

Optimal Interregional Redistribution under Migration and Asymmetric Information

Darong Dai* Liqun Liu[†] Guoqiang Tian[‡]

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Abstract

This paper studies optimal interregional redistribution by enriching the model of Huber and Runkel (2008) with inter-jurisdictional migrations. In the first-best optimum under symmetric information, their prediction continues to hold when migration intensity is low. In the second-best optimum under asymmetric information, their prediction continues to hold when either migration intensity is low or migration intensity is high and the regional difference is small. Otherwise, their prediction is reversed, and for both the symmetric and asymmetric information case, we obtain that the patient region should borrow more than the impatient region borrows, and the center should redistribute from the patient to the impatient region.

Keywords: Interregional redistribution; Asymmetric information; Inter-jurisdictional migration; Heterogeneous discounting.

JEL Codes: H23; H74; H77; D82.

*Institute for Advanced Research (IAR), Shanghai University of Finance and Economics, Shanghai 200433, P. R. China. E-mail: dai.darong@mail.shufe.edu.cn

[†]Private Enterprise Research Center, Texas A&M University, College Station, TX 77843, USA. E-mail: lliu@tamu.edu

[‡]Department of Economics, Texas A&M University, College Station, TX 77843, USA. E-mail: gtian@tamu.edu

1 Introduction

Interregional redistribution is implemented in developing countries such as China as an important policy tool to achieve balanced development (see, *The Economist*, 2016), and is also of policy relevance in developed economies such as Canada, France, the UK and the US (see, Mélitz and Zumer, 2002). The early literature either focuses on the cross-region spillover effects due to interregional migrations (e.g., Wildasin, 1991; Manasse and Schultz, 1999; Breuillé and Gary-Bobo, 2007) or focuses on the information asymmetries between the federal government and local governments (e.g., Oates, 1999; Bordignon, Manasse and Tabellini, 2001; Cornes and Silva, 2002; Huber and Runkel, 2008), but not much is known about their *joint effect* on the optimal mechanism to redistribute resources among regional governments. This paper represents an attempt to fill this gap and focuses on pure redistribution among two regional governments in the presence of both asymmetric information between the center and regions and the interregional migrations.

To this end, we develop a two-period model of a federation consisting of a benevolent federal government and two regions that differ in the rate of time preference and between which individuals migrate with a probability. Assuming that the federal government cannot observe the discount factor of each region and taking into account inter-jurisdictional migration possibilities, we extend the analysis of Huber and Runkel (2008) and obtain the following three results.

First, in the first-best optimum under full information, the prediction of Huber and Runkel (2008), namely that the impatient region should borrow more than the patient region borrows and the federal government should redistribute from the impatient region to the patient region, holds only if the intensity of mutual migration is low; otherwise the patient region should borrow more than does the impatient region and the redistribution should be from the patient region to the impatient region. Second, in the second-best optimum under asymmetric information, their prediction that the impatient region should borrow more than does the patient region and the redistribution should be from the former to the latter holds only if either migration intensity is low or migration intensity is high and the regional difference in discounting is small; otherwise the patient region should borrow more than does the impatient region and the redistribution should be from the patient region to the impatient region. Third, regardless of whether the intensity of mutual migration is low or high, the second-best optimum achieves less

interregional redistribution than does the first-best optimum. That is, information asymmetry limits the ability of the federal government to adopt a tax-transfer system to redistribute resources across heterogeneous regions. As such, the optimal interregional redistribution policy obtained by Huber and Runkel remains optimal only in special cases of our more general model with interregional migrations. Importantly, we establish under reasonable circumstances an optimal redistribution policy that is exactly the opposite of theirs.

While we initially follow Huber and Runkel in assuming that there is no endogenous intertemporal savings, we also consider a model with endogenous private savings decisions. We identify the sufficient conditions under which the features of interregional redistribution obtained under exogenous and homogeneous savings decisions could be carried over to more general circumstances with endogenous and/or heterogeneous savings decisions. In particular, if migration occurs with some specific probability, then we find that there is a stark difference between the baseline model and this extension regarding the optimal redistribution policy.

We choose the discount factor as the source of heterogeneity among regions due to these two considerations. First, as argued by Huber and Runkel and empirically demonstrated by Evans and Sezer (2004), the discount factor is indeed difficult to observe and is more likely to be the private information of local regions. Second, it is a key factor affecting intertemporal resource allocation, and hence is relevant in causing individual welfare disparity among regions. In fact, it seems to be trivial to claim that interregional redistribution should be from rich regions to poor regions, whereas it is nontrivial to judge, *ceteris paribus*, whether patient regions or impatient regions should be the contributor of interregional redistribution. We hence argue in a two-period life-cycle model that, relative to observable initial income disparities among regions, the heterogeneity of the preference for period-2 consumption (or consumption when old) might be a deeper reason motivating redistribution in period 1.

The rest of the paper is organized as follows. Section 2 describes the baseline model. In Section 3, we derive the optimal redistribution policies under exogenous and homogeneous savings decisions. As an extension, we derive the optimal redistribution policies under endogenous private savings decisions in Section 4. Section 5 concludes. Proofs are relegated to Appendix.

2 Baseline Model

We consider a federation consisting of a federal government (also referred to as the center) and two regions (also referred to as jurisdictions), denoted A and B , respectively. They have the same initial population size that is normalized to one for notational simplicity. These individuals live for two periods with identical income y_1 in period 1, identical income y_2 in period 2, and a lifetime utility function

$$u_1(c_1) + g_1(G_1) + \delta^R[u_2(c_2) + g_2(G_2)], \quad (1)$$

in which $\delta^R > 0$ denotes the discount factor of individuals born in region R ($R = A$ or B), c_1 and c_2 are respectively private consumptions in periods 1 and 2, G_1 and G_2 are respectively the public goods¹ in periods 1 and 2, and all four functions are strictly increasing and strictly concave.

Throughout, we impose the following assumption without loss of generality.

Assumption 2.1 $\delta^A < \delta^B$.

That is, individuals born in jurisdiction A are less patient than those born in jurisdiction B . Following Huber and Runkel (2008), individuals' discount factor that postulates how the future utility is compared to the present utility is assumed to be the only source of heterogeneity between these two regions. In what follows, we may simply call region A the impatient region and region B the patient region.²

For expositional convenience, we focus on the decisions of the representative individual and the local government in region A , because those of the representative individual and the local government in region B are symmetric.

¹The formal results in Section 3 hold regardless of whether they are interpreted as publicly-provided public goods or publicly-provided private goods.

²An equivalent specification is to assume that individuals born in both regions are of the same degree of patience, but local government A is less patient than is local government B . It can be interpreted as that the politician in local government A is more shortsighted relative to that in local government B , or the politician in local government B has a higher probability to be reelected in a democracy. As usually assumed in the political economy literature (e.g., Acemoglu, Golosov and Tsyvinski, 2010), a politician's discount factor is potentially different from that of citizens. In particular, we are likely to observe that politicians in local governments tend to discount the future more heavily than do citizens.

The representative individual in region A has private budget constraints $c_1^A + \tau_1^A = y_1$ and $c_2^A + \tau_2^A = y_2$ in periods 1 and 2, respectively. The lump sum taxes τ_1^A and τ_2^A are collected by the local government in region A to finance local public good provision. In period 1, the local government receives a transfer z^A from the center and issues debt b^A . Debt plus interest has to be repayed in period 2, taking as given the common interest rate $r > 0$.³ The fiscal budget constraints of local government- A in periods 1 and 2 can be written as $G_1^A = \tau_1^A + b^A + z^A$ and $G_2^A = \tau_2^A - (1 + r)b^A$, respectively. If the transfer from the center is negative, then it means that the local government has to pay a tax to the center. Under pure redistribution, the budget constraint of the federal government is

$$z^A + z^B = 0, \quad (2)$$

which means that the center collects resources from one region to finance transfers to the other region.

At the beginning of period 2, the representative individual in region A migrates to region B with the probability $p \in [0, 1)$ due to exogenous reasons, such as schooling, marriage, social network, or geographic preference.⁴ By symmetry, the number of individuals in each region will not change after the process of mutual migration.

Combining the private budget constraints with the public budget constraints, the expected utility maximization problem of the representative individual in region A is thus written as

$$\begin{aligned} \max_{c_1^A, c_2^A} \quad & u_1(c_1^A) + g_1(y_1 + b^A + z^A - c_1^A) \\ & + \delta^A [(1 - p)u_2(c_2^A) + (1 - p)g_2(y_2 - b^A(1 + r) - c_2^A) \\ & + pu_2(c_2^B) + pg_2(y_2 - b^B(1 + r) - c_2^B)], \end{aligned}$$

taking as given b^A, z^A, b^B and c_2^B . A higher value of p , namely a higher migration possibility, implies that the representative individual imposes a larger weight on the period-2 utility he obtains from migrating to the opponent region and a smaller weight on that he obtains from staying up.

The first-order conditions are thus written as

$$u'_1(c_1^A) = g'_1(G_1^A) \quad \text{and} \quad u'_2(c_2^A) = g'_2(G_2^A). \quad (3)$$

³It seems to be reasonable to assume that there is a common capital market within a federation. So, there is a single price level of capital to eliminate arbitrage opportunities.

⁴See, e.g., Dai, Jansen and Liu (2018).

By symmetry, $u'_1(c_1^B) = g'_1(G_1^B)$ and $u'_2(c_2^B) = g'_2(G_2^B)$ are the first-order conditions from the corresponding problem of the representative individual in region B . We denote the solutions by $\hat{c}_1^A, \hat{c}_2^A, \hat{c}_1^B$ and \hat{c}_2^B . The value function of region A is therefore expressed as

$$\begin{aligned} V(b^A, z^A, \delta^A; b^B) &\equiv u_1(\hat{c}_1^A) + g_1(y_1 + b^A + z^A - \hat{c}_1^A) \\ &\quad + \delta^A[(1-p)u_2(\hat{c}_2^A) + (1-p)g_2(y_2 - b^A(1+r) - \hat{c}_2^A) \\ &\quad + pu_2(\hat{c}_2^B) + pg_2(y_2 - b^B(1+r) - \hat{c}_2^B)]. \end{aligned} \tag{4}$$

3 Optimal Interregional Redistribution with Mutual Migration

We assume that the federal government cannot observe the degree of patience of each region. Applying revelation principle, the center offers each local government a contract stipulating the federal transfer and the region's debt. Formally, the center solves the maximization problem⁵

$$\max_{b^A, z^A, b^B, z^B} V(b^A, z^A, \delta^A; b^B) + V(b^B, z^B, \delta^B; b^A)$$

subject to budget constraint (2) and these incentive-compatibility constraints

$$\begin{aligned} V(b^A, z^A, \delta^A; b^B) &\geq V(b^B, z^B, \delta^A; b^B) \quad (\text{IC}_A); \\ V(b^B, z^B, \delta^B; b^A) &\geq V(b^A, z^A, \delta^B; b^A) \quad (\text{IC}_B). \end{aligned}$$

By using (3) and (4), it is easy to show that the corresponding single-crossing property is satisfied. The Lagrangian can thus be written as

$$\begin{aligned} \mathcal{L}(b^A, z^A, b^B, z^B; \mu_A, \mu_B, \lambda) &= (1 + \mu_A)V(b^A, z^A, \delta^A; b^B) - \mu_A V(b^B, z^B, \delta^A; b^B) \\ &\quad + (1 + \mu_B)V(b^B, z^B, \delta^B; b^A) - \mu_B V(b^A, z^A, \delta^B; b^A) \\ &\quad - \lambda(z^A + z^B), \end{aligned} \tag{5}$$

⁵Following the common practice in the related literature, participation constraints are ignored. In practice, it may be politically and/or economically costly for a region to leave the federation, and hence it is not quite restrictive to ignore these constraints. In fact, exploring the breakup possibility of a nation is of independent interest (e.g., Bolton and Roland, 1997).

in which μ_A, μ_B and λ are Lagrangian multipliers. Without loss of generality, we let the federal budget constraint (2) be binding so that $\lambda > 0$.

As a standard benchmark, we consider first the case with symmetric information between the center and local governments, and hence $\mu_A = \mu_B = 0$. We index the first-best allocation by the superscript FB .

Proposition 3.1 *Under Assumption 2.1 and symmetric information, the following statements are true.*

- (i) $G_1^{A,FB} = G_1^{B,FB}$ and $c_1^{A,FB} = c_1^{B,FB}$ for $\forall p \in [0, 1]$.
- (ii) If $p < 1/2$, then $G_2^{A,FB} < G_2^{B,FB}$, $c_2^{A,FB} < c_2^{B,FB}$, $b^{A,FB} > b^{B,FB}$, $z^{A,FB} < 0 < z^{B,FB}$, IC_A is violated, and IC_B is satisfied.
- (iii) If $p > 1/2$, then $G_2^{A,FB} > G_2^{B,FB}$, $c_2^{A,FB} > c_2^{B,FB}$, $b^{A,FB} < b^{B,FB}$, $z^{B,FB} < 0 < z^{A,FB}$, IC_A is satisfied, and IC_B is violated.
- (iv) If $p = 1/2$, then $G_2^{A,FB} = G_2^{B,FB}$, $c_2^{A,FB} = c_2^{B,FB}$, $b^{A,FB} = b^{B,FB}$, $z^{A,FB} = z^{B,FB} = 0$, and both IC_A and IC_B are satisfied.

In the first-best allocation, period-1 private and public consumptions are the same for both regions. If the intensity of mutual migration is low, namely $p < 1/2$, period-2 private and public consumptions are higher in the patient region than in the impatient region, the impatient region borrows more than does the patient region, and interregional redistribution is from the impatient region to the patient region. These characteristics are exactly the same as those found by Huber and Runkel (2008) in a federation without interregional migrations.

However, as shown in (iii), if the intensity of mutual migration is high, namely $p > 1/2$, period-2 private and public consumptions are lower in the patient region than in the impatient region, the patient region borrows more than does the impatient region, and interregional redistribution is from the patient region to the impatient region. This novel redistribution policy hence *overturns* that obtained by Huber and Runkel (2008). Also, if $p = 1/2$, then the patient region and the impatient region are treated in the same way in the first-best allocation, and hence interregional redistribution is no longer in need.

Under asymmetric information, the discount factor is private information so that the local government of a given region can mimic the local government

of the other region in order to obtain transfers. We now index the second-best allocation by the superscript $*$.

Proposition 3.2 *Under Assumption 2.1 and asymmetric information, the following statements are true.*

- (i) *If $\mu_A > \mu_B \geq 0$, then (i-a) truth-telling requires that either $p \leq p_{A/B}^*$, or $p > p_{A/B}^*$ and $\delta^A/\delta^B > \delta^*$ should hold for thresholds $p_{A/B}^* \in (1/2, 1)$ and $\delta^* \in (0, 1)$, (i-b) the second-best optimum satisfies $G_1^{A*} > G_1^{B*}$, $c_1^{A*} > c_1^{B*}$, $G_2^{A*} < G_2^{B*}$, $c_2^{A*} < c_2^{B*}$ and $b^{A*} > b^{B*}$, and (i-c) optimal interregional redistribution exhibits $z^{A*} < 0 < z^{B*}$ for $p \geq \frac{\mu_B(\delta^B - \delta^A)}{\delta^B + \mu_B(\delta^B - \delta^A)}$.*
- (ii) *If $\mu_B > \mu_A \geq 0$, then (ii-a) truth-telling requires that either $p \geq p_{B/A}^*$, or $p < p_{B/A}^*$ and $\delta^A/\delta^B > \delta^*$ should hold for thresholds $p_{B/A}^* \in (1/2, 1)$ and $\delta^* \in (0, 1)$, and (ii-b) the second-best optimum satisfies $G_1^{A*} < G_1^{B*}$, $c_1^{A*} < c_1^{B*}$, $G_2^{A*} > G_2^{B*}$, $c_2^{A*} > c_2^{B*}$, $b^{A*} < b^{B*}$ and $z^{B*} < 0 < z^{A*}$.*
- (iii) *If $\mu_A = \mu_B > 0$, then (iii-a) truth-telling requires that $p = p_{A/A}^*$ for threshold $p_{A/A}^* \in (1/2, 1)$, and (iii-b) the second-best optimum satisfies $G_1^{A*} = G_1^{B*}$, $c_1^{A*} = c_1^{B*}$, $G_2^{A*} = G_2^{B*}$, $c_2^{A*} = c_2^{B*}$, $b^{A*} = b^{B*}$ and $z^{A*} = z^{B*} = 0$.*

The second-best optimum derived by Huber and Runkel (2008) is just a special case of our part (i) via imposing that $\mu_B = 0$ and $p = 0$.

Part (i) of Proposition 3.2 studies the case in which the shadow price (μ_A) of incentive-compatibility constraint concerning the impatient region A is higher than that (μ_B) concerning the patient region B . As such, the impatient region obtains an information rent as their period-1 private and public consumptions are higher than those of the patient region. While the impatient region borrows more than does the patient region, the center redistributes from the former to the latter. The self-selection problem is resolved when either the intensity of mutual migration is not too high, or if it is high, the difference of the degree of patience between these two regions is not too large.

Part (ii) of Proposition 3.2 studies the case in which the shadow price (μ_A) of incentive-compatibility constraint concerning the impatient region is lower than that (μ_B) concerning the patient region. As such, the patient region

obtains an information rent as their period-1 private and public consumptions are higher than those of the impatient region. While the patient region borrows more than does the impatient region, the center redistributes from the former to the latter. The self-selection problem is resolved when either the intensity of mutual migration is high, or if it is not high, the difference of the degree of patience between these two regions is not too large.

Part (iii) of Proposition 3.2 studies the case in which the shadow price (μ_A) of incentive-compatibility constraint concerning the impatient region is the same as that (μ_B) concerning the patient region. The self-selection problem is resolved when the migration probability is equal to a specific value higher than $1/2$. We find in this special context that these two heterogeneous regions should be treated in the same way, and hence no interregional redistribution should be implemented.

To identify the effect of the asymmetric information between the center and local governments on optimal debt and interregional redistribution policies, we give the following:

Proposition 3.3 *Under Assumption 2.1, the following statements are true.*

- (i) *If $\mu_A > \mu_B$ and $\delta^A/\delta^B > \mu_B/\mu_A$, then $b^{B,FB} < b^{B*} < b^{A*} < b^{A,FB}$, $0 > z^{A*} > z^{A,FB}$ and $0 < z^{B*} < z^{B,FB}$.*
- (ii) *If $\mu_B > \mu_A$, then $b^{A,FB} < b^{A*} < b^{B*} < b^{B,FB}$, $0 > z^{B*} > z^{B,FB}$ and $0 < z^{A*} < z^{A,FB}$.*

In general, the second-best allocation achieves less interregional redistribution than does the first-best allocation. That is, everything else being equal, information asymmetry limits the ability of the center to redistribute resources across different types of regions (or local governments). In terms of the optimal debt policy for these local governments, the contributor region borrows less in the second-best allocation than it does in the first-best allocation, whereas the recipient region borrows more in the second-best allocation than it does in the first-best allocation. The reason is that the contributor pays less tax to and the recipient receives less transfer from the center under asymmetric information than they do under symmetric information.

4 An Extension with Endogenous Savings

To focus on the primary concern, we assume exogenous private savings decisions in the baseline model. There we can equivalently rewrite individual

incomes, y_1 and y_2 , as $y_1 \equiv y - k$ and $y_2 \equiv k(1 + r)$ for a given initial income $y > 0$ and a fixed amount of private savings/investment $k \in (0, y)$. Here we consider a natural extension with endogenous savings, and the individual expected utility maximization problem in region A reads as:

$$\max_{c_1^A, c_2^{A,S}, c_2^{A,M}, k^A} u_1(c_1^A) + g_1(y + b^A + z^A - k^A - c_1^A) + \delta^A[(1 - p)u_2(c_2^{A,S}) + (1 - p)g_2(G_2^A) + pu_2(c_2^{A,M}) + pg_2(G_2^B)],$$

in which period-2 public consumptions are given by⁶

$$\begin{aligned} G_2^A &= (1 - p)[k^A(1 + r) - c_2^{A,S}] + p[k^B(1 + r) - c_2^{B,M}] - (1 + r)b^A \\ G_2^B &= (1 - p)[k^B(1 + r) - c_2^{B,S}] + p[k^A(1 + r) - c_2^{A,M}] - (1 + r)b^B \end{aligned} \quad (6)$$

with $c_2^{A,S}$, $c_2^{B,S}$ and $c_2^{A,M}$, $c_2^{B,M}$ denoting, respectively, period-2 private consumptions when staying up and migrating. We thus obtain these FOCs:

$$\begin{aligned} u_1'(c_1^A) &= g_1'(G_1^A) \\ u_2'(c_2^{A,S}) &= (1 - p)g_2'(G_2^A) \\ u_2'(c_2^{A,M}) &= pg_2'(G_2^B) \\ g_1'(G_1^A) &= \delta^A(1 + r)[(1 - p)^2g_2'(G_2^A) + p^2g_2'(G_2^B)]. \end{aligned} \quad (7)$$

The corresponding FOCs of the representative individual in region B can be obtained by symmetry.

Using Lagrangian (5) and FOCs (7), we obtain the following proposition under symmetric information between the center and regions.

Proposition 4.1 *Under Assumption 2.1, symmetric information and endogenous savings, the following statements are true.*

- (i) $G_1^{A,FB} = G_1^{B,FB}$ and $c_1^{A,FB} = c_1^{B,FB}$ for $\forall p \in [0, 1]$.
- (ii) If $p < 1/2$, then $G_2^{A,FB} < G_2^{B,FB}$, $c_2^{A,S,FB} < c_2^{B,S,FB}$, $c_2^{A,M,FB} > c_2^{B,M,FB}$, IC_A is violated, and IC_B is satisfied. If, in addition, $k^{A,FB} \leq k^{B,FB}$, then $b^{A,FB} - k^{A,FB} > b^{B,FB} - k^{B,FB}$ and $z^{A,FB} < 0 < z^{B,FB}$.

⁶Noting that $c_2^{A,S} = c_2^{B,M} = y_2 - \tau_2^A \equiv c_2^A$ and $c_2^{B,S} = c_2^{A,M} = y_2 - \tau_2^B \equiv c_2^B$ in the baseline model, it is indeed endogenous savings that make things complicated here.

- (iii) If $p > 1/2$, then $G_2^{A,FB} > G_2^{B,FB}$, $c_2^{A,S,FB} > c_2^{B,S,FB}$, $c_2^{A,M,FB} < c_2^{B,M,FB}$, IC_A is satisfied, and IC_B is violated. If, in addition, $k^{A,FB} \geq k^{B,FB}$, then $b^{A,FB} - k^{A,FB} < b^{B,FB} - k^{B,FB}$ and $z^{B,FB} < 0 < z^{A,FB}$.
- (iv) If $p = 1/2$, then $G_2^{A,FB} = G_2^{B,FB}$, $c_2^{A,S,FB} = c_2^{B,S,FB}$, $c_2^{A,M,FB} = c_2^{B,M,FB}$, $b^{A,FB} = b^{B,FB}$, $k^{A,FB} < k^{B,FB}$, $z^{A,FB} < 0 < z^{B,FB}$, and both IC_A and IC_B are satisfied.

Allowing for endogenous private savings decisions, it follows from the last equation of (7) that the optimal amount of savings (and hence period-2 income) is likely to be different between these two regions and also the relative magnitude of $k^{A,FB}$ and $k^{B,FB}$ is in general indeterminate.

If migration intensity is low and the first-best amount of savings in the patient region is no smaller than that in the impatient region, then the first-best interregional redistribution is from the impatient region to the patient region, as established in part (ii) of Proposition 3.1. If, in contrast, migration intensity is high and the first-best amount of savings in the patient region is no greater than that in the impatient region, then the first-best interregional redistribution is from the patient region to the impatient region, as established in part (iii) of Proposition 3.1. As such, the feature of interregional redistribution obtained under exogenous and homogeneous savings decisions could be carried over to more general circumstances with endogenous and/or heterogeneous savings decisions.

In particular, if migration occurs with a probability of exactly 50%, then there is a stark difference between the baseline model and the current extension. In the first-best optimum of the baseline model, there should be no need to implement redistribution, whereas here redistribution is in need and should be from the impatient region to the patient region. The reason is that, departing from the baseline model in which these two regions have the same amount of savings and hence the same amount of period-2 income, here the patient region saves more than does the impatient region under $p = 1/2$, which yields that the net debt in the patient region should be smaller than that in the impatient region, namely $b^{A,FB} - k^{A,FB} > b^{B,FB} - k^{B,FB}$, provided that period-2 public consumption should be the same between these two regions. Therefore, given as shown in part (i) that the first-best period-1 public consumption is the same between these two regions, the unbiased center should redistribute resources from the impatient region to the patient region in period 1.

Under asymmetric information, we obtain the following:

Proposition 4.2 *Under Assumption 2.1, asymmetric information and endogenous savings, the following statements are true.*

- (i) *If $\mu_A > \mu_B \geq 0$, then (i-a) truth-telling requires that either $p \leq p_{A/B}^*$, or $p > p_{A/B}^*$ and $\delta^A/\delta^B > \delta^*$ should hold for thresholds $p_{A/B}^* \in (1/2, 1)$ and $\delta^* \in (0, 1)$, (i-b) the second-best optimum satisfies $G_1^{A*} > G_1^{B*}$, $c_1^{A*} > c_1^{B*}$, $G_2^{A*} < G_2^{B*}$, $c_2^{A,S*} < c_2^{B,S*}$ and $c_2^{A,M*} > c_2^{B,M*}$, and (i-c) if, in addition, $k^{A*} \geq k^{B*}$ for $p \leq 1/2$ or $k^{A*} \leq k^{B*}$ for $p \geq 1/2$, then $b^{A*} > b^{B*}$ holds true with $z^{A*} < 0 < z^{B*}$ for $p \geq \frac{\mu_B(\delta^B - \delta^A)}{\delta^B + \mu_B(\delta^B - \delta^A)}$.*
- (ii) *If $\mu_B > \mu_A \geq 0$, then (ii-a) truth-telling requires that either $p \geq p_{B/A}^*$, or $p < p_{B/A}^*$ and $\delta^A/\delta^B > \delta^*$ should hold for thresholds $p_{B/A}^* \in (1/2, 1)$ and $\delta^* \in (0, 1)$, and (ii-b) the second-best optimum satisfies $G_1^{A*} < G_1^{B*}$, $c_1^{A*} < c_1^{B*}$, $G_2^{A*} > G_2^{B*}$, $c_2^{A,S*} > c_2^{B,S*}$ and $c_2^{A,M*} < c_2^{B,M*}$, and (ii-c) if, in addition, $k^{A*} \geq k^{B*}$ for $p \geq 1/2$ or $k^{A*} \leq k^{B*}$ for $p \leq 1/2$, then $b^{A*} < b^{B*}$ holds true with $z^{B*} < 0 < z^{A*}$.*
- (iii) *If $\mu_A = \mu_B > 0$, then (iii-a) truth-telling requires that $p = p_{A/A}^*$ for threshold $p_{A/A}^* \in (1/2, 1)$, and (iii-b) the second-best optimum satisfies $G_1^{A*} = G_1^{B*}$, $c_1^{A*} = c_1^{B*}$, $G_2^{A*} = G_2^{B*}$, $c_2^{A,S*} = c_2^{B,S*}$, $c_2^{A,M*} = c_2^{B,M*}$, $k^{A*} < k^{B*}$, $b^{A*} > b^{B*}$ and $z^{A*} < 0 < z^{B*}$.*

The proof of this proposition is quite similar to those of Propositions 3.2 and 4.1, and hence is omitted to economize on the space.

We can compare Proposition 4.2 to Proposition 3.2 by adopting the similar logic behind the comparison between Proposition 4.1 and Proposition 3.1. Parts (i)-(ii) identify the sufficient conditions such that the features of second-best interregional redistribution shown in parts (i)-(ii) of Proposition 3.2 carry over to the current extension. These conditions mainly result from the indeterminacy of the relative magnitude of k^{A*} and k^{B*} . Analogous to part (iv) of Proposition 4.1, part (iii) considers the special case with $p = p_{A/A}^*$. It shows that the center should redistribute resources from the impatient region to the patient region, which is in stark contrast to that obtained in part (iii) of Proposition 3.2. As already shown in part (iii), the reason of such

a difference mainly lies in the endogenous heterogeneity of period-1 private savings between these two regions.

5 Concluding Summary

We enrich the two-region, two-period model of Huber and Runkel (2008) by allowing for between-jurisdiction mutual migrations to study optimal interregional redistribution policies under a benevolent federal government. Following Huber and Runkel, these two regions are assumed to differ only in the discount factor for period-2 utility (or utility when old). The federal government cannot observe the value of each region's discount factor, and hence the second-best interregional redistribution scheme is established by solving a mechanism-design problem.

We obtain three main findings in this context. First, in the benchmark of the first-best optimum, Huber and Runkel's results regarding the relative optimal debt level in the two regions and the direction of interregional redistribution continue to hold when migration intensity is low, but are reversed when migration intensity is high. Second, in the second-best optimum, their results continue to hold when either migration intensity is low or migration intensity is high and the regional difference in discounting is small; otherwise, their results are reversed. And third, their result that the optimal level of redistribution is smaller in the second-best optimum than in the first-best optimum continues to hold in the presence of mutual migration, regardless of the migration intensity.

We also consider a further extension with endogenous private savings in both regions. This consideration is indeed natural in a two-period life-cycle model. In particular, the regional difference in discount factor can now be interpreted as the difference in the spirit of capitalism à la Max Weber.⁷ However, due to the indeterminacy of the relative magnitude of endogenous private savings between these two regions, some additional assumptions are needed to establish clear-cut results. These additional assumptions could be interpreted as sufficient conditions such that the findings established in the baseline model carry over to the more general setting. To focus on the primary concern, we leave the discussion of the implementation of these optimal redistribution schemes to future research.

⁷For example, people born in some regions of China are regarded as having a stronger spirit of capitalism than do people born in other regions.

Finally, we argue that the novel result, namely that the patient region should borrow more than does the impatient region and the benevolent and unbiased center should redistribute from the patient region to the impatient region in period 1, is reasonable in the two-period life-cycle environment. Since the patient region values period-2 utility more than does the impatient region and the debt plus interest must be repayed in period 2, allowing the former to borrow more than does the latter is a fiscal-budget institutional arrangement that is likely to avoid over-borrowing of local governments. To achieve the social optimum, the center, whose goal is to maximize the sum of two heterogeneously discounted two-period utilities, should thus compensate the impatient region in period 1.

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Appendix: Proofs

Proof of Proposition 3.1. Using (4), (5) and Envelope Theorem, we have these FOCs:

$$\begin{aligned} g'_1(G_1^A) &= [\delta^A(1-p) + \delta^B p](1+r)g'_2(G_2^A) \\ g'_1(G_1^A) &= \lambda. \end{aligned} \tag{8}$$

By symmetry, we have another two FOCs:

$$\begin{aligned} g'_1(G_1^B) &= [\delta^B(1-p) + \delta^A p](1+r)g'_2(G_2^B) \\ g'_1(G_1^B) &= \lambda. \end{aligned} \tag{9}$$

It is immediate by (8) and (9) that $G_1^{A,FB} = G_1^{B,FB}$. This combined with (3) yields the desired assertion (i). Note that

$$[\delta^A(1-p) + \delta^B p] - [\delta^B(1-p) + \delta^A p] = (1-2p)(\delta^A - \delta^B),$$

then $p < 1/2$ combined with Assumption 2.1, (8) and (9) implies that $G_2^{A,FB} < G_2^{B,FB}$. This combined with (3) yields $c_2^{A,FB} < c_2^{B,FB}$. It follows from period-2 public budget constraint that $0 > G_2^A - G_2^B = (b^B - b^A)(1+r) + (c_2^B - c_2^A)$, which yields $(b^A - b^B)(1+r) > (c_2^B - c_2^A) > 0$, and hence $b^{A,FB} > b^{B,FB}$. This combined with period-1 public budget constraint, assertion (i) and federal budget constraint (2) implies that $z^{A,FB} < 0 < z^{B,FB}$. Therefore, we get from (4) and assertion (i) that

$$\begin{aligned} &V(b^{A,FB}, z^{A,FB}, \delta^A; b^{B,FB}) - V(b^{B,FB}, z^{B,FB}, \delta^A; b^{B,FB}) \\ &= \delta^A(1-p) \left[u_2(c_2^{A,FB}) + g_2(G_2^{A,FB}) - u_2(c_2^{B,FB}) - g_2(G_2^{B,FB}) \right] < 0 \end{aligned}$$

and by symmetry,

$$\begin{aligned} &V(b^{B,FB}, z^{B,FB}, \delta^B; b^{A,FB}) - V(b^{A,FB}, z^{A,FB}, \delta^B; b^{A,FB}) \\ &= \delta^B(1-p) \left[u_2(c_2^{B,FB}) + g_2(G_2^{B,FB}) - u_2(c_2^{A,FB}) - g_2(G_2^{A,FB}) \right] > 0. \end{aligned}$$

The proof of assertion (ii) is thus complete. Since assertions (iii)-(iv) can be similarly proved, we thus omit them to economize on the space. ■

Proof of Proposition 3.2. Using (4), (5) and Envelope Theorem, we have these FOCs:

$$\begin{aligned}\frac{g'_1(G_1^A)}{(1+r)g'_2(G_2^A)} &= \frac{(1+\mu_A)(1-p)\delta^A + [(1+\mu_B)p - \mu_B]\delta^B}{1+\mu_A - \mu_B} \\ g'_1(G_1^A) &= \frac{\lambda}{1+\mu_A - \mu_B}.\end{aligned}\tag{10}$$

By symmetry, we have another two FOCs:

$$\begin{aligned}\frac{g'_1(G_1^B)}{(1+r)g'_2(G_2^B)} &= \frac{(1+\mu_B)(1-p)\delta^B + [(1+\mu_A)p - \mu_A]\delta^A}{1+\mu_B - \mu_A} \\ g'_1(G_1^B) &= \frac{\lambda}{1+\mu_B - \mu_A}.\end{aligned}\tag{11}$$

It follows from Proposition 3.1 that truth-telling generally requires that μ_A and μ_B cannot be zero at the same time. We thus just need to consider these three cases shown in Proposition 3.2.

Consider first the case with $\mu_A > \mu_B \geq 0$, then it is immediate by (10), (11) and $\lambda > 0$ that $\mu_A - \mu_B < 1$ and $G_1^{A*} > G_1^{B*}$. Then, using (3) produces $c_1^{A*} > c_1^{B*}$. These results combined with (4) reveal that

$$\begin{aligned}0 &= V(b^{A*}, z^{A*}, \delta^A; b^{B*}) - V(b^{B*}, z^{B*}, \delta^A; b^{B*}) \\ &= \underbrace{[u_1(c_1^{A*}) - u_1(c_1^{B*})] + [g_1(G_1^{A*}) - g_1(G_1^{B*})]}_{>0} \\ &\quad + \delta^A(1-p) [u_2(c_2^{A*}) + g_2(G_2^{A*}) - u_2(c_2^{B*}) - g_2(G_2^{B*})].\end{aligned}\tag{12}$$

By symmetry, we have

$$\begin{aligned}0 &\leq V(b^{B*}, z^{B*}, \delta^B; b^{A*}) - V(b^{A*}, z^{A*}, \delta^B; b^{A*}) \\ &= \underbrace{[u_1(c_1^{B*}) - u_1(c_1^{A*})] + [g_1(G_1^{B*}) - g_1(G_1^{A*})]}_{<0} \\ &\quad + \delta^B(1-p) [u_2(c_2^{B*}) + g_2(G_2^{B*}) - u_2(c_2^{A*}) - g_2(G_2^{A*})].\end{aligned}\tag{13}$$

We thus get from (12) and (13) that $G_2^{A*} < G_2^{B*}$. By (3), we have $c_2^{A*} < c_2^{B*}$. In addition, we must have

$$\frac{(1+\mu_A)(1-p)\delta^A + [(1+\mu_B)p - \mu_B]\delta^B}{1+\mu_A - \mu_B} < \frac{(1+\mu_B)(1-p)\delta^B + [(1+\mu_A)p - \mu_A]\delta^A}{1+\mu_B - \mu_A}$$

by (10) and (11). This inequality can be equivalently simplified as

$$[(1 - 2p)(1 + \mu_A) + \mu_B] \delta^A < [(1 - 2p)(1 + \mu_B) + \mu_A] \delta^B. \quad (14)$$

We just need to figure out the conditions such that (14) holds. Note that

$$(1 - 2p)(1 + \mu_B) + \mu_A - [(1 - 2p)(1 + \mu_A) + \mu_B] = 2p(\mu_A - \mu_B) > 0$$

and

$$(1 - 2p)(1 + \mu_A) + \mu_B \geq 0 \Leftrightarrow p \leq \frac{1}{2} + \frac{\mu_B}{2(1 + \mu_A)}, \quad (15)$$

thus (14) automatically holds true under (15) and Assumption 2.1. If, otherwise, $p > \frac{1}{2} + \frac{\mu_B}{2(1 + \mu_A)}$, then using (14) yields

$$\frac{\delta^A}{\delta^B} > \frac{1 + \mu_A + \mu_B - 2p(1 + \mu_B)}{1 + \mu_A + \mu_B - 2p(1 + \mu_A)} \equiv \delta^*.$$

Note that

$$1 + \mu_A + \mu_B - 2p(1 + \mu_B) \geq 0 \Leftrightarrow p \leq \frac{1}{2} + \frac{\mu_A}{2(1 + \mu_B)} \equiv p_{A/B}^*$$

and

$$p_{A/B}^* > \frac{1}{2} + \frac{\mu_B}{2(1 + \mu_A)},$$

so $\delta^* \leq 0$ and hence (14) automatically holds true whenever $p \leq p_{A/B}^*$. If $p > p_{A/B}^*$, then $\delta^* \in (0, 1)$ and (14) holds true under another condition, namely $\delta^A/\delta^B > \delta^*$, as desired in part (i) of this proposition. Also, it is easy to get $b^{A^*} > b^{B^*}$ from $0 > G_2^{A^*} - G_2^{B^*} = (b^{B^*} - b^{A^*})(1 + r) + (c_2^{B^*} - c_2^{A^*})$.

To complete the proof of part (i), we need to characterize the second-best interregional redistribution. First, using Envelope Theorem and (4), we get

$$\left. \frac{dz^A}{db^A} \right|_{dV=0} = \frac{\delta^A(1 - p)(1 + r)g_2'(G_2^A) - g_1'(G_1^A)}{g_1'(G_1^A)}.$$

And also, using (3) and Implicit Function Theorem gives that

$$\frac{\partial c_1^A}{\partial b^A} = \frac{\partial c_1^A}{\partial z^A} = \frac{g_1''(G_1^A)}{u_1''(c_1^A) + g_1''(G_1^A)} \quad \text{and} \quad \frac{\partial c_2^A}{\partial b^A} = -\frac{(1 + r)g_2''(G_2^A)}{u_2''(c_2^A) + g_2''(G_2^A)}.$$

We thus obtain by simplifying the algebra that

$$\left. \frac{d^2 z^A}{d(b^A)^2} \right|_{dV=0} = - \frac{\delta^A(1-p)(1+r)^2 g_2''(G_2^A) u_2''(c_2^A)}{g_1'(G_1^A)[u_2''(c_2^A) + g_2''(G_2^A)]} - \frac{[\delta^A(1-p)(1+r)g_2'(G_2^A)]^2 g_1''(G_1^A) u_1''(c_1^A)}{[g_1'(G_1^A)]^3 [u_1''(c_1^A) + g_1''(G_1^A)]} > 0, \quad (16)$$

so the welfare indifference curve of region A is strictly convex and U-shaped in the (z, b) -space, and the minimum is achieved at $\delta^A(1-p)(1+r)g_2'(G_2^A) = g_1'(G_1^A)$. Second, it follows from (12) that (z^{A*}, b^{A*}) and (z^{B*}, b^{B*}) lie on the same welfare indifference curve of region A . Third, noting from (10) that

$$\begin{aligned} & \frac{(1 + \mu_A)(1 - p)\delta^A + [(1 + \mu_B)p - \mu_B]\delta^B}{1 + \mu_A - \mu_B} - (1 - p)\delta^A \\ &= \frac{\mu_B(1 - p)(\delta^A - \delta^B) + p\delta^B}{1 + \mu_A - \mu_B} \geq 0 \Leftrightarrow p \geq \frac{\mu_B(\delta^B - \delta^A)}{\delta^B + \mu_B(\delta^B - \delta^A)}, \end{aligned}$$

so (16) implies that both (z^{A*}, b^{A*}) and (z^{B*}, b^{B*}) lie on the decreasing part of the welfare indifference curve of region A whenever $p \geq \frac{\mu_B(\delta^B - \delta^A)}{\delta^B + \mu_B(\delta^B - \delta^A)}$. We, therefore, obtain $z^{A*} < 0 < z^{B*}$ by using $b^{A*} > b^{B*}$.

Assertions in parts (ii) and (iii) can be similarly proved with the thresholds given by

$$p_{B/A}^* \equiv \frac{1}{2} + \frac{\mu_B}{2(1 + \mu_A)} \quad \text{and} \quad p_{A/A}^* \equiv \frac{1}{2} + \frac{\mu_A}{2(1 + \mu_A)},$$

as exactly desired in parts (ii) and (iii), respectively. In particular, to obtain the characterization of optimal interregional redistribution shown in part (ii), we just need to note from (11) that

$$\begin{aligned} & \frac{(1 + \mu_B)(1 - p)\delta^B + [(1 + \mu_A)p - \mu_A]\delta^A}{1 + \mu_B - \mu_A} - (1 - p)\delta^B \\ &= \frac{\mu_A(1 - p)(\delta^B - \delta^A) + p\delta^A}{1 + \mu_B - \mu_A} > 0 \end{aligned}$$

under Assumption 2.1. That is, an application of the counterpart of (16) yields that both (z^{A*}, b^{A*}) and (z^{B*}, b^{B*}) lie on the decreasing part of the welfare indifference curve of region B , which immediately produces the desired interregional redistribution in part (ii). ■

Proof of Proposition 3.3. We just show the proof of part (i) because that of part (ii) can be similarly obtained. If $\mu_A > \mu_B$, then we get from (8) and (10) that

$$g'_1(G_1^{A*}) = \frac{\lambda}{1 + \mu_A - \mu_B} < \lambda = g'_1(G_1^{A,FB}) \Rightarrow G_1^{A*} > G_1^{A,FB},$$

which combined with (3) yields $c_1^{A*} > c_1^{A,FB}$. Making use of (8) and (10) again reveals that

$$\begin{aligned} \frac{\lambda}{(1+r)g'_2(G_2^{A,FB})} &= \delta^A(1-p) + \delta^B p \\ \frac{\lambda}{(1+r)g'_2(G_2^{A*})} &= \delta^A(1-p) + \delta^B p + (1-p)(\mu_A \delta^A - \mu_B \delta^B). \end{aligned}$$

Letting $\mu_A \delta^A > \mu_B \delta^B$, then it is straightforward that $G_2^{A*} > G_2^{A,FB}$ and $c_2^{A*} > c_2^{A,FB}$. Note that $0 < G_2^{A*} - G_2^{A,FB} = (b^{A,FB} - b^{A*})(1+r) + (c_2^{A,FB} - c_2^{A*})$, we thus have $(b^{A,FB} - b^{A*})(1+r) > c_2^{A*} - c_2^{A,FB} > 0$, as desired. Similarly, note that $0 < G_1^{A*} - G_1^{A,FB} = (b^{A*} - b^{A,FB}) + (z^{A*} - z^{A,FB}) + (c_1^{A,FB} - c_1^{A*})$, we thus immediately obtain $z^{A*} - z^{A,FB} > b^{A,FB} - b^{A*} + c_1^{A*} - c_1^{A,FB} > 0$, which combined with Propositions 3.1 and 3.2 gives rise to the desired assertion.

It follows from (9) and (11) that

$$g'_1(G_1^{B*}) = \frac{\lambda}{1 - (\mu_A - \mu_B)} > \lambda = g'_1(G_1^{B,FB}) \Rightarrow G_1^{B*} < G_1^{B,FB},$$

which combined with (3) yields $c_1^{B*} < c_1^{B,FB}$. Making use of (9) and (11) again reveals that

$$\begin{aligned} \frac{\lambda}{(1+r)g'_2(G_2^{B,FB})} &= \delta^B(1-p) + \delta^A p \\ \frac{\lambda}{(1+r)g'_2(G_2^{B*})} &= \delta^B(1-p) + \delta^A p + (1-p)(\mu_B \delta^B - \mu_A \delta^A). \end{aligned}$$

Under the restriction that $\mu_A \delta^A > \mu_B \delta^B$, we obtain $G_2^{B*} < G_2^{B,FB}$ and $c_2^{B*} < c_2^{B,FB}$. Note that $0 > G_2^{B*} - G_2^{B,FB} = (b^{B,FB} - b^{B*})(1+r) + (c_2^{B,FB} - c_2^{B*})$, we thus have $(b^{B,FB} - b^{B*})(1+r) < c_2^{B*} - c_2^{B,FB} < 0$, as desired. Similarly, note that $0 > G_1^{B*} - G_1^{B,FB} = (b^{B*} - b^{B,FB}) + (z^{B*} - z^{B,FB}) + (c_1^{B,FB} - c_1^{B*})$, we thus immediately obtain $z^{B*} - z^{B,FB} < b^{B,FB} - b^{B*} + c_1^{B*} - c_1^{B,FB} < 0$, which

combined with Propositions 3.1 and 3.2 gives rise to the desired assertion. ■

Proof of Proposition 4.1. Letting $\mu_A = \mu_B = 0$ in the Lagrangian (5). Using FOCs (7), (8) and (9), part (i) is immediate.

If $p < 1/2$, then $G_2^{A,FB} < G_2^{B,FB}$ follows from applying Assumption 2.1 to (8) and (9), and also $c_2^{A,S,FB} < c_2^{B,S,FB}$ and $c_2^{A,M,FB} > c_2^{B,M,FB}$ follow from applying $G_2^{A,FB} < G_2^{B,FB}$ to (7). These combined with (6) yield that

$$\begin{aligned}
0 &> G_2^{A,FB} - G_2^{B,FB} \\
&= (1 - 2p)(1 + r)(k^{A,FB} - k^{B,FB}) + (1 + r)(b^{B,FB} - b^{A,FB}) \\
&\quad + (1 - p)\underbrace{(c_2^{B,S,FB} - c_2^{A,S,FB})}_{>0} + p\underbrace{(c_2^{A,M,FB} - c_2^{B,M,FB})}_{>0} \\
&= (1 + r) [(b^{B,FB} - k^{B,FB}) - (b^{A,FB} - k^{A,FB})] + 2p(1 + r)(k^{B,FB} - k^{A,FB}) \\
&\quad + (1 - p)\underbrace{(c_2^{B,S,FB} - c_2^{A,S,FB})}_{>0} + p\underbrace{(c_2^{A,M,FB} - c_2^{B,M,FB})}_{>0},
\end{aligned} \tag{17}$$

which implies that $b^{B,FB} - k^{B,FB} < b^{A,FB} - k^{A,FB}$ whenever $k^{B,FB} \geq k^{A,FB}$ holds. Noting from part (i) that

$$0 = G_1^{A,FB} - G_1^{B,FB} = [(b^{A,FB} - k^{A,FB}) - (b^{B,FB} - k^{B,FB})] + (z^{A,FB} - z^{B,FB}), \tag{18}$$

we thus have $z^{A,FB} < 0 < z^{B,FB}$ whenever $k^{B,FB} \geq k^{A,FB}$ holds. Moreover, we can get

$$\begin{aligned}
&V(b^{A,FB}, z^{A,FB}, \delta^A; b^{B,FB}) - V(b^{B,FB}, z^{B,FB}, \delta^A; b^{B,FB}) \\
&= \delta^A p \underbrace{[u_2(c_2^{A,M,FB}) - u_2(c_2^{\tilde{A},M,FB})]}_{=0} \\
&\quad + \delta^A (1 - p) \underbrace{[u_2(c_2^{A,S,FB}) + g_2(G_2^{A,FB}) - u_2(c_2^{B,S,FB}) - g_2(G_2^{\tilde{A},S,FB})]}_{<0},
\end{aligned}$$

in which \tilde{A} denotes region A under misreporting/mimicing and we have used individual FOCs, namely

$$\begin{aligned}
u_2'(c_2^{A,S,FB}) &= (1 - p)g_2'(G_2^{A,FB}) > (1 - p)g_2'(G_2^{B,FB}) = u_2'(c_2^{\tilde{A},S,FB}), \\
u_2'(c_2^{A,M,FB}) &= pg_2'(G_2^{B,FB}) = pg_2'(G_2^{A,FB}) = u_2'(c_2^{\tilde{A},M,FB}).
\end{aligned}$$

By symmetry, we obtain

$$\begin{aligned} & V(b^{B,FB}, z^{B,FB}, \delta^B; b^{A,FB}) - V(b^{A,FB}, z^{A,FB}, \delta^B; b^{A,FB}) \\ &= \delta^B(1-p) \left[u_2(c_2^{B,S,FB}) + g_2(G_2^{B,FB}) - u_2(c_2^{\tilde{B},S,FB}) - g_2(G_2^{A,FB}) \right] > 0. \end{aligned}$$

The proof of part (ii) is thus complete.

The proof of part (iii) is omitted due to similarity. For part (iv), we just need to show that $k^{B,FB} > k^{A,FB}$. We get from the last equation of (7) that $4g_1'(G_1^{A,FB}) = \delta^A(1+r)[g_2'(G_2^{A,FB}) + g_2'(G_2^{B,FB})]$. By symmetry, we obtain $4g_1'(G_1^{B,FB}) = \delta^B(1+r)[g_2'(G_2^{A,FB}) + g_2'(G_2^{B,FB})]$ for region B . By part (i), we thus obtain $4g_1'(G_1^{A,FB}) = \delta^B(1+r)[g_2'(G_2^{A,FB}) + g_2'(G_2^{B,FB})]$ for region B . Differentiating on both sides of the equation for region A with respect to δ^A shows that

$$\frac{dk^{A,FB}}{d\delta^A} = -\frac{(1+r)[g_2'(G_2^{A,FB}) + g_2'(G_2^{B,FB})]}{4g_1''(G_1^{A,FB}) + [\delta^A(1+r)^2/2][g_2''(G_2^{A,FB}) + g_2''(G_2^{B,FB})]} > 0,$$

by which and Assumption 2.1 it is immediate that $k^{B,FB} > k^{A,FB}$, as desired. Applying (17) and (18) again with setting $G_2^{A,FB} = G_2^{B,FB}$, then the proof of part (iv) is complete. ■