Priority-Based Affirmative Action in School Choice

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Abstract

This paper investigates the affirmative action in school choice problems. We show that the student-proposing deferred acceptance (henceforth, DA) algorithm is (stable and) *minimally responsive* to the priority-based affirmative action if schools have the same priority orders over students, while no such a mechanism exists for quota-based or reserve-based affirmative action even if all schools have the same priorities. We also show that the DA algorithm makes *no minority student* worse off under a stronger priority-based affirmative action policy where the minority is given full priority. Moreover, under the DA algorithm, if a market gives full priority to the minority, then the market and a stronger priority-based (and reserve-based) affirmative action produce the *same* assignment outcome. Finally, we consider two efficiency improved mechanisms–Kesten's efficiency-adjusted deferred acceptance mechanism (EADAM) and DA-TTC mechanism. We show that neither EADAM nor DA-TTC respects improvements (in the sense of Balinski and Sönmez (1999)), and consequently, neither of the two mechanisms is minimally responsive to the priority-based affirmative action policy.

Keywords: School choice; Affirmative action; Minimal responsiveness; Deferred acceptance algorithm

JEL classification: C78, D61, D71, D78

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1 Introduction

School choice programs aim to give students the option to choose their school. At the same time, underrepresented students should be favored to close the opportunity gap. Affirmative action policies that give minority students higher chances to attend their desired schools have been playing an important role in achieving this goal in the Unites States and many other countries.

There are three popular affirmative action policies in school choice: *quota-based, reserve-based, and priority-based* affirmative actions. The *quota-based* (or *majority-quotas* type) affirmative action policy in school choice gives minority students higher chances to attend more preferred schools by *limiting the number of admitted majority students* at some schools. There are many examples of majority-quotas type public school admission policies in the United States.[1] The *reserve-based* affirmative action policy proposed by Hafalir et al. (2013) is to *reserve some seats at each school for the minority students*, and to require that a reserved seat at a school be assigned to a majority student only if no minority student prefers that school to her assignment. The *priority-based* affirmative action favors minority students by means of *promoting their priorities at schools*. In Chinese college admissions, the minority students are favored by a priority-based affirmative action policy that awards bonus points to minority students in the national college entrance exam. However, each of the three types of policies has perverse consequence under stable matching mechanism. On the full domain of school choice problem, every stable assignment mechanism suffers from the following difficulty: a stronger affirmative action policy may not benefit any minority student but hurt some minority students.

As for many other matching problems, it is primarily important to study the *stable* assignment outcome for school choice problem. The stable assignment problems for school choice without affirmative action policies have been extensively studied since a seminal paper of Abdulkadiroğlu and Sönmez (2003). The stable assignment requires that it be *individually rational, non-wasteful and satisfy the no-justified-envy property*. Specifically, individual rationality means that no student is assigned to an unacceptable school. Non-wastefulness means that there should be no student-school pair such that the student prefers the school to her assignment and the school has a vacant seat. The no-justified-

[1] See, for instance, Hafalir et al. (2013), Ehlers et al. (2014) and Doğan (2016) for detailed discussions, including disadvantages of the policy, such as avoidable inefficiency.

envy property requires that there should be no student-school pair (s, c) such that student s prefers school c to her assignment and school c admits another student who has lower priority than s at school c.

Since the affirmative action policies in school choice aim to improve the welfare of the minority students, the *minimal* requirement for a satisfactory assignment mechanism should be: running such an assignment mechanism, a stronger affirmative action makes *at least one* minority student better off in case there is a minority student who becomes worse off. In other words, a stronger affirmative action should not hurt a minority student without benefiting other minority students. If an assignment mechanism satisfies this property, then we say that it is *minimally responsive to the affirmative action*, or simply *minimally responsive*.[2]

For school choice with affirmative action, it is comparatively ideal to find a stable and minimally responsive assignment mechanism from the mechanism-design perspective. Unfortunately, it turns out that no assignment mechanism is *stable and minimally responsive* for any of the three types of affirmative actions on full domain. Kojima (2012) shows that there is no stable and minimally responsive assignment mechanism for quota-based or priority-based affirmative action problem. Hafalir et al. (2013) obtain the impossibility result for school choice with reserve-based affirmative action. Of course, these impossibility results are on the full domain of school choice problems. One can expect positive result if the domain of school choice problems is restricted in certain ways. Indeed, Doğan (2016) considers a restricted domain of school choice problem where a market *gives full priority to the minority* (in the sense that, at each school c, either each minority student is one of its q_c highest-priority students, or each minority student has higher priority than each majority student). He shows that, for school choice with quota-based affirmative action, a stable and minimally responsive mechanism exists if and only if the market gives full priority to the minority. We follow along this direction and study the *minimal responsiveness of the DA algorithm* for school choice with affirmative action under different restricted domains. For mechanisms satisfying relatively weak stability, we obtain two negative results by constructing a counterexample.

[2]Kojima (2012) first introduces this concept. He calls it "respecting the spirit of affirmative action". Recently, Doğan (2016) starts to use the notion of "minimal responsiveness".

1.1 Main results

This paper mainly investigates school choice with priority-based affirmative action. Our first result (Theorem 1) shows that the DA algorithm (proposed by Gale and Shapley (1962)) is *minimally responsive* to the priority-based affirmative action if all schools have the *same* priority orders over students. It is well known that the DA algorithm is a stable mechanism. We note that, for quota-based or reserve-based affirmative action, there is no stable and minimally responsive mechanism even if all schools have the same priorities over students. In some countries, such as China (Chen and Kesten (2011)), India (Bertrand et al. (2010)) and Turkey (Balinski and Sönmez (1999)), and many schools in the United States (Abdulkadiroğlu et al. (2005)), students take a centralized exam that determines common school priorities over students. In such matching environments, the priority-based affirmative action is more suitable to achieve the goal of favoring the minority students (at least in the sense of minimal responsiveness).

Our second result (Theorem 2) is concerned with the priority-based affirmative action on the restricted domain where the matching market *gives full priority to the minority*. As mentioned above, Doğan (2016) proposes the condition "market giving full priority to the minority" and shows that, for quota-based affirmative action, this condition is necessary and sufficient for the existence of a stable and minimally responsive assignment mechanism. For priority-based affirmative action, we show that, if a stronger affirmative action market gives full priority to the minority, then under the DA algorithm the stronger affirmative action policy makes *each minority student* weakly better off. That is, under the DA algorithm, the stronger affirmative action policy makes no minority student worse off. This property is undoubtedly attractive when an affirmative action aims to improve the Pareto welfare of the minority students.

Our third result (Theorem 3) considers the case that a market gives full priority to the minority. We show that, under the DA algorithm, the given market and a stronger priority-based (reserve-based) affirmative action produce the *same* assignment outcome. Moreover, under the DA algorithm, the given market and a stronger quota-based affirmative action produce the *same* assignment outcome for the *minority* students, while the stronger quota-based affirmative action makes the majority students weakly worse off (than the assignment outcome of the given market). Thus, a quota-based affirmative action would result in avoidable efficiency loss and, both priority-based and reserve-based affirmative actions do not play an actual role under the DA algorithm.

According to Kojima (2012) and Hafalir et al. (2013), we know that, on the full domain of school choice problems, there exists no stable mechanism that is minimally responsive to any type of the three popular affirmative action policies. A natural question is: If we relax the stability requirement, can we obtain a desirable result. Afacan and Salman (2016) show that the Boston mechanism (which is not stable) is minimally responsive to the priority-based and reserve-based affirmative action policies. We partly answer this question. We consider two unstable mechanisms–Kesten's EADAM and DA-TTC mechanism. The prominent advantage of these two mechanisms is that they can improve the efficiency based on the DA matching. We show that neither EADAM nor DA-TTC mechanism respects improvements (Theorems 4 and 5). And consequently, these two mechanisms are not minimally responsive to the priority-based affirmative action. Furthermore, DA-TTC mechanism is also not minimally responsive to the quota-based (resp. reserve-based) affirmative action.

1.2 Related literature

For school choice with type-specific quotas, Abdulkadiroğlu and Sönmez (2003) propose a modified version of the DA algorithm and discuss the related fairness and strategic properties. These results can be applied to school choice with quota-based affirmative action. However, the quota-based affirmative action policy may result in avoidable efficiency loss. To overcome this difficulty, Hafalir et al. (2013) propose reserve-based affirmative action and a solution called "the deferred acceptance algorithm with minority reserves". More generally, Ehlers et al. (2014) study quota-based affirmative action policies when there are both upper and lower type-specific bounds, and allowing for more than two types of students. For the case of soft bounds, they propose a solution that is similar to the DA algorithm with minority reserves. For problems with hard upper and lower type-specific bounds, Fragiadakis and Troyan (2017) introduce the dynamic quotas DA algorithms (DQDA) that result in Pareto superior allocations. The DQDA dynamically incorporates the information of the submitted preferences into the procedure of the algorithm. On the experimental side, Klijn et al. (2016) study the strategic behavior of students under quota-based and reserve-based affirmative action policies. There are a series of papers, such as Echenique and Yenmez (2015), Erdil and Kumano (2012), Kominers and Sönmez (2016), and Westkamp (2013) that investigate the affirmative action problem by considering the priorities or choices of the schools.

Kojima (2012) first deals with the minimal responsiveness of affirmative actions and presents impossibility results for quota-based and priority-based cases. Specifically, Kojima (2012) shows that there exists no stable and minimally responsive mechanism for quota-based or priority-based affirmative action. To avoid the inefficiency caused by majority quotas, Hafalir et al. (2013) propose the reserve-based affirmative action. However, Hafalir et al. (2013) provide an example to illustrate that there is no stable and minimally responsive mechanism for the reserve-based affirmative action. Given these impossibility results, the further research on minimal responsiveness of affirmative action problem follows along two directions. One is to relax the stability requirement of assignment mechanism, and the other is to restrict the domain to some particular matching environment. For the first direction, Afacan and Salman (2016) show that the Boston mechanism is minimally responsive to reserve-based and priority-based affirmative actions. However, it is well known that the Boston mechanism satisfies neither the strategy-proofness nor the no-justified-envy property. More recently, Doğan (2016) proposes a modified DA algorithm with minority reserves. He also shows that the proposed algorithm is minimally responsive to the reserve-based affirmative action and satisfies certain fairness property. For the second direction, Doğan (2016) proposes a restricted domain on school choice problem where schools give full priority to the minority. He shows that, for quota-based affirmative action problem, there is a stable and minimally responsive assignment mechanism if and only if the market gives full priority to the minority. This paper mainly follows along the second direction and investigates the minimal responsiveness of assignment mechanisms on certain restricted domains. For the first direction, we check the efficiency-adjusted deferred acceptance mechanism (EADAM) proposed by Kesten (2010) and DA-TTC mechanism and show that both of them are not minimally responsive to the priority-based affirmative action.

The remainder of the paper is organized as follows. We present some preliminaries on the formal model in the next section. Section 3 investigates the minimal responsiveness of the DA algorithms on particular restricted domains. Section 4 presents further impossibility results by providing a counterexample. Section 5 concludes. Several technical proofs are provided in the Appendix.

2 The Model

2.1 Settings

Let S and C be finite and disjoint sets of students and schools. There are two types of students: minority students and majority students. Let S^m and S^M denote the sets of minority and majority students, respectively. They are nonempty sets such that $S^m \cup S^M = S$ and $S^m \cap S^M = \emptyset$. Suppose that $|C|, |S| \geq 2$.

For each student $s \in S$, $\succ_s$ is a strict (i.e., complete, transitive, and anti-symmetric) preference relation over $C \cup \{s\}$, where s denotes the outside option, which can be attending a private school or being home-schooled. School c is *acceptable* to student s if $c \succ_s s$. The list of preferences for a group of students S' is denoted by $\succ_{S'} = (\succ_s)_{s \in S'}$. For each school $c \in C$, $\succ_c$ is a strict priority order over S. The list of priorities for a group of schools C' is denoted by $\succ_{C'} = (\succ_c)_{c \in C'}$. For agents i, j in C (resp. S) and k in S (resp. C), $i \succeq_k j$ denotes either $i \succ_k j$ or $i = j$.

For each $c \in C$, q_c is the capacity of c or the number of seats in c. We assume that there are enough seats for all students, so $\sum_{c \in C} q_c \geq |S|$. Let $q = (q_c)_{c \in C}$ be the capacity profile.

A *school choice problem* or simply a *problem* is a tuple $G = (S^m, S^M, C, \succ_S, \succ_C, q)$. Since S^m, S^M, C and q will be fixed, we also denote a problem by $(\succ_S, \succ_C)$. For school choice with *quota-based affirmative action*, there is an affirmative action parameter vector $q^M = (q_c^M)_{c \in C}$, where q_c^M represents the type-specific capacity for majority students at school c and $q_c^M \leq q_c$ for each $c \in C$. We denote such a problem by $G^q = (S^m, S^M, C, \succ_S, \succ_C, q, q^M)$, or $G^q = (\succ_S, \succ_C, q^M)$ in short. If $q_c^M = q_c$ for every $c \in C$, then the problem $G^q = (S^m, S^M, C, \succ_S, \succ_C, q, q^M = q)$ reduces to the problem without affirmative action $G = (S^m, S^M, C, \succ_S, \succ_C, q)$. For two affirmative action parameter vectors $q^M = (q_c^M)_{c \in C}$ and $\tilde{q}^M = (\tilde{q}_c^M)_{c \in C}$, if $\tilde{q}_c^M \leq q_c^M$ for every $c \in C$, then problem $\tilde{G}^q = (S^m, S^M, C, \succ_S, \succ_C, q, \tilde{q}^M)$ is said to *have a stronger quota-based affirmative action policy* than $G^q = (S^m, S^M, C, \succ_S, \succ_C, q, q^M)$.

For school choice with *reserve-based affirmative action*, there is an affirmative action parameter vector $r^m = (r_c^m)_{c \in C}$, where r_c^m represents the number of seats at school c at which the minority students are favored, and $r_c^m \leq q_c$ for each $c \in C$. We denote such a problem by $G^r = (S^m, S^M, C, \succ_S$

$, \succ_C, q, r^m)$, or $G^r = (\succ_S, \succ_C, r^m)$. For this problem, each school c is assigned a minority reserve r_c^m such that if the number of minority students admitted to c is less than r_c^m, then any minority student has higher priority than any majority student in c. If $r_c^m = 0$ for every $c \in C$, then the problem $G^r = (S^m, S^M, C, \succ_S, \succ_C, q, r^m = 0)$ reduces to the problem without affirmative action $G = (S^m, S^M, C, \succ_S, \succ_C, q)$. For two affirmative action parameter vectors $r^m = (r_c^m)_{c \in C}$ and $\tilde{r}^m = (\tilde{r}_c^m)_{c \in C}$, if $\tilde{r}_c^m \geq r_c^m$ for every $c \in C$, then problem $\tilde{G}^r = (S^m, S^M, C, \succ_S, \succ_C, q, \tilde{r}^m)$ is said to *have a stronger reserve-based affirmative action policy* than $G^r = (S^m, S^M, C, \succ_S, \succ_C, q, r^m)$.

For *priority-based affirmative action* problem, we say that $G' = (S^m, S^M, C, \succ_S, \succ'_C, q)$ *has a stronger priority-based affirmative action policy* than $G = (S^m, S^M, C, \succ_S, \succ_C, q)$ if, for every $c \in C$ and $s, s' \in S$, (1) if $s \succeq_c s'$ and $s \in S^m$ then $s \succeq'_c s'$, (2) if $s, s' \in S^M$ and $s \succeq_c s'$, then $s \succeq'_c s'$. In other words, a stronger priority-based affirmative action policy promotes the ranking of a minority student at schools relative to majority students while keeping the relative ranking of each student within her own group fixed.

A matching is an assignment of students to schools such that each student is assigned to a school or to her outside option, and no more students are assigned to a school than its capacity. Formally, a *matching* μ is a mapping from $C \cup S$ to the subsets of $C \cup S$ such that

(1) $\mu(s) \in C \cup \{s\}$ for every $s \in S$,

(2) $\mu(c) \subseteq S$ and $|\mu(c)| \leq q_c$ for every $c \in C$,

(3) $\mu(s) = c$ if and only if $s \in \mu(c)$ for every $c \in C$ and $s \in S$.

Let μ and μ' be matchings. We write $\mu \succ_s \mu'$ (resp. $\mu \succeq_s \mu'$) if $\mu(s) \succ_s \mu'(s)$ (resp. $\mu(s) \succeq_s \mu'(s)$) for $s \in S$. Let $\succ_S$ be a preference profile. If $\mu \succeq_s \mu'$ for all $s \in S$ and $\mu \succ_s \mu'$ for at least one $s \in S$, we say that μ *Pareto dominates* μ'. Affirmative action policies are implemented to improve the assignment of minority students, sometimes at the expense of majority students. Therefore, we also need an efficiency concept to analyze the welfare of the minority students. If $\mu \succeq_s \mu'$ for all $s \in S^m$ and $\mu \succ_s \mu'$ for at least one $s \in S^m$, then we say that μ *Pareto dominates* μ' *for the minority*.

2.2 Stable matching

We consider the notion of stable matching for three cases: no affirmative action, quota-based and reserve-based affirmative actions.

For school choice problem without affirmative action $G = (S^m, S^M, C, \succ_S, \succ_C, q)$, a matching is *stable* if it is individually rational, non-wasteful and satisfies no-justified-envy property. Formally, a matching μ is *stable* if the following conditions are satisfied:

(1) Individual rationality: $\mu(s) \succeq_s s$ for all $s \in S$.

(2) Non-wastefulness: If there are $s \in S$ and $c \in C$ such that $c \succ_s \mu(s)$, then $|\mu(c)| = q_c$.

(3) No-justified-envy: If there are $s \in S$ and $c \in C$ such that $c \succ_s \mu(s)$, then $s' \succ_c s$ for all $s' \in \mu(c)$.

For school choice problem with quota-based affirmative action $G^q = (S^m, S^M, C, \succ_S, \succ_C, q, q^M)$, it is implemented by prohibiting school c to admit more than q_c^M majority students. A matching μ^q is stable with respect to quota-based affirmative action, or simply *stable with quota*, if the following conditions are satisfied:

(1) Individual rationality: $\mu^q(s) \succeq_s s$ for all $s \in S$.

(2) Non-wastefulness: If there are $s \in S$ and $c \in C$ such that $c \succ_s \mu^q(s)$, then either $s \in S^m$ and $|\mu^q(c)| = q_c$ or $s \in S^M$ and $|\mu^q(c) \cap S^M| = q_c^M$.

(3) No-justified-envy within type: If there are $s, s' \in S$ and $c \in C$ such that $s' \in \mu^q(c), s \succ_c s'$ and $c \succ_s \mu^q(s)$, then $s \in S^M$, $s' \in S^m$ and $|\mu^q(c) \cap S^M| = q_c^M$.

(4) Feasibility: $|\mu^q(c) \cap S^M| \leq q_c^M$ for all $c \in C$.

For school choice problem with reserve-based affirmative action $G^r = (S^m, S^M, C, \succ_S, \succ_C, q, r^m)$, a matching μ^r is stable with respect to reserve-based affirmative action, or simply *stable with reserve*, if the following conditions are satisfied:

(1) Individual rationality: $\mu^r(s) \succeq_s s$ for all $s \in S$.

(2) Non-wastefulness: If there are $s \in S$ and $c \in C$ such that $c \succ_s \mu^r(s)$, then $|\mu^r(c)| = q_c$.

(3) No-justified-envy within type: If there are $s, s' \in S$ and $c \in C$ such that $s' \in \mu^r(c), s \succ_c s'$ and $c \succ_s \mu^r(s)$, then $s \in S^M$, $s' \in S^m$ and $|\mu^r(c) \cap S^m| \leq r_c^m$.

(4) Feasibility: There are no $s \in S^m$ and $c \in C$ such that $c \succ_s \mu^r(s)$ and $|\mu^r(c) \cap S^m| < r_c^m$.

2.3 DA algorithm

A *mechanism* is a mapping ϕ that, for each school choice problem G, associates a matching $\phi(G)$. A mechanism ϕ is *stable* (resp. *stable with quota or stable with reserve*) if $\phi(G)$ is a stable (resp. stable with quota or stable with reserve) matching in G for any given G. It is well known that the DA algorithm has been an important matching mechanism since it was proposed by Gale and Shapley (1962).

As a benchmark, we first present the DA algorithm (Gale and Shapley, 1962) for problems without affirmative action:

Step 1: Start with a matching in which no student is matched. Each student proposes to her first choice. Each school tentatively assigns its seats to its proposers one at a time following their priority order. Any remaining proposers are rejected.

In general, at

Step k: Each student who was rejected in the previous step proposes to her next choice. Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time following their priority order. Any remaining proposers are rejected.

The algorithm terminates when no student proposal is rejected and each student is assigned her final tentative assignment. We also refer to the induced direct mechanism as Gale-Shapley student optimal stable mechanism. For a problem $G = (S^m, S^M, C, \succ_S, \succ_C, q)$, we denote the matching produced by DA algorithm by $DA(S^m, S^M, C, \succ_S, \succ_C, q)$.

Next, we introduce the following DA algorithm (Gale and Shapley, 1962; adapted to controlled school choice by Abdulkadiroğlu and Sönmez, 2003) for problem with quota-based affirmative action (henceforth, DA^q):

Step 1: Start with a matching in which no student is matched. Each student s applies to her first choice school (call it c). The school c rejects s if (i) q_c seats are filled by students who have higher priority than s at c or (ii) $s \in S^M$ and q_c^M seats are filled by students in S^M who have higher priority than s at c. Each school c keeps all other students who applied to c.

In general, at

Step k: Each student s who was rejected in the previous step proposes to her first choice school (call it c) among all schools that have not rejected s before. The school c rejects s if (i) q_c seats are filled by students who have higher priority than s at c or (ii) $s \in S^M$ and q_c^M seats are filled by students in S^M who have higher priority than s at c. Each school c keeps all other students who applied to c.

The algorithm terminates at a step in which no rejection occurs, and the tentative matching at that step is finalized. Since no student applies to a school that has rejected her again and at least one rejection occurs in each step as long as the algorithm does not terminate, the algorithm stops in a finite number of steps. Modifying the argument by Gale and Shapley (1962), Abdulkadiroğlu and Sönmez (2003) show that the outcome of the DA algorithm is the student-optimal stable (with quota) matching. For a problem $G^q = (S^m, S^M, C, \succ_S, \succ_C, q, q^M)$, we denote the matching produced by DA^q algorithm by $DA^q(S^m, S^M, C, \succ_S, \succ_C, q, q^M)$.

For school choice with reserve-based affirmative action, we introduce the DA algorithm adapted by Hafalir et al. (2013) (henceforth, DA^r).

Step 1: Start with the matching in which no student is matched. Each student s applies to her first choice school. Each school c first accepts as many as r_c^m minority applicants with the highest priorities if there are enough minority applicants. Then it accepts applicants with the highest priorities from the remaining applicants until its capacity is filled or the applicants are exhausted. The rest of the applicants, if any remains, are rejected by c.

In general, at

Step k: Start with the tentative matching obtained at the end of *Step* $k-1$. Each student s who got rejected at *Step* $k-1$ applies to her first choice school (call it c) among all schools that have not rejected s before. The school c considers the new applicants and students admitted tentatively at *Step* $k-1$. Among these students, school c first accepts as many as r_c^m minority students with the highest priorities if there are enough minority students. Then it accepts students with the highest priorities from the remaining students. The rest of the students, if any remains, are rejected by c. If there are no rejections, then the algorithm stops.

The algorithm terminates when no rejection occurs and the tentative matching at that step is finalized. Since no student reapplies to a school that has rejected her and at least one rejection occurs in each step, the algorithm stops in finite time. Hafalir et al. (2013) show that the outcome of the DA algorithm is the student-optimal stable (with reserve) matching. For a problem $G^r = (S^m, S^M, C, \succ_S, \succ_C, q, r^m)$, we denote the matching produced by DA^r algorithm by $DA^r(S^m, S^M, C, \succ_S, \succ_C, q, r^m)$.

3 Minimal responsiveness of DA algorithms

For school choice with affirmative action, the minimal requirement for a matching mechanism is: under such a mechanism, a stronger affirmative action should not hurt some minority students without benefiting any minority student. If a mechanism satisfies this property, then we say that it is minimally responsive to the affirmative action, or simply minimally responsive. Formally, a mechanism ϕ is *minimally responsive to quota-based affirmative action* if there are no problems G^q and $\tilde{G}^q$ such that $\tilde{G}^q$ has a stronger affirmative action policy than G^q, and $\phi(G^q)$ Pareto dominates $\phi(\tilde{G}^q)$ for the minority. Similarly, we can define minimal responsiveness to reserve-based (priority-based) affirmative action.

It may be remarked that, on the full domain of school choice problems, there exists no mechanism that is stable and minimally responsive to the priority-based affirmative action. For the case of $\sum_c q_c < |S|$, Kojima (2012) illustrates this point by providing an example. For the case of $\sum_c q_c \geq |S|$, consider the following example.

Example 1. Let $C = \{c_1, c_2, c_3\}$, $S = \{s_1, s_2, s_3\}$, $S^m = \{s_1, s_2\}$, and $S^M = \{s_3\}$. All schools have a capacity of 1: $q = (1, 1, 1)$. Students' preferences and schools' priorities are given below

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$
c_1	c_2	c_2	s_3	s_1	s_3
c_2	c_1	c_1	s_1	s_3	s_2
c_3	c_3	c_3	s_2	s_2	s_1

In this market, there are two stable matchings μ and $\tilde{\mu}$, which are given by

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 \\ s_1 & s_3 & s_2 \end{pmatrix},$$

and

$$\tilde{\mu} = \begin{pmatrix} c_1 & c_2 & c_3 \\ s_3 & s_1 & s_2 \end{pmatrix}.$$

Consider the following two cases.

Case 1. Suppose that $\phi(S^m, S^M, C, \succ_S, \succ_C, q) = \mu$ for some stable mechanism ϕ. Consider $\succ'_C$ that changes c_2's priority as: $\succ'_{c_2}$: s_1, s_2, s_3. Then market $(S^m, S^M, C, \succ_S, \succ'_C, q)$ has a stronger priority-based affirmative action policy than $(S^m, S^M, C, \succ_S, \succ_C, q)$. In market $(S^m, S^M, C, \succ_S, \succ'_C, q)$, there is a unique stable matching μ' given by

$$\mu' = \begin{pmatrix} c_1 & c_2 & c_3 \\ s_3 & s_1 & s_2 \end{pmatrix}.$$

Clearly, μ Pareto dominates μ' for the minority.

Case 2. Suppose that $\phi(S^m, S^M, C, \succ_S, \succ_C, q) = \tilde{\mu}$ for some stable mechanism ϕ. Consider $\succ''_C$ that changes c_2's priority as: $\succ''_{c_2}$: s_3, s_1, s_2. Then market $(S^m, S^M, C, \succ_S, \succ_C, q)$ has a stronger priority-based affirmative action policy than $(S^m, S^M, C, \succ_S, \succ''_C, q)$. In market $(S^m, S^M, C, \succ_S, \succ''_C, q)$, there is a unique stable matching μ'' given by

$$\mu'' = \begin{pmatrix} c_1 & c_2 & c_3 \\ s_1 & s_3 & s_2 \end{pmatrix}.$$

Clearly, μ'' Pareto dominates $\tilde{\mu}$ for the minority.

We will first study priority-based affirmative action on a restricted domain where all schools have the same priority orders over students. In real life, if schools determine their priorities according to students' scores in a centralized exam, such as the national entrance exam for college admission in China, then all schools have the common priorities.

As preparation, we present the following result, which says that a stronger affirmative action policy makes the highest-priority minority student weakly better off under the DA algorithm. Formally, we have the following proposition.

Proposition 1. *Suppose that all schools have the same priority orders over students. Let s_1^m denote the highest-priority minority student for all schools, that is, s_1^m is a minority student and has higher priority than any other minority student. Let $\succ_C$ and $\succ'_C$ be two schools' priority profiles such that $(S^m, S^M, C, \succ_S, \succ'_C, q)$ has a stronger affirmative action policy than $(S^m, S^M, C, \succ_S, \succ_C, q)$. Then $DA(S^m, S^M, C, \succ_S, \succ'_C, q) \equiv \mu' \succeq_{s_1^m} \mu \equiv DA(S^m, S^M, C, \succ_S, \succ_C, q)$.*

Since all schools have the same priority orders over students, it is easy to check that the DA algorithm is equivalent to serial dictatorship (for students). This equivalence suggests that the stronger affirmative action policy provides the best minority student with a weakly higher ranking and under serial dictatorship, weakly higher ranking means the student selects a school from a weakly larger set which means the student cannot be worse off.

It may be remarked that one cannot expect to obtain the corresponding property for other minority students even under common schools' priority. Specifically, we consider the following example.

Example 2. Let $C = \{c_1, c_2, c_3\}$, $S^M = \{s_1\}$, and $S^m = \{s_2, s_3\}$. All schools have a capacity of 1: $q = (1, 1, 1)$. Students' preferences and schools' priorities are given by the following table.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{c_1}=\succ_{c_2}=\succ_{c_3}$	$\succ'_{c_1}=\succ'_{c_2}=\succ'_{c_3}$
c_1	c_1	c_2	s_1	s_2
c_2	c_3	c_1	s_2	s_1
c_3	c_2	c_3	s_3	s_3

For schools' priority profiles, $\succ'_C$ has a stronger affirmative action policy than $\succ_C$. The stable matchings produced by the DA algorithm under $(S^m, S^M, C, \succ_S, \succ_C, q)$ and $(S^m, S^M, C, \succ_S, \succ'_C, q)$ are, respectively

$$DA(S^m, S^M, C, \succ_S, \succ_C, q) = \begin{pmatrix} c_1 & c_2 & c_3 \\ s_1 & s_3 & s_2 \end{pmatrix},$$

and

$$DA(S^m, S^M, C, \succ_S, \succ'_C, q) = \begin{pmatrix} c_1 & c_2 & c_3 \\ s_2 & s_1 & s_3 \end{pmatrix}.$$

One can see that under a stronger affirmative action policy $\succ'_C$, the minority student s_3 becomes strictly worse off than under $\succ_C$.

Note that, for majority quotas type or minority reserves type affirmative action policies, the above property does not hold even if all schools have the same priority over students. In fact, there is no stable (with quota or with reserve) mechanism that can ensure the highest-priority student is weakly better off under a stronger (quota-based or reserve-based) affirmative action policy. For the case of minority reserves policy, readers can refer to Example 1 in Hafalir et al. (2013). For the case of majority quotas policy, we consider the following examples.

Example 3. Let $C = \{c_1, c_2\}$, $S^M = \{s_1, s_2\}$, $S^m = \{s_3, s_4\}$, and $S = S^M \cup S^m$. Schools' capacity profile is: $q = (3, 1)$. Students' preferences and schools' priorities are given by the following table.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$	$\succ_{c_1}=\succ_{c_2}$
c_1	c_1	c_2	c_1	s_1
c_2	c_2	c_1	c_2	s_2
				s_3
				s_4

Let $q^M = (2, 1)$ and $\tilde{q}^M = (1, 1)$ be different schools' majority quotas profiles. It is easy to see that $\tilde{q}^M$ has a stronger affirmative action policy than q^M. The only stable (with quota) matchings under $(S^m, S^M, C, \succ_S, \succ_C, q, q^M)$ and $(S^m, S^M, C, \succ_S, \succ_C, q, \tilde{q}^M)$ respectively are

$$\mu^q = \begin{pmatrix} c_1 & c_2 \\ \{s_1, s_2, s_4\} & s_3 \end{pmatrix},$$

and

$$\tilde{\mu}^q = \begin{pmatrix} c_1 & c_2 \\ \{s_1, s_3, s_4\} & s_2 \end{pmatrix}.$$

One can see that under a stronger affirmative action policy $\tilde{q}^M$, the highest-priority minority student s_3 becomes strictly worse off than under q^M.

On the restricted domain where all schools have the same priority orders over students, we obtain the following minimal responsiveness result for the priority-based affirmative action.

Theorem 1. *Suppose that all schools have the same priority orders over students. Then the DA algorithm is minimally responsive to the priority-based affirmative action.*

For majority quotas type or minority reserves type affirmative actions, no stable mechanism is minimally responsive even if all schools have the same priority over students. For the case of minority reserves policy, readers can refer to Example 1 in Hafalir et al. (2013). For the case of majority quotas type, consider the following example.

Example 4. Let $C = \{c_1, c_2\}$, $S^M = \{s_1\}$, $S^m = \{s_2\}$, and $S = S^M \cup S^m$. Schools' capacity profile is: $q = (1, 1)$. Students' preferences and schools' priorities are given by the following table.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{c_1} = \succ_{c_2}$
c_1	c_2	s_1
c_2	c_1	s_2

Let $q^M = (1, 1)$ and $\tilde{q}^M = (0, 1)$ be different schools' majority quotas profiles. $\tilde{q}^M$ has a stronger affirmative action policy than q^M. The only stable (with quota) matchings under $(S^m, S^M, C, \succ_S, \succ_C, q, q^M)$ and $(S^m, S^M, C, \succ_S, \succ_C, q, \tilde{q}^M)$ respectively are

$$\mu^q = \begin{pmatrix} c_1 & c_2 \\ s_1 & s_2 \end{pmatrix},$$

and

$$\tilde{\mu}^q = \begin{pmatrix} c_1 & c_2 \\ s_2 & s_1 \end{pmatrix}.$$

One can see that under a stronger affirmative action policy $\tilde{q}^M$, the minority student s_2 becomes strictly worse off than under q^M.

Now we return to the usual setting for school choice problem that imposes no restriction on students' preferences. In order to study majority quota type affirmative action problem, Doğan (2016) proposes the condition that the market gives full priority to the minority. He obtains that, for majority quota type affirmative action problems, a stable (with quota) and minimally responsive matching mechanism exists if and only if the market gives full priority to the minority. Loosely speaking, a market $(S^m, S^M, C, \succ_S, \succ_C, q)$ giving full priority to the minority means that, at each school c, either each minority student is ranked above each majority student, or each minority student is one of the q_c highest-priority students. Formally, a market $(S^m, S^M, C, \succ_S, \succ_C, q)$ *gives full priority to the minority* if there are no $m \in S^m$, $M \in S^M$, and $c \in C$ such that $[M \succ_c m$ and $|\{s \in S : s \succeq_c m\}| > q_c]$.

For priority-based affirmative action, we can obtain a more desirable result which says that, if the stronger affirmative action problem $(S^m, S^M, C, \succ_S, \succ'_C, q)$ gives full priority to the minority, then the stronger affirmative action policy makes each minority student weakly better off under the DA algorithm. Formally, we have the following theorem.

Theorem 2. *For priority-based affirmative action problems, let* $\succ_C$ *and* $\succ'_C$ *be two schools' priority profiles such that* $G' = (S^m, S^M, C, \succ_S, \succ'_C, q)$ *has a stronger affirmative action policy than* $G = (S^m, S^M, C, \succ_S, \succ_C, q)$ *. If the stronger affirmative action problem* $G' = (S^m, S^M, C, \succ_S, \succ'_C, q)$ *gives full priority to the minority, then* $DA(S^m, S^M, C, \succ_S, \succ'_C, q) \succeq_s DA(S^m, S^M, C, \succ_S, \succ_C, q)$ *for each minority student* $s \in S^m$*.*

Note that the condition that the stronger affirmative action problem $G' = (S^m, S^M, C, \succ_S, \succ'_C, q)$ gives full priority to the minority is not necessary for the property obtained in Theorem 2. Specifically, we consider the following example.

Example 5. Let $C = \{c_1, c_2\}, S^M = \{s_1\}$, $S^m = \{s_2\}$, and $S = S^M \cup S^m$. All schools have a capacity of 1: $q = (1, 1)$. Students' preferences and schools' priorities are given by the following table.

$\succ_{s_1}$ and $\succ_{s_2}$	$\succ_{c_1}=\succ_{c_2}$	$\succ'_{c_1}$	$\succ'_{c_2}$
arbitrary	s_1	s_2	s_1
	s_2	s_1	s_2

One can see that $G' = (S^m, S^M, C, \succ_S, \succ'_C, q)$ has a stronger affirmative action policy than $G = (S^m, S^M, C, \succ_S, \succ_C, q)$ and $G' = (S^m, S^M, C, \succ_S, \succ'_C, q)$ does not give full priority to the minority. However, for arbitrary students' preferences, it is easy to check that $DA(S^m, S^M, C, \succ_S, \succ'_C, q) \succeq_{s_2} DA(S^m, S^M, C, \succ_S, \succ_C, q)$.

More generally, this property must hold if $|S^m| = 1$. It is an immediate consequence of respecting improvements obtained by Balinski and Sönmez (1999).[3] Formally, we state this result as follows.

[3] A mechanism respecting improvements means that, if a student's priority at each school (weakly) improves, then running the given matching mechanism the student will become (weakly) better off. Specifically, we assume that, for each $c \in C$, $\succ'_c$ (weakly) improves the priority of some student s with respect to $\succ_c$ and leaves other students' order unchanged. A mechanism φ respects improvements if $\varphi(\succ'_C, \succ_S)(s) \succeq_s \varphi(\succ_C, \succ_S)(s)$.

Proposition 2. *For priority-based affirmative action problems, let $\succ_C$ and $\succ'_C$ be two schools' priority profiles such that $(S^m, S^M, C, \succ_S, \succ'_C, q)$ has a stronger affirmative action policy than $(S^m, S^M, C, \succ_S, \succ_C, q)$. If $S^m = \{s\}$, then $DA(S^m, S^M, C, \succ_S, \succ'_C, q) \succeq_s DA(S^m, S^M, C, \succ_S, \succ_C, q)$. That is, if $|S^m| = 1$, then a stronger priority-based affirmative action policy makes the minority student weakly better off under the DA algorithm.*

Next we study the comparison between different affirmative action policies when market $(S^m, S^M, C, \succ_S, \succ_C, q)$ gives full priority to the minority. Let $(S^m, S^M, C, \succ_S, \succ'_C, q)$ (resp. $(S^m, S^M, C, \succ_S, \succ_C, q, r^m)$ and $(S^m, S^M, C, \succ_S, \succ_C, q, q^M)$) be a problem that has a stronger priority-based (resp. minority reserves type and majority quotas type) affirmative action policy than $(S^m, S^M, C, \succ_S, \succ_C, q)$. For a given market $(S^m, S^M, C, \succ_S, \succ_C, q)$, if all of the majority students leave the market, then we denote the corresponding problem by $(S^m, C, \succ_{S^m}, \succ_C, q)$. Denote $DA(S^m, S^M, C, \succ_S, \succ_C, q) \equiv \mu$, $DA(S^m, C, \succ_{S^m}, \succ_C, q) \equiv \mu'$, $DA(S^m, S^M, C, \succ_S, \succ'_C, q) \equiv \mu^p$, $DA^q(S^m, S^M, C, \succ_S, \succ_C, q, q^M) \equiv \mu^q$, and $DA^r(S^m, S^M, C, \succ_S, \succ_C, q, r^m) \equiv \mu^r$. Then we present our next result.

Theorem 3. *Suppose that a problem $(S^m, S^M, C, \succ_S, \succ_C, q)$ gives full priority to the minority students. Then*

(1). $\mu(s) = \mu'(s)$ for every $s \in S^m$;

(2). $\mu(s) = \mu^q(s)$ for every $s \in S^m$ and $\mu(s) \succeq_s \mu^q(s)$ for each $s \in S^M$;

(3). $\mu(s) = \mu^p(s) = \mu^r(s)$ for every $s \in S$.

According to Theorem 3, one can see that, if a market gives full priority to the minority, stronger priority-based and reserve-based affirmative action policies do not play an actual role under the DA algorithm. Moreover, a stronger quota-based affirmative action just probably makes majority students worse off and results in avoidable efficiency loss.

4 Further impossibility results

In this section, we study the minimal responsiveness of two efficiency-improved mechanisms–Kesten's EADAM and DA-TTC mechanism. We show that the EADAM does not respect improvements.[4] That

[4] In the sense of Balinski and Sönmez (1999), see footnote 3 above.

is, under EADAM, even if all schools (weakly) improve the priority of student s, it may happen that student s becomes worse off. Then as an immediate consequence, we obtain that EADAM is not minimally responsive to the priority-based affirmative action. For DA-TTC mechanism, we show that it is not minimally responsive to any type of the three popular affirmative action policies.

4.1 Kesten's EADAM

For reserve-based affirmative action policy, Doğan (2016) proposes a modified DA algorithm with minority reserves. His algorithm is minimally responsive to the reserve-based affirmative action and actually satisfies a relatively weak stability requirement. Doğan's algorithm is inspired by the efficiency-adjusted deferred acceptance mechanism (EADAM) in Kesten (2010). In fact, for school choice problems, Kesten (2010) proposes EADAM with consent idea to improve the efficiency of DA mechanism.[5] As is well known in the literature (see e.g., Ergin (2002), Kesten (2010), Tang and Zhang (2017)), the inefficiency of DA may arise when there exists an interrupter during the DA algorithm procedure.[6] Specifically, in the procedure of DA algorithm, it may happen that, some student s proposes to school c and is tentatively accepted, but her tentative acceptance at c initiates a chain of rejections that eventually lead c to reject student s. One can see that, by proposing to school c, student s gains nothing, but blocks trading among other students. In Kesten (2010), student s is called an "interrupter" at school c. To improve the efficiency of DA algorithm, Kesten proposes EADAM, which iteratively reruns DA after removing the last interruptions caused by consenting interrupters in the DA procedure. He also obtains that no student has incentive to not consent under EADAM and EADAM is Pareto efficient when all students consent. Recently, Klooster and Troyan (2016) define the concept called essential stability.[7] They show that EADAM (with all students consenting) is essentially stable. Tang and Zhang

[5]Recently, Tang and Yu (2014) design a new mechanism, which simplifies EADAM and produces the same matching outcome as EADAM.

[6] Formally, in the DA procedure of a school choice problem, if student s is tentatively accepted by school c at some step t and is later rejected by school c at some step $t' > t$, and at least one other student is rejected by school c at some step r such that $t \leq r < t'$, then student s is an *interrupter* at school c and the pair (s, c) is an interrupting pair of step t'.

[7]For a matching μ and any of its blocking pair (s, c), consider the following reassignment process: if s claims a seat at c and crowds out a lowest priority student s' admitted to c, then s' claims her most favorite school that wishes to block with her and crowds out another student from that school. Let this student claim as usual. If at any step some student

(2017) propose the concept of weak stability, which says that none of the blocking pairs of a weakly stable matching can be matched by a more stable matching (i.e., a matching with a weakly smaller set of blocking pairs). Then they obtain that EADAM is a weakly stable mechanism. They also note that the Boston mechanism is not weakly stable. Then among DA, EADAM and the Boston mechanism, DA is the most stable, but is not minimally responsive to the priority-based affirmative action; the Boston mechanism satisfies the weakest stability and minimal responsiveness (to the priority-based affirmative action). Then we have a natural question: whether EADAM (which has the medium stability among these three mechanisms) is minimally responsive to the priority-based affirmative action.

For completeness, we first present the EADAM with all students consenting. For any school choice problem G, Kesten's EADAM with all students consenting operates as follows:

Round 0 Run DA for the problem G.

Round $k, k \geq 1$ Identify the last step of the round-$(k-1)$ DA procedure in which there is (are) some interrupter(s) being rejected, and then identify all interrupting pairs of this step and, for each pair, remove the respective school from the interrupter's preference. After that, rerun DA (round-k DA) with the new preference profile.

Stop when there are no more interrupters.

According to the procedure of EADAM, we know that EADAM iteratively removes the interrupters in DA. Based on this observation, one can probably expect that EADAM is minimally responsive to the priority-based affirmative action. For Example 1 (Case 1) in the present paper, it is easy to check that EADAM does not bring perverse consequence under the given stronger priority-based affirmative action policy, while DA does. Then it would be interesting to check whether EADAM is indeed minimally responsive to the priority-based affirmative action. Unfortunately, we will show that EADAM does not respect improvements. In other words, when there is only one minority student, a stronger priority-based affirmative action policy may make the minority student worse off. Specifically, for EADAM, we obtain the following result.

claims c and crowds s out of it, then the initial claim of student s to school c is viewed as vacuous. A matching is said to be essentially stable if all of its blocking pairs are vacuous.

Theorem 4. *EADAM (with all students consenting) does not respect improvements. Consequently, EADAM is not minimally responsive to the priority-based affirmative action policy.*

Proof. We consider the following example. Let $C = \{c_1, c_2, c_3, c_4\}, S = \{s_1, s_2, s_3, s_4\}$. All schools have a capacity of 1: $q = (1, 1, 1, 1)$. Students' preferences and schools' priorities are given by the following table:

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$	$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$	$\succ_{c_4}$
c_1	c_1	c_2	c_3	s_4	s_3	s_2	$\vdots$
c_4	c_2	c_4	c_1	s_1	s_2	s_4	
$\vdots$	c_3	$\vdots$	$\vdots$	s_2	$\vdots$	$\vdots$	
	c_4			s_3			

Then the procedure of the DA mechanism as follows:

step	c_1	c_2	c_3	c_4
1	$\boxed{s_1}, s_2$	$\boxed{s_3}$	$\boxed{s_4}$	
2		$s_2, \boxed{s_3}$		
3			$\boxed{s_2}, s_4$	
4	$s_1, \boxed{s_4}$			
5	$\boxed{s_4}$	$\boxed{s_3}$	$\boxed{s_2}$	$\boxed{s_1}$

One can see that s_1 is the only interrupter (at c_1) in the procedure of DA algorithm. We remove c_1 from s_1's preference and rerun DA with the new preference profile. Then the matching produced by EADAM is

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_2 & s_3 & s_4 & s_1 \end{pmatrix}.$$

If we change the schools' priorities as follows: $\succ'_{c_2}$: $s_2, s_3, \cdots$, and $\succ'_{c_i} = \succ_{c_i}$ for other schools. Then the procedure of the DA mechanism under $(\succ'_C, \succ_S)$ as follows:

step	c_1	c_2	c_3	c_4
1	$\boxed{s_1}$,s_2	$\boxed{s_3}$	$\boxed{s_4}$	
2		$\boxed{s_2}$,s_3		
3	$\boxed{s_1}$	$\boxed{s_2}$	$\boxed{s_4}$	$\boxed{s_3}$

One can see that there exists no interrupter in the procedure of DA algorithm. Then DA and EADAM produce the same matching as follows:

$$\mu' = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_1 & s_2 & s_4 & s_3 \end{pmatrix}.$$

One can check that $\succ'_C$ improves s_2 with respect to $\succ_C$.[8] However, running EADAM, student s_2 under $(\succ'_C, \succ_S)$ is worse than under $(\succ_C, \succ_S)$. Then EADAM does not satisfy the property of respecting improvements. In this example, if we assume that s_1, s_3 and s_4 are majority students, and s_2 is minority student, then $\succ'_C$ is a stronger priority-based affirmative action policy than $\succ_C$. Under EADAM, the stronger affirmative action policy makes all minority student (only one- s_2) worse off. Thus, EADAM is not minimally responsive to the priority-based affirmative action policy. □

4.2 DA-TTC mechanism

For efficiency improvements in school choice, Erdil and Ergin (2008) propose the stable improvement cycles algorithm, which is closely related to DA-TTC mechanism (deferred acceptance then top trading cycles mechanism). It is easy to check that DA-TTC mechanism is not stable. Furthermore, Tang and Zhang (2017) and Klooster and Troyan (2016) show that DA-TTC mechanism is not weakly stable or essentially stable, respectively. An obvious advantage of DA-TTC mechanism is that it improves the efficiency by trading among students at TTC stage. Then it is interesting to check whether DA-TTC mechanism is minimally responsive to affirmative action policies. We first specify the procedure of the DA-TTC mechanism as follows:

[8]Priority profile $\succ'_C$ improving s with respect to $\succ_C$ means that, for each $c \in C$ and any $s_1, s_2 \in S \setminus s$, if $s \succ_c s_1$ then $s \succ'_c s_1$, and $s_1 \succ_c s_2$ if and only if $s_1 \succ'_c s_2$.

Round 1. For any school choice problem G, run DA algorithm, denote the matching by

$\mu = DA(G)$.

Round 2. Run TTC as follows:[9]

Step 1. Let each student point to her most favorite school and let each school c point to the highest priority student among students in $\mu(c)$. There will be cycles. For each cycle, assign each student in the cycle with a seat of her most favorite school, then remove her with her assignment.

Step $k, k \geq 2$. Repeat Step 1 for the remaining school seats and students.

It is easy to show that DA-TTC mechanism will guarantee that students always receive an assignment that is no worse than their DA school. In fact, DA-TTC mechanism improves the efficiency by trading among students after DA algorithm. One might hope that minority students are not hurt by priority-based affirmative action policy under the DA-TTC mechanism. Unfortunately, it is not the case.

Specifically, we consider the example in the proof of Theorem 4 above. It is easy to check that, with respect to $(\succ_C, \succ_S)$, the assignment produced by DA-TTC is

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_2 & s_3 & s_4 & s_1 \end{pmatrix}.$$

And for $(\succ'_C, \succ_S)$, the assignment produced by DA-TTC is

$$\mu' = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_1 & s_2 & s_4 & s_3 \end{pmatrix}.$$

Then DA-TTC mechanism does not respect improvements (for student s_2). If we assume that s_1, s_3 and s_4 are majority students, and s_2 is minority student, then $\succ'_C$ is a stronger priority-based affirmative action policy than $\succ_C$. Under DA-TTC, the stronger affirmative action policy makes all minority student (only one- s_2) worse off. Thus, DA-TTC is not minimally responsive to the priority-based affirmative action policy. We conclude this result as follows.

[9] Gale's top-trading cycles algorithm was originally introduced in Shapley and Scarf (1974) and later studied in detail by Pápai (2000) and Abdulkadiroğlu and Sönmez (2003). Here we adopt a variation version. See, for instance, Klooster and Troyan (2016) and Cantala and Pápai (2014) for a similar presentation.

Theorem 5. *DA-TTC mechanism does not respect improvements. Consequently, DA-TTC mechanism is not minimally responsive to the priority-based affirmative action policy.*[10]

5 Conclusion

Affirmative action policies in school choice aim to improve the welfare of the minority students. This requires that a given matching mechanism should satisfy the minimal responsiveness. In view of fairness and efficiency issues, it is comparatively ideal to find a stable and minimally responsive assignment mechanism from the mechanism-design perspective. Unfortunately, no assignment mechanism is stable and minimally responsive to the popular affirmative action policies in the full domain of school choice problems.

We study the minimal responsiveness of the DA algorithm for school choice with affirmative action on some restricted domains. We first show that the DA algorithm is stable and minimally responsive to the priority-based affirmative action if all schools have the same priority orders over students. However, for quota-based or reserve-based affirmative action, there is no stable and minimally responsive mechanism even if all schools have the same priority over students. Then for such matching environments, both quota-based and reserve-based affirmative action policies have their limitation. In contrast, the priority-based affirmative action is more suitable to achieve the goal of favoring the minority students at least in the sense of minimal responsiveness. Additionally, the DA algorithm is not only stable and minimally responsive but also strategy-proof (Roth (1985)). Strategy-proofness is regarded as a very important property for a matching mechanism to be successful because it encourages students to reveal their true preferences and avoids sophisticatedly strategic behavior.

We then consider the priority-based affirmative action on the restricted domain that each minority student has higher priority than each majority student at each school. We show that, if a stronger affirmative action market fulfills this requirement, then under the DA algorithm the stronger affirmative action policy makes each minority student weakly better off. If minimal responsiveness is the minimal requirement on a satisfactory assignment mechanism, then the DA algorithm reaches a much stronger

[10]We also show that DA-TTC mechanism is not minimally responsive to the quota-based (resp. reserve-based) affirmative action policy. See Appendix for details.

requirement of responsiveness to the priority-based affirmative action when the stronger policy gives full priority to the minority.

We also consider the case that a market gives full priority to the minority. We show that a stronger quota-based affirmative action just makes majority students worse off and results in avoidable efficiency loss, while stronger priority-based and reserve-based affirmative actions do not play an actual role under the algorithm. Then for this kind of problems, taking no affirmative action is already the best policy for the minority.

Finally we study Kesten's EADAM with all students consenting. EADAM is Pareto efficient and weakly stable. However, we obtain that EADAM does not respect improvements, and consequently, EADAM is not minimally responsive to the priority-based affirmative action. For another efficiency-improved mechanism-DA-TTC, we show that it is not minimally responsive to any type of the three popular affirmative action policies.

A Appendix

A.1 Proof of Theorem 1. We argue by contradiction. Suppose that there are two markets $(\succ_S, \succ_C)$ and $(\succ_S, \succ'_C)$ such that $(\succ_S, \succ'_C)$ has a stronger affirmative action policy than $(\succ_S, \succ_C)$, but $DA(\succ_S, \succ_C)$ Pareto dominates $DA(\succ_S, \succ'_C)$ for the minority students with respect to $\succ_S$. We impose an order on the minority students such that $S^m = \{s_1^m, s_2^m, \cdots, s_k^m\}$ and $s_1^m \succ_c s_2^m \succ_c \cdots \succ_c s_k^m$. By Proposition 1 we know $DA(\succ_S, \succ'_C) \succeq_{s_1^m} DA(\succ_S, \succ_C)$. Then s_1^m is assigned to the same school under both $DA(\succ_S, \succ_C)$ and $DA(\succ_S, \succ'_C)$. We can choose a minority student $s_r^m \in S^m$ such that $DA(\succ_S, \succ_C) \succ_{s_r^m} DA(\succ_S, \succ'_C)$ and, for all $1 \leq i < r$, s_i^m is assigned to the same school under both $DA(\succ_S, \succ_C)$ and $DA(\succ_S, \succ'_C)$. We denote $DA(\succ_S, \succ_C) \equiv \mu$, $DA(\succ_S, \succ'_C) \equiv \mu'$ and $\mu(s_r^m) \equiv c_0$. According to the DA algorithm under $(\succ_S, \succ'_C)$, $DA(\succ_S, \succ_C) \succ_{s_r^m} DA(\succ_S, \succ'_C)$ implies that s_r^m must ever propose to c_0 and c_0 rejects her at some step. Then it is easy to see that $|\mu'(c_0)| = q_{c_0}$ and each member in $\mu'(c_0)$ has higher priority than s_r^m at c_0 (with respect to $\succ'_{c_0}$). Combining conditions $|\mu'(c_0)| = q_{c_0}$, $|\mu(c_0)| \leq q_{c_0}$ and $s_r^m \in \mu(c_0) \setminus \mu'(c_0)$, one can infer that there is at least one student, say s_1^M, such that $s_1^M \in \mu'(c_0) \setminus \mu(c_0)$ and $s_1^M \succ'_{c_0} s_r^m$. Since $\mu(s_1^M) \neq \mu'(s_1^M)$, we have $s_1^M \notin \{s_i^m : 1 \leq i \leq r-1\}$. By $s_1^M \succ'_{c_0} s_r^m$ we obtain $s_1^M \notin \{s_i^m : r \leq i \leq k\}$. Therefore $s_1^M \in S^M$. Since $(\succ_S, \succ'_C)$ has a stronger affirmative action policy than $(\succ_S, \succ_C)$ and $s_1^M \succ'_{c_0} s_r^m$, one can obtain $s_1^M \succ_{c_0} s_r^m$. According to conditions $s_r^m \in \mu(c_0)$, $s_1^M \notin \mu(c_0)$ and $s_1^M \succ_{c_0} s_r^m$, one can infer that s_1^M has never proposed to c_0 in the process of the DA algorithm under $(\succ_S, \succ_C)$. Then $\mu(s_1^M) \equiv c_1 \succ_{s_1^M} c_0$.

Since $\mu(s_i^m) = \mu'(s_i^m)$ for all $1 \leq i < r$ and $s_1^M \succ'_{c_0} s_r^m$, every minority student who has higher priority than s_1^M is assigned to the same seat under both μ and μ'. According to serial dictatorship (for students), every majority student who has higher priority than s_1^M (including s_1^M) is assigned to the same seat under both $(\succ_S, \succ_C)$ and $(\succ_S, \succ'_C)$. Then by the equivalence of DA algorithm and serial dictatorship we have $\mu(s_1^M) = \mu'(s_1^M)$. We achieve a contradiction and complete the proof. □

A.2 Proof of Theorem 2. We argue by contradiction. Suppose that there exists some minority student $s_0 \in S^m$ such that $DA(\succ_S, \succ_C) \equiv \mu \succ_{s_0} \mu' \equiv DA(\succ_S, \succ'_C)$. We denote $\mu(s_0) \equiv c_0$. According to the DA algorithm under $(\succ_S, \succ'_C)$, $DA(\succ_S, \succ_C) \succ_{s_0} DA(\succ_S, \succ'_C)$ implies that s_0 must ever propose to c_0 and c_0 finally rejects her at some step, say step k_n. Then it is easy to see that $|\mu'(c_0)| = q_{c_0}$ and each member in $\mu'(c_0)$ has higher priority than s_0 at c_0(with respect to $\succ'_{c_0}$). We obtain that s_0 is not in the set of the q_{c_0} highest-priority students of c_0 (with respect to $\succ'_{c_0}$). Since the market $(\succ_S, \succ'_C)$ gives full priority to the minority, by definition it must be the case that each minority student is ranked above each majority student under $\succ'_{c_0}$. Then there is no majority student who has higher priority than s_0 with respect to $\succ'_{c_0}$. Therefore, one can infer that c_0 tentatively accepts q_{c_0} students when c_0 rejects s_0 at step k_n, and each of the q_{c_0} students is in S^m and has higher priority than s_0 at c_0 (with respect to $\succ'_{c_0}$). Since $|\mu(c_0)| \leq q_{c_0}$ and $s_0 \in \mu(c_0)$, it is easy to see that there exists at least one student, say s_1 (in S^m), among the q_{c_0} students tentatively accepted by c_0 at step k_n such that $s_1 \notin \mu(c_0)$. We can obtain that $s_1 \succ'_{c_0} s_0$ is equivalent to $s_1 \succ_{c_0} s_0$, as both s_0 and s_1 are minority students. Combining $s_0 \in \mu(c_0)$, $s_1 \notin \mu(c_0)$ and $s_1 \succ_{c_0} s_0$, one can infer that s_1 has never proposed to c_0 in the process of the DA algorithm under $(\succ_S, \succ_C)$. Then $\mu(s_1) \equiv c_1 \succ_{s_1} c_0$.

Since $c_1 \succ_{s_1} c_0$ and s_1 ever proposed to c_0 at some step, say step $k'_n(\leq k_n)$, in the DA process under $(\succ_S, \succ'_C)$, one can infer that s_1 must have proposed to c_1 and c_1 rejected her at another step, say step $k_{n-1}(< k'_n)$, in the DA process of $(\succ_S, \succ'_C)$. Then it is exactly similar to the analysis above, and one can obtain that $|\mu'(c_1)| = q_{c_1}$ and each member in $\mu'(c_1)$ has higher priority than s_1 at c_1 (with respect to $\succ'_{c_1}$). Then s_1 is not in the set of the q_{c_1} highest-priority students of c_1 (with respect to $\succ'_{c_1}$). Since the market $(\succ_S, \succ'_C)$ gives full priority to the minority, by definition it must be the case that each minority student is ranked above each majority student under $\succ'_{c_1}$. Then there is no majority student who has higher priority than s_1 under $\succ'_{c_1}$. Therefore, one can infer that c_1 tentatively accepts q_{c_1} students when c_1 rejects s_1 at step k_{n-1}, and each of the q_{c_1} students is in S^m and has higher priority than s_1 at c_1 (with respect to $\succ'_{c_1}$). Since $|\mu(c_1)| \leq q_{c_1}$ and $s_1 \in \mu(c_1)$, it is easy to see that there exists at least one student, say s_2 (in S^m), among the q_{c_1} students tentatively accepted by c_1 at step k_{n-1} such that $s_2 \notin \mu(c_1)$. We can infer that $s_2 \succ'_{c_1} s_1$ implies $s_2 \succ_{c_1} s_1$, as both s_1 and s_2 are minority students. Combining $s_1 \in \mu(c_1)$, $s_2 \notin \mu(c_1)$ and $s_2 \succ_{c_1} s_1$, one can infer that s_2 has never proposed to c_1 in the process of the DA algorithm under $(\succ_S, \succ_C)$. Then $\mu(s_2) \equiv c_2 \succ_{s_2} c_1$.

Taking a repeated argument procedure as above, we can obtain a sequence of students and schools $s_1, c_1, \cdots, s_i, c_i \cdots$ and a infinite sequence of steps of DA algorithm process $k_n, k_{n-1}, \cdots, k_{n+1-i}, \cdots$ such that $k_m > k_{m-1}$ for all $m \leq n$. Then $k_{n+1-i} < 0$ when i is sufficiently large. Since k_{n+1-i} is some step of the DA algorithm process under$(\succ_S, \succ'_C)$, $k_{n+1-i} \geq 1$. We reach a contradiction and complete the proof. □

A.3 Proof of Theorem 3. (1). By a classical result of Gale and Sotomayor (1985) (see also Theorem 5.35 in Roth and Sotomayor (1990)), one can obtain that $\mu'(s) \succeq_s \mu(s)$ for each $s \in S^m$. We only need to show that $\mu(s) \succeq_s \mu'(s)$ for each $s \in S^m$. Suppose not, then there exists some $s \in S^m$ such that $\mu'(s) \succ_s \mu(s)$. One can repeat a procedure as in the proof of Theorem 2 and complete the proof.

(2). We first show that $\mu(s) \succeq_s \mu^q(s)$ for each $s \in S^m$. Suppose not. Then there exists some $s \in S^m$ such that $\mu^q(s) \succ_s \mu(s)$. One can take a similar procedure as in the proof of Theorem 2 and reach a contradiction. Symmetrically, we can show that $\mu^q(s) \succeq_s \mu(s)$. Then $\mu(s) = \mu^q(s)$ for each $s \in S^m$. For the second part, we suppose there exists some $s \in S^M$ such that $\mu^q(s) \succ_s \mu(s)$. We denote $\mu^q(s) \equiv c_0$. According to the DA algorithm under $(\succ_S, \succ_C)$, $c_0 \succ_s \mu(s)$ implies that s must ever propose to c_0 and c_0 finally rejects her at some step, say step k_n. Then one can infer that c_0 tentatively accepts q_{c_0} students when c_0 rejects s at step k_n, and each of the q_{c_0} students has higher priority than s_0 at c_0 (with respect to $\succ_{c_0}$). As $\mu(s) = \mu^q(s)$ for each $s \in S^m$, we obtain $\{s \in \mu^q(c_0) : s \in S^m\} = \{s \in \mu(c_0) : s \in S^m\}$. Since this market gives full priority to the minority, c_0 tentatively accepts no more than $|\{s \in \mu^q(c_0) : s \in S^m\}|$ minority students at step k_n. Otherwise, it will result in $|\{s \in \mu^q(c_0) : s \in S^m\}| < |\{s \in \mu(c_0) : s \in S^m\}|$. Since c_0 rejects s at step k_n, one can infer that there exists at least one student, say $s_1 \in S^M$, among the q_{c_0} students tentatively accepted by c_0 at step k_n such that $s_1 \notin \mu^q(c_0)$. Then $s_1 \succ_{c_0} s$. Combining $s \in \mu^q(c_0)$, $s_1 \notin \mu^q(c_0)$ and $s_1 \succ_{c_0} s$, we can infer that s_1 has never proposed to c_0 in the process of the DA algorithm under $(\succ_S, \succ_C, q^M)$. Then $\mu^q(s_1) \equiv c_1 \succ_{s_1} c_0$.

Since $c_1 \succ_{s_1} c_0$ and s_1 ever proposes to c_0 at some step, say step $k'_n(\leq k_n)$, in the DA process under $(\succ_S, \succ_C)$, one can infer that s_1 must ever propose to c_1 and c_1 rejects her at another step, say step $k_{n-1}(< k'_n)$, in the DA process of $(\succ_S, \succ_C)$. Then it is exactly similar to the analysis given above, and one can infer that c_1 tentatively accepts q_{c_1} students when c_1 rejects s_1 at step k_{n-1}, and each of the q_{c_1} students has higher priority than s_1 at c_1. As $\mu(s) = \mu^q(s)$ for each $s \in S^m$, we obtain $\{s \in \mu^q(c_1) : s \in S^m\} = \{s \in \mu(c_1) : s \in S^m\}$. Since this market gives full priority to the minority, c_1 tentatively accepts no more than $|\{s \in \mu^q(c_1) : s \in S^m\}|$ minority students at step k_{n-1}. Otherwise, it will result in $|\{s \in \mu^q(c_1) : s \in S^m\}| < |\{s \in \mu(c_1) : s \in S^m\}|$. Since c_1 rejects s_1 at step k_{n-1}, one can infer that there exists at least one student, say $s_2 \in S^M$, among the q_{c_1} students tentatively accepted by c_1 at step k_{n-1} such that $s_2 \notin \mu^q(c_1)$. Then $s_2 \succ_{c_1} s_1$. Combining $s_1 \in \mu^q(c_1)$, $s_2 \notin \mu^q(c_1)$

and $s_2 \succ_{c_1} s_1$, we can infer that s_2 has never proposed to c_1 in the process of the DA algorithm under $(\succ_S, \succ_C, q^M)$. Then $\mu^q(s_2) \equiv c_2 \succ_{s_2} c_1$.

Taking a repeated argument process as above, we can obtain a sequence of students and schools $s_1, c_1, \cdots, s_i, c_i \cdots$ and a sequence of steps of DA algorithm procedure $k_n, k_{n-1}, \cdots, k_{n+1-i}, \cdots$ such that $k_m > k_{m-1}$ for all $m \leq n$. Then $k_{n+1-i} < 0$ when i is sufficiently large. Since k_{n+1-i} is some step of the DA algorithm process under $(\succ_S, \succ_C, q^M)$, $k_{n+1-i} \geq 1$. We reach a contradiction and complete the proof.

(3). It is easy to see that the stronger policy problem $(\succ_S, \succ'_C)$ also gives full priority to the minority. We only need to show that, for each $s \in S$, $\mu(s) = \mu^p(s)$ and $\mu(s) = \mu^r(s)$, respectively. For the case of minority students we can take a similar argument as in the proof of Theorem 2, and for the case of majority students we can take a similar argument as in the proof of (2) of this theorem. □

A.4 DA-TTC mechanism for quota-based and reserve-based affirmative action

For the quota-based affirmative action, we only need to replace DA in Round 1 with DA^q, then obtain DA^q-TTC.[11] Similarly, we replace DA in Round 1 with DA^r, then obtain DA^r-TTC for reserve-based affirmative action.

For DA^q-TTC mechanism, we consider the problem as given by the example in the proof of Theorem 4. We assume that s_1, s_3 and s_4 are majority students, and s_2 is minority student. Let $q^M = (1,1,1,1)$ and $\tilde{q}^M = (1,0,1,1)$ be two quota-based affirmative action policies. Then $\tilde{q}^M$ is a stronger policy than q^M. For the case of $q^M = (1,1,1,1)$, It is easy to check that the assignment produced by DA^q-TTC is

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_2 & s_3 & s_4 & s_1 \end{pmatrix}.$$

And for $\tilde{q}^M = (1,0,1,1)$, the procedure of the DA ^{q} mechanism as follows:

[11]For DA^q-TTC mechanism, it may happen that a school admits more majority students than its majority quota at TTC stage. Such case can be regarded as "soft" requirement on majority quota. This is based on the consideration of efficiency improvements with sacrifice of majority quota requirement. One can check that, for Example 4 in this paper, DA^q-TTC mechanism does not produce perverse effect under the stronger affirmative action policy, while DA^q does.

step	c_1	c_2	c_3	c_4
1	$\boxed{s_1}, s_2$	s_3	$\boxed{s_4}$	
2	$\boxed{s_1}$	$\boxed{s_2}$	$\boxed{s_4}$	$\boxed{s_3}$

Then DA^q-TTC mechanism produces the following assignment:

$$\mu' = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_1 & s_2 & s_4 & s_3 \end{pmatrix}.$$

Then under DA^q-TTC, the stronger affirmative action policy makes all minority student (only one-s_2) worse off. Thus, DA^q-TTC is not minimally responsive to the quota-based affirmative action policy.

For DA^r-TTC mechanism, we also consider the example in the proof of Theorem 4. We assume that s_1, s_3 and s_4 are majority students, and s_2 is minority student. Let $r^m = (0, 0, 0, 0)$ and $\tilde{r}^m = (0, 1, 0, 0)$ be two reserve-based affirmative action policies. Then $\tilde{r}^m$ is a stronger policy than r^m. For the case of $r^m = (0, 0, 0, 0)$, it is easy to check that the assignment produced by DA^r-TTC is

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_2 & s_3 & s_4 & s_1 \end{pmatrix}.$$

And for $\tilde{r}^m = (0, 1, 0, 0)$, one can check that DA^r-TTC mechanism produces the following assignment:

$$\mu' = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_1 & s_2 & s_4 & s_3 \end{pmatrix}.$$

Then under DA^r-TTC, the stronger affirmative action policy makes all minority student (only s_2) worse off. Thus, DA^r-TTC is not minimally responsive to the reserve-based affirmative action policy.

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