Relativity, mobility, and optimal nonlinear income taxation in an open economy

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Abstract

Recent evidence suggests that globalization has reduced the barriers to international labor mobility and induced more cross-country social comparisons. In an open economy with tax-driven migration and consumption externalities (relativity), we derive an optimal tax formula that subsumes existing ones obtained under a maximin social objective and additively separable utility and sign the second-best (Mirrleesian) marginal tax rates for all skill levels. We establish the thresholds of the elasticity and level of migration to determine when relativity and inequality act as complements or substitutes in shaping the optimal tax rate. Under both Nash and Stackelberg tax competition if the migration probability of top-income workers is approximately 50%, numerical calculation using parameter estimates from empirical studies shows that the country with labor inflow (outflow) implements over 10% lower (higher) marginal tax rates than suggested by the autarky equilibrium of Kanbur and Tuomala (2013).

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1. Introduction

Since Veblen (1899), economists have recognized that the well-being of economic agents depends on relative consumption (“relativity”) in addition to absolute consumption; hence, taxing consumption externalities seems to be welfare enhancing, as with any other Pigouvian tax.¹ Additionally, evidence reported by Senik (2009) and Clark and Senik (2010) shows

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¹ See, e.g., Duesenberry (1949), Layard (1980), Ireland (1998, 2001), Abel (2005), Alpizar et al. (2005), Luttmer (2005), Andersson (2008), Clark et al. (2008), Frank (2008), Aronsson and Johannson-Stenman (2013), Mujcic and Frijters (2015), and Dai (2020). Indeed, whenever there are externalities, govern-
that social comparisons increase the demand for income redistribution, and thus the proper tax policy should play both externality-correcting and income-redistributing roles. Kanbur and Tuomala (2013) (K&T hereafter) was the first paper to examine whether a nonlinear income tax is an effective tool for reducing inequalities and attenuating possible externalities arising from relative income concerns. Nevertheless, they focus on the case of a closed economy. Recent evidence suggests that globalization has reduced the barriers to international labor mobility. For example, Kleven et al. (2013), Kleven et al. (2014) and Akcigit et al. (2016) estimate large migration elasticities with respect to the tax rate for highly skilled workers. In addition, as mentioned in Piketty and Saez (2013), the migration elasticity for all top earners is likely to increase over time as labor markets become better integrated and the fraction of foreign workers grows. The mobility of taxpayers thus induces tax competition between countries. In addition, globalization also induces more cross-country social comparisons; that is, the welfare status of an individual also depends on how it compares to that of individuals living in other countries. For example, using survey data for countries in Western Europe, Becchetti et al. (2013) find that the contribution of cross-country comparisons to well-being increased from the early 1970s to 2002. Piketty (2014) even argues that cross-country social comparisons constitute an important part of the motivation behind Thatcher’s and Reagan’s drastic income tax reductions in the early 1980s.

The goal of this paper is, therefore, to address the following questions. How do the externality-correcting and income-redistributing roles of an income taxation policy interact in an open economy with competing governments? How does the tax competition induced by cross-border labor mobility affect the interplay of relativity and inequality in determining the optimal structure of income taxes? Moreover, how might the effect change across alternative forms of tax competition? To the best of our knowledge, none of the papers in the literature answers these questions in the optimal nonlinear income taxation framework inspired by Mirrlees (1971).

We focus on income tax schedules that competing governments find optimal to implement in two types of non-cooperative equilibria: Nash and Stackelberg. We begin with the Nash solution in which each country takes the strategy (or policy choice) of the opponent country as given. Each government fully internalizes consumption externalities affecting workers within its own country but ignores the externalities affecting the opponent country. However, as argued by Aronsson and Johansson-Stenman (2015), Nash competition is not necessarily the most realistic assumption since the ability to commit to public policy may differ across countries, which may arise from the differentials in political institutions or the size of government (or fiscal capacity). Thus, we further analyze a Stackelberg equilibrium in which one country acts as the leader with the opponent country as the follower. As is canonical, the leader shall recognize the behavioral responses of the follower and account for the externalities it imposes on the follower. This, accordingly, implies that optimal tax schedules in these two types of equilibrium are generally different for the leader country.

In each country, workers differ in both skill levels and migration costs. The distribution functions of skills and migration costs are continuously differentiable and assumed to be public knowledge, while their values are the private information of each worker. We thus follow the mechanism design approach. Taking as given the income taxation policies implemented in both countries, privately informed workers make individual decisions along two margins: the allocation of one unit of time between work and leisure on the intensive margin and the location choice on the extensive margin. The endogenous residence choice, which allows workers to have a reservation utility that depends on the tax policy of the opponent country, induces mobile tax bases. As such, individual participation constraints characterized as choosing to stay rather to migrate shall be incorporated into the government budget constraints through the endogenous tax bases (or ex post skill distributions). To make the analysis more transparent, we restrict attention to the most redistributive social objective, maximin, in the spirit of Rawls (1971). After accounting for individual responses, each government designs incentive-compatible allocations such that the utility of the worst-off workers is maximized and the public-sector budget constraint is satisfied. Throughout, taxes can be conditioned only on the observable income and are levied according to the residence principle.

We characterize the best response of each government and obtain a formula determining optimal (second-best) marginal tax rates (MTRs) for all skill levels. The optimal tax formula obtained by Oswald (1983) and K&T for closed economies is augmented by a migration effect that changes the Pigouvian tax term and the Mirrleesian tax term, leading to a more comprehensive formula. In addition, as in Lehmann et al. (2014), we derive an optimal tax formula under the useful benchmark

\footnote{In fact, this kind of research is of considerable practical relevance. For instance, given that income inequality has risen significantly in both the U.S. and China (e.g., Piketty, 2014; Huang, 2019), it is reasonable to conjecture that social status concerns measured in relative income/consumption will become a more serious social-economic issue.}

\footnote{Specifically, an individual who exhibits jealousy is subjectively worse off (or less happy) when she realizes that individuals (especially with similar skills) living in other countries turn out to earn more. As globallization is the process of the world becoming more connected and interdependent, such knowledge about foreigners is available through media report, traveling, and social networking sites.}

\footnote{This historical event of tax competition may be interpreted from the following counterfactual perspective. That is, if only the U.S. had reduced income taxes, citizens in the U.K. might have envied their counterparts in the U.S. and those with high skills might even have chosen to migrate to the U.S., holding everything else constant. However, if one interprets this as citizens being proud of their country of origin, then domestic average income should enter the objective function positively, as suggested by a referee. The current paper focuses on the former interpretation (or the effect of jealousy on income tax design), and interested readers can refer to Dai (2018), which has investigated the latter interpretation.}

\footnote{As is demonstrated by Broadway and Jacquet (2008), focusing on the maximin objective significantly simplifies the analytical analysis of the optimal income tax structure.}
called the Tiebout-best, in which workers’ skills are assumed to be public knowledge, while migration costs remain private. By eliminating incentive-compatibility constraints, the maximization problem of tax design becomes much simpler. In fact, we explicitly solve for the Tiebout-best tax liabilities and MTRs for all skill levels. Interpreting the Tiebout-best as the usual first-best (or the complete information) benchmark, the constrained optimum exhibits a version of the no-distortion-at-the-top property for the highest skilled workers but with a downward distortion relative to the Tiebout-best for the lowest skilled workers.\footnote{See, e.g., Laffont and Martimort (2002).}

In the optimal income taxation literature, economists either focus on how consumption relativity and income inequality jointly shape the optimal tax schedules under a single government in the case of a closed economy (e.g., Alvarez-Cuadrado and Long, 2012; K&T; Aronsson and Johansson-Stenman, 2014) or focus on how labor mobility and income inequality jointly shape the optimal tax schedules with two competing governments (e.g., Simula and Trannoy, 2012; Lehmann et al., 2014; Lipatov and Weichenrieder, 2015). Although these studies have provided some useful insights, the first strand of literature neglects cross-border effects and assumes away the possibility that people may have endogenous outside options, and hence, their reservation utilities should be endogenously determined in the optimal tax design problem. As people do care about social status that creates positional externalities (see, for example, Fong, 2001, and Heffetz and Frank, 2011 for evidence), the social welfare function adopted by the second strand of literature that fails to internalize such externalities is biased and hence misleading for public policy suggestions. Indeed, such social welfare functions are not well defined in terms of the design of optimal redistributive taxation policies. By analyzing optimal income taxation in an environment with both competing governments under the threat of migration, and individuals that have concerns for relative consumption, the present paper contributes to the existing literature where these two features have only been considered in isolation (respectively by Lehmann et al., 2014 and K&T).

For the income taxation policy, relativity calls for an externality-corrective role while inequality calls for an income-redistributive role. To analyze how relativity and inequality jointly shape the MTR facing top-income workers, we obtain a closed-form formula for the optimal asymptotic MTR that is shown to be strictly decreasing in the elasticity of migration. That is, the migration threat from the highest skilled workers has been taken into account seriously by both governments. We show that the elasticity and level of migration are two key variables in determining whether relativity and inequality play a complementary role in shaping the optimal top tax rate. Under both Nash and Stackelberg competition, if the migration elasticity is no greater than one (or the migration threat is weak), then the greater inequality is, the smaller the effect of relativity in raising the top tax rate, or the stronger the relative consumption concern is, the smaller the effect of inequality in raising the top tax rate. Thus, relativity and inequality play a substitutive role under small migration intensities; when migration threat or competition pressure is weak, the externality-correcting and income-re-distributing roles of the taxation policy act as substitutes in terms of raising the top MTR. Our result subsumes the prediction of K&T regarding top-income workers as a special case with a zero migration elasticity. If the migration elasticity is greater than one (or the migration threat is strong), then the greater inequality is, the larger the effect of relativity in raising the top tax rate, or the stronger relative consumption concern is, the larger the effect of inequality in raising the top tax rate, as long as the ex post mass of top-income workers is below some threshold; otherwise, relativity and inequality play a substitutive role. Kleven et al. (2013), Kleven et al. (2014) and Acgigt et al. (2016) find empirical evidence that the migration elasticities for highly paid foreigners with respect to the tax rate can be larger than one, so we establish under reasonable migration probabilities a complementary relationship that is exactly the opposite of the prediction obtained by K&T. In addition, the form of strategic tax competition turns out to be qualitatively neutral.

By using realistic parameter values from empirical studies, we simulate these tax rates in both Nash and Stackelberg equilibria and compare them to those calculated using the K&T formula. In both types of equilibria, our calculation shows that the country with a large labor inflow imposes much lower (less redistributive) tax rates than suggested by K&T, while the country with a large labor outflow imposes much higher (more redistributive) ones. By and large, a less redistributive top tax rate attracts the inflow of residents with the highest skill level, which in turn contributes to the ex post tax base. In addition, there are combinations of parameter values measuring relativity, inequality and mobility such that tax competition induces higher MTRs than in autarky. Consequently, depending on the sizes of within-country and cross-country relative consumption concerns, it may be optimal to lose or drive out some individuals of the highest skill through the migration channel to reduce the domestic average consumption/income, thereby making the lowest skilled workers better off through the social comparison effect. This finding, on the one hand, is reasonable in a sense given that the goal of governments is to maximize the welfare of the lowest skilled. On the other hand, it is technically due to the potential multiplicity of equilibria, and hence, it represents only one equilibrium prediction among many. As an implication for open economies, normative public policy design on income taxes must take between-country strategic tax competition, cross-border migration and relative consumption concerns as relevant factors; otherwise, workers are likely to face welfare loss and/or the economy could face efficiency loss.\footnote{In fact, these concerns seem to be the major motive behind Trump’s massive tax cuts, although tax reform per se may not be an optimal solution to the political economic issues facing the United States.}

Our study is related to the literature studying optimal nonlinear income taxation in an open economy, such as Mirrlees (1982), Simula and Trannoy (2010), Bierbrauer et al. (2013), Lehmann et al. (2014), Aronsson and Johansson-Stenman (2015),
and Blumkin et al. (2015). The major distinction between these studies and our paper is that we focus on examining how the interplay of relativity and inequality determines the optimal nonlinear income tax schedule and, moreover, how the joint effect of externality correction and inequality attenuation on the optimal top tax rate is modified by tax-driven migration. The interesting study of Aronsson and Johansson-Stenman (2015) also considers both tax competition and relative consumption concerns, but it assumes away cross-border labor mobility for simplicity. We show that migration can shape the tax—competition effect on equilibrium tax rates in a novel way, i.e., between-country competition does not necessarily lead to lower (or less redistributive) equilibrium tax rates, and hence the conventional wisdom that the presence of labor mobility shall limit the ability of governments to redistribute incomes via a tax-transfer system is not always true for any country under the globalization of labor markets. More important, we show that both the elasticity and level of migration are crucial variables for judging whether relativity and inequality act as complements or substitutes in shaping the optimal top tax rate. In sum, to address the classical equity-efficiency tradeoff when designing optimal nonlinear income tax schemes in a more realistic setting, our study demonstrates the qualitative and quantitative relevance of simultaneously accounting for tax-driven migration and relative consumption (or social status) concerns.

The remainder of the paper is organized as follows. Section 2 establishes the model. Sections 3 and 4 derive and characterize the optimal tax formulas in Nash and Stackelberg equilibria, respectively. Section 5 provides some numerical examples of the optimal asymptotic MTRs and compares our results to those calculated using the K&T formula. Section 6 concludes the paper. Proofs are relegated to Appendix A.

2. The model

We consider an economy consisting of two countries, indexed by $i \in \{A, B\}$. The measure of workers in country $i$ is normalized to 1, while that of the opponent country $i^{-} = \{A, B\}$ is denoted by $n_{i}^{-}$, with $0 < n_{i}^{-} \leq 1$. Each worker is characterized by three characteristics: her native country $i \in \{A, B\}$, her skill type $w \in [w, \overline{w}]$ with $0 < w < \overline{w} \leq \infty$, and the migration cost $m \in \mathbb{R}^{+}$ she supports if deciding to live abroad. If a worker faces an infinitely large migration cost, then she is immobile. Following Lehmann et al. (2014) and Dai and Tian (2020), we do not impose any restriction on the correlation between skill types and migration costs.

The skill density function in country $i$, $f_{i}(w) = \hat{f}_{i}^{\prime}(w) > 0$, is assumed to be differentiable for all $w \in [w, \overline{w}]$ and is single-peaked, with a mode at $w_{m}$. For each skill $w$, $g_{i}(m|w)$ denotes the conditional density of the migration cost, and $G_{i}(m|w) = \int_{0}^{m} g_{i}(x|w)dx$ denotes the conditional cumulative distribution function. The initial joint density of $(m, w)$ is thus $g_{i}(m|w)f_{i}(w)$, while $G_{i}(m|w)\hat{f}_{i}(w)$ is the mass of workers of skill $w$ with migration costs lower than $m$.

Following Mirrlees (1971), governments do not observe workers’ types $(w, m)$ and can only condition transfers on earnings $y$ via a nonlinear income tax function, $T_{i}(\cdot)$, for $i = A, B$. By assumption, taxes are levied according to the residence principle. In an open economy with international labor mobility, the migration threat actually induces tax competition between these two governments, and we shall consider both Nash and Stackelberg competition.

2.1. Individual choices

Assume that all workers have the same additively separable utility function. Therefore, for a worker of type $(w, m)$ in country $i$, her utility function is written as:

$$u(c_{i}(w), l_{i}(w); \mu_{i}, \mu_{i^{-}}; m) = v(c_{i}(w)) - h(l_{i}(w)) + \psi(\mu_{i}, \mu_{i^{-}}) - \ln m,$$

(1)

where $c_{i}$ is consumption, $l_{i}$ is labor (and $1 - l_{i}$ is leisure), $\ln$ is equal to 1 if she decides to migrate and to 0 otherwise, $\mu_{i}$ is a domestic comparison consumption level, and $\mu_{i^{-}}$ is a cross-country comparison consumption level, with $v^{'} > 0 \geq v''$, $h^{'} > 0$ and $h'' > 0$. Following common practice,\(^8\) comparison consumption levels are constructed as follows:

$$\mu_{i} = \frac{1}{w} \int_{w}^{\overline{w}} c_{i}(w)f_{i}(w)dw,$$

(2)

for $i \in \{A, B\}$. Here, we assume that individuals make social comparisons before they make migration decisions. As a result, their migration decisions are affected by individual jealousy.\(^9\) This is a reasonable assumption because many people choose to relocate from developing countries of origin, such as China, India and Mexico, to developed countries, such as the U.S., the U.K. and Canada, because they envy their counterparts in these destination countries. Elaborating further, the complex interaction between mobility and social comparison concerns in equilibrium proceeds as follows. On the one hand, individual migration decisions are affected by relative consumption concerns, which depend on average after-tax incomes and hence are affected by tax policies. On the other hand, individual participation constraints characterized as that an individual prefers staying in the country of origin to migrating to the opponent country, and the effects on the tax base of each country should be considered by governments when designing optimal income tax policies, thereby affecting the average after-tax incomes and social comparison concerns.


\(^9\) In Appendix B, we also derive the tax formula for the circumstance in which individuals make relative consumption comparisons based on ex post (i.e., after migration occurs) rather than the ex ante skill distributions.
For later use, we further give the following:

**Assumption 2.1 (Bounded Jealousy).** For \( \psi_i(\mu_i, \mu_{-i}) = \partial \psi_i / \partial \mu_i < 0 \), \( \psi_{-i}(\mu_i, \mu_{-i}) = \partial \psi_i / \partial \mu_{-i} < 0 \), we have \( \max \{\psi_i(\mu_i, \mu_{-i}), \psi_{-i}(\mu_i, \mu_{-i})\} = v'(c_i(w)) \) for \( i \in \{A, B\} \) and \( w \in [\overline{w}, \overline{w}] \).

Under the assumption of \( \partial \psi_i / \partial \mu_i < 0 \) and \( \partial \psi_i / \partial \mu_{-i} < 0 \), individuals exhibit jealousy. That is, by (1) and everything else equal, the utility of any individual becomes smaller (or an individual is subjectively worse off) when the consumptions of other individuals become larger (and hence the average consumption levels, \( \mu_i \) and \( \mu_{-i} \), become higher). Assumption 2.1 also states that the utility contribution of relative consumption is strictly smaller than that of absolute consumption. This assumption is consistent with general intuition and real data (see Clark et al., 2008).

The worker obtains her income from wages, with the before-tax income denoted by \( y_i = w_i(w) \geq 0 \). Her budget constraint is thus:

\[
c_i(w) = y_i(w) - T_i(y_i(w)). \tag{3}
\]

Each worker is assumed to be small relative to the whole economy, and hence, she takes \( \mu_i \) and \( \mu_{-i} \) as exogenously given. If she stays in country \( i \), she maximizes (1) subject to \( I = 0 \) and (3), yielding the first-order condition:

\[
\frac{h'(l_i(w))}{w T'(y_i(w))} = 1 - T'_i(y_i(w)). \tag{4}
\]

We denote by \( U_i(w) \) her indirect utility.

We now proceed to her migration decision. We assume that migration occurs if and only if \( m < U_i(w) - U_{-i}(w) \). As in Lehmann et al. (2014) and Dai and Tian (2020), after combining the migration decisions made by workers born in both countries, the ex post density of residents of skill \( w \) in country \( i \) can be written as:

\[
\phi_i(\Delta_i(w); w) = \begin{cases} f_i(w) + G_{-i}(\Delta_i(w)|w)f_{-i}(w)n_{-i} & \text{for } \Delta_i(w) \geq 0, \\ (1 - G_i(-\Delta_i(w)|w))f_i(w) & \text{for } \Delta_i(w) \leq 0. \end{cases} \tag{5}
\]

with \( \Delta_i(w) \equiv U_i(w) - U_{-i}(w) \). To ensure that \( \phi_i(\cdot; w) \) is differentiable, we impose the technical restriction that \( g_i(0|w)f_i(w) = g_{-i}(0|w)f_{-i}(w)n_{-i} \), which is verified when the two countries are symmetric or when there is a fixed cost of migration, namely \( g_i(0|w) = g_{-i}(0|w) = 0 \). We can then define the semi-elasticity of migration and the elasticity of migration, respectively, as:

\[
\eta_i(\Delta_i(w); w) \equiv \frac{\partial \phi_i(\Delta_i(w); w)}{\partial \Delta_i} \frac{1}{\phi_i(\Delta_i(w); w)} \tag{6}
\]

and

\[
\theta_i(\Delta_i; w) \equiv c_i(w)\eta_i(\Delta_i; w). \tag{7}
\]

For later use, and to save on notation, we let \( \tilde{f}_i(w) \equiv \phi_i(\Delta_i(w); w) \), \( \tilde{\eta}_i(w) \equiv \eta_i(\Delta_i(w); w) \) and \( \tilde{\theta}_i(w) \equiv \theta_i(\Delta_i; w) \).

2.2. Governments

In country \( i \in \{A, B\} \), a benevolent government designs the tax system to maximize the welfare of the worst-off workers. By using (1) and (4), it is easy to show that \( U_i(w) = \min[U_i(w) : w \in [\overline{w}, \overline{w}]] \). That is, the worst-off workers are exactly those with a wage rate \( w \) at the bottom of the skill distribution.

We choose maximin as the social objective due to the following considerations. First, many jobs held by workers with the lowest skills are at the bottom of the global value chain and are characterized as low-paid, insecure and dangerous (Gerffendi and Luo, 2014). Second, they have the lowest migration (or foot-voting) ability, as migration rates increase in skill (Docquier and Marfouk, 2006). Third, especially in developed countries, such workers are worse off in an open economy because they may lose jobs in the global competition with those workers with the lowest skills in developing countries. For example, as argued by Stiglitz (2018), the demand for unskilled labor in advanced countries declines with the opening of trade and with that its wage, as workers’ bargaining power is weakened as firms threaten to relocate if workers do not accept lower wages or worse working conditions. Fourth, as a normative criterion, maximin is a crucial principle in achieving social justice as suggested by Rawls (1971).

As is canonical, each government faces two types of constraints. The first is the fiscal budget constraint:

\[
\int_{\overline{w}}^{\overline{w}} T_i(y_i(w))\phi_i(U_i(w) - U_{-i}(w); w)dw \geq R, \tag{8}
\]

where \( R \geq 0 \) is an exogenous revenue requirement. As \( v_i(\cdot) > 0 \), (8) must be binding. In particular, here, the participation constraint has been incorporated into the fiscal budget constraint (8) through the ex post skill density \( \phi_i \) that endogenizes the tax base. As \( \phi_i \) is a function of the difference in gross utilities, \( U_i(w) - U_{-i}(w) \), which depends on the taxation policies of both jurisdictions, the strategic interaction plays a role through the fiscal budget constraint of each government.

The second is the set of incentive-compatibility constraints:

\[
v(c_i(w)) - h(y_i(w)/w) \geq v(c_i(w')) - h(y_i(w')/w) \quad \forall w, w' \in [\overline{w}, \overline{w}]. \tag{9}
\]
The necessary conditions for (9) to be satisfied are as follows:

\[ \dot{U}_i(w) = h'(l_i(w)) \frac{\dot{f}_i(w)}{w} \quad \forall w \in [w^L, w^R]. \]  

(10)

which gives the first-order incentive compatibility (FOIC) conditions. Sufficiency is guaranteed by the second-order incentive compatibility (SOIC) conditions, \( \dot{y}_i(w) \geq 0 \) for all \( w \). If the SOIC conditions are verified to be not binding for all \( w \), then the first-order approach is appropriate.

As a result, the optimal tax design is equivalent to solving the following maximization problem:

\[
\max_{[U_i(w), l_i(w), \mu_i]} U_i(w)
\]

subject to (2), (8), (10) and \( \dot{y}_i(w) \geq 0 \) for all \( w \).

3. Nash equilibria

3.1. Optimal tax formula

We state the solution to the maximization problem above under Nash competition in the following theorem.

**Theorem 3.1.** In a Nash equilibrium with \( \dot{y}_i(w) > 0 \) for all \( w \), the second-best MTRs verify:

\[
\frac{T^*_i(y_i(w))}{1 - T^*_i(y_i(w))} = \frac{\gamma_i f_i(w)}{\lambda_i f_i(w)} + \frac{\mathcal{A}_i(w) B_i(w) C_i(w)}{\tilde{f}_i(w) - \tilde{f}_i(w)}.
\]

(11)

where: \( \mathcal{A}_i(w) \equiv 1 + [l_i(w) h''(l_i(w))/h'(l_i(w))] \). \( B_i(w) \equiv \left[ \tilde{f}_i(w) - \tilde{f}_i(w) \right] / w \tilde{f}_i(w) \).

\[
C_i(w) = \left[ v'(c_i(w)) \int_w^{w^R} \frac{1}{\psi(c(t), \mu_i, \mu_{i-1})} \left( 1 + \frac{\mu_i}{\lambda_i} \right) \left( T_i(y_i(t)) \tilde{n}_i(t) \right) \tilde{f}_i(t) dt \right]
\]

(12)

and

\[
\frac{\gamma_i}{\lambda_i} = -\frac{\int_w^{w^R} \psi(c(t), \mu_i, \mu_{i-1}) \tilde{f}_i(w) dw}{1 + \int_w^{w^R} \psi(c(t), \mu_i, \mu_{i-1}) \tilde{f}_i(w) dw}
\]

(13)

with \( \tilde{f}_i(w) \equiv \int_w^w \tilde{f}_i(t) dt \) denoting the ex post skill distribution in country \( i \in \{A, B\} \). Moreover, if \( T^*_i(y_i(w)) \) is non-increasing in \( w \), then the SOIC conditions are not binding, namely \( \dot{y}_i(w) > 0 \) holds for all \( w \).

**Proof.** See Appendix A. □

Our optimal tax formula (11) differs from the classic one derived by Diamond (1998) and Saez (2001) in three ways: (i) the ex post density \( \tilde{f}_i(\cdot) \) of taxpayers replaces the ex ante density \( f_i(\cdot) \); (ii) tax liability \( T_i(y_i(\cdot)) \) enters the term \( C_i(w) \) as a tax level effect, and (iii) a Pigouvian tax is used to correct for consumption externalities. Additionally, (i) and (ii) constitute new features relative to K&T.

To intuitively interpret the optimal tax formula (11), we investigate the effects of a small tax reform, as shown in Fig. 1, in a unilaterally deviating country \( i \): the second-best MTRs \( T'_i(y_i(w)) \) are uniformly increased by a small amount \( \tau > 0 \) on the income interval \( [y_i(w) - \delta, y_i(w)] \) for some small constant \( \delta > 0 \). As a consequence, tax liabilities above \( y_i(w) \) are uniformly increased by \( \delta \tau \). This gives rise to the following effects.

First, a worker with income in \( [y_i(w) - \delta, y_i(w)] \) responds to the rise in the MTR by a substitution effect between leisure and labor, which hence reduces the taxes she pays. Second, each worker with skills above \( w \) faces a lump-sum increase \( \delta \tau \) in her tax liability, which is referred to as the mechanical effect in the literature (e.g., Saez, 2001). Since the unilateral rise in tax liability reduces her indirect utility in the deviating country, relative to its competitor, the amount of labor outflow increases, and hence, the number of taxpayers with skills above \( w \) decreases. Following Lehmann et al. (2014), we define the tax liability effect as the sum of the mechanical and migration effects for all skill levels above \( w \). Third, the increase in tax liability tightens the consumption budget, and hence it follows from (12) that income effect will in turn reduce the positive mechanical effect. Since the optimal tax formula (11) is derived based on the Nash equilibrium, any unilateral deviation we consider cannot induce any first-order effect on the tax revenue of the deviating country; otherwise, the policy choice in the deviating country would not be a best response.\(^\text{10}\) This implies that the tax liability effect must be positive such that the substitution effect is offset by the tax liability effect.

\(^\text{10}\) As in Lehmann et al. (2014), the best response allocations given in Theorem 3.1 can also be characterized by solving a dual problem, that is, maximizing tax revenues subject to the FOIC condition, the constraint with the indirect utility of the lowest skills being greater than or equal to some exogenous value, and the equation for average consumption.
In Theorem 3.1, the average consumption of country $-i$ is taken as exogenous, and the equation for $\mu_{-i}$ has not been taken into account as a constraint facing the government of country $i$. The following lemma shows how the second-best MTRs derived in Theorem 3.1 change after taking into account the equation for $\mu_{-i}$.

**Lemma 3.1.** If $\mu_{-i} = \int w c_{-i}(w) f_{-i}(w) dw$ is taken as a constraint facing the government in country $i$, then the tax formula derived in Theorem 3.1 does not change except that $\gamma_i/\lambda_i$ is now given by:

$$\gamma_i/\lambda_i = -\int w \phi(\mu_{-i}/\lambda_{-i}) f_i(w)dw / \int w \psi(\mu_{-i}/\lambda_{-i}) f_i(w)dw + q_i/\lambda_i - \int w \phi(\mu_{-i}/\lambda_{-i}) f_i(w)dw / \int w \psi(\mu_{-i}/\lambda_{-i}) f_i(w)dw,$$

in which $q_i > 0$ denotes the Lagrangian multiplier on the constraint $\mu_{-i} = \int w c_{-i}(w) f_{-i}(w) dw$. Under Assumption 2.1 and the new formula for $\gamma_i/\lambda_i$, the change in second-best MTRs is ambiguous, i.e., they may become higher or lower.

**Proof.** See Appendix A. □

It is easy to verify that incorporating the equation for $\mu_{-i}$ as a constraint will not change the following formal results, which do not depend on the specific value of $\gamma_i/\lambda_i$ takes, although doing so tends to generate a nontrivial quantitative effect on the equilibrium MTRs.

To see how relativity changes the MTR and the average tax rate (ATR), we also numerically solve the optimal tax formula (11) under the following assumptions (see Fig. 2). First, these two countries are assumed to be symmetric. Second, following Jacquet et al. (2013), we select the mode skill level $w_m = 19,800$ and the highest skill level $w = 340,748$ and assume that workers within this income interval have a Pareto income distribution, with a density function $f(w) = aw^a/w^{a+1}$ for $w_m \leq w \leq w$. Third, we use quasilinear-in-consumption preferences with a constant elasticity of labor supply, formally $u_i = c_i - [1^{1+1/2}/(1 + 1/2)] + \sigma_D\mu_i + \sigma_F\mu_{-i}$ with $\sigma_D, \sigma_F \in (-1, 0)$. Fourth, similar to the distribution assumption used by Jacquet et al. (2013), we let the conditional distribution of migration costs be logistic:

$$G(0|w) = \frac{\exp(-\chi w)}{1 + \exp(-\chi w)} \text{ for } \chi \in (0, 1).$$

Parameter values for simulation are given by $\varepsilon = 0.25$, $a = 2$, $\chi = 0.5$, $l = 0.33$ and $\sigma_D \in (-1, 0)$. It follows from Fig. 2 that both the MTR and ATR increase as the degree of relative consumption concern $[\sigma_D]$ increases, specifically from 0.25 to 0.5 and then to 0.75, for any $w \in [w_m, w]$.

---

11 We thank a referee for suggesting that we consider this case.
12 Since it is impossible to solve for a formula for the ATR in the current context, we rely on numerical simulation to determine the shape of the ATR and how it changes with respect to the change in the degree of consumption relativity.
13 See, e.g., Diamond and Saez (2011), Kanbur and Tsoumala (2013) and Jacquet et al. (2013).
14 Applying the utility function $u_i = c_i - [1^{1+1/2}/(1 + 1/2)] + \sigma_D\mu_i + \sigma_F\mu_{-i}$ to Eq. (13) gives rise to $\gamma_i/\lambda_i = -[\sigma_D/(1 + \sigma_F)]f_i(w)$; hence, the optimal tax formula does not depend on $\sigma_F$ in this case. In fact, as long as $\partial^2 u_i/\partial \mu_i \partial \mu_{-i} = 0$, the optimal tax formula for country $i$ does not depend on $\sigma_F$. We adopt...
3.2. Qualitative properties

To derive the qualitative properties of the optimal tax formula established in Theorem 3.1, we follow the approach developed by Jacquet et al. (2013) and begin by considering the same problem as in the second best, except that skills \( w \) are common knowledge, and thus migration costs \( m \) remain private. Using the same terminology as Lehmann et al. (2014), we call this benchmark the Tiebout-best.

**Lemma 3.2.** In a Nash equilibrium, we have the following predictions:

(i) The Tiebout-best tax liabilities are given by

\[
T_i^*(y_i(w)) = \frac{1}{\nu_i(c_i(w))\eta_i(w)} \left[ 1 + \frac{\gamma_i}{\lambda_i} \frac{f_i(w)}{\gamma_i f_i(w)} \right]
\]

for \( \forall i \in \{A, B\}, \forall w \in [w, \bar{w}] \) with an upward jump discontinuity at \( w \).

(ii) The Tiebout-best MTRs verify:

\[
\frac{T_i^*(y_i(w))}{1 - T_i^*(y_i(w))} = \frac{\gamma_i}{\lambda_i} \frac{f_i(w)}{\gamma_i f_i(w)} \quad \forall w \in [w, \bar{w}]
\]

with \( \gamma_i/\lambda_i \) given in Theorem 3.1.

**Proof.** See Appendix A. □

Under jealousy-type consumption comparisons, it follows from (13) that \( \gamma_i/\lambda_i > 0 \). Thus, for all but the lowest skilled, the Tiebout-best tax liabilities are strictly decreasing in the semi-elasticity of migration, as shown in part (i). The intuition for this result is straightforward. In addition, if the revenue requirement \( R \) is sufficiently small, then it follows from the fiscal budget constraint (8) that the worst-off workers receive net transfers in the Tiebout-best economy. As shown in part (ii), the Tiebout-best MTRs are used to correct for consumption externalities and attract labor inflow. In particular, these MTRs are positive in equilibrium. Additionally, the ex ante to ex post density ratio, \( f_i(w)/\gamma_i f_i(w) \), and the jealousy comparison term, \( \gamma_i/\lambda_i \), impose a complementary effect on the Tiebout-best tax liabilities and MTRs.

Assuming quasilinear-in-consumption preferences, the formula for the Tiebout-best tax liability obtained by Lehmann et al. (2014), namely, the tax liability required from the residents with skill levels above \( w \) is equal to the inverse of their semi-elasticity of migration \( \eta_i(w) \), is augmented by the positive externality-corrective tax component \( \gamma_i f_i(w)/[\lambda_i f_i(w)] \). In particular, if attention is restricted to the pure redistributive taxation under \( R = 0 \), then the worst-off residents receive more transfers than in the economy considered by Lehmann et al. (2014). As such, the relativity concern leads the Tiebout-best tax schedule to have a larger income redistributive effect. This is in some ways consistent with the empirical finding that social comparisons increase the demand for income redistribution (e.g., Senik, 2009; Clark and Senik, 2010).

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**Fig. 2.** MTR and ATR in a Symmetric Nash equilibrium.

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this linear specification with respect to relative consumption mainly because of its advantage that the degrees of relative consumption concerns can be totally measured by exogenous constants, namely \( \sigma_D \) and \( \sigma_F \); otherwise, we are unable to draw figures such as Fig. 2 to explicitly and intuitively see how the change in the degree of relativity concerns changes marginal and average tax rates in a symmetric Nash equilibrium. Moreover, we can let \( \sigma_F \) and \( \sigma_D > 0 \) such that Assumption 2.1 is fulfilled.
To characterize the second-best tax schedules, we offer the following proposition:

**Proposition 3.1.** Suppose that Assumption 2.1 holds; then, the optimal tax structure in the Nash equilibrium has the following characteristics:

(i) If \( T_i(y_i(w)) \leq T^*(y_i(w)) \) for \( \forall w \in (w, \bar{w}) \), then \( T'_i(y_i(w)) > 0 \) for \( \forall w \in (w, \bar{w}) \).

(ii) If \( T'_i(y_i(w)) > 0 \) and \( T'_i(y_i(w)) = T^*_i(y_i(w)) > 0 \) for \( w < \infty \).

(iii) If \( \theta_i(y_i(w)) \) is iselastic, \( f_i(w) / f_i(w) \) is decreasing in \( w \), and \( T_i(y_i(w)) \leq T^*_i(y_i(w)) \) for \( \forall w \in [w, \bar{w}] \), we then have the following:

(a) \( T'_i(y_i(w)) \) is decreasing for \( w \leq \bar{w} \);  
(b) \( T'_i(y_i(w)) \) is decreasing for \( w > \bar{w} \) when \( w_1(w) \) is non-decreasing in \( w \).

(iv) If \( f_i(w) / f_i(w) \) is non-increasing in \( w \), \( \nu / \nu' \leq \bar{\theta}_i(w) = \bar{\theta}_i(w) \) and there exists a \( w \in (w, \bar{w}) \) such that \( T'_i(y_i(w)) \geq 0 \), then

\[
T_i(y_i(w)) = \begin{cases} 
T^*_i(y_i(w)) & \text{for } w < w \leq \bar{w}, \\
\frac{1}{\nu'} \left[ 1 + \frac{\nu}{\nu'} \bar{\theta}_i(w) \right] & \text{for } w = \bar{w}.
\end{cases}
\]

**Proof.** See Appendix A. □

Parts (i)-(ii) identify the (sufficient) conditions such that the second-best MTRs are positive over the entire income distribution. In particular, if tax liabilities are bounded above by the Tiebout-best tax liabilities, then the second-best MTRs are positive for nearly all skills. Part (iv) identifies the sufficient conditions such that the second-best tax liabilities are, indeed, bounded above by the Tiebout-best tax liabilities. Part (iii) identifies the conditions such that the SOIC conditions are not binding, namely, the first-order approach is reliable.

The Nash equilibrium tax schedule is thus featured as follows. For the lowest skilled workers, the second-best MTR is positive and is also higher than that under the Tiebout-best, implying a downward distortion relative to the Tiebout-best. However, for the highest skilled workers, the MTR is the same under the second-best and the Tiebout-best, which may be regarded as a version of “no distortion at the top” when the Tiebout-best is interpreted as the usual first-best.

The following proposition establishes a closed-form formula for the optimal asymptotic tax rates (or the tax rates placed on top-income workers).

**Proposition 3.2.** Suppose that the economic environments satisfy the following conditions:

(a) \( \nu(\cdot) = 1 \), namely quasi-linear-in-consumption preferences;  
(b) \( h(\cdot) \) is iselastic with elasticity coefficient \( \varepsilon > 0 \); and  
(c) \( F_i(w) \) is a Pareto distribution with \( \bar{W} = \infty \) and Pareto index \( \alpha_i > 1 \).

Then, the optimal asymptotic marginal tax rate (AMTR) in a Nash equilibrium is:

\[
T^*_i(\nu_i(\infty)) = -\frac{\nu_i(\infty)}{1 + \frac{(1 + \varepsilon)(1 + \alpha_i)}{2\alpha_i(1 + 1/a_i) + \bar{\theta}_i(\infty)}} \frac{1}{1 + \frac{(1 + \varepsilon)(1 + \alpha_i)}{2\alpha_i(1 + 1/a_i) + \bar{\theta}_i(\infty)}}
\]

with \( \bar{\theta}_i(\infty) \equiv \lim_{w \uparrow \infty} \bar{\theta}_i(w) \geq 0 \) and \( \alpha_i(\infty) \equiv \lim_{w \uparrow \infty} \frac{f_i(w)}{f_i(w)} \geq 0 \).

**Proof.** See Appendix A. □

Given that the optimal tax formula (11) is quite complicated, restrictions (a)-(c) must be tolerated for explicitly solving for the optimal AMTR. In fact, conditions (a) and (b) are widely used in the literature on optimal taxation, and the Pareto distribution is an empirically supported assumption for high-income workers. In the current context, the optimal AMTR is a continuously differentiable function of five important variables: the degree of consumption comparison \( \gamma_i / \lambda_i \), the measure of labor flow \( \alpha_i(\infty) \), the elasticity of labor supply \( \varepsilon \), the degree of income inequality \( 1/a_i \), and the elasticity of migration \( \bar{\theta}_i(\infty) \). In particular, AMTR is strictly decreasing in the elasticity of migration.

The following proposition characterizes the composition effect of consumption relativity and income inequality on the optimal AMTR.

**Proposition 3.3.** Suppose that \( \psi(\mu_i, \mu_i) = \sigma_D \mu_i + \bar{\psi}(\mu_i) \) for a constant \( \sigma_D \in (-1, 0) \), \( F_i(w) = F_i(w) \) and \( \partial \bar{f}_i(\infty)/\partial \alpha_i = 0 \); then, we have the following predictions.

(i) If \( \bar{\theta}_i(\infty) \leq 1 \), then

\[
\frac{\partial^2 T_i^*(y_i(\infty))}{\partial (\sigma_D) \partial (1/a_i)} < 0.
\]

(ii) If \( \bar{\theta}_i(\infty) > 1 \), then

\[
\frac{\partial^2 T_i^*(y_i(\infty))}{\partial (\sigma_D) \partial (1/a_i)} = \begin{cases} 
< 0 & \text{for } \alpha_i(\infty) < \left( \frac{\bar{\theta}_i(\infty)}{1 + \bar{\theta}_i(\infty)} \right)^{1 + \bar{\theta}_i(\infty)}, \cr
> 0 & \text{for } \alpha_i(\infty) > \left( \frac{\bar{\theta}_i(\infty)}{1 + \bar{\theta}_i(\infty)} \right)^{1 + \bar{\theta}_i(\infty)}.
\end{cases}
\]
Proof. See Appendix A. □

If migration elasticity is no greater than one, then relativity and inequality play a substitutive role in shaping AMTR. Specifically, the higher inequality is, the lower the effect of relativity in raising AMTR; similarly, the higher relativity is, the lower the effect of inequality in raising AMTR. In other words, if the migration threat from the highest skilled workers is weak, which may be due to a high migration cost facing these workers, then the externality-corrective and income-redistributive roles act as substitutes in shaping the optimal tax rate. However, if the elasticity of migration is greater than one, then relativity and inequality play a substitutive role only when the ex post mass of top-income workers is greater than some threshold; otherwise, relativity and inequality play a complementary role in shaping the optimal AMTR. Therefore, both the elasticity and level of migration are key determinants of predicting when relativity and inequality act as substitutes or complements in shaping the optimal top tax rate.

The substitution pattern implies that the demand for income redistribution driven by social comparisons is weakened by ability inequality, or the demand for redistribution driven by ability inequality is weakened by social comparisons. This holds in two cases: (1) the migration elasticity is small or the intensity of tax competition is low, and (2) the migration elasticity is large while the ex post mass of residents of the top skill level is large. In the former case, as the tax base is somehow immobile, if relativity and inequality enhance one another in motivating redistribution, the equity concern is overemphasized and efficiency losses arise due to the weakened incentive of labor supply, thus violating the best balance between equity and efficiency. In the latter case, as the high-skill tax base is sufficiently mobile, if the redistributive motive is too strong, then the ex post mass of high-skill workers would not be large. The case for the complementary pattern can be analogously analyzed. In a word, the composition effect of consumption relativity and income inequality on optimal AMTRs reflects how the fundamental equity-efficiency tradeoff is addressed in an open economy with competing governments. Under similar assumptions, K&T show that relativity and inequality always play a substitutive role in a closed economy. We show that such a conclusion depends on the elasticity of migration and the level of migration in an open economy. Thus, Proposition 3.3 subsumes the corresponding prediction of K&T as a special case with \( \bar{\theta}_1(\infty) = 0 \) and \( \alpha_1(\infty) = 1 \).

4. Stackelberg equilibria

4.1. Optimal tax formula

Without any loss of generality, we denote by \( i \) the leader country and by \(-i\) the follower country in the current Stackelberg game. We thus state the solution to the optimal tax design problem under Stackelberg competition in the following theorem.

**Theorem 4.1.** In a Stackelberg equilibrium, the optimal tax formula is the same as that in the Nash equilibrium, except that:

\[
\frac{\gamma_i}{\lambda_i} = \frac{\int_W \left( \frac{\partial c_i(w)}{\partial \mu_i} + \frac{\partial c_i(w)}{\partial \mu_{-i}} \frac{\partial \mu_{-i}}{\partial \mu_i} \right) f_i(w) dw}{1 - \int_W \left( \frac{\partial c_i(w)}{\partial \mu_i} + \frac{\partial c_i(w)}{\partial \mu_{-i}} \right) f_i(w) dw}
\]

with

\[
\frac{\partial \mu_{-i}}{\partial \mu_i} = \frac{\int_W \frac{\partial c_{-i}(w)}{\partial \mu_i} f_{-i}(w) dw}{1 - \int_W \frac{\partial c_{-i}(w)}{\partial \mu_{-i}} f_{-i}(w) dw}
\]

for the leader country \( i \).

**Proof.** See Appendix A. □

Theorems 4.1 and 3.1 combined demonstrate how the form of tax competition might affect optimal tax rates. Intuitively, since the leader country accounts for the behavioral response of the follower country in the Stackelberg game, it partially internalizes cross-country consumption externalities, namely, the additional term \( \frac{\partial \mu_{-i}}{\partial \mu_i} \) is generally different from zero.

4.2. Qualitative properties

Using Theorem 4.1, the following corollary is immediate.

**Corollary 4.1.** If \( |\psi_i(\mu_i, \mu_{-i})| > |\psi_{-i}(\mu_i, \mu_{-i})| \) for country \( i \), then the qualitative properties (of Nash equilibrium) established in Propositions 3.1 and 3.2 carry over to the current Stackelberg equilibrium.

---

15 Kleven et al. (2013), Kleven et al. (2014) and Akcigit et al. (2016), indeed, find empirical evidence that the migration elasticities for highly paid foreigners with respect to the tax rate can be larger than one.
For additively separable functional forms of $\psi(\mu_i, \mu_{-i})$, condition $|\psi_i(\mu_i, \mu_{-i})| > |\psi_{-i}(\mu_i, \mu_{-i})|$ means that the degree of domestic consumption comparison is greater than that of cross-country consumption comparison for workers in country $i$. Given the real-life observation that people more often make status comparisons with people who live in their social networks, this restriction can be regarded as reasonable.

**Proposition 4.1.** If economic environments satisfy the following conditions, the predictions established in Proposition 3.3 carry over to the current equilibrium:

(a) the utility function of relative consumption has the following form

$$\psi(\mu_i, \mu_{-i}) = \begin{cases} \sigma_D \mu_i + \sigma_F \mu_{-i} & \text{for country } i, \\ \sigma_D \mu_{-i} + \sigma_F \mu_i & \text{for country } -i \end{cases}$$

with coefficients $\sigma_D, \sigma_F \in (-1, 0)$ and $|\sigma_F| + |\sigma_D| < 1$;

(b) $f_i(w) = F_i(w)$; and

(c) $\partial^2 F_i(\infty)/\partial a_i = 0$

for the optimal AMTR of leader country $i$ in a Stackelberg equilibrium.

**Proof.** See Appendix A. □

Provided that we have assumed quasilinear-in-consumption preferences in solving for the optimal AMTR, condition (a) is thus a natural restriction. Condition (b) simplifies our analysis by eliminating asymmetry between these two countries, which is not an essential requirement for establishing the current prediction. Condition (c) is a technical assumption mainly for the purpose of simplicity. The main message conveyed by Proposition 4.1 is that the composition effect of relativity and inequality imposed on the optimal AMTR is generally the same under both forms of tax competition, even though the corresponding AMTRs are generally different in magnitude.

**Proposition 4.2.** If Assumption 2.1 holds, then the government of the leader country imposes a higher MTR in the Stackelberg equilibrium than that in the Nash equilibrium.

**Proof.** See Appendix A. □

Intuitively, since jealousy implies negative consumption externality and MTRs strictly increase as externality increases, the leader country that (partially) internalizes cross-country consumption externalities imposes a higher tax rate than that it may impose in a simultaneous-move static game where no one internalizes cross-country consumption externalities.

5. Numerical illustration

In this section, we provide some numerical examples on the optimal AMTR established in Proposition 3.2. Although these exercises are very coarse, they enable us to quantitatively see how large the difference in the optimal AMTR can be made by the effects of strategic tax competition and cross-country consumption comparisons.

For simplicity, we use the linear utility function of relative consumption given in condition (a) of Proposition 4.1. The following tables present AMTRs for different parameter values, when the Pareto index $a_i = 2$ and $3$, the coefficient of domestic relative consumption $\sigma_D = -0.5$ and 0, and the elasticity of labor supply $\varepsilon = 0.25, 0.33, \text{and } 0.5$. We consider three elasticity scenarios. The first two with $\varepsilon = 0.25$ and 0.33 are realistic, midrange estimates (see Saez et al., 2012), while $\varepsilon = 0.5$ is slightly larger than the average empirical estimate. We consider two inequality scenarios. The first with $a_i = 2$ is based on the 2005 U.S. empirical income distribution (see Diamond and Saez, 2011), while $a_i = 3$ is chosen to be larger than this realistic number to represent an experimental scenario with a more equal income distribution.

We consider two relativity scenarios. $\sigma_D = 0$ denotes the benchmark case without relative consumption, whereas $\sigma_D = -0.5$ measures the degree of jealousy. One of the key findings of the empirical research on relativity is that the estimated coefficients on income (consumption) and income comparison are statistically almost equal in absolute value and of the opposite sign (see, e.g., Luttmer, 2005). Given the assumption of quasi-linear preferences, $\sigma_D = -0.5$ seems to be reasonable. In fact, it is consistent with the finding of Alpizar et al. (2005), who use survey-experimental methods to determine the extent to which people care about absolute versus relative income and consumption. By assumption, the degree of cross-country social comparison is smaller than that of domestic social comparison, so we let $\sigma^x = 0.04$ in what follows. Following Piketty and Saez (2013), we let the value of the elasticity of migration be 0.25, i.e., $\theta_i(\infty) = 0.25$. We summarize all realistic parameter values in Table 1.

If $\Delta_i(\infty) \geq 0$, namely, top-income workers obtain an indirect utility in country $i$ that is no less than what they can obtain in the other country $-i$, then the density ratio $\alpha_i(\infty)$ must not be greater than 1. That is, country $i$ has the potential to attract more high-skill workers from the opponent country $-i$. Similarly, if $\Delta_i(\infty) \leq 0$, then the density ratio $\alpha_i(\infty)$ must not be smaller than 1. The following tables consider both cases.

\[\text{In fact, we were unable to find any realistic estimates of this parameter in the empirical literature.}\]
Specifically, effect level flow. If between cross-country economy concerns. We consider the case, in which a large labor inflow is induced through the substitution effect, thus benefiting the lowest skilled (or worst off) workers through the consumption comparison effect. Third, for each given elasticity and given degree of inequality, AMTR increases as elasticity increases. Our observation is also reasonable in the present context because increasing AMTR when labor-supply elasticity becomes larger could effectively reduce the income earned by the highest skilled through the substitution effect, thus benefiting the lowest skilled (or worst off) workers through the consumption comparison effect. Third, for each given elasticity and given degree of inequality, AMTR increases as elasticity increases. This observation is also reasonable in the present context because increasing AMTR when labor-supply elasticity becomes larger could effectively reduce the income earned by the highest skilled through the substitution effect, thus benefiting the lowest skilled (or worst off) workers through the consumption comparison effect.

We summarize the economic mechanism in Table 2, in which the superscripts of $MTR^O$ and $MTR^C$ denote an open economy and a closed economy, respectively. In particular, we consider only the MTR of the leader country under Stackelberg tax competition. Essentially, as shown in Table 2, relativity and migration are the determinant factors in the comparison.

Since no one internalizes the cross-country consumption externality under Nash competition, the relativity effect on MTR is the same between an open economy and a closed economy. In contrast, as the leader country (partly) internalizes the cross-country externality under Stackelberg competition, the relativity effect on MTR implemented by the leader country in an open economy should be greater than the counterpart in a closed economy. Therefore, if there is no migration between countries, only the MTR implemented under Stackelberg competition should be higher than the corresponding MTR implemented in a closed economy.

When comparing $MTR^O$ and $MTR^C$ in the Nash case, the migration effect is shown to dominate the relativity effect. If the magnitude of equilibrium labor flow is small, regardless of whether it is an inflow or an outflow facing the country under consideration, Nash competition implies a smaller MTR than that in a closed economy without any migration threat imposed on the government. This result seems to be consistent with the classical wisdom that the presence of geographical mobility shall limit the ability of a government to redistribute incomes via a tax-transfer system. Nevertheless, if the magnitude of equilibrium labor flow is large, and hence the Nash equilibrium is likely to be more asymmetric, then the migration effect is heterogeneous between the case of labor inflow and the case of labor outflow. Specifically, a large labor inflow must be induced by a much lower MTR than its $MTR^C$, while a large labor outflow must be induced by a much higher MTR than its $MTR^C$. In consequence, in an asymmetric migration equilibrium in which a country faces large labor outflow whereas the opponent country faces large labor inflow, this country under tax competition induced by cross-border labor mobility actually implements a much higher tax rate on the top skill level than what it may implement in the case of autarky. An asymmetric equilibrium emerges because the underlying environment of the two countries, such as the distribution functions of skills and migration costs, is allowed to be heterogenous.

When comparing $MTR^O$ and $MTR^C$ in the Stackelberg case, the relative dominance of the relativity effect and migration effect turns out to be ambiguous. If the magnitude of equilibrium labor flow is small, then the relativity effect dominates the migration effect irrespective of labor inflows or outflows facing the leader country, implying that $MTR^O$ under Stackelberg competition should be higher than its $MTR^C$. However, if the magnitude of equilibrium labor flow is large, then the migration effect dominates the relativity effect and it is heterogeneous between the case of labor inflow and that of outflow. Specifically, a large labor inflow must be induced by a much lower MTR than its $MTR^C$, while a large labor outflow must be

---

**Table 1**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Source/target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta(\infty)$</td>
<td>0.25</td>
<td>Migration elasticity</td>
<td>Piketty and Saez (2013)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.1-0.4</td>
<td>Labor-supply elasticity</td>
<td>Saez et al. (2012)</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>Pareto index</td>
<td>Diamond and Saez (2011)</td>
</tr>
<tr>
<td>$\sigma_a$</td>
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<td>Domestic relativity</td>
<td>Clark et al. (2008)</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>-0.2</td>
<td>Cross-country relativity</td>
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</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>Economic mechanism governing quantitative findings.</th>
<th>Nash</th>
<th>Nash</th>
<th>Stack</th>
<th>Stack</th>
</tr>
</thead>
<tbody>
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<td>=</td>
<td>&gt;</td>
<td>&gt;</td>
</tr>
<tr>
<td>Migration effect</td>
<td>Small</td>
<td>Large</td>
<td>Small</td>
<td>Large</td>
</tr>
<tr>
<td>$MTR^O$ Labor inflow</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&gt;</td>
<td>&lt;</td>
</tr>
<tr>
<td>$MTR^O$ Labor outflow</td>
<td>&lt;</td>
<td>&gt;</td>
<td>&gt;</td>
<td>&gt;</td>
</tr>
</tbody>
</table>

5.1. A comparison with K&T

In the following tables, we use red numbers to denote the optimal AMTRs calculated using the formula of K&T. In both types of equilibrium, we obtain three main findings under different values of $\alpha(\infty)$.

First, for each given labor-supply elasticity and given degree of relative consumption, AMTR increases as inequality increases. This is reasonable as the governments adopt the most redistributive social objective. Second, for each given degree of inequality and given degree of relative consumption, AMTR increases as elasticity increases. This observation is also reasonable in the present context because increasing AMTR when labor-supply elasticity becomes larger could effectively reduce the income earned by the highest skilled through the substitution effect, thus benefiting the lowest skilled (or worst off) workers through the consumption comparison effect. Third, for each given elasticity and given degree of inequality, AMTR increases as elasticity increases. This observation is also reasonable in the present context because increasing AMTR when labor-supply elasticity becomes larger could effectively reduce the income earned by the highest skilled through the substitution effect, thus benefiting the lowest skilled (or worst off) workers through the consumption comparison effect.

We summarize the economic mechanism in Table 2, in which the superscripts of $MTR^O$ and $MTR^C$ denote an open economy and a closed economy, respectively. In particular, we consider only the MTR of the leader country under Stackelberg tax competition. Essentially, as shown in Table 2, relativity and migration are the determinant factors in the comparison.

Since no one internalizes the cross-country consumption externality under Nash competition, the relativity effect on MTR is the same between an open economy and a closed economy. In contrast, as the leader country (partly) internalizes the cross-country externality under Stackelberg competition, the relativity effect on MTR implemented by the leader country in an open economy should be greater than the counterpart in a closed economy. Therefore, if there is no migration between countries, only the MTR implemented under Stackelberg competition should be higher than the corresponding MTR implemented in a closed economy.

When comparing $MTR^O$ and $MTR^C$ in the Nash case, the migration effect is shown to dominate the relativity effect. If the magnitude of equilibrium labor flow is small, regardless of whether it is an inflow or an outflow facing the country under consideration, Nash competition implies a smaller MTR than that in a closed economy without any migration threat imposed on the government. This result seems to be consistent with the classical wisdom that the presence of geographical mobility shall limit the ability of a government to redistribute incomes via a tax-transfer system. Nevertheless, if the magnitude of equilibrium labor flow is large, and hence the Nash equilibrium is likely to be more asymmetric, then the migration effect is heterogeneous between the case of labor inflow and the case of labor outflow. Specifically, a large labor inflow must be induced by a much lower MTR than its $MTR^C$, while a large labor outflow must be induced by a much higher MTR than its $MTR^C$. In consequence, in an asymmetric migration equilibrium in which a country faces large labor outflow whereas the opponent country faces large labor inflow, this country under tax competition induced by cross-border labor mobility actually implements a much higher tax rate on the top skill level than what it may implement in the case of autarky. An asymmetric equilibrium emerges because the underlying environment of the two countries, such as the distribution functions of skills and migration costs, is allowed to be heterogenous.

When comparing $MTR^O$ and $MTR^C$ in the Stackelberg case, the relative dominance of the relativity effect and migration effect turns out to be ambiguous. If the magnitude of equilibrium labor flow is small, then the relativity effect dominates the migration effect irrespective of labor inflows or outflows facing the leader country, implying that $MTR^O$ under Stackelberg competition should be higher than its $MTR^C$. However, if the magnitude of equilibrium labor flow is large, then the migration effect dominates the relativity effect and it is heterogeneous between the case of labor inflow and that of outflow. Specifically, a large labor inflow must be induced by a much lower MTR than its $MTR^C$, while a large labor outflow must be
induced by a much higher MTR than its MTR\textsuperscript{C}. As a result, under a large equilibrium labor flow, such qualitative features seem to be analogous across Nash and Stackelberg tax competition. Once again, such comparisons verify the important role of the level of migration in terms of determining the relative degree of income redistributions implemented in an open economy and a closed economy.

5.1.1. Nash vs. K&T

Tables 3–6 compare the optimal AMTRs in a Nash equilibrium to those in the autarky equilibrium of K&T. The numerical findings reveal that the difference in AMTRs (or the departure of these two types of equilibrium) increases as the net level of migration increases, precisely as $\alpha_i(\infty)$ declines under $\Delta_i(\infty) \geq 0$ or as $\alpha_i(\infty)$ increases under $\Delta_i(\infty) \leq 0$. In particular, imposing symmetry between the two countries, we can obtain that $\alpha_i(\infty) = 0.67 \iff Pr(m < U_i(\infty) - U_{-i}(\infty)) = 49\%$ and $\alpha_i(\infty) = 2.00 \iff Pr(m < U_i(\infty) - U_{-i}(\infty)) = 50\%$, namely, the migration probability of the highest skilled workers is approximately 50\% at these values of $\alpha_i(\infty)$. Under the jealousy-type relativity with $\Delta_i(\infty) \geq 0$, the Nash AMTRs are always smaller than those of K&T (see Fig. 3). However, when $\Delta_i(\infty) \leq 0$, they are greater than those of K&T when $\alpha_i(\infty)$ is larger than some critical value (see Fig. 4). Additionally, if there is no relative consumption concern, then they are always smaller than those of K&T. Moreover, the migration effect on taxation and redistribution is magnified by the consumption comparison effect.

5.1.2. Stackelberg vs. K&T

Tables 7–10 compare the optimal AMTRs in a Stackelberg equilibrium, denoted by black numbers, to those in the autarky equilibrium of K&T. The difference in AMTRs increases as the net level of migration increases, precisely as $\alpha_i(\infty)$ declines under $\Delta_i(\infty) \geq 0$ or as $\alpha_i(\infty)$ increases under $\Delta_i(\infty) \leq 0$. Under the jealousy-type relativity with $\Delta_i(\infty) \leq 0$, Stackelberg
AMTRs are generally larger than those of K&T (see Fig. 6). However, if $\Delta(\infty) \geq 0$, they are smaller than those of K&T for $\alpha_i(\infty)$ smaller than some critical value (see Fig. 5).

5.2. Nash vs. Stackelberg for the leader country

Tables 11 and 12 illustrate Proposition 4.2 by comparing AMTRs under these two types of equilibrium. As is obvious, regardless of whether $\Delta_i(\infty) \geq 0$ or $\Delta_i(\infty) \leq 0$, these AMTRs in a Nash equilibrium are generally smaller than those implemented as the leader country in a Stackelberg equilibrium (see also Figs. 7 and 8), denoted by blue numbers in these

---

We conjecture that this interesting finding is technically due to potential multiplicity of equilibria. Elaborating further, the leader country that also internalizes cross-country consumption externalities loses a mass of high-skill workers through the migration channel, making the lowest skilled better off through the social comparison effect.
Table 8
AMTR (%) under $\Delta_i(\infty) \geq 0$ with $a_i(\infty) = 0.55$.

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\varepsilon = 0.25$</th>
<th>$\varepsilon = 0.25$</th>
<th>$\varepsilon = 0.33$</th>
<th>$\varepsilon = 0.33$</th>
<th>$\varepsilon = 0.5$</th>
<th>$\varepsilon = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$</td>
<td>$a_i = 2$</td>
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<td>$a_i = 2$</td>
<td>$a_i = 3$</td>
<td>$a_i = 2$</td>
<td>$a_i = 3$</td>
</tr>
<tr>
<td>$\sigma_0 = -0.5$</td>
<td>61.7, 69.2</td>
<td>57.5, 64.7</td>
<td>62.3, 70.0</td>
<td>58.1, 65.4</td>
<td>63.5, 71.4</td>
<td>59.3, 66.7</td>
</tr>
<tr>
<td>$\sigma_0 = 0$</td>
<td>36.4, 38.5</td>
<td>28.9, 29.4</td>
<td>38.5, 39.9</td>
<td>31.1, 30.7</td>
<td>40.8, 42.9</td>
<td>33.2, 33.3</td>
</tr>
</tbody>
</table>

Table 9
AMTR (%) under $\Delta_i(\infty) \leq 0$ with $a_i(\infty) = 1.05$.

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\varepsilon = 0.25$</th>
<th>$\varepsilon = 0.25$</th>
<th>$\varepsilon = 0.33$</th>
<th>$\varepsilon = 0.33$</th>
<th>$\varepsilon = 0.5$</th>
<th>$\varepsilon = 0.5$</th>
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<tr>
<td>$\sigma_0$</td>
<td>$a_i = 2$</td>
<td>$a_i = 3$</td>
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<td>$a_i = 3$</td>
<td>$a_i = 2$</td>
<td>$a_i = 3$</td>
</tr>
<tr>
<td>$\sigma_0 = -0.5$</td>
<td>72.1, 69.2</td>
<td>69.1, 64.7</td>
<td>72.5, 70.0</td>
<td>69.5, 65.4</td>
<td>73.5, 71.4</td>
<td>70.4, 66.7</td>
</tr>
<tr>
<td>$\sigma_0 = 0$</td>
<td>37.6, 38.5</td>
<td>30.2, 29.4</td>
<td>38.8, 39.9</td>
<td>31.3, 30.7</td>
<td>41.0, 42.9</td>
<td>33.5, 33.3</td>
</tr>
</tbody>
</table>

Table 10
AMTR (%) under $\Delta_i(\infty) \leq 0$ with $a_i(\infty) = 1.55$.

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\varepsilon = 0.25$</th>
<th>$\varepsilon = 0.25$</th>
<th>$\varepsilon = 0.33$</th>
<th>$\varepsilon = 0.33$</th>
<th>$\varepsilon = 0.5$</th>
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<td>$\sigma_0$</td>
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<td>$a_i = 3$</td>
<td>$a_i = 2$</td>
<td>$a_i = 3$</td>
</tr>
<tr>
<td>$\sigma_0 = -0.5$</td>
<td>78.0, 69.2</td>
<td>75.7, 64.7</td>
<td>78.4, 70.0</td>
<td>76.1, 65.4</td>
<td>79.1, 71.4</td>
<td>76.7, 66.7</td>
</tr>
<tr>
<td>$\sigma_0 = 0$</td>
<td>38.7, 38.5</td>
<td>31.5, 29.4</td>
<td>38.8, 39.9</td>
<td>31.3, 30.7</td>
<td>41.0, 42.9</td>
<td>33.5, 33.3</td>
</tr>
</tbody>
</table>

Table 11
AMTR (%) under $\Delta_i(\infty) \geq 0$ with $a_i(\infty) = 0.95$.

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\varepsilon = 0.25$</th>
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<th>$\varepsilon = 0.33$</th>
<th>$\varepsilon = 0.33$</th>
<th>$\varepsilon = 0.5$</th>
<th>$\varepsilon = 0.5$</th>
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<tbody>
<tr>
<td>$\sigma_0$</td>
<td>$a_i = 2$</td>
<td>$a_i = 3$</td>
<td>$a_i = 2$</td>
<td>$a_i = 3$</td>
<td>$a_i = 2$</td>
<td>$a_i = 3$</td>
</tr>
<tr>
<td>$\sigma_0 = -0.5$</td>
<td>65.2, 70.5</td>
<td>61.5, 67.3</td>
<td>65.8, 71.0</td>
<td>62.0, 67.8</td>
<td>67.0, 71.9</td>
<td>63.1, 68.7</td>
</tr>
<tr>
<td>$\sigma_0 = 0$</td>
<td>35.1, 37.4</td>
<td>27.4, 30.0</td>
<td>36.3, 38.5</td>
<td>28.5, 31.1</td>
<td>38.7, 40.8</td>
<td>30.8, 33.2</td>
</tr>
</tbody>
</table>

Table 12
AMTR (%) under $\Delta_i(\infty) \leq 0$ with $a_i(\infty) = 1.05$.

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\varepsilon = 0.25$</th>
<th>$\varepsilon = 0.25$</th>
<th>$\varepsilon = 0.33$</th>
<th>$\varepsilon = 0.33$</th>
<th>$\varepsilon = 0.5$</th>
<th>$\varepsilon = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$</td>
<td>$a_i = 2$</td>
<td>$a_i = 3$</td>
<td>$a_i = 2$</td>
<td>$a_i = 3$</td>
<td>$a_i = 2$</td>
<td>$a_i = 3$</td>
</tr>
<tr>
<td>$\sigma_0 = -0.5$</td>
<td>68.8, 72.1</td>
<td>63.3, 69.1</td>
<td>67.4, 72.5</td>
<td>63.8, 69.5</td>
<td>68.5, 73.5</td>
<td>64.8, 70.4</td>
</tr>
<tr>
<td>$\sigma_0 = 0$</td>
<td>35.1, 37.6</td>
<td>27.4, 30.2</td>
<td>36.3, 38.8</td>
<td>28.5, 31.3</td>
<td>38.7, 41.0</td>
<td>30.8, 33.5</td>
</tr>
</tbody>
</table>
two tables. The differences in MTRs under any given degree of relativity concerns may not be particularly large simply because we let $\alpha_i(\infty) = 0.95$ and $\alpha_i(\infty) = 1.05$. Both are close to one and hence the equilibrium net level of labor flows is low, which may be interpreted as that the migration effect is small. Therefore, as the migration equilibrium becomes more asymmetric or $\alpha_i(\infty)$ deviates further from one, regardless of whether from the below or the above, the differences in the AMTRs generated by these two types of tax competition would increase.
6. Conclusion

In this paper, we develop a theoretical framework to analyze how the interplay of relative consumption concern and income inequality determines optimal income taxes in an international setting with two competing countries. The taxation policy thus features externality-correcting and income-redistributing effects. We establish and qualitatively characterize nonlinear labor income tax schedules that competing Rawlsian governments should implement when workers with private information on skills and migration costs decide where to live and how much time to work. Following the mechanism design approach, the benevolent governments must design incentive compatible, individually rational and resource feasible allocations. In addition to the standard Nash, we also examine the scenario wherein the two governments play Stackelberg.

The main results are summarized as follows. First, we obtain an optimal tax formula that can be interpreted as a non-trivial generalization of those formulas obtained by Diamond (1998), Saez (2001), K&T, Lehmann et al. (2014) and Aronsson and Johansson-Stenman (2015). Second, we numerically calculate the optimal AMTRs under both types of equilibrium and compare them to those AMTRs of K&T, finding that the country with a large labor inflow imposes much lower ones and that the country with a large labor outflow imposes much higher ones, than suggested by K&T. This finding holds for various combinations of parameter values measuring relative consumption, labor mobility and income inequality. Third, we show that the leader country that accounts for the follower country’s response and partly internalizes cross-country externalities imposes higher MTRs in Stackelberg equilibrium than in Nash equilibrium. Fourth, we provide a complete characterization of how relativity and inequality jointly determine the optimal AMTR under both Nash and Stackelberg tax competition and find that both the elasticity and level of migration are key determinants of predicting when relativity and inequality act as complements (or substitutes) in shaping the optimal AMTR.

We, therefore, show that the optimal redistributive taxation policy for countries involved in globalization should not ignore these important effects resulting from tax-driven migration or the interplay of relativity and inequality. Since, quantitatively, alternative forms of tax competition do generate heterogeneous degrees of impact on optimal MTRs, the identification of the form of tax competition should be of practical relevance, which awaits future research.

Declaration of Competing Interest

No conflict of interest.

Appendix A. Proofs

Proof of Theorem 3.1. We shall complete the proof in 3 steps.

Step 1. Given the FOC (4) of individual choice, the indirect utility of a type-\(w\) worker in country \(i \in \{A, B\}\) can be written as

\[
U_i(w) = v(\varphi_i(U_i(w), l_i(w), \mu_i, \mu_{-i})) - h(l_i(w)) + \psi(\mu_i), \quad \mu_{-i}
\]

(14)
where we treat individual consumption $c_i(w)$ as an implicit function of $U_i(w)$, $l_i(w)$, $\mu_i$, $\mu_{-i}$, and equivalently rewrite it as $\varphi_i(\cdot)$. By applying the Implicit Function Theorem, we get from (14) that

$$
\frac{\partial \varphi_i}{\partial l_i} = \frac{h'(l_i(w))}{v'(c_i(w))}, \quad \frac{\partial \varphi_i}{\partial U_i} = \frac{1}{v'(c_i(w))}, \quad \frac{\partial \varphi_i}{\partial \mu_i} = -\frac{\psi_i(\mu_i, \mu_{-i})}{v'(c_i(w))} \quad \text{and} \quad \frac{\partial \varphi_i}{\partial \mu_{-i}} = -\frac{\psi_{-i}(\mu_i, \mu_{-i})}{v'(c_i(w))}.
$$

(15)

Step 2. For expository purposes, we follow the first-order approach and ignore the SOIC conditions. After deriving the solutions, then we can verify whether the SOIC conditions are binding or not. The corresponding Lagrangian is written as follows:

$$
\mathcal{L}_i\{U_i(w), l_i(w)\}_{w \in [\underline{w}, \bar{w}]}, \mu_i; \lambda_i, \gamma_i, \{\xi_i(w)\}_{w \in [\underline{w}, \bar{w}]}
$$

$$
= U_i(w) + \lambda_i \int_{\underline{w}}^{\bar{w}} \left\{ [W_i(w) - \varphi_i(U_i(w), l_i(w), \mu_i, \mu_{-i})] \phi_i(U_i(w) - U_{-i}(w); w) - \frac{R}{\underline{w} - \bar{w}} \right\} dw
$$

$$
+ \xi_i(w) \int_{\underline{w}}^{\bar{w}} h'(l_i(w)) \frac{l_i(w)}{w} - \bar{U}_i(w) \right\} dw + \gamma_i \left[ \mu_i - \int_{\underline{w}}^{\bar{w}} \varphi_i(U_i(w), l_i(w), \mu_i, \mu_{-i}) f_i(w) dw \right]
$$

(16)

where $\lambda_i > 0$ is the multiplier associated with the binding budget constraint (8), $\xi_i(w)$ is the multiplier associated with the FOIC conditions (10), and $\gamma_i$ is the multiplier associated with the comparison consumption constraint (2). Integrating by parts, we obtain

$$
\int_{\underline{w}}^{\bar{w}} \xi_i(w) \bar{U}_i(w) dw = \xi_i(\bar{w}) \bar{U}_i(\bar{w}) - \xi_i(\underline{w}) U_i(\underline{w}) - \int_{\underline{w}}^{\bar{w}} \dot{\xi}_i(w) U_i(w) dw.
$$

(17)

Plugging (17) in (16) gives rise to

$$
\mathcal{L}_i\{U_i(w), l_i(w)\}_{w \in [\underline{w}, \bar{w}]}, \mu_i; \lambda_i, \gamma_i, \{\xi_i(w)\}_{w \in [\underline{w}, \bar{w}]}
$$

$$
= U_i(w) + \lambda_i \int_{\underline{w}}^{\bar{w}} \left\{ [W_i(w) - \varphi_i(U_i(w), l_i(w), \mu_i, \mu_{-i})] \phi_i(U_i(w) - U_{-i}(w); w) - \frac{R}{\underline{w} - \bar{w}} \right\} dw
$$

$$
+ \xi_i(w) U_i(w) - \xi_i(\bar{w}) U_i(\bar{w}) + \int_{\underline{w}}^{\bar{w}} \left[ \xi_i(w) h'(l_i(w)) \frac{l_i(w)}{w} + \dot{\xi}_i(w) U_i(w) \right] dw
$$

$$
+ \gamma_i \left[ \mu_i - \int_{\underline{w}}^{\bar{w}} \varphi_i(U_i(w), l_i(w), \mu_i, \mu_{-i}) f_i(w) dw \right].
$$

Assuming that there is no bunching of workers of different skills and the existence of an interior solution, applying (15) shows that the necessary conditions can be written as follows:

$$
\frac{\partial \mathcal{L}_i}{\partial l_i(w)} = \lambda_i \left[ w - \frac{h'(l_i(w))}{v'(c_i(w))} \right] \tilde{f}_i(w) - \gamma_i \frac{h'(l_i(w))}{v'(c_i(w))} f_i(w) + \frac{\xi_i(w) h'(l_i(w))}{w} \left[ 1 + \frac{l_i(w) h''(l_i(w))}{h'(l_i(w))} \right] = 0 \quad \forall w \in [\underline{w}, \bar{w}].
$$

(18)

$$
\frac{\partial \mathcal{L}_i}{\partial U_i(w)} = -\frac{\lambda_i \tilde{f}_i(w)}{v'(c_i(w))} + \lambda_i \bar{T}_i(y_i(w)) \tilde{n}_i(w) \tilde{f}_i(w) - \gamma_i \frac{f_i(w)}{v'(c_i(w))} + \dot{\xi}_i(w) = 0 \quad \forall w \in (\underline{w}, \bar{w}).
$$

(19)

$$
\frac{\partial \mathcal{L}_i}{\partial U_i(\bar{w})} = 1 + \xi_i(\bar{w}) = 0,
$$

(20)

$$
\frac{\partial \mathcal{L}_i}{\partial U_i(\underline{w})} = -\xi_i(\underline{w}) = 0,
$$

(21)

$$
\frac{\partial \mathcal{L}_i}{\partial \mu_i} = \lambda_i \int_{\underline{w}}^{\bar{w}} \psi_i(\mu_i, \mu_{-i}) \tilde{f}_i(w) dw + \gamma_i \left[ 1 + \int_{\underline{w}}^{\bar{w}} \frac{\psi_i(\mu_i, \mu_{-i})}{v'(c_i(w))} f_i(w) dw \right] = 0.
$$

(22)

Using (19), we get

$$
\frac{\dot{\xi}_i(w)}{\lambda_i} = \frac{\gamma_i}{\lambda_i} \frac{f_i(w)}{v'(c_i(w))} + \frac{\tilde{f}_i(w)}{v'(c_i(w))} - \bar{T}_i(y_i(w)) \tilde{n}_i(w) \tilde{f}_i(w).
$$

Integrating on both sides of this equation and using the transversality condition (21), we obtain

$$
-\frac{\xi_i(w)}{\lambda_i} = \frac{\gamma_i}{\lambda_i} \int_{\underline{w}}^{\bar{w}} \frac{f_i(t)}{v'(c_i(t))} dt + \int_{\underline{w}}^{\bar{w}} \left[ \frac{1}{v'(c_i(t))} - \bar{T}_i(y_i(t)) \tilde{n}_i(t) \right] \tilde{f}_i(t) dt.
$$

(23)
Rearranging (18) via using FOC (4), we have
\[
\frac{T'_i(y_i(w))}{1 - T'_i(y_i(w))} = \frac{\gamma_i f_i(w)}{\lambda_i} \frac{\zeta_i(w) v'(c_i(w))}{\lambda_i \cdot \frac{l_i(w) h''(l_i(w))}{w f_i(w)}} \bigg[1 + \frac{l_i(w) h''(l_i(w))}{h'(l_i(w))}\bigg] \quad \forall w \in [\underline{w}, \overline{w}].
\] (24)

Substituting (23) into (24) gives the first-order conditions characterizing the optimum MTRs, with \(\gamma_i/\lambda_i\) determined by solving (22).

Step 3. To derive a sufficient condition for the optimal marginal tax profile to satisfy the SOIC conditions, we rewrite the FOC (4) as
\[
\frac{v'(c_i(w))}{h'(y_i(w)/w)} = \frac{1}{w[1 - T'_i(y_i(w))]}. \quad \text{LHS} \quad \text{RHS}
\] (25)

Noting that
\[
d\text{LHS} = \frac{v''(c_i(w)) \zeta_i(w)}{h'(y_i(w)/w)} - \frac{v'(c_i(w)) h''(y_i(w)/w)[w \dot{y}_i(w) - y_i(w)]}{[wh'(y_i(w)/w)]^2}
\]
and \(c_i(w) = y_i(w) - T_i(y_i(w)) \Rightarrow \dot{c}_i(w) = \dot{y}_i(w)[1 - T'_i(y_i(w))]\), thus
\[
d\text{LHS} dw < 0 \implies \dot{y}_i(w) > 0. \quad \text{(26)}
\]

Also, noting that
\[
d\text{RHS} = -\{w[1 - T'_i(y_i(w))]\}^2 \left[1 - T'_i(y_i(w)) - w \frac{dT'_i(y_i(w))}{dw}\right].
\]
we thus arrive at
\[
\frac{dT'_i(y_i(w))}{dw} \leq 0 \implies \frac{d\text{RHS}}{dw} < 0. \quad \text{(27)}
\]

Therefore, (25) combined with (26) and (27) implies that
\[
\frac{dT'_i(y_i(w))}{dw} \leq 0 \implies \dot{y}_i(w) > 0,
\]
as desired. \(\square\)

**Proof of Lemma 3.1.** By symmetry we can, as in the proof of Theorem 3.1, treat individual consumption \(c_{-i}(w)\) in country \(-i\) as an implicit function of \(U_{-i}(w)\), \(L_{-i}(w)\), \(\mu_{-i}\), \(\mu_{i}\), and equivalently rewrite it as \(\varphi_{-i}(\cdot)\). In consequence, \(\mu_{-i}\) can be formally expressed as
\[
\mu_{-i} = \int_w^\Pi \varphi_{-i}(U_{-i}(w), L_{-i}(w), \mu_{-i}, \mu_{i}) f_{-i}(w)dw.
\]

Taking into account this equation as a constraint, the Lagrangian given by (16) should be rewritten as follows:
\[
\mathcal{L}_i\left\{U_i(w), l_i(w)\}_{w \in [\underline{w}, \overline{w}]}, \lambda_i; \lambda_{-i}, \gamma_i, q_i, \{\zeta_i(w)\}_{w \in [\underline{w}, \overline{w}]}
\]
\[
= U_i(w) + \lambda_i \int_w^\Pi \left\{[l_i(w) - \varphi_i(U_i(w), l_i(w), \mu_i, \mu_{-i})] \phi_i(U_i(w) - U_{-i}(w); w) - \frac{R}{\overline{w} - \underline{w}}\right\}dw
\]
\[
+ \int_w^\Pi \zeta_i(w) \left[h'(l_i(w)) \frac{l_i(w)}{w} - U_i(w)\right] dw + \gamma_i \left[\mu_{-i} - \int_w^\Pi \varphi_{-i}(U_{-i}(w), L_{-i}(w), \mu_{-i}, \mu_{i}) f_{-i}(w)dw\right]
\]
\[
+ q_i \left[\mu_{-i} - \int_w^\Pi \varphi_{-i}(U_{-i}(w), L_{-i}(w), \mu_{-i}, \mu_{i}) f_{-i}(w)dw\right],
\]
in which \(q_i > 0\) denotes the corresponding Lagrangian multiplier. It is easy to verify that the FOCs given by Eqs. (18)-(21) remain true while the FOC (22) changes to
\[
\frac{\partial \mathcal{L}_i}{\partial \mu_{i}} = \lambda_i \int_w^\Pi \psi_i(\mu_{i}, \mu_{-i}) f_i(w)dw + \gamma_i \left[1 + \int_w^\Psi \frac{\psi_i(\mu_{i}, \mu_{-i})}{v'(c_{i}(w))} f_i(w)dw\right] + q_i \left[\int_w^\Psi \frac{\psi_i(\mu_{-i}, \mu_{i})}{v'(c_{-i}(w))} f_{-i}(w)dw\right] = 0,
\]
by which the new formula for \(\gamma_i/\lambda_i\) is established. Under Assumption 2.1, it is immediate to obtain that
\[
\frac{q_i}{\lambda_i} \cdot \frac{\int_w^\Pi \frac{\psi_i(\mu_{-i}, \mu_{i})}{v'(c_{-i}(w))} f_{-i}(w)dw}{\int_w^\Pi \frac{\psi_i(\mu_{i}, \mu_{-i})}{v'(c_{i}(w))} f_i(w)dw} > 0.
\]
by which it seems true that the $\gamma_i/\lambda_i$, obtained here is larger (and hence the equilibrium MTRs are higher) than that obtained in Theorem 3.1. However, as it follows from the FOCs (18) and (19) that equilibrium $(l_i(w))_{w \in [W, \bar{W}]}$ and $(U_i(w))_{w \in [W, \bar{W}]}$ rely on $\gamma_i/\lambda_i$ in a complex nonlinear way, so does equilibrium $\mu_i$, we thus cannot have a clear-cut prediction on whether $\gamma_i/\lambda_i$ becomes larger or smaller after taking into account the equation for $\mu_i$ as a constraint. □

Proof of Lemma 3.2. The Lagrangian of government $i$’s problem can be expressed as

$$
\mathcal{L}_i\left(\{U_i(w), l_i(w)\}_{w \in [W, \bar{W}]}, \mu_i, \lambda_i, \gamma_i\right) = U_i(w) + \lambda_i \int_{\underline{w}}^{\bar{w}} \left\{ w \phi_i(l_i(w), l_i(w), \mu_i, \mu_{i-1}) \phi_i(U_i(w) - U_{i-1}(w); w) - \frac{R}{\bar{W} - w} \right\} dw + \gamma_i \left[ \mu_i - \int_{\underline{w}}^{\bar{w}} \phi_i(U_i(w), l_i(w), \mu_i, \mu_{i-1}) f_i(w) dw \right].
$$

where $\lambda_i > 0$ is the multiplier associated with the binding budget constraint (8) and $\gamma_i$ is the multiplier associated with the comparison consumption constraint (2). Assuming that there is no bunching of workers of different skills and the existence of an interior solution, applying (15) gives these necessary conditions:

$$
\frac{\partial \mathcal{L}_i}{\partial U_i(w)} = \lambda_i \left[ T_i(y_i(w)) \tilde{h}_i(w) - \frac{1}{v'(C_i(w))} \right] f_i(w) - \gamma_i \frac{f_i(w)}{v'(C_i(w))} = 0 \quad \forall w \in (\underline{w}, \bar{W}], \quad \text{(28)}
$$

$$
\frac{\partial \mathcal{L}_i}{\partial l_i(w)} = \lambda_i \left( w - \frac{h'(l_i(w))}{v'(C_i(w))} \right) f_i(w) - \gamma_i \frac{h'(l_i(w)) f_i(w)}{v'(C_i(w))} = 0 \quad \forall w \in [\underline{w}, \bar{W}], \quad \text{(29)}
$$

and

$$
\frac{\partial \mathcal{L}_i}{\partial \mu_i} = \lambda_i \int_{\underline{w}}^{\bar{w}} \frac{\psi_i(\mu_i, \mu_{i-1})}{v'(C_i(w))} f_i(w) dw + \gamma_i \left[ 1 + \int_{\underline{w}}^{\bar{w}} \frac{\psi_i(\mu_i, \mu_{i-1})}{v'(C_i(w))} f_i(w) dw \right] = 0. \quad \text{(30)}
$$

By using (28), we obtain the Tiebout-best tax liabilities. By using (29) and the FOC (4), we obtain the Tiebout-best MTRs. The ratio $\gamma_i/\lambda_i$ is determined by (30). As is obvious, the least productive workers receive a transfer determined by the government’s budget constraint. Therefore, the optimal tax function is discontinuous at $w = \underline{w}$. □

Proof of Proposition 3.1. We shall complete the proof in 6 steps.

Step 1. By Assumption 2.1, $\gamma_i/\lambda_i > 0$ is guaranteed. Given that $v(\cdot)$ is strictly increasing and $h(\cdot)$ is strictly increasing and convex, for (i) to hold it suffices to show that $C_i(w) \geq 0$ for $\forall w \in (\underline{w}, \bar{W}]$. Therefore, by directly comparing the formulas of MTRs established in Theorem 3.1 and Lemma 3.2, claim (i) is immediate.

Step 2. By applying the transversality condition (20) to Eq. (24), it is easy to see that

$$
\frac{T_i'(y_i(\underline{w}))}{1 - T_i'(y_i(\underline{w}))} > \frac{\gamma_i}{\lambda_i} \frac{f_i(\underline{w})}{f_i(\underline{w})} > 0
$$

under Assumption 2.1. Similarly, for a bounded skill distribution with $\underline{w} < \bar{W}$, applying the transversality condition (21) to Eq. (24) gives

$$
\frac{T_i'(y_i(\bar{W}))}{1 - T_i'(y_i(\bar{W}))} = \frac{\gamma_i}{\lambda_i} \frac{f_i(\bar{W})}{f_i(\bar{W})} > 0
$$

under Assumption 2.1. By using Lemma 3.2 again, the required assertion (ii) follows.

Step 3. Suppose $h(\cdot)$ takes the isoelastic form, then $A_i(w)$ is a positive constant. Suppose the first-order approach is valid, namely the SOIC conditions are not binding in the Nash equilibrium, then we have that $v(\cdot)$ is strictly decreasing in $w$ as $v(\cdot)$ is assumed to be strictly concave. With single-peaked skill distributions, $1/wf_i(w)$ always decreases before the mode $w_m$. Beyond the mode, it either increases or decreases, depending on how rapidly $f_i(w)$ falls with $w$. A sufficient, though not necessary, condition for decreasing $T_i(\cdot)$ over the entire skill distribution is that aggregate skills $wf_i(w)$ are non-decreasing beyond the mode $w_m$. Also, noting from term $C_i(w)$ that

$$
\frac{d}{dw} \int_{\underline{w}}^{\bar{w}} \left\{ \frac{1}{v'(C_i(t))} \left[ 1 + \frac{\gamma_i}{\lambda_i} \frac{f_i(t)}{f_i(\bar{w})} \right] - T_i(y_i(t)) \tilde{h}_i(t) \right\} f_i(t) dt
$$

$$
= \left\{ T_i(y_i(w)) - \frac{1}{v'(C_i(w))} \tilde{h}_i(w) \right\} \left[ 1 + \frac{\gamma_i}{\lambda_i} \frac{f_i(w)}{f_i(\bar{w})} \right] \tilde{h}_i(w) f_i(w),
$$

hence an application of Lemma 3.2 completes the proof of claim (iii).

Step 4. Define

$$
\Sigma_i(w) = - \frac{C_i(w)}{\lambda_i} \frac{\gamma_i}{\lambda_i} \int_{\underline{w}}^{\bar{w}} \frac{1}{v'(C_i(t))} f_i(t) dt + \int_{\underline{w}}^{\bar{w}} \left[ \frac{1}{v'(C_i(t))} - T_i(y_i(t)) \tilde{h}_i(t) \right] f_i(t) dt.
$$
then the signs of $\Sigma_i(w)$ and $\xi_i(w)$ are opposite. As $\xi_i(w)$ is differentiable, $\Sigma_i(w)$ is differentiable as well and we have

$$
\Sigma_i'(w) = \left\{ T_i(y_i(w)) - \frac{1}{v'(c_i(w)) \eta_i(w)} \left[ 1 + \frac{\gamma_i f_i(w)}{\lambda_i f_i(w)} \right] \right\} \eta_i(w) \tilde{f}_i(w) = \xi_i(w) \eta_i(w) \tilde{f}_i(w),
$$

which implies that $\Sigma_i'(w)$ and $\xi_i(w)$ have the same sign. Note that

$$
\frac{d}{dw} \left\{ \frac{1}{v'(c_i(w)) \eta_i(w)} \left[ 1 + \frac{\gamma_i f_i(w)}{\lambda_i f_i(w)} \right] \right\} = -\frac{v''(c_i(w)) c_i(w)}{(v'(c_i(w)))^2 \eta_i(w)} \left[ 1 + \frac{\gamma_i f_i(w)}{\lambda_i f_i(w)} \right] - \frac{\dot{\eta}_i(w)}{v'(c_i(w)) \eta_i(w)} \left[ 1 + \frac{\gamma_i f_i(w)}{\lambda_i f_i(w)} \right] + \frac{1}{v'(c_i(w)) \eta_i(w)} \frac{\gamma_i f_i(w)}{\lambda_i f_i(w)} \frac{d}{dw} \left[ f_i(w) \tilde{f}_i(w) \right]
$$

under Assumption 2.1 and the assumptions that $\dot{y}_i(w) > 0$ and $f_i(w)/\tilde{f}_i(w)$ is non-increasing in $w$, thus the condition

$$
-\frac{v''(c_i(w)) c_i(w)}{(v'(c_i(w)))^2 \eta_i(w)} \left[ 1 + \frac{\gamma_i f_i(w)}{\lambda_i f_i(w)} \right] \leq \frac{\dot{\eta}_i(w)}{v'(c_i(w)) \eta_i(w)} \left[ 1 + \frac{\gamma_i f_i(w)}{\lambda_i f_i(w)} \right] + \frac{1}{v'(c_i(w)) \eta_i(w)} \frac{\gamma_i f_i(w)}{\lambda_i f_i(w)} \frac{d}{dw} \left( f_i(w) \tilde{f}_i(w) \right)
$$

is sufficient for

$$
\frac{d}{dw} \left\{ \frac{1}{v'(c_i(w)) \eta_i(w)} \left[ 1 + \frac{\gamma_i f_i(w)}{\lambda_i f_i(w)} \right] \right\} \leq 0.
$$

If we assume that $\Sigma_i(w) \geq 0$, then we get from the optimal tax formula in Theorem 3.1 that $T'_i(y_i(w)) > 0$. Then applying (31) and (32) shows that $\xi_i'(w) > 0$ given $\Sigma_i(w) > 0$.

Step 5. Assume that there exists $w \in (w_1, w_2)$ such that $\Sigma_i(w) > 0$. Then we have two cases to consider in what follows, namely either $\Sigma_i(w) > 0$ or $\Sigma_i'(w) < 0$. If $\Sigma_i(w) > 0$, then we have both $\xi_i(w) > 0$ and $\xi_i'(w) > 0$. So the continuity of $\xi_i(w)$ with respect to $w$ implies that there is an open interval with lower bound $\tilde{w}$ such that $\xi_i(\tilde{w}) > 0$, and hence $\Sigma_i'(\tilde{w}) > 0$. On this interval, $\Sigma_i(\cdot)$ is thus positive and strictly increasing on this interval. Without loss of generality, let $(\tilde{w}, \tilde{w})$ be a maximal interval on which $\Sigma_i(w) > 0$ with $\tilde{w} < \tilde{w} < \tilde{w}$. As a consequence, $0 \leq \xi_i(\tilde{w}) < \xi_i'(\tilde{w})$, which implies that $\xi_i'(w) > 0$ for $\forall w \in (\tilde{w}, \tilde{w})$. As a result, $0 \leq \xi_i(\tilde{w}) < \xi_i'(\tilde{w})$, which leads us to $\Sigma_i'(\tilde{w}) > 0$ by using (31). Therefore, $\Sigma_i(\cdot)$ is increasing on $[\tilde{w}, \tilde{w}]$ given that $\Sigma_i'(\tilde{w}) \geq 0$. We know from the transversality condition (21) that $\Sigma_i(\tilde{w}) = -\xi_i'(\tilde{w})/\lambda_i > 0$. As we have already shown that $0 \leq \Sigma_i(\tilde{w}) < \Sigma_i(\tilde{w})$, an immediate contradiction occurs. We, accordingly, claim that $\Sigma_i'(\tilde{w}) \geq 0$ does not hold true.

Step 6. Given that we have shown that $\Sigma_i'(\tilde{w}) < 0$ for the chosen $\tilde{w}$, we thus have $\xi_i'(\tilde{w}) < 0$ by (31) and $\xi_i'(\tilde{w}) > 0$. Similarly, the continuity of $\xi_i'(w)$ with respect to $w$ implies that there is an open interval with upper bound $\tilde{w}$ such that $\xi_i'(\tilde{w}) < 0$, and hence $\Sigma_i'(\tilde{w}) < 0$. On this interval, $\Sigma_i(\cdot)$ is thus positive and strictly decreasing on this interval. Without loss of generality, let $(\tilde{w}', \tilde{w})$ be a maximal interval on which $\Sigma_i(\tilde{w}) < 0$ with $\tilde{w} \leq \tilde{w}' < \tilde{w}$. In consequence, $0 \leq \xi_i(\tilde{w}) < \xi_i'(\tilde{w})$, which implies that $\xi_i'(\tilde{w}) < 0$ for $\forall w \in [\tilde{w}', \tilde{w}]$. As a result, $0 \leq \xi_i(\tilde{w}) < \xi_i'(\tilde{w})$, which leads us to $\Sigma_i'(\tilde{w}) < 0$ by using (31). Therefore, $\Sigma_i(\cdot)$ will not stop decreasing until reaching the lower bound $\tilde{w}$, namely $\Sigma_i(\cdot)$ will be decreasing on $[\tilde{w}, \tilde{w}]$. We know from the transversality condition (20) that $\Sigma_i(w) = -\xi_i(\tilde{w})/\lambda_i > 0$. Since we have already shown that $0 \leq \Sigma_i(\tilde{w}) < \Sigma_i(\tilde{w})$, thus the transversality condition is fulfilled in this case. By using (31) and Lemma 3.2 again, the required assertion (iv) follows.

**Proof of Proposition 3.2.** It follows from condition (b) that $A_4(w) = 1 + \epsilon$, a fixed positive constant. The ex post skill distribution term $B_i(w)$ can be decomposed through

$$
B_i(w) = \frac{1 - f_i(w)}{w_1 f_i(w)} \cdot \frac{\bar{f}_i(\infty) - \bar{f}_i(w)}{1 - f_i(w)},
$$

By condition (c), we have $\frac{1 - f_i(w)}{w_1 f_i(w)} = 1/a_i$. By l’Hôpital’s rule, we obtain

$$
\lim_{w \to \infty} \frac{\bar{f}_i(\infty) - \bar{f}_i(w)}{1 - f_i(w)} = \lim_{w \to \infty} \frac{\tilde{f}_i(w)}{f_i(w)}.
$$

As a result, $\lim_{w \to \infty} B_i(w) = 1/a_i$. By using the definition of the elasticity of migration and conditions (a) and (c), term $C_i(w)$ can be rewritten as

$$
C_i(w) = \frac{\int_0^\infty \left[ 1 + \frac{\gamma_i \xi_i(t)}{\lambda_i f_i(t)} - \frac{T_i(y_i(t))}{T_i(y_i(t)) - \bar{y}_i(t)} \right] \tilde{f}_i(t) dt}{\bar{f}_i(\infty) - \bar{f}_i(w)}.
$$

Thus, making use of the l’Hôpital’s rule again shows that

$$
\lim_{w \to \infty} C_i(w) = 1 + \frac{\gamma_i}{\lambda_i} \gamma_4(\infty) - \frac{T_i'(y_i(\infty))}{1 - T_i'(y_i(\infty))} \bar{y}_i(\infty).
$$
So, we get from the optimal tax formula derived in Theorem 3.1 that
\[
\frac{T_i^*(y_i(\infty))}{1-T_i^*(y_i(\infty))} = \frac{\gamma_i}{\lambda_i} \alpha_i(\infty) + (1 + \varepsilon) \frac{1}{a_i} \left[ 1 + \frac{\gamma_i}{\lambda_i} \alpha_i(\infty) - \frac{T_i^*(y_i(\infty))}{1-T_i^*(y_i(\infty))} \tilde{\theta}_i(\infty) \right],
\]
rearranging the algebra of which gives the desired optimal asymptotic tax rate. □

Proof of Proposition 3.3. We shall complete the proof in 3 steps.

Step 1. By applying condition (a) assumed in Proposition 3.2 and the assumption \(\psi(\mu_i, \mu_{-i}) = \sigma_D \in (-1, 0)\) to Eq. (13) produces
\[
\frac{\gamma_i}{\lambda_i} = \frac{-\sigma_D}{1 + \sigma_D} \tilde{f}_i(\infty) > 0,
\]
in which it is unnecessary that \(\tilde{f}_i(\infty) = 1\). Also, if \(f_i(w) = F_i(w)\), then by using the definition of \(f_i(w)\) we obtain \(\partial f_i(\infty)/\partial(1/a_i) = 0\). Therefore, as long as \(\partial f_i(\infty)/\partial(1/a_i) = 0\), we must have \(\partial(f_i^2/\lambda_i)/\partial(1/a_i) = 0\). In addition, it follows from the definition of \(\tilde{\theta}_i(w)\) that \(\partial \tilde{\theta}_i(\infty)/\partial(1/a_i) = 0\). Finally, it is straightforward that \(\partial(\gamma_i/\lambda_i)/\partial(-\sigma_D) > 0\).

Step 2. Using the established formula of \(T_i^*(y_i(\infty))\), we have
\[
\frac{\partial^2 T_i^*(y_i(\infty))}{\partial \left(\frac{\gamma_i}{\lambda_i} \alpha_i(\infty) \right) \partial (1/a_i)} = \frac{1 + \varepsilon}{\left\{ 1 + \frac{\gamma_i}{\lambda_i} \alpha_i(\infty) + (1 + \varepsilon)(1/a_i) \tilde{\theta}_i(\infty) \right\}^2} \times \left( 1 + \frac{\gamma_i}{\lambda_i} \alpha_i(\infty) + \tilde{\theta}_i(\infty) \right) \left( 1 + \frac{1 + \varepsilon}{a_i} \left[ \tilde{\theta}_i(\infty) - 1 \right] - \tilde{\theta}_i(\infty) \left( 1 + \frac{2(1 + \varepsilon)}{a_i} \right) \right) \right).
\]

Thus if the following condition holds true:
\[
\left[ 1 + \frac{\gamma_i}{\lambda_i} \alpha_i(\infty) + \tilde{\theta}_i(\infty) \right] \left[ \tilde{\theta}_i(\infty) - 1 \right] \leq 0,
\]
then the cross-partial derivative is negative for any \(\tilde{\theta}_i(\infty) \in (0, 1]\). It is easy to verify that this condition holds for \(\tilde{\theta}_i(\infty) \leq 1\) with any \(\sigma_D \in (-1, 0)\), as desired in part (i).

Step 3. If, however, \(\tilde{\theta}_i(\infty) > 1\), then we see that
\[
\left[ 1 + \frac{\gamma_i}{\lambda_i} \alpha_i(\infty) + \tilde{\theta}_i(\infty) \right] \left( 1 + \frac{1 + \varepsilon}{a_i} \left[ \tilde{\theta}_i(\infty) - 1 \right] - \tilde{\theta}_i(\infty) \left( 1 + \frac{2(1 + \varepsilon)}{a_i} \right) \right)
\]
is equivalent to
\[
1 + \frac{\gamma_i}{\lambda_i} \alpha_i(\infty) < \frac{\tilde{\theta}_i(\infty) \left[ 2 + \frac{1 + \varepsilon}{a_i} \left[ 1 + \tilde{\theta}_i(\infty) \right] \right]}{\left( 1 + \frac{1 + \varepsilon}{a_i} \right) \left( \tilde{\theta}_i(\infty) - 1 \right)}.
\]

Also, noting that
\[
\frac{\tilde{\theta}_i(\infty) \left[ 2 + \frac{1 + \varepsilon}{a_i} \left[ 1 + \tilde{\theta}_i(\infty) \right] \right]}{\left( 1 + \frac{1 + \varepsilon}{a_i} \right) \left( \tilde{\theta}_i(\infty) - 1 \right)} = 1 + \tilde{\theta}_i(\infty) + \frac{1 + \varepsilon}{a_i} \left[ \tilde{\theta}_i(\infty) - 1 \right] \geq 0,
\]
the desired assertion in part (ii) follows. □

Proof of Theorem 4.1. As usual, we derive the Stackelberg equilibrium by using backward induction. Thus, the Lagrangian of the follower country \(-i\) is the same as in the case when these two countries play Nash, while the Lagrangian for the leader country \(i\) is different and reads as follows:
\[
L_i[U_i(w), l_i(w)]_{w \in [w_1, w_2]}, \mu_i, \lambda_i, \gamma_i, \{s_i(w)\}_{w \in [w_1, w_2]} = U_i(w) + \lambda_i \int_{w_2}^{w_1} \left[ w_i(w) - \psi_i(U_i(w), l_i(w), \mu_i, \mu_{-i} + \mu_i) \right] \phi_i(U_i(w) - U_{-i}(w); w) - \frac{R}{w - w} \right] dw.
\]
\[ + \zeta_i(w)U_i(w) - \zeta_i(\overline{w})U_i(\overline{w}) + \int_\mathbb{W} \left[ \zeta_i(w) h'(l_i(w)) \frac{l_i(w)}{w} + \zeta_i(w)U_i(w) \right] dw \\
+ \gamma_i \left[ \mu_i - \int_\mathbb{W} \varphi_i(U_i(w), l_i(w), \mu_i, \mu_{\gamma}(\mu_i)) f_i(w) dw \right]. \]

Note that
\[ \mu_{\gamma} = \int_\mathbb{W} \varphi_{\gamma}(U_{\gamma}(w), l_{\gamma}(w), \mu_{\gamma}, \mu_{\gamma}, \mu_i) f_i(w) dw, \]

making use of the Implicit Function Theorem produces
\[ \frac{\partial \mu_{\gamma}}{\partial \mu_i} = \frac{f_i(w)}{1 - \int_\mathbb{W} \frac{\partial \varphi_i}{\partial \mu_{\gamma}} f_i(w) dw}. \tag{33} \]

Assuming that there is no bunching of workers of different skills and the existence of an interior solution, then all of these first-order necessary conditions of Lagrangian \( \mathcal{L} \) are the same as those in the proof of Theorem 3.1 but
\[ \frac{\partial \mathcal{L}}{\partial \mu_i} = -\lambda_i \int_\mathbb{W} \left[ \frac{\partial \varphi_i}{\partial \mu_i} + \frac{\partial \varphi_i}{\partial \mu_{\gamma}} \frac{\partial \mu_{\gamma}}{\partial \mu_i} \right] f_i(w) dw + \gamma_i \left[ 1 - \int_\mathbb{W} \left( \frac{\partial \varphi_i}{\partial \mu_i} + \frac{\partial \varphi_i}{\partial \mu_{\gamma}} \frac{\partial \mu_{\gamma}}{\partial \mu_i} \right) f_i(w) dw \right] = 0. \]

where \( \frac{\partial \mu_{\gamma}}{\partial \mu_i} \) is given by Eq. (33). The proof is thus complete. \( \square \)

**Proof of Proposition 4.1.** Applying condition (a) and (15) to Eq. (33) shows that
\[ \frac{\partial \mu_{\gamma}}{\partial \mu_i} = -\frac{\sigma_f}{1 + \sigma_D}, \]

substituting which into the formula of \( \gamma_i/\lambda_i \) shown in Theorem 4.1 reveals that
\[ \frac{\gamma_i}{\lambda_i} = \frac{\sigma_f^2 - (1 + \sigma_D)^2 \lambda_i}{(1 + \sigma_D)^2 - \sigma_f^2} \bar{f}(\overline{w}). \]

We thus obtain
\[ \frac{\partial (\gamma_i/\lambda_i)}{\partial \sigma_D} = \frac{-\sigma_f^2 + (1 + \sigma_D)^2 \lambda_i}{(1 + \sigma_D)^2 - \sigma_f^2} \bar{f}(\overline{w}) < 0 \]

and
\[ \frac{\partial (\gamma_i/\lambda_i)}{\partial \sigma_f^2} = \frac{1 + \sigma_D}{(1 + \sigma_D)^2 - \sigma_f^2} \bar{f}(\overline{w}) > 0. \]

As a result, using chain rule, Corollary 4.1 and Proposition 3.2 gives rise to
\[ \frac{\partial^2 T'_{\gamma}(y_i(\infty))}{\partial \sigma_D \partial (1/\lambda_i)} = \frac{\partial^2 T'_{\gamma}(y_i(\infty))}{\partial (\frac{\lambda_i}{\lambda_i} \alpha_i(\infty)) \partial (1/\lambda_i)} \cdot \alpha_i(\infty) \frac{\partial (\gamma_i/\lambda_i)}{\partial \sigma_D} > 0 \]

and
\[ \frac{\partial^2 T'_{\gamma}(y_i(\infty))}{\partial \sigma_f^2 \partial (1/\lambda_i)} = \frac{\partial^2 T'_{\gamma}(y_i(\infty))}{\partial (\frac{\lambda_i}{\lambda_i} \alpha_i(\infty)) \partial (1/\lambda_i)} \cdot \alpha_i(\infty) \frac{\partial (\gamma_i/\lambda_i)}{\partial \sigma_f^2} < 0 \]

for \( \forall \sigma_D \in (-1, 0) \) and \( \hat{\theta}(\infty) < 1 \), as desired. For the other cases, we can similarly show that the predictions of Nash equilibrium carry over to the current Stackelberg equilibrium. \( \square \)

**Proof of Proposition 4.2.** It follows from Theorem 4.1 that
\[ \frac{\partial (\gamma_i/\lambda_i)}{\partial (\mu_{\gamma}/\mu_i)} = \left[ 1 - \int_\mathbb{W} \left( \frac{\partial \varphi_i}{\partial \mu_i} \frac{\partial \mu_{\gamma}}{\partial \mu_i} + \frac{\partial \varphi_i}{\partial \mu_{\gamma}} \frac{\partial \mu_{\gamma}}{\partial \mu_i} \right) f_i(w) dw \right]^2 \]

where
\[ \Lambda = \left[ 1 - \int_\mathbb{W} \frac{\partial \varphi_i}{\partial \mu_i} f_i(w) dw \right] \int_\mathbb{W} \frac{\partial \varphi_i}{\partial \mu_{\gamma}} f_i(w) dw + \int_\mathbb{W} \frac{\partial \varphi_i}{\partial \mu_{\gamma}} f_i(w) dw \int_\mathbb{W} \frac{\partial \varphi_i}{\partial \mu_i} f_i(w) dw. \]
If Assumption 2.1 holds, then $\Lambda > 0$, and hence
\[
\frac{\partial (\gamma_i/\lambda_i)}{\partial (\partial \mu_{-i}/\partial \mu_i)} > 0.
\]
Also, using Theorem 4.1 and Assumption 2.1 again gives rise to $\partial \mu_{-i}/\partial \mu_i > 0$. Since we get from the optimal tax formula in Theorem 3.1 that optimal MTRs strictly increase in $\gamma_i/\lambda_i$ and $\partial \mu_{-i}/\partial \mu_i = 0$ in the Nash equilibrium, the required assertion accordingly follows. \(\Box\)

Appendix B

In the text, we assume that relative consumption comparisons are made before individuals make migration decisions. As such, the average consumptions, namely $\mu_i$ and $\mu_{-i}$, are defined over the ex ante skill densities of these two countries, namely $f_i(w)$ and $f_{-i}(w)$, respectively. If, instead, relative consumption concerns are based on ex post and hence endogenous skill densities, then, as correctly pointed out by a referee, the choice of tax rate of country $i$ can affect the level of consumption $\mu_{-i}$ in country $-i$ by affecting the distribution of skills. For example, if country $i$ chooses a very regressive tax rate to attract high-skilled workers, the average consumption of country $-i$ might decrease even if the indirect utility and labor supply of each type are fixed, simply by the fact that there are less high-skilled workers who have high consumption. Hence, in deriving the optimal tax formula in a Nash equilibrium, we need to take into account also the equation for $\mu_{-i}$ as a constraint, differently from e.g. Aronsson and Johansson-Stenman (2013, 2015), where the average consumption in the other country is rightfully taken as exogenous because there is no migration.\(^{18}\)

Now, the Lagrangian given in the proof of Theorem 3.1 should be rewritten as:
\[
\mathcal{L}_i \left( \{U_i(w), l_i(w)\}_{w \in [\underline{w}, \overline{w}]}, \mu_i, \lambda_i, \gamma_i, q_i, \{\xi_i(w)\}_{w \in [\underline{w}, \overline{w}]} \right) = U_i(w) + \lambda_i \int_\underline{w}^{\overline{w}} \left\{ [w f_i(w) - \psi_i(U_i(w), l_i(w), \mu_i, \mu_{-i})] \phi_i(U_i(w) - U_{-i}(w); w) - \frac{R}{W(w)} \right\} dw
\]
\[
+ \frac{\partial \mathcal{L}_i}{\partial U_i(w)} = \lambda_i \left[ w - \frac{h'(l_i(w))}{v'(\xi_i(w))} \right] f_i(w) - \gamma_i \frac{h'(l_i(w))}{v'(\xi_i(w))} \tilde{f}_i(w) + \frac{\xi_i(w) h'(l_i(w))}{w} \int_\underline{w}^{\overline{w}} \left[ 1 + \frac{l_i(w) h''(l_i(w))}{h'(l_i(w))} \right] \tilde{l}_i(w) dw = 0 \quad \forall w \in [\underline{w}, \overline{w}].
\]
(34)

Assuming that there is no bunching of workers of different skills and the existence of an interior solution, applying (15) shows that the necessary conditions can be written as follows:
\[
\frac{\partial \mathcal{L}_i}{\partial l_i(w)} = \lambda_i \left[ w - \frac{h'(l_i(w))}{v'(\xi_i(w))} \right] f_i(w) - \gamma_i \frac{h'(l_i(w))}{v'(\xi_i(w))} \tilde{f}_i(w) + \frac{\xi_i(w) h'(l_i(w))}{w} \int_\underline{w}^{\overline{w}} \left[ 1 + \frac{l_i(w) h''(l_i(w))}{h'(l_i(w))} \right] \tilde{l}_i(w) dw = 0 \quad \forall w \in (\underline{w}, \overline{w}),
\]
(35)

\[
\frac{\partial \mathcal{L}_i}{\partial U_i(w)} = 1 + \frac{\xi_i(w)}{\overline{w} - \underline{w}} = 0,
\]
(36)

\[
\frac{\partial \mathcal{L}_i}{\partial \mu_i} = -\frac{\xi_i(w)}{\overline{w} - \underline{w}} = 0,
\]
(37)

\[
\frac{\partial \mathcal{L}_i}{\partial \mu_{-i}} = \lambda_i \int_\underline{w}^{\overline{w}} \frac{\psi_i(\mu_i, \mu_{-i})}{v'(\xi_i(w))} \tilde{f}_i(w) dw + \gamma_i \left[ 1 + \int_\underline{w}^{\overline{w}} \frac{\psi_i(\mu_i, \mu_{-i})}{v'(\xi_i(w))} \tilde{l}_i(w) dw \right] + q_i \left[ \int_\underline{w}^{\overline{w}} \psi_i(\mu_{-i}) \tilde{f}_i(w) dw \right] = 0.
\]
(38)

Using (29), we get
\[
\frac{\xi_i(w)}{\lambda_i} = \left( 1 + \frac{\gamma_i}{\lambda_i} \right) \frac{\tilde{f}_i(w)}{v'(\xi_i(w))} + \left[ \gamma_i \frac{\xi_i(w)}{\lambda_i} - T_i(y_i(w)) \right] \tilde{\eta}_i(w) - \frac{q_i c_{-i}(w)}{\lambda_i} \tilde{\eta}_{-i}(w) \tilde{f}_{-i}(w).
\]
\(^{18}\) We wish to thank a referee for pointing out this interesting departure of our paper to these insightful studies.
Integrating on both sides of this equation and using the transversality condition \( (37) \), we obtain

\[
- \frac{\gamma_i}{\lambda_i} = \left( 1 + \frac{\gamma_i}{\lambda_i} \right) \int_w^\pi \tilde{f}_i(t) \frac{\nu_i(c_i(t) - T_i(y_i(t)))}{\nu_i(c_i(t) - T_i(y_i(t)))} dt + \int_w^\pi \left[ \frac{\nu_i(c_i(t) - T_i(y_i(t)))}{\lambda_i} \right] \tilde{e}_i(t) dt - \frac{q_i}{\lambda_i} \int_w^\pi c_{-i}(t) \tilde{e}_{-i}(t) dt.
\]

(39)

Rearranging \( (28) \) via using FOC \( (4) \), we have

\[
\frac{T_i'(y_i(w))}{1 - T_i'(y_i(w))} = \frac{\gamma_i}{\lambda_i} = - \frac{\gamma_i}{\lambda_i} \frac{\nu_i(c_i(w))}{\nu_i(c_i(w))} \int_w^\pi \left[ \frac{1}{\nu_i(c_i(w))} \left( 1 + \frac{\gamma_i}{\lambda_i} \right) \left( \frac{\nu_i(c_i(t) - T_i(y_i(t)))}{\lambda_i} \right) \tilde{e}_i(t) dt \right] \tilde{f}_i(w) - \tilde{f}_i(w)
\]

\[\frac{\gamma_i}{\lambda_i} = - \frac{\nu_i(c_i(w))}{\nu_i(c_i(w))} \int_w^\pi c_{-i}(t) \tilde{e}_{-i}(t) \tilde{f}_i(t) dt + \frac{q_i}{\lambda_i} \int_w^\pi c_{-i}(t) \tilde{e}_{-i}(t) \tilde{f}_i(t) dt \]

(42)

and

\[
\frac{\gamma_i}{\lambda_i} = - \frac{\nu_i(c_i(w))}{\nu_i(c_i(w))} \int_w^\pi c_{-i}(t) \tilde{e}_{-i}(t) \tilde{f}_i(t) dt + \frac{q_i}{\lambda_i} \int_w^\pi c_{-i}(t) \tilde{e}_{-i}(t) \tilde{f}_i(t) dt
\]

\[\frac{\gamma_i}{\lambda_i} = \frac{\gamma_i}{\lambda_i} \frac{\nu_i(c_i(w))}{\nu_i(c_i(w))} \int_w^\pi \left( \frac{\nu_i(c_i(t) - T_i(y_i(t)))}{\lambda_i} \right) \tilde{e}_i(t) dt \tilde{f}_i(w) = \tilde{f}_i(w)
\]

(43)

with \( \tilde{f}_i(w) = \int_w^\pi \tilde{f}_i(t) dt \) denoting the ex post skill distribution in country \( i \in \{A, B\} \).

Comparing the tax formula \( (41) \) to the tax formula \( (11) \), it is easy to see the difference in the Pigouvian-type tax component, which is skill independent in the former while is skill dependent in the latter except for the extreme case with \( \tilde{f}_i(w) = f_i(w) \). The intuition for this difference is easy to understand. If individuals make social comparisons before they make migration decisions, as assumed in the text, but their migration decisions do affect the tax base and hence the ex post consumption externality facing each government. As such, in the Nash equilibrium in which governments take into account individual migration responses on the extensive margin, the Pigouvian type tax in formula \( (11) \) not only corrects for, directly, the status-seeking externality, measured by \( \gamma_i/\lambda_i \) as in the autarky equilibrium of Kanbur and Tuomola (2013), but also corrects for the indirect multiplier effect of individual migration decisions on the status-seeking externality, measured by \( f_i(w)/f_i(w) \). Elaborating further, for any skill level \( w \), if \( f_i(w) > f_i(w) \), namely, country \( i \) faces a tax-driven labor outflow of skill \( w \), then the multiplier induced by tax-driven migration is greater than one, which is consistent with the equilibrium prediction that government actually imposes a higher Pigouvian tax than in the autarky equilibrium, namely, \( \gamma_i/\lambda_i \) \( f_i(w) > f_i(w) \) > \( \gamma_i/\lambda_i \). The other possibilities with \( f_i(w) < f_i(w) \) or \( f_i(w) = f_i(w) \) can be analyzed analogically. If, however, individuals make social comparisons based on ex post skill densities, then the multiplier effect disappears in the tax formula \( (41) \) because this case could be interpreted as individuals having internalized such an effect when making individual responses to possible tax policies.

The Mirrleesian-type tax in \( (41) \) is decomposed into an efficiency term \( \lambda_i(w) \), a skill distribution term \( B_i(w) \), and the average of mechanical income, participation and status-seeking effects above skill \( w \) denoted by \( C_i(w) \). It is immediate that \( A_i(w) \) and \( B_i(w) \) are the same in both \( (41) \) and \( (11) \), whereas \( C_i(w) \) in \( (42) \) is quite different from that in \( (12) \) by using also \( (43) \) and \( (13) \).

Finally, similar to Theorem 4.1, we can also derive an optimal tax formula for the leader country in a Stackelberg equilibrium. The formal derivation is, nevertheless, omitted to economize on the space. Here, we need to clarify one more point, as rightfully pointed out by a referee. Assuming that average consumptions are defined over ex post skill distributions, in both Nash and Stackelberg equilibria the countries internalize the cross-country consumption externality, but in a different way: under Nash just through the migration channel, while in the Stackelberg case, the leader country internalizes also the change in the taxes chosen by the follower.

References

Abel, A.B., 2005. Optimal taxation when consumers have endogenous benchmark levels of consumption. Rev. Econ. Stud. 72 (1), 21–42.


