde{\varepsilon}]$.

Within EU, from the viewpoint of marginal utility analysis, we know that a prudent DM prefers to receive the risk compensation in the bad state when she is risk averse, while she prefers to receive the risk compensation in the good state when she is risk loving. In other words, the DM always prefers to receive the risk compensation in the state where its marginal utility is higher. This is because: for all $x, k \geq 0$ and $x-k \geq 0$, for the risk averter, $u''(x) < 0$ implies $u'(x) < u'(x-k)$, under the assumption of prudence, i.e., $u''' > 0$, we have $u'(x-k) < Eu'(x-k+\tilde{\varepsilon})$ (by Jensen's inequality), thus $u'(x) < Eu'(x-k+\tilde{\varepsilon})$, indeed the DM prefers to receive the risk compensation in the bad state; In contrast, for the risk lover, $u''(x) > 0$ implies $u'(x) > u'(x-k)$, under the assumption of prudence, i.e., $u''' > 0$, we have $Eu'(x-k+\tilde{\varepsilon}) > u'(x-k)$, under proper conditions, [3] it implies that $u'(x) > Eu'(x-k+\tilde{\varepsilon})$, thus the DM prefers to receive the risk compensation in the good state.

3 Risk Compensation to Prudent Decision Maker

Eeckhoudt and Schlesinger (2006) and Crainich et al. (2013) have shown that a DM is prudent if she always prefers lottery B to lottery A in Figure 1.

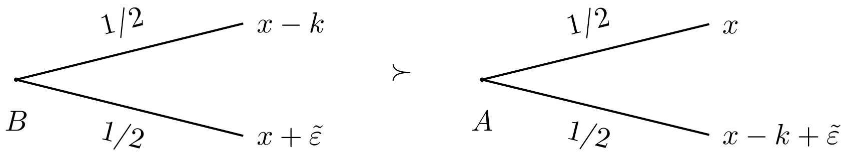

Figure 1: Lottery preference as prudence

Within EU, for the prudent DM with the utility function $u(x)$, we have:

$$\frac{1}{2}u(x-k) + \frac{1}{2}Eu(x+\tilde{\varepsilon}) > \frac{1}{2}u(x) + \frac{1}{2}Eu(x-k+\tilde{\varepsilon})$$

where x is a positive initial wealth, k is a positive constant and $\tilde{\varepsilon}$ is a zero mean risk.

In order to maintain the utility level when the pains ($-k$ and $\tilde{\varepsilon}$) are disaggregated, i.e., $1/2u(x-k) + 1/2Eu(x+\tilde{\varepsilon})$, Crainich and Eeckhoudt (2008) indicate that it needs to provide the risk compensation to the prudent DM for the risk misapportionment.

To this end, for the prudent DM, let m_G and m_B denote respectively the risk compensation received in the good and bad state such that she is indifferent between B and A' or A'' in Figure 2 and Figure 3.

[3] Without loss of generality, given the zero mean risk $\tilde{\varepsilon} = [p, a; 1-p, b]$, thus $u'(x) > Eu'(x-k+\tilde{\varepsilon})$ is equivalent to

$$\frac{u'(x) - u'(x-k+a)}{a} > (<) \frac{u'(x) - u'(x-k+b)}{b}$$

if $a > (<) 0$.

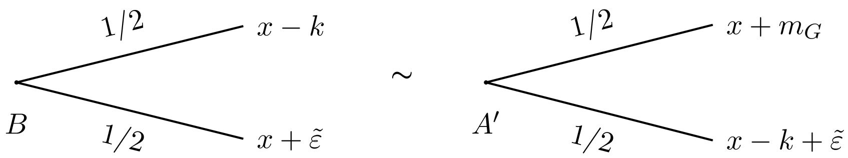

Figure 2: The risk compensation m_G such that B is indifferent to A'

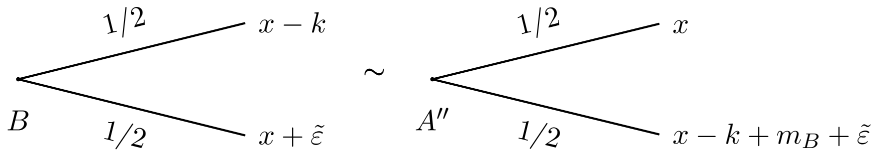

Figure 3: The risk compensation m_B such that B is indifferent to A''

Accordingly, within EU, $B \sim A'$ and $B \sim A''$ are equivalent respectively to

$$\frac{1}{2}u(x-k) + \frac{1}{2}Eu(x+\tilde{\varepsilon}) = \frac{1}{2}u(x+m_G) + \frac{1}{2}Eu(x-k+\tilde{\varepsilon}) \tag{1}$$

and

$$\frac{1}{2}u(x-k) + \frac{1}{2}Eu(x+\tilde{\varepsilon}) = \frac{1}{2}u(x) + \frac{1}{2}Eu(x-k+m_B+\tilde{\varepsilon}) \tag{2}$$

However, for the risk averse DM, what will happen if the same risk compensation m_B is received in the good state instead of in the bad state? Obviously, according to the idea of Eeckhoudt et al. (2009), we know that the following inequality holds when the DM is prudent:

$$\frac{1}{2}u(x) + \frac{1}{2}Eu(x-k+m_B+\tilde{\varepsilon}) > \frac{1}{2}u(x+m_B) + \frac{1}{2}Eu(x-k+\tilde{\varepsilon}) \tag{3}$$

Inequality (3) implies that if the prudent DM has a chance to make decision that the risk compensation is added to either the good state or the bad state, she prefers to receive the risk compensation in the bad state rather than in the good state when she is risk averse. Associating equation (1) and (2) with inequality (3), we have:

$$\frac{1}{2}u(x+m_G) + \frac{1}{2}Eu(x-k+\tilde{\varepsilon}) > \frac{1}{2}u(x+m_B) + \frac{1}{2}Eu(x-k+\tilde{\varepsilon}) \tag{4}$$

Thus, inequality (4) implies $m_G > m_B$ due to monotonicity of u. In order to measure the intensity of prudence, Ebert and Wiesen (2014) add the risk compensation m^{PR} to lottery A such that the prudent DM is indifferent between lottery B and lottery $A''' = A + m^{PR}$ in Figure 4.

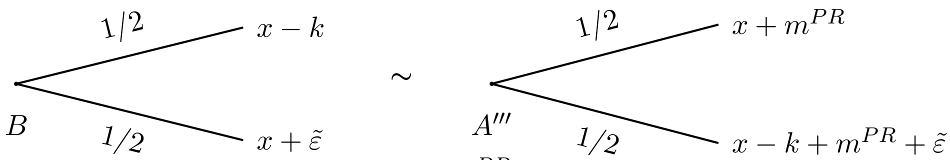

Figure 4: The risk compensation m^{PR} such that B is indifferent to A'''

Accordingly, within EU, we have:

$$\frac{1}{2}u(x-k)+\frac{1}{2}Eu(x+\tilde{\varepsilon})=\frac{1}{2}u(x+m^{PR})+\frac{1}{2}Eu(x-k+m^{PR}+\tilde{\varepsilon})$$

From analysis above, we know that $m_B < m^{PR} < m_G$. [4] As described in section 2, for the risk averse and prudent DM, indeed the decision to be made that the risk compensation should be received in the bad state is in line with the idea of disaggregation of pains, which is in fact advocated all along by Eeckhoudt and Schlesinger (2006) and Eeckhoudt et al. (2009).

Analogous to the analysis of the risk averter, however, the risk loving and prudent DM have the opposite decision. Thus we obtain the following result.

PROPOSITION 3.1 *Assume that the decision maker is prudent, to maintain the same utility level when the pains are disaggregated,*

(i) if the decision maker is risk averse, then she prefers to receive the risk compensation in the bad state, and $m_B < m_G$*;*

(ii) if the decision maker is risk loving, then she prefers to receive the risk compensation in the good state, and $m_G < m_B$*,*

where m_G *and* m_B *satisfy the equation (1) and (2), respectively.*

In order to measure the intensity of prudence, we here use the same technique in Crainich and Eeckhoudt (2008), i.e., a second order approximation à la Arrow-Pratt approach, the approximate expression of the risk compensation to the risk averter, m_B, is obtained :

$$m_B \cong \frac{\frac{\sigma^2}{2}\frac{u'''(x)}{u'(x)}}{1+\frac{\sigma^2}{2}\frac{u'''(x)}{u'(x)}}k$$

while Crainich and Eeckhoudt (2008) obtain the approximate expression of the risk compensation to the risk averter, m_G :

$$m_G \cong \frac{\sigma^2}{2}\frac{u'''(x)}{u'(x)}k$$

[4] Under the assumption of prudence, we know that m^{PR} (m_B or m_G) is always larger than 0 and less than k, i.e., $0 < m < k$, otherwise,

$$\frac{1}{2}u(x)+\frac{1}{2}Eu(x-k+m+\tilde{\varepsilon}) > \frac{1}{2}u(x-k)+\frac{1}{2}Eu(x+\tilde{\varepsilon})$$

where m represents the risk compensation m^{PR} in Ebert and Wiesen (2014), m_G in Crainich and Eeckhoudt (2008) and m_B in this note, respectively.

Obviously, $m_B < m_G$.

Proposition 3.1 shows that it is optimal decision to the prudent DM that the risk compensation should be received in the bad state when she is risk averse, while the risk compensation should be received in the good state when she is risk loving. As shown in section 2, indeed the intuition behind the result is that the marginal utility is decreasing in wealth level when the DM is risk averse, while the marginal utility is increasing in wealth level when the DM is risk loving.

Let now us consider the case of two decision makers, assume that $u(x)$ and $v(x)$ are their respective utility function. For the prudent DM with utility function $v(x)$, let n_G and n_B denote respectively the risk compensation received in the good and bad state such that the following equations hold:

$$\frac{1}{2}v(x-k) + \frac{1}{2}Ev(x+\tilde{\varepsilon}) = \frac{1}{2}v(x+n_G) + \frac{1}{2}Ev(x-k+\tilde{\varepsilon}) \tag{5}$$

and

$$\frac{1}{2}v(x-k) + \frac{1}{2}Ev(x+\tilde{\varepsilon}) = \frac{1}{2}v(x) + \frac{1}{2}Ev(x-k+n_B+\tilde{\varepsilon}) \tag{6}$$

PROPOSITION 3.2 *Assume that $u(x)$ and $v(x)$ are all prudent decision makers, if the intensity of downside risk aversion on $u(x)$ is larger than that on $v(x)$, i.e.,*

$$\frac{u'''(x)}{u'(x)} \geq \frac{v'''(x)}{v'(x)}$$

(i) if $u(x)$ and $v(x)$ are all risk averse, then $m_B \geq n_B$, where m_B and n_B satisfy the equation (2) and (6), respectively;

(ii) if $u(x)$ and $v(x)$ are all risk loving, then $m_G \geq n_G$, where m_G and n_G satisfy the equation (1) and (5), respectively.

Consequently, Proposition 3.2 reveals the relation between the risk compensation and the intensity of downside risk aversion. In other words, Proposition 3.2 shows that the DM is whether risk averse or not risk loving, the more the intensity of downside risk aversion, the larger the risk compensation to the DM. [5]

Following Ebert and Wiesen's (2014) experiment design, [6] according to Proposition 3.1 and Proposition 3.2, we can indeed make a experiment design measure the intensity of prudence.

[5] It needs to stress that the result maybe only hold for the small risk and loss, however it maybe not hold for the large risk.

[6] Ebert and Wiesen (2014) provide subjects with a menu of pairwise lottery choices in order to identify subjects' degrees of prudence. Specifically, subjects make 60 decisions, the 60 decisions divide into 20 decisions on 3 different

Specifically, the risk compensation can only be received in either the good state or the bad state rather than both states. In such experiment design, we not only identify the intensity of prudence, but meantime test for the direction of second order risk preferences, i.e., the subject is risk aversion or risk loving. Theoretically, the subject prefers to receive the risk compensation in the bad state when she is risk averse, while the subject prefers to receive the risk compensation in the good state when she is risk loving. In particular, we can verify that whether the behavior the subject behaves is consistent with the risk preference observed in testing for the direction of second order risk preferences. In this regard, our experiment design seems feasible.

4 Conclusion

This note is mainly devoted to an analysis of the risk compensation to the prudent decision maker. Unlike Crainich and Eekhoudt (2008) and Ebert and Wiesen (2014), this note considers a different form of risk compensation. Undoubtedly, our results have a certain instructive significance to measure the intensity of prudence in a laboratory setting.

References

[1] Arrow, K. (1971). Essays in the theory of risk-bearing, North-Holland, New York.

[2] Crainich, D. and Eeckhoudt, L. (2008). On the intensity of downside risk aversion, Journal of Risk and Uncertainty 36(3): 267-276.

[3] Crainich, D., Eeckhoudt, L. and Trannoy, A. (2013). Even (mixed) risk lovers are prudent, American Economic Review 103(4): 1529-1535.

[4] Deck, C. and Schlesinger, H. (2010). Exploring higher order risk effects. Review of Economic Studies 77(4): 1403-1420.

[5] Deck, C. and Schlesinger, H. (2014). Consistency of higher-order risk preferences. Econometrica 82(5): 1913-1943.

[6] Denuit, M. and Eeckhoudt, L. (2010). Stronger measures of higher-order risk attitudes, Journal of Economic Theory 145(5): 2027-2036.

screens for prudence (tasks PR1, PR2, PR3), each decision is for a different value of the compensation m varying from -2.50 to 2.50 with a step size of 0.25. In their experiment, the risk compensations m^{PR} is the smallest amount such that subject prefers $A + m^{PR} = [x + m^{PR}, x - k + \tilde{\varepsilon} + m^{PR}]$ over $B = [x - k, x + \tilde{\varepsilon}]$. More details about experiment design refer to Ebert and Wiesen (2014).

[7] Dréze, J. and Modigliani, F. (1972). Consumption decisions under uncertainty, Journal of Economic Theory 5(3): 308-335.

[8] Ebert, S. and Wiesen, D. (2011). Testing for prudence and skewness seeking. Management Science 57(7): 1334-1349.

[9] Ebert, S. and Wiesen, D. (2014). Joint measurement of risk aversion, prudence, and temperance, Journal of Risk and Uncertainty 48(3): 231-252.

[10] Eeckhoudt, L. and Schlesinger, H. (2006). Putting risk in its proper place, American Economic Review 96(1): 280-289.

[11] Eeckhoudt, L., Schlesinger, H. and Tsetlin, I. (2009). Apportioning of risks via stochastic dominance, Journal of Economic Theory 144(3): 994-1003.

[12] Friedman, M. and Savage, L. (1948). The utility analysis of choices involving risk, Journal of Political Economy 56(4): 279-304.

[13] Jindapon, P. and Neilson, W. (2007). Higher-order generalizations of Arrow-Pratt and Ross risk aversion: A comparative statics approach, Journal of Economic Theory 136(1): 719-728.

[14] Kimball, M. (1990). Precautionary saving in the small and in the large, Econometrica 58(1): 53-73.

[15] Leland, H. (1968). Saving and uncertainty: The precautionary demand for saving, Quarterly Journal of Economics 82(3): 465-473.

[16] Li, J. (2009). Comparative higher-degree Ross risk aversion, Insurance: Mathematics and Economics 45(3): 333-336.

[17] Menezes, C., Geiss, C. and Tressler, J. (1980). Increasing downside risk, American Economic Review 70(5): 921-932.

[18] Modica, S. and Scarsini, M. (2005). A note on comparative downside risk aversion, Journal of Economic Theory 122(1): 267-271.

[19] Noussair, C., Trautmann, S. and van de Kuilen, G. (2014). Higher order risk attitudes, demographics, and financial decisions. Review of Economic Studies 81(1): 325-355.

[20] Pratt, J. (1964). Risk aversion in the small and in the large, Econometrica 32(12): 122-136.

[21] Ross, S. (1981). Some stronger measures of risk aversion in the small and in the large with applications, Econometrica 49(3): 621-663.

[22] Sandmo, A. (1970). The effect of uncertainty on saving decisions, Review of Economic Studies 37(3): 353-360.