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Toward Longer Investment: Is an inclusive regime always better than an authoritarian one?☆

Darong Dai [a,*], Guoqiang Tian [b,c]

[a] Institute for Advanced Research, Key Laboratory of Mathematical Economics (SUFE), Ministry of Education of China, Shanghai University of Finance and Economics, Shanghai, 200433, People's Republic of China
[b] Department of Economics, Texas A&M University, College Station, TX, 77843, USA
[c] Institute for Advanced Studies in Finance and Economics, Hubei University of Economics, Wuhan, 430205, People's Republic of China

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ABSTRACT

Recent evidence shows that large-scale capital outflows have occurred in emerging economies, such as China and Russia. We thus conduct a theoretical analysis comparing the performance of authoritarian and inclusive regimes in keeping international capital within borders as long as possible. We arrive at the following conclusions, revealing that institutional factors play a more critical role than tax policy: for an authoritarian regime to dominate an inclusive one, a lower degree of government transparency must be accompanied by a lower degree of capital mobility; an inclusive regime dominates an authoritarian one as long as capital is sufficiently mobile. Moreover, we provide a rationale for the following order of institutional change in open economies: before liberalizing capital accounts, an inclusive, transparent government should be established; otherwise, an authoritarian, opaque government could prevail when the exit cost is sufficiently high or the degree of capital mobility is sufficiently low.

1. Introduction

In market economies, tax authorities face the constraint that capitalists can "vote with their feet" by means of capital flight.[1] The issue of capital flight is especially negative for developing countries. In China, for example, 2.8 trillion renminbi (RMB) were transferred overseas in 2011; emerging markets in 2015 saw an estimated $735 billion in net capital outflows with all but $59 billion of that total coming from China.[2] Likewise, Russia warns of capital flight. According to the Central Bank of Russia, capital outflow reached $151.5 billion in 2014, 2.5 times greater than in 2013.[3] A 2012 report for Global Financial Integrity estimated that between 2001 and 2010, capital flight from developing countries increased from $477.1 to $1138 billion, registering a trend growth rate of 12.6% *per annum*.[4]

Such a scale of capital flight is detrimental to investment and thereby to the sustainable economic growth of these economies.[5] Other things remaining equal, the capability of sustaining international capital investment is desirable for promoting investment-driven economic development as well as for enlarging the tax base along the temporal dimension and is hence beneficial to tax revenue collection that is of critical importance in building state capacity (e.g., Besley and Persson, 2009). The question addressed here is: what kind of political regime

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* Corresponding author.
E-mail addresses: darongdai@hotmail.com (D. Dai), gtian@tamu.edu (G. Tian).

[1] It is a realization of the exit choice emphasized by Hirschman (1970).
[2] See *Financial Times*, January 20, 2016.
[3] See *Forbes*/Investing, March 2, 2015.
[4] For more details, please refer to the link: http://www.gfintegrity.org/the-adverse-economic-consequences-of-capital-flight-and-illicit-flows-from-developing-countries/.
[5] See, e.g., Cuddington (1986) and Pastor (1990) for similar arguments.

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can incentivize international investors to invest in host countries for a longer time?[6] Or, what kind of relationship connecting governments and capitalists is more desirable for keeping international investors within borders as long as possible?[7]

Although it is consistent with our intuition that political regimes should be relevant in affecting the choice of capital investment horizon, by reviewing the existing literature, we find that such a connection is left unexplored in theory. The related papers (e.g., Lensink et al., 2000; Collier et al., 2001; Hermes and Lensink, 2001; Le and Zak, 2006) focused on estimating how political-economic risks and policy uncertainties affected capital flight. They were silent on how the endogenous investment horizon changed across alternative political regimes. In terms of discouraging capital flight and incentivizing sustainable investment, one should design incentive-compatible regimes rather than impose direct punishments, as suggested by Segal and Vincent (1998). As such, we were motivated to offer a theory that helps us understand how political regimes shape endogenous investment endurance.

In addition to the authoritarian regime, which is characterized as a top-down hierarchy of authority, we consider another type of political regime that features inclusive governance arrangements. Absolute political authority supports the government in unilaterally determining the tax rate, whereas inclusive regimes allow for a bargaining table on which capitalists and the government may reach a mutually beneficial tax rate. We derive the equilibrium capital income tax rate as a component of a subgame perfect equilibrium under authoritarian conditions, while deriving it as a component of a cooperative equilibrium under bilateral bargaining. We demonstrate that the cooperative equilibrium satisfies individual rationality, group rationality, subgame consistency, and Pareto efficiency, and that no one unilaterally deviates from cooperation, regardless of whether the Nash bargaining solution, the Shapley value, or a proportional distribution is adopted as the allocation principle (or, the payoff distribution procedure). The inclusive regime is somehow justified by these properties.

Among many policy variables, the capital income tax is the one that we choose to compare in these two types of regimes. Everything else being equal, a linear capital income tax rate distorts capitalists' intertemporal savings motive and hence the path of capital accumulation, thereby making it a relevant policy affecting the sustainability of private capital investment. Alternative regimes may induce varied levels of distortion and therefore affect the endurance of investment differently. To characterize such difference, we formalize these two regimes as alternative game forms with the government and a representative capitalist as the players. We then derive the equilibrium outcomes under alternative game forms with the same information structure as well as the equilibrium outcomes under the same game form with different information structures.

As well-known institutional factors, government transparency and the exit cost are incorporated into the analysis as follows. To characterize the degree of government transparency, we assume that the capitalist may exhibit delayed information availability relative to the government, namely, the government may exhibit opaqueness in delivering information to international investors. We focus first on the case of an authoritarian, opaque government and an inclusive, transparent government. In other words, we normalize the size of the delayed information facing the capitalist to zero under the inclusive regime, but let the size of the delayed information become positive under the authoritarian regime.[8] This specification captures, in part, the information constraints that exist in reality and also enables us to focus on the primary concern of this paper. To complement the analysis under such specifications, we also consider the case of an authoritarian, transparent government and an inclusive, opaque government.

We use the exit cost, which is assumed to be an exogenously given parameter, to capture the degree of market failure or institutional friction associated with cross-border capital mobility. We can, for example, interpret it as a kind of transaction cost resulting from the imperfectness of the capital market, or as an exogenous "exit tax" imposed by the government.[9] Intuitively, a higher exit cost facing the capitalist implies a lower degree of capital mobility.

We obtain two main results in the case of an authoritarian, opaque government and an inclusive, transparent government. First, the higher the degree of government transparency, the longer the expected investment horizon will be. In consequence, *ceteris paribus*, strengthening government transparency is desirable even under the authoritarian regime. Second, there is an endogenous threshold of the degree of capital mobility such that the following conclusion holds. Below the threshold (namely, the point at which capital becomes relatively immobile), the authoritarian regime dominates the inclusive regime when its degree of opaqueness is lower than some critical value; otherwise, the inclusive regime dominates the authoritarian regime. Above the threshold (namely, the point at which capital becomes relatively mobile), the inclusive regime dominates the authoritarian regime even if the authoritarian government is transparent, implying that the inclusive regime dominates the authoritarian regime whenever capital is sufficiently mobile.

Therefore, to identify their relative advantage in sustaining international investment, both *the degree of government transparency* and *the degree of capital mobility* are relevant institutional factors. In numerical experiments, we also find that the lower the degree of government transparency under the authoritarian regime, the higher is the threshold for the degree of capital immobility required such that the authoritarian regime dominates the inclusive regime beyond this threshold. Moreover, we perform the comparative statics of the equilibrium exit times with respect to the relevant parameters, namely, the constant capital return rate, the subjective discount factor, and the fraction of benevolent politicians in the government. We find that only when the exit cost is below some threshold does a higher capital return rate imply a longer investment horizon; otherwise, a higher capital return rate could lead to an even shorter investment horizon, regardless of the political regime and information structure. Similar findings apply to the comparative statics with respect to the subjective discount factor and the fraction of benevolent politicians. These results further show the critical role of capital mobility in shaping the capitalist's optimal choice of investment horizon.

We obtain another two main results in the case of an authoritarian, transparent government and an inclusive, opaque government.

[6] For future research, total welfare and the total value of output are alternative yardsticks used to compare the relative performance of inclusive and authoritarian regimes.

[7] Given the importance of foreign investment and capital formation for emerging economies, this question is quite relevant for many transitional economies. To illustrate the importance of building a healthy and sustainable relationship between governments and capitalists, we take China as an example. In fact, two extreme relationships have emerged over the past few decades. In Mao's time, especially during the decade of the Great Cultural Revolution, the ideology of the "Left" was carried to its extreme and capitalists were put in great danger. After implementing the "Reform and Opening-up Policy" by Deng Xiaoping, capitalists were welcomed by the Chinese government, although many corrupt relationships can be observed. We hence believe that the issue addressed here has practical implications for contemporary China.

[8] As shall be further explained in the model, this normalization is imposed for simplicity as it is not essential for deriving our formal results.

[9] For example, Hillary Clinton planned to impose an exit tax on businesses that relocated outside of the United States (see *Wall Street Journal*, August 21, 2016); Japan's government targets wealthy individuals with an exit tax in hope of preventing them from moving to a location where the taxes are low (see *Wall Street Journal*, December 18, 2014); and in China, an article published in the state-run *People's Daily* (November 2011) entitled "We Should Make it Harder for the Wealthy to Emigrate" proposes an exit tax on wealthy Chinese leaving the country (see *The Atlantic*, April 11, 2013).

First, if the exit cost and size of information delay under the inclusive regime are relatively small, particularly, smaller than some identified threshold, then the capitalist would choose a longer investment horizon when facing an inclusive, opaque government than the horizon he would choose when facing an authoritarian, transparent government. Second, if the exit cost under both political regimes is large, particularly, larger than some threshold, then the authoritarian, transparent government dominates the inclusive, opaque government in sustaining capital investment. As a result, everything else being equal, lowering the exit cost (or increasing the degree of cross-border capital mobility) can strengthen the relative advantage of the inclusive regime, which holds true even if the authoritarian government is more transparent than the inclusive government. In addition, the relative advantage of authority can be strengthened by increasing the exit cost (or lowering the degree of capital mobility).

Our work is related to three branches of the literature. Concerning the mechanism of "voting with one's feet," our paper is related to Tiebout (1956), Qian and Roland (1998), Cai and Treisman (2005), and Bai et al. (2016), to name just a few. Departing from scholars who use static models, we solve for the optimal exit strategy in a dynamic, stochastic environment. We use a dynamic model for two considerations. First, capital investment activity is dynamic in nature, and therefore a dynamic model represents a better approach than any static model. Second, analyzing the effect of capital taxation on intertemporal savings decisions generally calls for a dynamic model (see Saez, 2013). More importantly, while the literature listed above analyzed how the threat of voting with one's feet may constrain governmental behavior, we studied how political regimes, in turn, shape the equilibrium feature of voting by feet. Therefore, our paper complements the existing literature by exploring the interplay between political regimes and foot-voting mechanisms.

Concerning the occurrence of capital flight, existing studies have provided varied explanations. For example, Alesina and Tabellini (1989) developed a model to argue that it is the uncertainty over the fiscal policies of future governments that generates capital flight. In Tornell and Velasco (1992), it emerged as a response to the poor protection of property rights, whereas Svensson (1998) argued that political instability and polarization held back private investment. To complement these arguments, we show that capital flight could be rationalized as an equilibrium choice of sustainable capital accumulation under uncertain conditions. In addition, our theory predicts not only the magnitude of capital outflows but also *when* capital flight is supposed to occur, the latter perspective being ignored by the literature. Based on cross-country data from 40 countries over 7 years, Zhao et al. (2003) presented empirical evidence that a low level of government transparency is likely to significantly reduce the magnitude of capital inflows to host countries. As a theoretical complement, our results show that, *ceteris paribus*, a low level of government transparency is also likely to hurt the sustainability of capital inflows.

Our paper is also related to the literature using stochastic differential games to analyze strategic interactions between governments and firms. For example, Lancaster (1973) and Kaitala and Pohjola (1990) adopted a deterministic differential game to prove that cooperation between governments and firms will be more beneficial compared to the Nash equilibrium. Seierstad (1993) used an extension of the Lancaster model to show the dynamic efficiency of capitalism. Leong and Huang (2010) developed a stochastic differential game of capitalism to analyze the role of uncertainty and demonstrated that cooperation is Pareto-optimal relative to the non-cooperative Markovian Nash equilibrium. Exploiting the AK model of economic growth in an infinite-horizon economy, Dai (2013) comparatively studied a dynamic sequential game and a cooperative stochastic differential game between a representative household and a typical revenue-maximizing politician in terms of the performance of the equilibrium capital income tax rate, the savings rate, as well as the resulting economic growth rate. Recently, Dai et al. (2019) comparatively studied two differential games between rent-seeking politicians and capitalists, which also implied that the two relationships, namely, top-down authority and rational cooperation, connected governments and capitalists, but their goal was to identify the one having an advantage in reducing rent-seeking distortions and promoting economic growth.

In sum, our paper distinguishes itself from the differential game literature previously mentioned in three respects. First, it is obvious that our motivation and research questions depart from those of these papers. Specifically, instead of exploring the perspectives of economic growth, welfare loss, class conflict, and income redistribution considered in the existing literature, our goal is to compare the performance of cooperative and non-cooperative differential games under alternative information structures in keeping international capital within a country's borders as long as possible. To the best of our knowledge, the present paper adds a novel perspective to the related literature. Second, none of the existing studies accounts for the possible differences in the underlying information structures of these two political regimes, and therefore none of them could analyze the policy implications generated by such a difference in the critical dimension of government transparency. Third, although in the literature, economic growth is assumed to be driven by capital accumulation, the previous studies all focus on a sort of autarky equilibrium; that is, none of the models in the literature allows for the exit mechanism emphasized in this paper. As is obvious, cross-border capital mobility is indeed a constraint of practical relevance facing the governments of open economies.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 derives the equilibrium outcomes under authoritarian and inclusive regimes. Section 4 establishes the major results. Section 5 offers further discussions concerning alternative information structures under each political regime, the case of time-varying tax policy under the authoritarian regime, as well as the endogenous effects of government transparency and the exit cost on the equilibrium welfare of the authoritarian government. Section 6 concludes the paper. All proofs appear in the Appendix.

2. The model

To avoid nonessential complications, we consider an economy populated by a representative capitalist and a government consisting of heterogeneous politicians.

2.1. Capitalist

The capitalist owns initial capital stock of $k(0) \equiv k_0 > 0$, a deterministic constant, and accumulates it in accordance with the following stochastic differential equation[10]

$$dk(t) = [(1-\tau_k)(r-\delta)k(t) - c(t)]dt + \sigma k(t)dB(t), \tag{1}$$

where $\sigma > 0$ is a constant percentage volatility measuring a set of unpredictable events occurring during this motion, and $B(t)$ is a standard Brownian motion defined on the filtered probability space $(\Omega, \mathfrak{F}, \{\mathfrak{F}_t\}_{0\le t\le \tau}, P)$ with $0 < \tau \le \infty$, $B(0) = 0$ a.s.-P and with the usual conditions fulfilled. τ_k is the capital income tax rate, $r > 0$ is the constant capital return rate, $0 < \delta < 1$ is the constant depreciation rate, and c represents consumption. The economy is characterized by three parameters: r, δ and σ. The capitalist is assumed to initially invest k_0 amount of capital into the economy (through either the production or financial sectors).

[10] Stochastic differential equations are often used to characterize capital accumulation under uncertainty (see, e.g., Merton, 1975; Leong and Huang, 2010; Wälde, 2011; Dai, 2012, 2013, 2014; and Dai et al., 2019). They all assumed that the source of uncertainty was population growth, whereas here, as in Dai (2018), we do not restrict our attention to any specific source of uncertainty.

By imposing log preferences,[11] the intertemporal objective reads as follows[12]

$$\mathbb{E}_{t_0}\left[\int_0^{\tau} e^{-\rho(t_0+t)}\ln c(t)dt\right],$$

where $\mathbb{E}_{t_0}$ is the expectation operator depending on information flow up to time $t_0 \geq 0$, $0<\rho<1$ is a subjective discount factor, and τ is the exit time that determines the investment horizon. We define two sets of admissible exit times by $\mathcal{T}_0 \equiv \{t \geq 0; \mathfrak{F}_t - \text{adapted exit times}, P - \text{almost surely finite}\}$ and $\mathcal{T}_\Delta \equiv \{t \geq \Delta; \mathfrak{F}_{t-\Delta} - \text{adapted exit times for a constant } \Delta > 0, P - \text{almost surely finite}\}$, meaning that the capitalist has **overall information**, denoted by filtration $\mathfrak{F}_t$, and **Δ-delayed information**, denoted by filtration $\mathfrak{F}_{t-\Delta}$, at time t, respectively. To focus on the key issue, we assume that Δ is a commonly known constant. We can relax this assumption by, for example, incorporating uncertainty into the size of the delayed information. However, this assumption is not essential for establishing the following formal results, as we can use an expected value of Δ to replace the current Δ and our main theoretical results still hold true.

The capitalist first chooses an exit time τ, namely, **the timing of terminating (or withdrawing) investment**, based on a sustainability consideration, then chooses an optimal consumption plan during $[0,\tau)$. In particular, if $\mathbb{E}_{t_0}(\tau)=0$ at equilibrium, then the capitalist will not invest in the current economy in the first place. This specification of the decision-making procedure is consistent with our intuition and can be interpreted as a natural extension of classic intertemporal optimization by endogenizing the planning horizon. Here the **sustainability** of capital accumulation requires that τ be a solution to the following optimal stopping problem[13]

$$\Phi_{\cdot}(t_0,k_0) \equiv \sup_{\tau\in\mathcal{T}} \mathbb{E}^{t_0,k_0}\left[e^{-\rho(t_0+\tau)}(k(\tau)-\varpi)\right], \tag{2}$$

subject to (1). The subscript $\cdot = 0$ or Δ for $\mathcal{T}=\mathcal{T}_0$ or $\mathcal{T}_\Delta$, $\mathbb{E}^{t_0,k_0}$ is the expectation with respect to probability law P^{t_0,k_0} of time-space process $dZ(t) \equiv (dt, dk(t))'$ with initial state $Z(0) \equiv (t_0,k_0)'$ and transpose operation $'$, and $\varpi > 0$ is a constant **exit cost** that measures the barriers to interjurisdictional or international capital mobility as well as the associated transaction cost. For any given τ, the optimal consumption plan solves the following problem:

$$\max_{c(t)>0} \mathbb{E}_{t_0}\left[\int_0^{\tau} e^{-\rho(t_0+t)}\ln c(t)dt\right], \tag{3}$$

subject to (1).

2.2. Government

To be as realistic as possible, the government is assumed to consist of both benevolent and selfish politicians.[14] The measure of politicians is normalized to one, with a constant fraction ε of the benevolent, who have the same utility as the capitalist, and the remaining $1-\varepsilon$ of the selfish, who maximize the utility generated by tax revenue.

We focus on two political regimes, implying two alternative tax rate-setting problems.

Definition 2.1. A political regime is called an **authoritarian regime** if the government moves first to unilaterally determine the tax rate.

Definition 2.2. A political regime is called an **inclusive regime** if the government bargains with the capitalist to cooperatively determine the tax rate.

From a real-life perspective, authoritarian regime is easy to comprehend. To demonstrate the validity of inclusive regime, we shall take China as an example.[15] In fact, political centralization, fiscal decentralization, interjurisdictional competition and the mechanism governing political turnover in China motivated local governments to form a sort of cooperative relationship with capitalists (or investors), hence attracting sufficient capital investment and guaranteeing fast economic growth during the past several decades (e.g., Montinola et al., 1995; Li and Zhou, 2005; Xu, 2011). Alternatively, a type of inclusive regime adopted by local governments laid out the self-enforced institutional foundation of China's growth miracle.

The fundamental difference between the two political regimes can be illustrated from the perspective of fiscal revenue collection. It is intuitive that taxation policy will affect both the speed of capital accumulation and the exit threshold chosen by the capitalist. As shall be formally shown subsequently, the inclusive regime always implies a lower capital income tax rate than the authoritarian regime, which yields that the speed of capital accumulation is always faster under inclusivity than under authority. In consequence, the negative effect on fiscal revenue collection under an inclusive regime is that, for any given level of capital investment, the total revenue generated is lower than that under authority. However, a lower tax rate may also provide a stronger incentive for international investors to stay, and thus the tax base may become larger along the temporal dimension, generating a positive effect on fiscal revenue collection under an inclusive regime. As the effect of taxation policy on the exit threshold is not clear-cut, the net effect on fiscal revenue collection is generally ambiguous. The corresponding analysis for an authoritarian regime can be performed analogously. In sum, the basic insight is that the inclusive regime does not necessarily dominate the authoritarian regime, and vice versa, in terms of collecting fiscal revenue in the present model of an endogenous investment horizon.

3. Equilibrium derivation

Prior to deriving the equilibrium, let us first provide the following assumption regarding the information structure under each political regime.

Assumption 3.1. When choosing an investment horizon, the capitalist has Δ-delayed information under an authoritarian regime while he has overall information under an inclusive regime.

The difference in information structure stems from the observation that strong top-down authoritarian regimes generally lead to a low degree of transparency, while an inclusive regime induces rational cooperation that calls for a relatively high degree of transparency.

[11] This assumption greatly simplifies the tractability of the model and enables us to derive formal results transparently. As a caveat, we admit that our results might not necessarily easily carry over to general utility functions.

[12] We have considered the alternative objective, $\mathbb{E}_{t_0}\left[\int_0^{\tau} e^{-\rho(t_0+t)}\ln c(t)dt + e^{-\rho(t_0+\tau)}\ln k(\tau)\right]$, with a terminal utility generated by capital wealth. In Appendix L, we provide the technical details of solving for the optimal stopping rule under the objective of this integral form. It turns out that our main theoretical predictions still hold. We thus use the current form for the sake of expositional simplicity.

[13] In the literature (e.g., Radner, 1961; Kurz, 1965; McKenzie, 1963, 1976) regarding optimal capital accumulation, sustainability is usually defined by maximizing terminal stocks (or final states). Departing from these studies, we use optimal stopping theory, which enables us to simultaneously determine optimal terminal stock and optimal exit time.

[14] We introduce selfish or rent-seeking politicians into the model because, as recently argued by Stiglitz (2019), these politicians' interests are typically not perfectly aligned with those whose interests they are supposed to represent.

[15] In the language of Olson (2000), an inclusive regime may be interpreted as maximizing the encompassing interests between the power and the citizens. We may also interpret it as a realization of open access orders respecting economically incentive-compatible requirements (see North et al., 2006). Intuitively, an inclusive regime utilizes a collective decision-making process that involves more people and attempts to make the most satisfactory decision for everyone.

That is, information sharing is the prerequisite condition for forging a sustainable relationship of incentive-compatible cooperation.

It is not necessary to let the capitalist have overall information under the inclusive regime, and thus we normalize his information delay to zero because only the difference between the information structures is significant when comparing the two regimes. In other words, we can still obtain our major results after relaxing this normalization imposed on the inclusive regime.

3.1. Equilibrium under the authoritarian regime

Under the authoritarian regime, events proceed as follows:

- **Stage 1.** The capitalist chooses an exit time, $\tau_\Delta \in \mathcal{T}_\Delta$, by solving problem (2).
- **Stage 2.** The government determines a capital income tax rate by solving this problem:

$$\max_{0\le\tau_k\le 1} \mathbb{E}_{t_0}\left(\int_0^{\tau_\Delta} e^{-\rho(t_0+t)}\left\{\underbrace{\varepsilon\ln c(t)}_{\text{utility for the benevolent}} + \underbrace{(1-\varepsilon)\ln[\tau_k(r-\delta)k(t)]}_{\text{utility for the selfish}}\right\}dt\right) \tag{4}$$

subject to (1).

- **Stage 3.** The capitalist chooses a consumption plan by solving the problem given in equation (3).

In contrast to the political polarization adopted by Alesina and Tabellini (1989) and the normative assumption of a benevolent government, we assume as shown in (4) that the government maximizes a weighted average of the utilities of both types of politicians. Indeed, one can interpret it as a kind of political power balance between these two conflicting groups of politicians, which is not a rare phenomenon in both democracies and non-democracies. For instance, it could represent a two-party bargaining equilibrium or the realization of a political compromise (e.g., Dixit et al., 2000) within the government. In addition, we focus on the taxation policy that *lacks commitment* in the sense that it is determined after the capitalist has chosen an exit time.[16]

Using backward induction, the equilibrium is derived and stated in the following lemma.

Lemma 3.1. *Suppose the economy is under an authoritarian regime and Assumption 3.1 holds. Then, the subgame perfect equilibrium is given by* $\left\{c^*(t), \tau_k^*\right\} = \left\{\rho k^*(t), \frac{\rho(1-\varepsilon)}{r-\delta}\right\}$, *in which the trajectory of capital accumulation along the subgame perfect equilibrium is expressed as*

$$k^*(t) = k_0 \exp\left\{\left[r-\delta-\rho(2-\varepsilon)-\frac{1}{2}\sigma^2\right]t+\sigma B(t)\right\}. \tag{5}$$

Proof. See the Appendix. □

[16] As suggested by a referee, it would be worthwhile revising the timing of game by letting the government determine the tax rate first because it may not always be the case that the government has less commitment power than the capitalist. Importantly, given that the exit only happens in the future and in a random fashion, it is reasonable to envisage that the capitalist would be willing to renege once the tax policy has already been chosen by the government. Nevertheless, we are unable to provide an explicit characterization of the corresponding equilibrium. Indeed, when we solve Problem (4) in Stage 1 by backward induction, taking as given the optimal exit time τ_Δ chosen by the capitalist in Stage 2, which is a highly non-linear function of the tax rate, it makes the optimization problem mathematically unmanageable. In addition, we cannot even guarantee that the objective function is concave or quasi-concave with respect to the choice variable. As such, we focus on the present timing of the game under the authoritarian regime because it is not only somehow reasonable but also allows for technical tractability.

It is easy to see from (5) that capital income taxation discourages the capitalist's consumption by negatively distorting his capital accumulation along the entire path. The larger the fraction of selfish politicians (or, equivalently, the smaller the fraction of benevolent politicians), the higher the equilibrium capital income tax rate will be.

Moreover, the optimal exit time in Stage 1 is stated in the following lemma.

Lemma 3.2. *Suppose the economy is under an authoritarian regime and Assumption 3.1 holds. If the constant capital return rate is restricted such that* $r \in (r_{\min}, r_{\max})$ *with* $r_{\min} \equiv \delta + \rho(2-\varepsilon) + \frac{1}{2}\sigma^2$ *and* $r_{\max} \equiv \delta + \rho(2-\varepsilon) + \rho$, *and* $\mu \equiv r - \delta - \rho(2-\varepsilon) > 0$, *then the optimal exit time is* $\tau_\Delta^* = \inf\left\{t > 0 : k^*(t) = \widetilde{k}^* \equiv \frac{\lambda_1\widetilde{\varpi}}{\lambda_1-1}\right\}$, *in which*

$$\lambda_1 = \frac{\sigma^2 - 2\mu + \sqrt{(2\mu-\sigma^2)^2 + 8\rho\sigma^2}}{2\sigma^2} \tag{6}$$

and

$$\widetilde{\varpi} = \varpi e^{-\mu\Delta}. \tag{7}$$

Proof. *See the Appendix.* □

Lemma 3.2 confirms the existence and uniqueness of an optimal exit time under mild assumptions. As an optimal stopping rule, the capitalist shall withdraw his capital investment by selling the asset or stock when the accumulation of capital stock reaches a certain threshold, denoted $\widetilde{k}^*$, during which the associated transaction cost has already been taken into account.

If the capitalist has overall information rather than Δ-delayed information in choosing an optimal exit time, then Lemma 3.2 needs to be revised as follows.

Lemma 3.3. *Suppose the economy is under an authoritarian regime with the information delay satisfying* $\Delta = 0$. *Then, the optimal exit time is* $\tau_0^* = \inf\left\{t > 0 : k^*(t) = k^* \equiv \frac{\lambda_1\varpi}{\lambda_1-1}\right\}$, *where* $k^*(t)$ *and* λ_1 *are given by equations (5) and (6), respectively.*

Proof. *Since the proof is similar to that of Lemma 3.2, we omit it to economize on space.* □

As is obvious, the optimal stopping rule is different from that in Lemma 3.2. The direct implication is that the factor of information availability does matter in determining the optimal exit time. Furthermore, this lemma offers a useful intermediate case in the sense that we can compare it with Lemma 3.2 to identify the equilibrium effect resulting from different information structures within the same game form, and compare it with the following Lemma 3.6 to identify the equilibrium effect resulting from different game forms under the same information structure.

3.2. Equilibrium under the inclusive regime

Under the inclusive regime, events proceed as follows:

- **Stage 1.** The capitalist chooses an exit time, denoted by $\tau_0 \in \mathcal{T}_0$, by solving Problem (2).
- **Stage 2.** Under rational cooperation, the maximization problem is given by

$$\max_{c(t)>0, 0\le\tau_k\le 1} \mathbb{E}_{t_0}\left(\int_0^{\tau_0} e^{-\rho(t_0+t)}\left\{\underbrace{\ln c(t)}_{\text{utility for the capitalist}} + \overbrace{\varepsilon\ln c(t) + (1-\varepsilon)\ln[\tau_k(r-\delta)k(t)]}^{\text{utility for the government}}\right\}dt\right) \tag{8}$$

subject to equation (1). That is, the expectation term in equation (8) defines the collective objective.

Using backward induction, the cooperative equilibrium is derived as follows.

Lemma 3.4. *Suppose the economy is under an inclusive regime and Assumption 3.1 holds. Then, the cooperative equilibrium is given by* $\left\{c^{**}(t), \tau_k^{**}\right\} = \left\{\frac{\rho(1+\varepsilon)}{2}k^{**}(t), \frac{\rho(1-\varepsilon)}{2(r-\delta)}\right\}$, *in which the trajectory of capital accumulation along the cooperative equilibrium is expressed as*

$$k^{**}(t) = k_0 \exp\left[\left(r-\delta-\rho-\frac{1}{2}\sigma^2\right)t + \sigma B(t)\right]. \tag{9}$$

Proof. *It is similar to that of Lemma 3.1 and, hence, the proof is omitted.* □

When compared to the non-cooperative equilibrium given in Lemma 3.1, at the cooperative equilibrium, the tax rate is smaller, the consumption rate is smaller, the savings rate is higher, and hence the expected growth rate of capital accumulation is higher. Nevertheless, note that the relationship between the optimal exit time and the equilibrium speed of capital accumulation is generally not monotone, that is, we cannot predict that one equilibrium definitely induces a longer investment horizon than does the alternative equilibrium. Another interesting observation arises from comparing (5) with (9), i.e., the composition of politicians matters for the equilibrium capital accumulation under the authoritarian regime, while it is irrelevant under the inclusive regime. This is because the composition of politicians only affects the equilibrium tax rate under the authoritarian regime, whereas it affects both equilibrium consumption and the equilibrium tax rate under cooperation, and the two effects offset each other along the equilibrium path of capital accumulation under the inclusive regime.

Before characterizing the cooperative equilibrium established in Lemma 3.4, let us consider first the non-cooperative differential game with given initial condition (t_0, k_0). The capitalist chooses precisely the best consumption strategy, $\widehat{c}(t)$, given the government's best-response strategy, $\widehat{\tau}_k$, and simultaneously, the government chooses the best taxation strategy, $\widehat{\tau}_k$, given the capitalist's best-response strategy, $\widehat{c}(t)$. In addition, we let $J^C(t,k(t))$ and $J^G(t,k(t))$ be value functions for the capitalist and the government, respectively.

Definition 3.1. A set of strategies $\left\{\widehat{c}(t), \widehat{\tau}_k\right\}$ constitutes a **Markovian-feedback Nash equilibrium** of the non-cooperative differential game if there exist continuously differentiable functions $J^C(t,k(t)) : \mathbb{R}_+^2 \to \mathbb{R}$ and $J^G(t,k(t)) : \mathbb{R}_+^2 \to \mathbb{R}$ satisfying Bellman equations:

$$\begin{aligned}&-J_t^C(t,k(t)) - \frac{1}{2}\sigma^2 k^2(t) J_{kk}^C(t,k(t))\\ &= \max_{c(t)>0}\left\{e^{-\rho(t_0+t)}\ln c(t) + J_k^C(t,k(t))[(1-\widehat{\tau}_k)(r-\delta)k(t) - c(t)]\right\}\end{aligned}$$

and

$$\begin{aligned}&-J_t^G(t,k(t)) - \frac{1}{2}\sigma^2 k^2(t) J_{kk}^G(t,k(t)) = \max_{0\le\tau_k\le 1}\left\{e^{-\rho(t_0+t)}\varepsilon\ln\widehat{c}(t)\right.\\ &+e^{-\rho(t_0+t)}(1-\varepsilon)\ln[\tau_k(r-\delta)k(t)]\\ &\left.+J_k^G(t,k(t))k(t)[(1-\tau_k)(r-\delta)k(t) - \widehat{c}(t)]\right\},\end{aligned}$$

respectively, with

$$J^C(t_0,k_0) \equiv \mathbb{E}_{t_0}\left(\int_0^{\tau_0} e^{-\rho(t+t_0)}\ln\widehat{c}(t)dt|k(t_0) = k_0\right)$$

and

$$\begin{aligned}&J^G(t_0,k_0) \equiv\\ &\mathbb{E}_{t_0}\left(\int_0^{\tau_0} e^{-\rho(t+t_0)}\{\varepsilon\ln\widehat{c}(t) + (1-\varepsilon)\ln[\widehat{\tau}_k(r-\delta)k(t)]\}dt|k(t_0) = k_0\right)\end{aligned}$$

representing the current - value payoffs for the capitalist and the government, respectively.

Then, the following lemma provides a characterization of the non-cooperative Markovian-feedback Nash equilibrium.

Lemma 3.5. *The Markovian-feedback Nash equilibrium is* $\left\{\widehat{c}(t), \widehat{\tau}_k\right\} = \left\{\rho k(t), \frac{\rho(1-\varepsilon)}{r-\delta}\right\}$ *with value functions* $J^C(t,k(t)) = e^{-\rho(t_0+t)}[C_7\ln k(t) + C_8]$ *and* $J^G(t,k(t)) = e^{-\rho(t_0+t)}[C_9\ln k(t) + C_{10}]$ *for the capitalist and government, respectively, in which* $C_7 = C_9 = \frac{1}{\rho}$, $C_8 = -\frac{\sigma^2}{2\rho^2} + \frac{1}{\rho}\ln\rho + \frac{r-\delta}{\rho^2} - \frac{2-\varepsilon}{\rho}$ *and* $C_{10} = -\frac{\sigma^2}{2\rho^2} + \frac{\varepsilon}{\rho}\ln\rho + \frac{r-\delta-\rho}{\rho^2} + \frac{1-\varepsilon}{\rho}\ln\left[\frac{\rho(1-\varepsilon)}{e}\right]$.

Proof. *Omitted, as it is essentially the same as that of Lemma 3.1.* □

In light of Lemma 3.5, we can give the following definition.

Definition 3.2. Group rationality is satisfied if $J(t,k^{**}(t)) > J^C(t,k^{**}(t)) + J^G(t,k^{**}(t))$ along the cooperative trajectory $\{k^{**}(t)\}_{t=0}^{\tau_0}$.

Let Γ_t^{**} denote the set of reliable values of $k^{**}(t)$ at time t generated by (9). For notational consistency, we use k_t^{**} to represent a generic element of set Γ_t^{**}. Also, let vector $\xi(t') \equiv \left[\xi^C(t'), \xi^G(t')\right]$ denote the instantaneous payoff at time $t' \in (0,\tau_0)$. In particular, along cooperative trajectory $\left\{k_t^{**}\right\}_{t=0}^{\tau_0}$ we put the following value functions:

$$v^{(t_0)i}\left(t', k_{t'}^{**}\right) \equiv \mathbb{E}_{t'}\left[\int_{t'}^{\tau_0} e^{-\rho(z-t')}\xi^i(z)dz|k(t') = k_{t'}^{**}\right]$$

and

$$v^{(t_0)i}\left(t, k_t^{**}\right) \equiv \mathbb{E}_t\left[\int_t^{\tau_0} e^{-\rho(z-t)}\xi^i(z)dz|k(t) = k_t^{**}\right]$$

for $i \in \{C,G\}$, $k_{t'}^{**} \in \Gamma_{t'}^{**}$, $k_t^{**} \in \Gamma_t^{**}$ and $t \ge t' \ge t_0 \ge 0$.

Definition 3.3. The vector $v^{(t_0)}\left(t', k_{t'}^{**}\right) \equiv \left[v^{(t_0)C}\left(t', k_{t'}^{**}\right), v^{(t_0)G}\left(t', k_{t'}^{**}\right)\right]$ is a **valid imputation** for $t' \in (0,\tau_0)$ and $k_{t'}^{**} \in \Gamma_{t'}^{**}$ if it satisfies the following two requirements:

(1) It is a **Pareto-optimal** imputation vector;
(2) **Individual rationality** is fulfilled, i.e., $v^{(t_0)i}\left(t', k_{t'}^{**}\right) \ge J^i\left(t', k_{t'}^{**}\right)$ for $i \in \{C,G\}$.

In particular, Pareto optimality is automatically satisfied by the cooperative maximization problem given by equation (8).

We need additional notation:

$$\gamma^{(t_0)i}\left(t'; t', k_{t'}^{**}\right) \equiv \mathbb{E}_{t'}\left[\int_{t'}^{\tau_0} e^{-\rho(z-t')}\xi^i(z)dz|k(t') = k_{t'}^{**}\right] = v^{(t_0)i}(t', k_{t'}^{**})$$

and

$$\gamma^{(t_0)i}\left(t'; t, k_t^{**}\right) \equiv \mathbb{E}_t\left[\int_t^{\tau_0} e^{-\rho(z-t')}\xi^i(z)dz|k(t) = k_t^{**}\right]$$

for $i \in \{C, G\}$ and $t \geq t' \geq t_0 \geq 0$. Noting the following property:

$$\begin{aligned}\gamma^{(t_0)i}\left(t';t,k_t^{**}\right) &\equiv e^{-\rho(t-t')}\mathbb{E}_t\left[\int_t^{\tau_0} e^{-\rho(z-t)}\xi^i(z)\,dz|k(t)=k_t^{**}\right] \\ &= e^{-\rho(t-t')}\gamma^{(t_0)i}(t;t,k_t^{**})\end{aligned} \tag{10}$$

for $i \in \{C, G\}$ and $k_t^{**} \in \Gamma_t^{**}$, which yields the following definition.

Definition 3.4. A solution imputation is said to satisfy **subgame consistency** if it satisfies condition (10).

That is, subgame consistency requires that the extension of the solution policy to a situation with a later starting time and any feasible state brought about by prior optimal behaviors would remain optimal.

We now define two commonly used payoff distribution procedures (PDPs).

Definition 3.5. An allocation principle is called a **Nash bargaining solution/Shapley value** if at time t_0 the imputation assigned to player i is

$$v^{(t_0)i}(t_0,k_0) = J^i(t_0,k_0) + \frac{1}{2}\left[J(t_0,k_0) - \sum_{j\in\{C,G\}} J^j(t_0,k_0)\right]$$

for $i \in \{C, G\}$; and at time $t' \in (0, \tau_0)$, the imputation assigned to player i is

$$v^{(t_0)i}\left(t',k_{t'}^{**}\right) = J^i\left(t',k_{t'}^{**}\right) + \frac{1}{2}\left[J\left(t',k_{t'}^{**}\right) - \sum_{j\in\{C,G\}} J^j\left(t',k_{t'}^{**}\right)\right]$$

for $i \in \{C, G\}$ and $k_{t'}^{**} \in \Gamma_{t'}^{**}$.

It is easy to verify that the Nash bargaining solution and Shapley value coincide with each other when there are just two players in the game.

Definition 3.6. An allocation principle is considered to be a **proportional distribution** if at time t_0 the imputation assigned to player i is

$$v^{(t_0)i}(t_0,k_0) = \frac{J^i(t_0,k_0)}{\sum_{j\in\{C,G\}} J^j(t_0,k_0)} J(t_0,k_0)$$

for $i \in \{C, G\}$; and at time $t' \in (0, \tau_0)$, the imputation assigned to player i is

$$v^{(t_0)i}\left(t',k_{t'}^{**}\right) = \frac{J^i\left(t',k_{t'}^{**}\right)}{\sum_{j\in\{C,G\}} J^j\left(t',k_{t'}^{**}\right)} J\left(t',k_{t'}^{**}\right)$$

for $i \in \{C, G\}$ and $k_{t'}^{**} \in \Gamma_{t'}^{**}$.

We now characterize the cooperative equilibrium by establishing some desirable features.

Lemma 3.6. *Suppose the economy is under an inclusive regime and Assumption 3.1 holds. Then, we have the following conclusions:*

(i) *Under both the Nash bargaining solution and the Shapley value as well as proportional distribution allocation principles, the cooperative equilibrium satisfies group rationality, individual rationality, Pareto efficiency and subgame consistency, and neither the capitalist nor the government unilaterally deviates from cooperation.*

(ii) *If* $r \in (\widetilde{r}_{\min}, \widetilde{r}_{\max}]$ *with* $\widetilde{r}_{\min} \equiv \delta + \rho + \frac{1}{2}\sigma^2$ *and* $\widetilde{r}_{\max} \equiv \delta + 2\rho - \frac{1}{2}\sigma^2$, *then the optimal exit time is* $\tau_0^{**} = \inf\left\{t > 0 : k^{**}(t) = k^{**} \equiv \frac{h_1\varpi}{h_1-1}\right\}$, *in which*

$$h_1 = \frac{\sigma^2 - 2(r-\delta-\rho) + \sqrt{[2(r-\delta-\rho)-\sigma^2]^2 + 8\rho\sigma^2}}{2\sigma^2}. \tag{11}$$

Proof. *See the Appendix.* □

4. Sustaining investment: authoritarian versus inclusive regimes

4.1. Theoretical predictions

Let $\mathbb{E}(\tau_\Delta^*)$[17] and $\mathbb{E}(\tau_0^*)$ denote the expected exit times under the authoritarian regime for $\Delta > 0$ and $\Delta = 0$, respectively. Let $\mathbb{E}(\tau_0^{**})$ denote the expected exit time under the inclusive regime. The following theorem analyzes the equilibrium choice between the authoritarian regime and the inclusive regime by using the standard of inducing a later exit time and hence a longer expected investment horizon, providing the same entry time, $t_0 = 0$. This theorem carries the central message of our paper.

Theorem 4.1. (***Authoritarian versus Inclusive Regimes***). *If Assumption 3.1 holds, then we have the following conclusions.*

(i) *If* $\frac{\varpi}{k_0} > \left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta}$, *then* $\mathbb{E}(\tau_\Delta^*) > 0$; *if* $\frac{\varpi}{k_0} > \frac{\lambda_1-1}{\lambda_1}$, *then* $\mathbb{E}(\tau_0^*) > 0$; *and if* $\frac{\varpi}{k_0} > \frac{h_1-1}{h_1}$, *then* $\mathbb{E}(\tau_0^{**}) > 0$.

(ii) $\mathbb{E}(\tau_0^*) > \mathbb{E}(\tau_\Delta^*)$ *for any* $\Delta > 0$ *and* $\mathbb{E}(\tau_\Delta^*)$ *is strictly decreasing in* Δ.

(iii) *If we have*

$$\Delta < \Delta_1^* \equiv \left(\frac{1}{\mu}\right)\left[\frac{\mu - \frac{1}{2}\sigma^2}{\rho(1-\varepsilon)}\right]\ln\left(\frac{\frac{h_1}{h_1-1}}{\frac{\lambda_1}{\lambda_1-1}}\right) > 0,$$

then there exists a finite upper bound, denoted by

$$\Xi^* \equiv \left(\frac{h_1}{h_1-1}\right)^{\frac{\mu-\frac{1}{2}\sigma^2}{\rho(1-\varepsilon)}}\left(\frac{\lambda_1-1}{\lambda_1}\right)^{\frac{r-\delta-\rho-\frac{1}{2}\sigma^2}{\rho(1-\varepsilon)}} > 0,$$

of $\frac{\varpi}{k_0}$ *such that* $\mathbb{E}(\tau_0^{**}) > \mathbb{E}(\tau_\Delta^*)$ *for any* $\frac{\varpi}{k_0} \leq \Xi^*$.

(iv) *If* $\frac{\varpi}{k_0} > \Xi^*$, *then there exists a threshold, denoted by*

$$\Delta_2^* \equiv \frac{\ln\left(\frac{k^*}{k_0}\right) - \ln\left(\frac{k^{**}}{k_0}\right)\left(\frac{\mu-\frac{1}{2}\sigma^2}{r-\delta-\rho-\frac{1}{2}\sigma^2}\right)}{\mu} > 0,$$

of Δ *such that*

$$\mathbb{E}(\tau_\Delta^*)\begin{cases} > \mathbb{E}(\tau_0^{**}) & \text{for } \Delta < \Delta_2^*, \\ = \mathbb{E}(\tau_0^{**}) & \text{for } \Delta = \Delta_2^*, \\ < \mathbb{E}(\tau_0^{**}) & \text{for } \Delta > \Delta_2^*. \end{cases}$$

(v) *For the threshold* $\Xi^* > 0$ *defined above, we have*

$$\mathbb{E}(\tau_0^*)\begin{cases} > \mathbb{E}(\tau_0^{**}) & \text{for } \frac{\varpi}{k_0} > \Xi^*, \\ = \mathbb{E}(\tau_0^{**}) & \text{for } \frac{\varpi}{k_0} = \Xi^*, \\ < \mathbb{E}(\tau_0^{**}) & \text{for } \frac{\varpi}{k_0} < \Xi^*. \end{cases}$$

Proof. *See the Appendix.* □

Part (i) provides conditions that guarantee positive exit times under alternative regimes. We have identified the conditions under which a political regime incentivizes the capitalist to sustain investment for a longer time than the competing regime. In what follows, the authoritarian regime is said to dominate the inclusive regime if it induces a strictly later exit time than does the inclusive regime, and vice versa; the authoritarian and inclusive regimes are said to be indifferent if they induce identical expected exit times.

By using parameter values given in Table 1, Figs. 1–2 graphically illustrate Theorem 4.1. As shown in these two figures, the equilibrium expected investment horizons are linear functions of the size of

[17] For notational simplicity, here we assume the initial time to be $t_0 = 0$, and hence $\mathbb{E}_{t_0}$ is simply written as $\mathbb{E}$.

Table 1
Parameter values.

Parameter	Value	Description
r	0.086	Capital return rate
δ	0.025	Depreciation rate
ρ	0.03	Subjective discount factor
σ	0.025	Percentage volatility
ε	0.146	Fraction of benevolent politicians

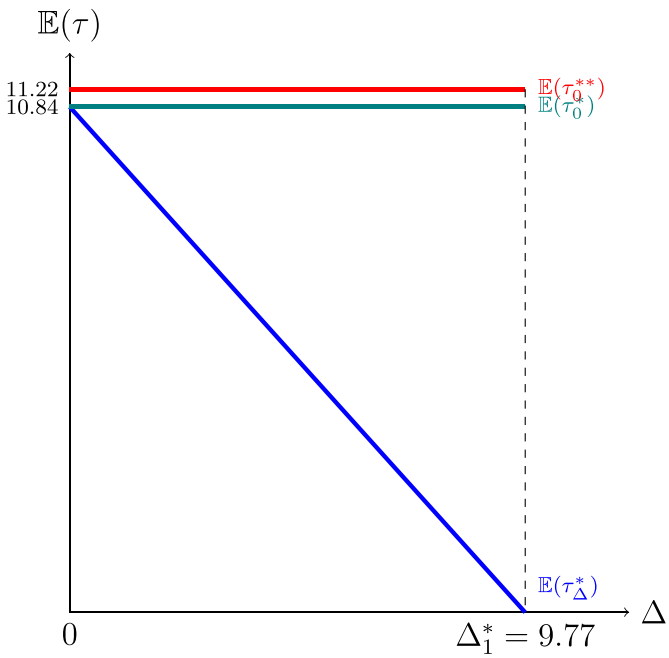

Fig. 1. Results (ii) and (iii) of Theorem 4.1: $\varpi/k_0 = 0.695 = \Xi^*$.

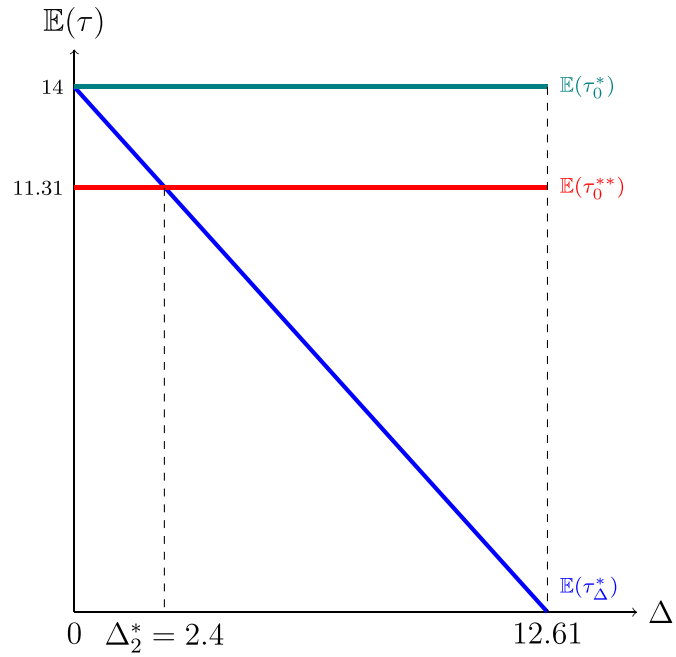

Fig. 2. Results (ii) and (iv) of Theorem 4.1: $\varpi/k_0 = 0.71 > 0.695 = \Xi^*$.

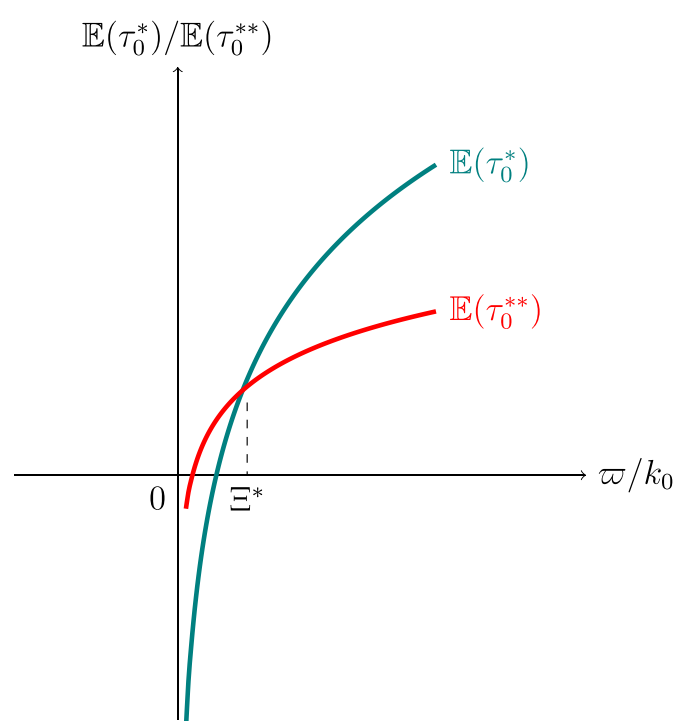

Fig. 3. Result (v) of Theorem 4.1.

the delayed information, Δ. Fig. 1 considers the case with ϖ/k_0 taking the threshold value $\Xi^* = 0.695$, while Fig. 2 considers the case with ϖ/k_0 taking a value greater than the threshold value. In both figures, $\mathbb{E}(\tau_0^*) > \mathbb{E}(\tau_\Delta^*)$ for all $\Delta > 0$, as desired in part (ii). In Fig. 1, $\mathbb{E}(\tau_0^{**}) > \mathbb{E}(\tau_\Delta^*)$ for any $\Delta < \Delta_1^*$, as desired in part (iii). Fig. 2 shows the critical value $\Delta_2^* = 2.4$ such that part (iv) follows. Fig. 3 considers the special case with $\Delta = 0$, in which there is no informational discrepancy between the two regimes. It demonstrates that the equilibrium expected investment horizons are strictly increasing and concave functions of the index of capital mobility. And there is a single crossing at threshold Ξ^*.

First, if the information delay is smaller than a critical value, then there is an upper bound of the exit cost such that the inclusive regime dominates the authoritarian regime within the bound. Second, if the exit cost is beyond the upper bound, then we can find another threshold of the information delay such that the authoritarian regime dominates the inclusive regime below the threshold, the authoritarian and inclusive regimes are equivalent at the threshold, and the inclusive regime dominates the authoritarian regime above the threshold. Third, for the special case in which the information delay under the authoritarian regime approaches zero, we find a threshold, which is exactly the upper bound of the exit cost identified above, such that the authoritarian regime dominates the inclusive regime above the threshold, the authoritarian and inclusive regimes are equivalent at the threshold, and the inclusive regime dominates the authoritarian regime below the threshold. Particularly, the greater the size of the delayed information under the authoritarian regime, the earlier the expected exit time will be, a discouragement effect originating from the delayed information availability. Loosely speaking, our result implies that, *ceteris paribus*, narrowing the information delay can increase the relative advantage of authority while lowering exit cost can increase the relative advantage of inclusivity. Theorem 4.1, accordingly, provides novel predictions about the connection between political regime and investment endurance.

Elaborating further, an inclusive regime does not always dominate an authoritarian one in the sense of inducing a longer investment horizon, because the comparison of equilibrium investment horizons calls for the *joint consideration* of the endogenous speed and goal of capital accumulation under the two political regimes. It follows from Lemmas 3.1 and 3.4 that the inclusive regime yields a lower capital income tax rate of the government while a higher savings rate of the capitalist than its counterparts under the authoritarian regime. As such, the capitalist under inclusive regime has a strictly higher speed of capital accumulation than he would have under authority. This fact implies that for any common initial stock of capital investment, the capitalist will actually choose a shorter investment horizon under the inclusive regime than that under the authoritarian regime if the threshold (or goal) of capital accumulation beyond which he will exit is the same under the two political regimes. Nevertheless, as shown in Lemmas 3.2, 3.3, and 3.6, the threshold of capital accumulation under the inclusive regime is strictly larger than that under the authoritarian one, which makes sense intuitively because the lower capital income tax rate provides an incentive for setting a higher goal of capital accumulation. Therefore, the capitalist has a higher speed of capital accumulation under an inclusive regime and also has a higher goal of capital accumulation than his counterparts under one of authority, which thus explains why the inclusive regime does not always produce a longer investment horizon than the authoritarian one. Immediately, we can provide the following claims: If the goal of capital accumulation under the inclusive regime is not quite higher than the goal under authority, then the authority might dominate the inclusive regime; if, however, the goal of capital accumulation under the inclusive regime is sufficiently higher than that under one of authority, then the inclusive regime will dominate the authority.

4.2. Comparative statics

For these expected exit times, $\mathbb{E}(\tau_\Delta^*)$, $\mathbb{E}(\tau_0^*)$ and $\mathbb{E}(\tau_0^{**})$, we perform comparative statics with respect to relevant parameters: the rate of capital return (r), the fraction of benevolent politicians (ε), and the subjective discount factor (ρ). We state them in the following three corollaries, which enable us to more comprehensively understand how the optimal expected exit time under a given political regime is shaped by the underlying economic environment.

Corollary 4.1. *With respect to the capital return rate we have the following comparative statics.*

(i) *Let* $\Upsilon_\Delta^* \equiv k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left[\left(\mu-\frac{1}{2}\sigma^2\right)\frac{1}{\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial r}+\frac{1}{2}\sigma^2\Delta\right]$, *then we have*

$$\frac{\partial\mathbb{E}(\tau_\Delta^*)}{\partial r}\begin{cases}>0 & \text{for } \varpi\in\left(k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta},\Upsilon_\Delta^*\right)\text{ and }\Delta<\frac{1}{\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial r},\\ =0 & \text{for } \varpi=\Upsilon_\Delta^*,\\ <0 & \text{for } \varpi>\max\left\{k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta},\Upsilon_\Delta^*\right\}.\end{cases}$$

(ii) *Let* $\Upsilon_0^* \equiv k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left[\left(\mu-\frac{1}{2}\sigma^2\right)\frac{1}{\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial r}\right]$, *then we have*

$$\frac{\partial\mathbb{E}(\tau_0^*)}{\partial r}\begin{cases}>0 & \text{for } \varpi\in\left(k_0\left(\frac{\lambda_1-1}{\lambda_1}\right),\Upsilon_0^*\right),\\ =0 & \text{for } \varpi=\Upsilon_0^*,\\ <0 & \text{for } \varpi>\Upsilon_0^*.\end{cases}$$

(iii) *Let* $\Upsilon_0^{**} \equiv k_0\left(\frac{h_1-1}{h_1}\right)\exp\left[\left(r-\delta-\rho-\frac{1}{2}\sigma^2\right)\frac{1}{h_1(1-h_1)}\frac{\partial h_1}{\partial r}\right]$, *then we have*

$$\frac{\partial\mathbb{E}(\tau_0^{**})}{\partial r}\begin{cases}>0 & \text{for } \varpi\in\left(k_0\left(\frac{h_1-1}{h_1}\right),\Upsilon_0^{**}\right),\\ =0 & \text{for } \varpi=\Upsilon_0^{**},\\ <0 & \text{for } \varpi>\Upsilon_0^{**}.\end{cases}$$

Proof. *See the Appendix.* □

Only when the exit cost is smaller than some threshold can we get that a higher capital return rate implies a longer investment horizon; otherwise, a higher capital return rate could lead to an even shorter investment horizon. The threshold varies between alternative political regimes as well as between alternative information structures under a given political regime.

To reduce capital flight in magnitude, Le and Zak (2006) find that, everything else being equal, capital flight is lower when investment return is high. We, in contrast, focus on the time dimension regarding capital flight and find that, *ceteris paribus*, only when the exit cost is small will a higher return rate result in a later occurrence of capital flight. In fact, the marginal effect of the return rate placed on the expected exit time is a strictly decreasing function of the exit cost. As such, *ceteris paribus*, exit cost dominates return rate as we can adjust the amount of exit cost to manipulate the effect of the capital return rate on the expected exit time (or expected investment horizon).

Corollary 4.2. *With respect to the fraction of benevolent politicians in the government we have the following comparative statics.*

(i) *Let* $\widetilde{\Upsilon}_\Delta^* \equiv k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left[\left(\mu-\frac{1}{2}\sigma^2\right)\frac{1}{\rho\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial\varepsilon}+\frac{1}{2}\sigma^2\Delta\right]$, *then we have*

$$\frac{\partial\mathbb{E}(\tau_\Delta^*)}{\partial\varepsilon}\begin{cases}>0 & \text{for } \varpi\in\left(k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta},\widetilde{\Upsilon}_\Delta^*\right)\text{ and }\Delta<\frac{1}{\rho\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial\varepsilon},\\ =0 & \text{for } \varpi=\widetilde{\Upsilon}_\Delta^*,\\ <0 & \text{for } \varpi>\max\left\{k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta},\widetilde{\Upsilon}_\Delta^*\right\}.\end{cases}$$

(ii) *Let* $\widetilde{\Upsilon}_0^* \equiv k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left[\left(\mu-\frac{1}{2}\sigma^2\right)\frac{1}{\rho\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial\varepsilon}\right]$, *then we have*

$$\frac{\partial\mathbb{E}(\tau_0^*)}{\partial\varepsilon}\begin{cases}>0 & \text{for } \varpi\in\left(k_0\left(\frac{\lambda_1-1}{\lambda_1}\right),\widetilde{\Upsilon}_0^*\right),\\ =0 & \text{for } \varpi=\widetilde{\Upsilon}_0^*,\\ <0 & \text{for } \varpi>\widetilde{\Upsilon}_0^*.\end{cases}$$

(iii) $\partial\mathbb{E}(\tau_0^{**})/\partial\varepsilon=0$.

Proof. *See the Appendix.* □

Under an authoritarian regime, only when the exit cost is smaller than some threshold does a larger fraction of benevolent politicians imply a longer investment horizon; otherwise, a larger fraction of benevolent politicians could lead to a shorter investment horizon. The threshold varies between alternative information structures. Under an inclusive regime, the expected investment horizon turns out to be independent of the political composition of the government.

Corollary 4.3. *With respect to the subjective discount factor we have the following comparative statics.*

(i) *Let* $\widehat{\Upsilon}_\Delta^* \equiv k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left[\left(\mu-\frac{1}{2}\sigma^2\right)\frac{1}{(2-\varepsilon)\lambda_1(\lambda_1-1)}\frac{\partial\lambda_1}{\partial\rho}+\frac{1}{2}\sigma^2\Delta\right]$, *then we have*

$$\frac{\partial \mathbb{E}(\tau_\Delta^*)}{\partial \rho}\begin{cases} > 0 & \text{for } \varpi > \max\left\{k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta}, \widehat{\Upsilon}_\Delta^*\right\}, \\ = 0 & \text{for } \varpi = \widehat{\Upsilon}_\Delta^*, \\ < 0 & \text{for } \varpi \in \left(k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta}, \widehat{\Upsilon}_\Delta^*\right) \text{ and } \Delta < \frac{1}{(2-\varepsilon)\lambda_1(\lambda_1-1)}\frac{\partial\lambda_1}{\partial\rho}. \end{cases}$$

(ii) *Let* $\widehat{\Upsilon}_0^* \equiv k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left[\left(\mu-\frac{1}{2}\sigma^2\right)\frac{1}{(2-\varepsilon)\lambda_1(\lambda_1-1)}\frac{\partial\lambda_1}{\partial\rho}\right]$, *then we have*

$$\frac{\partial \mathbb{E}(\tau_0^*)}{\partial \rho}\begin{cases} > 0 & \text{for } \varpi > \widehat{\Upsilon}_0^*, \\ = 0 & \text{for } \varpi = \widehat{\Upsilon}_0^*, \\ < 0 & \text{for } \varpi \in \left(k_0\left(\frac{\lambda_1-1}{\lambda_1}\right), \widehat{\Upsilon}_0^*\right). \end{cases}$$

(iii) *Let* $\widehat{\Upsilon}_0^{**} \equiv k_0\left(\frac{h_1-1}{h_1}\right)\exp\left[\left(r-\delta-\rho-\frac{1}{2}\sigma^2\right)\frac{1}{h_1(h_1-1)}\frac{\partial h_1}{\partial\rho}\right]$, *then we have*

$$\frac{\partial \mathbb{E}(\tau_0^{**})}{\partial \rho}\begin{cases} > 0 & \text{for } \varpi > \widehat{\Upsilon}_0^{**}, \\ = 0 & \text{for } \varpi = \widehat{\Upsilon}_0^{**}, \\ < 0 & \text{for } \varpi \in \left(k_0\left(\frac{h_1-1}{h_1}\right), \widehat{\Upsilon}_0^{**}\right). \end{cases}$$

Proof. *See the Appendix.* □

Only when the exit cost is smaller than some threshold does being more patient imply a longer investment horizon; otherwise, being more patient could result in a shorter investment horizon. The threshold varies between alternative political regimes as well as between alternative information structures under a given political regime.

4.3. Numerical illustration

We now carry out a numerical analysis of the model, which can help us understand Theorem 4.1. Although these exercises are very coarse and do not represent rigorous calibration, they indeed enable us to quantitatively observe how large the difference in the equilibrium expected investment horizon can be made due to the presence of alternative political regimes. Following the estimation of Poterba (1998), we set $r = 0.086$ which is the average pre-tax rate of return on capital for the 1990–1996 period. As commonly used in real-business-cycle models, we set $\delta = 0.025$, which corresponds to about 10% depreciation *per annum*, and the time-discount rate $\rho = 0.03$. Following Poterba (1998), we set the target $\tau_k = 0.42$, namely, the average tax rate during 1990–1996, and use the equilibrium capital income tax rate under an authoritarian regime to yield $\varepsilon = 0.146$. We summarize all parameter values in Table 1.

We next calculate the expected investment horizons for different values of information delay and the ratio of the exit cost to initial capital, which are reported in Tables 2–7 (and Figs. 4–9).

To guarantee $\mathbb{E}(\tau_\Delta^*) > 0$, it follows from part (i) of Theorem 4.1 that $\frac{\varpi}{k_0} > \left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta}$ must be satisfied. In fact, if this condition is satisfied, then $\mathbb{E}(\tau_0^*) > 0$ and $\mathbb{E}(\tau_0^{**}) > 0$. By using the parameter values in Table 1 and equation (6), we obtain $\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta} = e^{0.005\Delta}/1.5$. Also, we can have $e^{0.005\Delta}/1.5 \cong 0.67$ for all $\Delta \in [0,2]$, $e^{0.005\Delta}/1.5 \cong 0.68$ for $\Delta = 3$ or 5, $e^{0.005\Delta}/1.5 \cong 0.69$ for $\Delta = 7$, and $e^{0.005\Delta}/1.5 \cong 0.701$ for $\Delta = 10$. This is why we begin with $\varpi/k_0 = 0.68$, 0.69, 0.70 and 0.702 in Tables 2–7 These cases with different possible values of Δ are informative enough for quantitatively illustrating our theoretical predictions.

We thus obtain the following findings, which are consistent with Theorem 4.1. First, we always have $\mathbb{E}(\tau_\Delta^*) < \mathbb{E}(\tau_0^*)$, verifying the discouragement effect of delayed information availability on investment horizon. Second, for any given Δ, expected investment horizon increases as ϖ/k_0 increases, regardless of the political regime under consideration. Third, the expected investment horizon under authoritarian regime increases much faster with respect to the ratio ϖ/k_0 than that under inclusive regime. Fourth, there is a unique threshold of ϖ/k_0 such that $\mathbb{E}(\tau_0^{**}) > \mathbb{E}(\tau_0^*)$ below the threshold, while $\mathbb{E}(\tau_0^{**}) < \mathbb{E}(\tau_0^*)$ above the threshold. Fifth, there is another greater threshold of ϖ/k_0 such that $\mathbb{E}(\tau_0^{**}) > \mathbb{E}(\tau_\Delta^*)$ below the threshold, while $\mathbb{E}(\tau_0^{**}) < \mathbb{E}(\tau_\Delta^*)$ above the threshold. And sixth, in general, the larger the value of Δ, the higher the threshold of ϖ/k_0 required to obtain $\mathbb{E}(\tau_0^{**}) < \mathbb{E}(\tau_\Delta^*)$.

Consequently, we have two implications: (1) as the difference between the two thresholds (of ϖ/k_0) measures the equilibrium effect resulting from the informational difference between the two political regimes, the observation that the difference is non-decreasing in Δ means that a larger informational difference generally creates a greater equilibrium effect; (2) to make the authoritarian regime dominate the inclusive regime in inducing a longer expected investment horizon, a larger Δ must be accompanied by a larger ϖ/k_0, namely, a lower degree of government transparency under authority must be accompanied by a lower degree of capital mobility.

5. Further discussion

5.1. Revising the information structure under the two political regimes

Assumption 3.1 links an authoritarian government with opaqueness in delivering information to international investors while considering an inclusive government as being transparent. Nevertheless, it is worthwhile investigating how an inclusive but opaque government would shape the choice of investment horizon for the international investors, particularly given the very often prolonged democratic political procedures. In addition, since vested interest groups under authoritarian regime usually enjoy a wide range of advantages, including investment opportunities, there might be no information delay for the capitalist under this political regime. As such, in what follows, we shall compare the two political regimes in terms of sustaining capital investment under the new information structures.

Now, the optimal exit time established in Lemma 3.6 will be revised as follows.

Lemma 5.1. *Suppose the economy is under inclusive regime with the capitalist facing Δ-delayed information. Then, the results established in Lemma 3.6 still hold except for that the optimal exit time is now given by* $\tau_\Delta^{**} = \inf\left\{t > 0 : k^{**}(t) = \widetilde{k}^{**} \equiv \frac{h_1\varpi}{h_1-1}\cdot e^{-(r-\delta-\rho)\Delta}\right\}$, *for the same h_1 given by equation (11).*

Proof. *Omitted, as it is quite similar to that of* Lemma 3.6. □

Making use of Lemmas 3.1, 3.2, and 5.1, we obtain the following proposition.

Proposition 5.1. (***Authoritarian versus Inclusive Regimes***). *For the economic environment under the revised information structures, the following conclusions are true.*

(i) *For the case of an inclusive, opaque government and an authoritarian, transparent government, if* $\max\{(\lambda_1-1)/\lambda_1, [(h_1-1)/h_1]e^{(r-\delta-\rho)\Delta}\} < \varpi/k_0 < \Xi^*$,

Table 2
Expected investment horizons for $t_0 = 0$ and $\Delta = 0.5$

ϖ/k_0	0.68	0.69	0.70	0.705	0.706	0.707	0.708	0.709	0.71	0.72
$\mathbb{E}(\tau_\Delta^*)$	3.43	6.35	9.22	10.65	10.93	11.21	11.50	**11.78**	12.06	14.86
$\mathbb{E}(\tau_0^*)$	3.96	6.88	9.76	11.18	11.47	**11.75**	12.03	12.31	12.59	15.39
$\mathbb{E}(\tau_0^{**})$	10.25	10.74	11.22	11.45	11.50	**11.55**	11.59	**11.64**	11.69	12.15

Table 3
Expected investment horizons for $t_0 = 0$ and $\Delta = 1$.

ϖ/k_0	0.68	0.69	0.70	0.706	0.707	0.708	0.709	0.71	0.711	0.72
$\mathbb{E}(\tau_\Delta^*)$	3.03	5.94	8.82	10.53	10.81	11.10	11.38	11.66	**11.94**	14.46
$\mathbb{E}(\tau_0^*)$	3.96	6.88	9.76	11.47	**11.75**	12.03	12.31	12.59	12.88	15.39
$\mathbb{E}(\tau_0^{**})$	10.25	10.74	11.22	11.50	**11.55**	11.59	11.64	11.69	**11.74**	12.15

Table 4
Expected investment horizons for $t_0 = 0$ and $\Delta = 3$.

ϖ/k_0	0.69	0.70	0.707	0.71	0.711	0.712	0.713	0.717	0.718	0.72
$\mathbb{E}(\tau_\Delta^*)$	4.20	7.07	9.06	9.91	10.19	10.47	10.75	11.87	**12.15**	12.71
$\mathbb{E}(\tau_0^*)$	6.88	9.76	**11.75**	12.59	12.88	13.16	13.44	14.56	14.84	15.39
$\mathbb{E}(\tau_0^{**})$	10.74	11.22	**11.55**	11.69	11.74	11.78	11.83	12.02	**12.06**	12.15

Table 5
Expected investment horizons for $t_0 = 0$ and $\Delta = 5$.

ϖ/k_0	0.69	0.70	0.707	0.71	0.72	0.722	0.728	0.729	0.73	0.74
$\mathbb{E}(\tau_\Delta^*)$	1.47	4.35	6.34	7.19	9.99	10.54	12.20	12.47	**12.75**	15.47
$\mathbb{E}(\tau_0^*)$	6.88	9.76	**11.75**	12.59	15.39	15.95	17.60	17.88	18.15	20.87
$\mathbb{E}(\tau_0^{**})$	10.74	11.22	**11.55**	11.69	12.15	12.25	12.52	12.57	**12.61**	13.07

Table 6
Expected investment horizons for $t_0 = 0$ and $\Delta = 7$.

ϖ/k_0	0.70	0.707	0.71	0.72	0.73	0.735	0.736	0.739	0.74	0.75
$\mathbb{E}(\tau_\Delta^*)$	2.98	4.97	5.81	8.61	11.37	12.74	**13.01**	13.82	14.09	16.78
$\mathbb{E}(\tau_0^*)$	9.76	**11.75**	12.59	15.39	18.15	19.52	19.79	20.60	20.87	23.56
$\mathbb{E}(\tau_0^{**})$	11.22	**11.55**	11.69	12.15	12.61	12.84	**12.89**	13.02	13.07	13.52

Table 7
Expected investment horizons for $t_0 = 0$ and $\Delta = 10$.

ϖ/k_0	0.702	0.707	0.71	0.72	0.73	0.74	0.745	0.747	0.748	0.75
$\mathbb{E}(\tau_\Delta^*)$	0.77	2.19	3.04	5.83	8.59	11.31	12.66	13.20	**13.46**	14.00
$\mathbb{E}(\tau_0^*)$	10.33	**11.75**	12.59	15.39	18.15	20.87	22.22	22.76	23.02	23.56
$\mathbb{E}(\tau_0^{**})$	11.31	**11.55**	11.69	12.15	12.61	13.07	13.29	13.38	**13.43**	13.52

then we have $\mathbb{E}(\tau_0^*) < \mathbb{E}(\tau_\Delta^{**})$ *for* $\Delta < \min\{\Delta_3^*, \Delta_4^*\}$*; if, however,* $\varpi/k_0 > \max\{\Xi^*, [(h_1 - 1)/h_1]e^{(r-\delta-\rho)\Delta}\}$*, then we have* $\mathbb{E}(\tau_0^*) > \mathbb{E}(\tau_\Delta^{**})$*. Here,* Ξ^* *is given in Theorem 4.1 with*

$$\Delta_3^* \equiv \frac{\ln\left(\frac{k^{**}}{k_0}\right) - \ln\left(\frac{k^*}{k_0}\right)\left(\frac{r-\delta-\rho-\frac{1}{2}\sigma^2}{\mu-\frac{1}{2}\sigma^2}\right)}{r-\delta-\rho} > 0$$

and

$$\Delta_4^* \equiv \left(\frac{1}{r-\delta-\rho}\right)\left[\frac{r-\delta-\rho-\frac{1}{2}\sigma^2}{\rho(1-\varepsilon)}\right]\ln\left(\frac{\frac{h_1}{h_1-1}}{\frac{\lambda_1}{\lambda_1-1}}\right) > 0.$$

(ii) *Suppose the two political regimes feature the same degree of opaqueness, namely, the capitalist always faces* Δ*-delayed information. Then, if* $\Delta < \min\{\Delta_1^*, \Delta_4^*\}$ *and* $\max\{[(\lambda_1 - 1)/\lambda_1]e^{\mu\Delta}, [(h_1 - 1)/h_1]e^{(r-\delta-\rho)\Delta}\} < \varpi/k_0 \le \Xi^*$*, then we have* $\mathbb{E}(\tau_\Delta^*) < \mathbb{E}(\tau_\Delta^{**})$*. If, however,* $\varpi/k_0 > \max\{\Xi^*, [(\lambda_1 - 1)/\lambda_1]e^{\mu\Delta}, [(h_1 - 1)/h_1]e^{(r-\delta-\rho)\Delta}\}$*, then we have* $\mathbb{E}(\tau_\Delta^*) > \mathbb{E}(\tau_\Delta^{**})$ *for* $\Delta < \Delta_5^*$*, in which* Δ_1^* *is given in Theorem 4.1 with*

$$\Delta_5^* \equiv \frac{2\left(r-\delta-\rho-\frac{1}{2}\sigma^2\right)}{\rho(1-\varepsilon)\sigma^2}\left[\ln\left(\frac{k^*}{k_0}\right) - \ln\left(\frac{k^{**}}{k_0}\right)\left(\frac{\mu-\frac{1}{2}\sigma^2}{r-\delta-\rho-\frac{1}{2}\sigma^2}\right)\right] > 0.$$

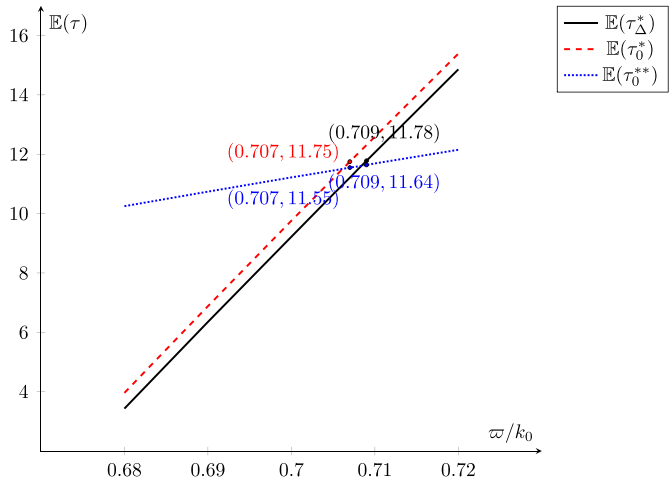

Fig. 4. Expected investment horizons for $t_0 = 0$ and $\Delta = 0.5$.

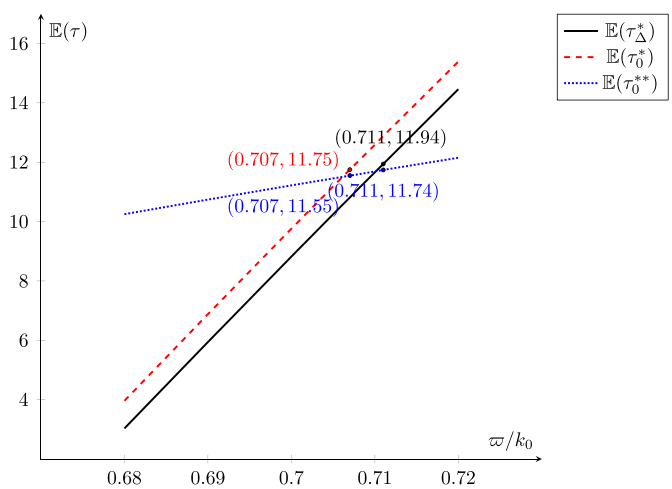

Fig. 5. Expected investment horizons for $t_0 = 0$ and $\Delta = 1$.

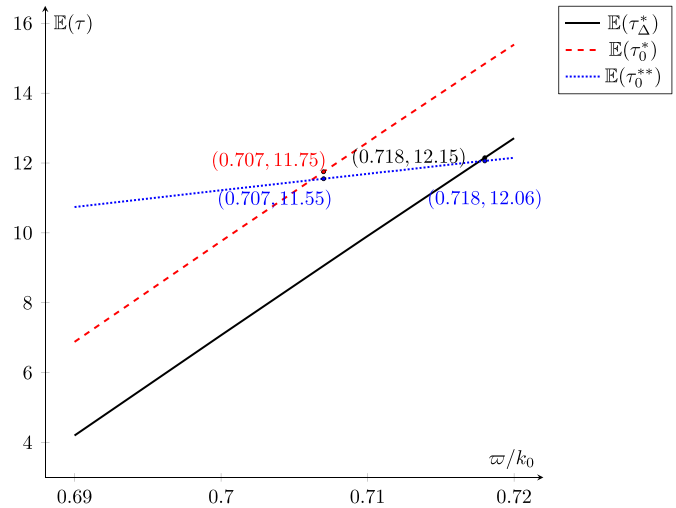

Fig. 6. Expected investment horizons for $t_0 = 0$ and $\Delta = 3$.

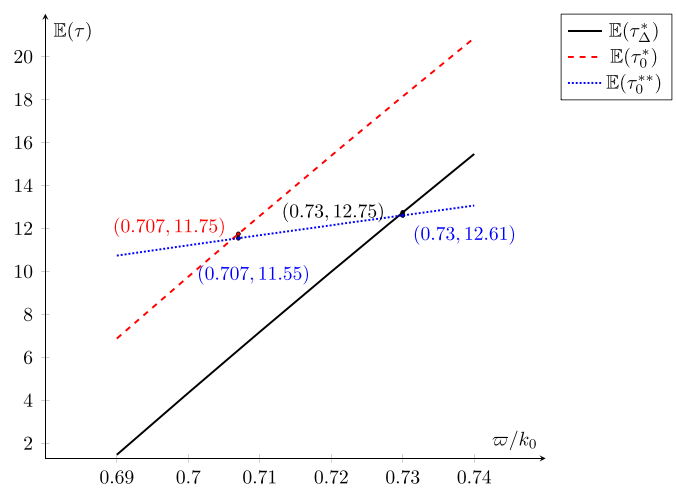

Fig. 7. Expected investment horizons for $t_0 = 0$ and $\Delta = 5$.

Proof. *See the Appendix.* □

Proposition 5.1 establishes, for any given initial level of capital investment, the following results regarding the comparison of the two political regimes in terms of sustaining capital investment. If the capitalist is not delayed in receiving information under the authoritarian regime while he is Δ-delayed under the inclusive regime, Proposition 5.1(i) provides two main results. First, if the exit cost (or capital mobility cost) and the size of information delay under the inclusive regime are relatively small, particularly, smaller than some thresholds, then the capitalist would choose a longer investment horizon when facing an inclusive, opaque government than that he would choose when facing an authoritarian, transparent government. Second, if the exit cost under both political regimes is large, particularly, larger than a certain threshold, then the authoritarian, transparent government dominates the inclusive, opaque government in sustaining capital investment. Proposition 5.1(ii) considers the case of controlling for the transparency difference between the two political regimes and provides another two results. The first is similar to the one obtained in Proposition 5.1(i); namely, the inclusive regime dominates the authoritarian one when both the exit cost and information delay are relatively small. The second result is that if exit cost is large, then the authoritarian regime dominates the inclusive regime when the common size of the information delay is small.

Elaborating further, these results imply that the key insights of Theorem 4.1 are still true. That is, all else remaining equal, lowering the exit cost can strengthen the relative advantage of the inclusive regime, which holds true even if the authoritarian government is more transparent than the inclusive government. The relative advantage of authority can be strengthened by increasing the exit cost, which is particularly true for the case of an authoritarian, transparent government and an inclusive, opaque government; in the case of no transparency (or opaqueness) difference between the two political regimes, this conclusion also requires that the degree of transparency is not too low (or, equivalently, that the degree of opaqueness is not too high).

5.2. *Time-varying taxation policy under an authoritarian regime*

In the model, it is assumed that τ_Δ belongs to the set of $\mathfrak{F}_{t-\Delta}$-adapted exit times under an authoritarian regime while both consumption $c(t)$ and the tax rate τ_k are $\mathfrak{F}_t$-adapted processes. We thus address the following question: can we establish some new implications when $c(t)$ is $\mathfrak{F}_{t-\Delta}$-adapted as well? For simplicity, we take τ_Δ as given and consider the following dynamic game:

Stage 1. The government determines $\tau_k(t)$, an $\mathfrak{F}_t$-adapted càdlàg process, by solving the following problem:

$$\max_{0\le\tau_k(t)\le 1} \mathbb{E}_{t_0}\left(\int_0^{\tau_\Delta} e^{-\rho(t_0+t)}\left\{\varepsilon\ln c(t)+(1-\varepsilon)\ln[\tau_k(t)(r-\delta)k(t)]\right\}dt\right)$$

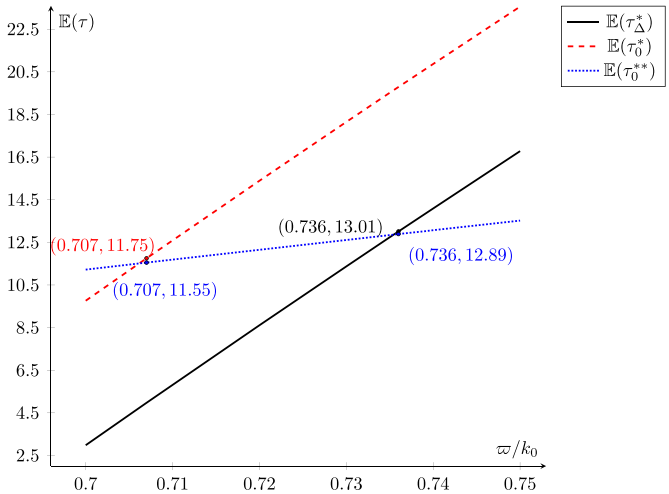

Fig. 8. Expected investment horizons for $t_0 = 0$ and $\Delta = 7$.

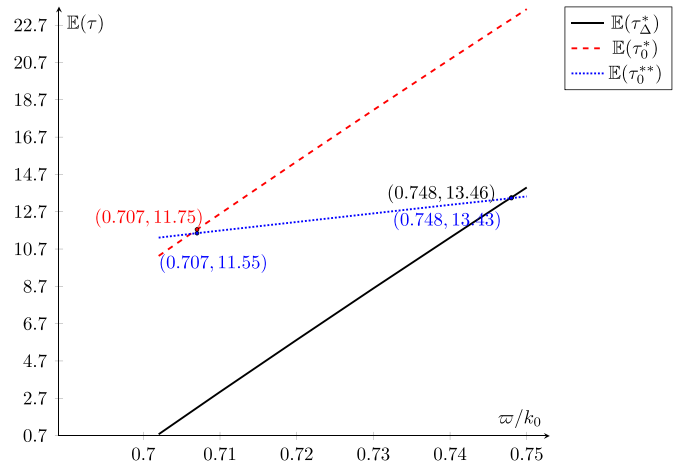

Fig. 9. Expected investment horizons for $t_0 = 0$ and $\Delta = 10$.

subject to $dk(t) = [(1 - \tau_k(t))(r - \delta)k(t) - c(t)]dt + \sigma k(t)dB(t)$.

Stage 2. The capitalist chooses $c(t)$, an $\mathfrak{F}_{t-\Delta}$-adapted càdlàg process, by solving the following problem:

$$\max_{c(t)>0} \mathbb{E}_{t_0}\left[\int_0^{\tau_\Delta} e^{-\rho(t_0+t)} \ln c(t)dt\right]$$

subject to $dk(t) = [(1 - \tau_k(t))(r - \delta)k(t) - c(t)]dt + \sigma k(t)dB(t)$. Without any loss of generality, we write consumption at time t as $c(t) \equiv r_c(t)k(t)$, with $r_c(t)$ denoting the fraction of capital allocated to consumption.

We state the equilibrium in the following lemma.

Lemma 5.2. *For the case of a time-varying tax rate under an authoritarian regime, the subgame perfect equilibrium at time t is given by*

$$\left\{r_c^{***}(t), \tau_k^{***}(t)\right\} = \left\{\frac{\rho}{1 - e^{\rho(t-\tau_\Delta)}}, \frac{\rho(1-\varepsilon)}{(r-\delta)[1 - e^{\rho(t-\tau_\Delta)}]}\right\}.$$

Proof. *See the Appendix.* □

In consequence, both the equilibrium consumption rate and equilibrium tax rate become time-varying processes. Immediately, we can give:

Corollary 5.1. *For the case of a time-varying tax rate under an authoritarian regime, the following results are true.*

(i) $r_c^{***}(t) > \rho$ *and* $\tau_k^{***}(t) > \tau_k^*$ *for any* $t \in (0, \min\left\{\tau_\Delta, \tau_\Delta^*\right\})$.

(ii) $\frac{\partial r_c^{***}(t)}{\partial t} > 0$ *and* $\frac{\partial r_c^{***}(t)}{\partial \tau_\Delta} < 0$.

(iii) $\frac{\partial \tau_k^{***}(t)}{\partial t} > 0$ *and* $\frac{\partial \tau_k^{***}(t)}{\partial \tau_\Delta} < 0$.

Proof. *Straightforward and hence omitted.* □

Compared to the model, here, the capitalist consumes more on the margin and the government taxes more on the margin. In a nutshell, the relative information advantage motivates the government to grab more and the relative information disadvantage weakens the capitalist's savings motive. The later the exit time, the smaller both the equilibrium consumption rate and tax rate are. Intuitively, delayed information availability combined with a small τ_Δ induces shortsighted behavior, resulting in higher equilibrium consumption rate and tax rate, which is strengthened by the fact that both $r_c^{***}(t)$ and $\tau_k^{***}(t)$ are strictly increasing as time t approaches the exit time τ_Δ. Therefore, sustainable investment can, in turn, promote the development of the spirit of capitalism *à la* Max Weber.

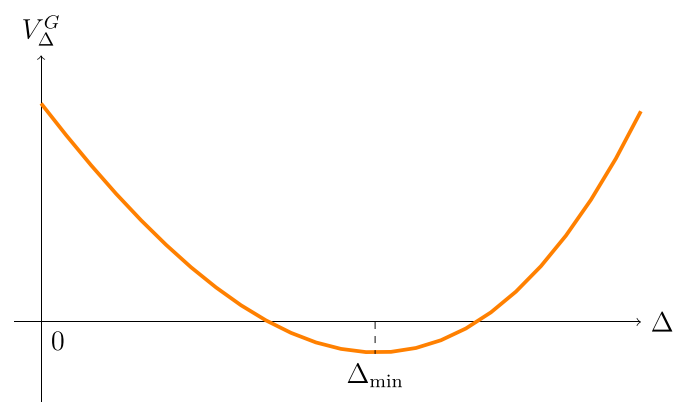

Fig. 10. U-shaped relationship between V_Δ^G and Δ

5.3. The choice of information delay and exit cost under an authoritarian regime

An authoritarian government is expected to have the power and motive to (partially) determine both the information delay and the exit cost. As we can use Δ to measure the government's hidden information, a logical question thus arises: what is the optimal choice of Δ for the government? To answer this question, we analyze how Δ affects the government's welfare during $(0, \mathbb{E}(\tau_\Delta^*))$:

$$V_\Delta^G \equiv \mathbb{E}_{t_0}\left[\int_0^{\mathbb{E}(\tau_\Delta^*)} V^G(t, k(t))dt\right],$$

where $V^G(t, k(t)) = e^{-\rho(t_0+t)}\left[\frac{1}{\rho}\ln k(t) + C_4\right]$ with C_4 and $k(t)$ given by (19) and (25), respectively.

Indeed, we obtain the following lemma.

Lemma 5.3. *For the current economic environment, the relationship between* V_Δ^G *and* Δ *can be characterized as follows.*

(i) *Suppose* $\varpi > \left(\frac{\lambda_1 - 1}{\lambda_1}\right)e^{-\rho C_4}$, *then there exists a unique critical value, written as* $\Delta_{\min} = \frac{1}{\mu}\ln\left(\frac{\lambda_1 \varpi}{\lambda_1 - 1}\right) + \frac{\rho}{\mu}C_4 > 0$, *of* Δ *such that*

$$\frac{\partial V_\Delta^G}{\partial \Delta}\begin{cases} > 0 & \text{for } \Delta > \Delta_{\min}, \\ = 0 & \text{for } \Delta = \Delta_{\min}, \\ < 0 & \text{for } \Delta < \Delta_{\min}. \end{cases}$$

(ii) *For* $\forall\Delta > 0$, *suppose* $k_0 < e^{-\rho C_4}$, *then*

$$\frac{\partial V_{\Delta}^{G}}{\partial \Delta}\begin{cases} > 0 & \text{for } \varpi \in \left(k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta}, \left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left(\mu\Delta-\rho C_4\right)\right), \\ = 0 & \text{for } \varpi = \left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left(\mu\Delta-\rho C_4\right), \\ < 0 & \text{for } \varpi > \left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left(\mu\Delta-\rho C_4\right). \end{cases}$$

Proof. *See the Appendix.* □

The U-shaped relationship implies that only when the information advantage is already larger than a certain threshold can the government benefit from further increasing its hidden information; otherwise, doing so hurts the government (see also Fig. 10). This argument relies on a lower bound being imposed on the exit cost. In fact, we can find another threshold of the exit cost such that the government benefits from further increasing its hidden information, but only when the exit cost is lower than the threshold; otherwise, doing so turns out to be harmful to the government.

If we interpret the exit cost as a kind of transaction cost determined by the government, then the immediate question is: what is the optimal choice of the exit cost for the government? Answering this question produces the following lemma.

Lemma 5.4. *The relationship between* V_{Δ}^{G} *and* ϖ *can be characterized as follows. Suppose* $k_0 < e^{-\rho C_4}$, *then there exists a unique critical value, written as* $\varpi_{\min} = \left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left(\mu\Delta-\rho C_4\right)$, *of* ϖ *such that*

$$\frac{\partial V_{\Delta}^{G}}{\partial \varpi}\begin{cases} > 0 & \text{for } \varpi > \varpi_{\min}, \\ = 0 & \text{for } \varpi = \varpi_{\min}, \\ < 0 & \text{for } \varpi \in \left(k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta}, \varpi_{\min}\right). \end{cases}$$

Proof. *See the Appendix.* □

The U-shaped relationship implies that only when the exit cost is already larger than a certain threshold can the government benefit from further increasing the difficulty of cross-border capital mobility; otherwise, doing so hurts the government. Even so, we argue that Δ and ϖ cannot both be set arbitrarily high. There generally exists an upper bound for each parameter, and these bounds are also endogenously determined. A possible approach is to introduce a shadow (or an underground) economy into the current framework and allow the capitalist to choose freely between investing in the formal economy and investing in the shadow economy. As documented by Schneider and Enste (2000), the shadow economy is not subject to government regulation and hence taxes are avoided. In addition, the corresponding returns and risks may differ. In consequence, these bounds can be set so that the capitalist is indifferent between the two options, namely, the shadow economy and the formal economy generate the same expected utility and the same amount of terminal capital stock.

6. Conclusion

We develop an analytical framework to comparatively study an authoritarian regime and an inclusive regime in providing incentives for longer investments. In terms of determining a capital income tax rate, authoritarian and inclusive regimes represent two types of political regimes connecting governments and international investors. We identify sharp conditions that enable us to predict when authority dominates inclusivity, when inclusivity dominates authority, and when they might be indifferent, in the sense that international capitalists are willing to invest in the economy for a longer time.

Controlling for the capital return rate and assuming that the inclusive government is more transparent than the authoritarian government, our results imply that for any given initial level of capital investment, the relative advantage of the inclusive regime can be strengthened by lowering the exit cost (or allowing for a higher degree of capital mobility), while the relative advantage of the authoritarian regime can be strengthened by increasing the degree of government transparency. Regardless of whether the inclusive government is more or less transparent than the authoritarian government, it is likely the case that authority dominates inclusivity when the exit cost under the two political regimes is sufficiently high.

Another important finding is that simply cutting the tax rate, namely, offering favorable tax policy support, is not sufficient to incentivize capitalists to invest in the host countries for a longer time. In fact, the inclusive regime is shown to induce a lower equilibrium tax rate than the authoritarian regime, which makes sense intuitively according to our definitions of the two political regimes, whereas authority can still dominate inclusivity in sustaining the international investment. The implication is that in addition to fiscal policy such as capital taxation, institutional factors, such as the degree of capital mobility and government transparency, are also relevant in shaping investors' choice of investment horizon.

As a final remark, our results seem to justify the following order of institutional change for open economies. Before liberalizing capital accounts, governments should first establish an inclusive (or business-friendly) political regime with strengthened transparency. This insight would be of practical relevance for emerging economies like China that face great external uncertainty, especially against the background of COVID-19, as recently highlighted by Huang et al. (2020) and Zhang (2020), in addressing some fundamental institutional obstacles that are harmful to China's sustainable economic development.

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Declaration of competing interest

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Appendix. Proofs

A. Proof of Lemma 3.1

We will complete it in two steps.

Step 1: Solving the problem in Stage 3 yields the following:

Claim 6.1. *The capitalist sets the consumption at time t to be* $c^*(t) = \rho k(t)$.

Proof. We apply the backward induction principle and assume that the capitalist takes the exit time as fixed, and also assumes it to be finite, in general. We will check later to verify that it is indeed finite, almost surely, at equilibrium. As such, the intertemporal utility-maximization problem can be solved using standard dynamic programming. We now prove that there is a continuously differentiable function $V^C(t, k(t))$ satisfying the Bellman-Isaacs-Fleming partial differential equation as follows:

$$\begin{aligned} &-V_t^C(t,k(t)) - \frac{1}{2}\sigma^2 k^2(t) V_{kk}^C(t,k(t)) \\ &= \max_{c(t)>0} \left\{ e^{-\rho(t_0+t)} \ln c(t) + V_k^C(t,k(t))[(1-\tau_k)(r-\delta)k(t) - c(t)] \right\}. \end{aligned} \tag{12}$$

Performing the maximization operator yields

$$\frac{1}{c(t)} = e^{\rho(t_0+t)} V_k^C(t,k(t)). \tag{13}$$

We follow the guess-and-verify approach and put that

$$V^C(t,k(t)) = e^{-\rho(t_0+t)}[C_1 \ln k(t) + C_2], \tag{14}$$

in which constants C_1 *and* C_2 *are to be determined. Applying* (13) *and* (14) *to* (12) *and rearranging the algebra result in* $C_1 = \frac{1}{\rho}$ *and*

$$C_2 = -\frac{\sigma^2}{2\rho^2} + \frac{1}{\rho}\ln\rho + \frac{1}{\rho^2}(1-\tau_k)(r-\delta) - \frac{1}{\rho}. \tag{15}$$

Finally, since by problem (3) we have ignored terminal utility in the current model, there is no need to consider the terminal boundary constraint(see, e.g., Yeung and Petrosyan, 2006). However, it is worthwhile emphasizing that this specification does not necessarily mean that the capitalist has zero terminal utility. That is, it does not imply that $V^C(\tau, k(\tau)) = 0$. □

Step 2: Solving the problem in Stage 2 yields:

Claim 6.2. *The government sets the tax rate to* $\tau_k^* = \frac{\rho(1-\varepsilon)}{r-\delta}$.

Proof. *As before, the Bellman equation reads as follows*:

$$\begin{aligned} &-V_t^G(t,k(t)) - \frac{1}{2}\sigma^2 k^2(t) V_{kk}^G(t,k(t)) = \max_{0\le\tau_k\le 1} \left\{ e^{-\rho(t_0+t)}\varepsilon \ln[\rho k(t)] \right. \\ &\left. + e^{-\rho(t_0+t)}(1-\varepsilon)\ln[\tau_k(r-\delta)k(t)] + V_k^G(t,k(t))k(t)[(1-\tau_k)(r-\delta) - \rho] \right\}. \end{aligned} \tag{16}$$

Performing the maximization operator gives rise to

$$e^{-\rho(t_0+t)}(1-\varepsilon) = V_k^C(t,k(t))k(t)(r-\delta)\tau_k. \tag{17}$$

If we try

$$V^G(t,k(t)) = e^{-\rho(t_0+t)}[C_3 \ln k(t) + C_4] \tag{18}$$

for constants C_3 and C_4 which are to be determined, then applying (17) and (18) to (16) produces $C_3 = \frac{1}{\rho}$ and

$$C_4 = -\frac{\sigma^2}{2\rho^2} + \frac{1}{\rho}\ln\rho + \frac{1}{\rho^2}(r-\delta-\rho) + \frac{1-\varepsilon}{\rho}\ln\left(\frac{1-\varepsilon}{e}\right). \tag{19}$$

Then, by making use of $C_3 = \frac{1}{\rho}$, (17) *and* (18), *we obtain* $\tau_k^* = \rho(1-\varepsilon)/(r-\delta)$, *as desired.* □

Therefore, we obtain the subgame perfect equilibrium outcome by combining these results. **QED**

B. Proof of Lemma 3.2

We will complete it in nine steps.

Step 1: To solve the problem in Stage 1, we first put $Z(t) \equiv (t_0 + t, k(t))'$ for $t \ge 0$. Then, it follows from (1) and Claims 6.1 and 6.2 that

$$dZ(t) = \begin{bmatrix} 1 \\ \left(\underbrace{r-\delta-\rho(2-\varepsilon)}_{\equiv\mu>0}\right)k(t) \end{bmatrix} dt + \begin{bmatrix} 0 \\ \sigma k(t) \end{bmatrix} dB(t), \quad Z(0) = \begin{bmatrix} t_0 \\ k_0 \end{bmatrix} \tag{20}$$

and the corresponding differential generator is

$$\mathcal{A}\phi(t_0,k_0)=\frac{\partial\phi}{\partial t_0}+\mu k_0\frac{\partial\phi}{\partial k_0}+\frac{1}{2}\sigma^2k_0^2\frac{\partial^2\phi}{\partial k_0^2},\quad \forall\phi\in C^2(\mathbb{R}^2). \tag{21}$$

If we try a function ϕ of the form $\phi(t_0,k_0)=e^{-\rho t_0}k_0^{\lambda}$ for some constant $\lambda\in\mathbb{R}$, we can then get $\mathcal{A}\phi(t_0,k_0)=e^{-\rho t_0}k_0^{\lambda}\left[-\rho+\mu\lambda+\frac{1}{2}\sigma^2\lambda(\lambda-1)\right]$. By solving equation $\sigma^2\lambda^2+(2\mu-\sigma^2)\lambda-2\rho=0$, we get the unique positive root as follows:

$$\lambda_1=\frac{\sigma^2-2\mu+\sqrt{(2\mu-\sigma^2)^2+8\rho\sigma^2}}{2\sigma^2}. \tag{22}$$

If we let $\lambda_1>1$, then we should rely on the additional assumption that

$$\rho>\mu. \tag{23}$$

Step 2: In what follows, we will suppose that condition (23) always holds true. With this value of λ_1 we put

$$\phi(t_0,k_0)=\begin{cases}e^{-\rho t_0}\widetilde{C}k_0^{\lambda_1} & \text{if } (t_0,k_0)\in D\\ \psi(t_0,k_0) & \text{if } (t_0,k_0)\notin D\end{cases} \tag{24}$$

for some constant $\widetilde{C}$, function $\psi(t_0,k_0)$ and continuation region D, remaining to be determined.

To find a reasonable guess for the continuation region D, we first obtain by using the Itô formula that

$$k(t)=k_0\exp\left[\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma B(t)\right], \tag{25}$$

which implies that

$$\mathbb{E}^{k_0}[k(\Delta)]=k_0\exp(\mu\Delta) \tag{26}$$

for $\Delta>0$. Hence, we can rewrite the objective function as

$$\begin{aligned}\psi(t_0,k_0)&\equiv\mathbb{E}^{t_0,k_0}\left[e^{-\rho(t_0+\Delta)}(k(\Delta)-\varpi)\right]\\&=e^{-\rho(t_0+\Delta)}\left\{\mathbb{E}^{k_0}[k(\Delta)]-\varpi\right\}\\&=e^{-\rho(t_0+\Delta)}\left(k_0e^{\mu\Delta}-\varpi\right)\\&=e^{-\rho t_0}\exp((\mu-\rho)\Delta)\left(k_0-\varpi e^{-\mu\Delta}\right)\\&\equiv e^{-\rho t_0}\Sigma\left(k_0-\widetilde{\varpi}\right),\end{aligned} \tag{27}$$

where we have used (26) and defined

$$\Sigma\equiv\exp((\mu-\rho)\Delta),\quad \widetilde{\varpi}\equiv\varpi e^{-\mu\Delta}. \tag{28}$$

Then, applying (21) to (27) yields $\mathcal{A}\psi(t_0,k_0)=e^{-\rho t_0}\Sigma[(\mu-\rho)k_0+\rho\widetilde{\varpi}]$. Therefore, we have

$$\begin{aligned}U&\equiv\{(t_0,k_0):\mathcal{A}\psi(t_0,k_0)>0\}\\&=\left\{(t_0,k_0):(\mu-\rho)k_0+\rho\widetilde{\varpi}>0\right\}\\&=\left\{(t_0,k_0):k_0<\frac{\rho\widetilde{\varpi}}{\rho-\mu}\right\},\end{aligned}$$

where we have used assumption (23).

Step 3: We now determine the associated continuation region denoted by D. First, note that for $\forall t'$,

$$\begin{aligned}\psi^*\left(t_0-t',k_0\right)&=\sup_{\tau}\mathbb{E}_{(t_0-t')}\left[e^{-\rho\tau}\Sigma\left(k(\tau)-\widetilde{\varpi}\right)\right]\\&=\sup_{\tau}\mathbb{E}\left[e^{-\rho(\tau+(t_0-t'))}\Sigma\left(k(\tau)-\widetilde{\varpi}\right)\right]\\&=e^{\rho t'}\sup_{\tau}\mathbb{E}\left[e^{-\rho(\tau+t_0)}\Sigma\left(k(\tau)-\widetilde{\varpi}\right)\right]\\&=e^{\rho t'}\sup_{\tau}\mathbb{E}_{t_0}\left[e^{-\rho\tau}\Sigma\left(k(\tau)-\widetilde{\varpi}\right)\right]=e^{\rho t'}\psi^*\left(t_0,k_0\right).\end{aligned}$$

Then, we obtain

$$\begin{aligned}D+\left(t',0\right)&=\left\{\left(t+t',k_0\right):(t,k_0)\in D\right\}\\&=\left\{(t_0,k_0):\left(t_0-t',k_0\right)\in D\right\}\\&=\left\{(t_0,k_0):\psi\left(t_0-t',k_0\right)<\psi^*\left(t_0-t',k_0\right)\right\}\\&=\left\{(t_0,k_0):e^{\rho t'}\psi\left(t_0,k_0\right)<e^{\rho t'}\psi^*\left(t_0,k_0\right)\right\}\\&=\left\{(t_0,k_0):\psi\left(t_0,k_0\right)<\psi^*\left(t_0,k_0\right)\right\}=D,\end{aligned}$$

which yields that the continuation region D is invariant with respect to t in the sense that $D + (t', 0) = D$ for $\forall t'$. In consequence, the connected component of D that contains U must have the form

$$D = \left\{ (t_0, k_0) : 0 < k_0 < \widetilde{k}^* \right\} \tag{29}$$

for some $\widetilde{k}^*$ such that $U \subseteq D$, i.e.,

$$\widetilde{k}^* \geq \frac{\rho\widetilde{\varpi}}{\rho - \mu}. \tag{30}$$

Step 4: Indeed, we can even argue that D cannot have any other components, and we prove this claim by means of contradiction. Suppose that U' is another component of D and it is disjointed from U, then we should have $\mathcal{A}\psi < 0$ in U' and so, if $Z(0) \in U'$, it follows from the Dynkin's Formula that

$$\mathbb{E}_{Z(0)}\left[\psi\left(Z(\tau)\right)\right] = \psi\left(Z(0)\right) + \mathbb{E}_{Z(0)}\left[\int_0^\tau \mathcal{A}\psi\left(Z(t)\right)dt\right] < \psi\left(Z(0)\right)$$

for all exit times τ, bounded by the exit time from a k-bounded strip in U'. According to this, we can then apply the Existence Theorem for Optimal Stopping (see, Øksendal, 2003) to conclude that $\psi^*\left(Z(0)\right) = \psi\left(Z(0)\right)$, which leads to $U' = \emptyset$, an empty set.

Hence, in view of (24), (27), and (29) we now put

$$\phi(t_0, k_0) = \begin{cases} e^{-\rho t_0}\widetilde{C}k_0^{\lambda_1} & \text{if } 0 < k_0 < \widetilde{k}^* \\ e^{-\rho t_0}\Sigma\left(k_0 - \widetilde{\varpi}\right) & \text{if } \widetilde{k}^* \leq k_0 \end{cases} \tag{31}$$

for some constant $\widetilde{C} > 0$, to be determined. We guess, without any loss of generality, that the value function is C^1 at $k_0 = \widetilde{k}^*$, which gives the following "high-contact" (or smooth-fit) conditions: $\widetilde{C}(\widetilde{k}^*)^{\lambda_1} = \Sigma\left(\widetilde{k}^* - \widetilde{\varpi}\right)$ (continuity at $k_0 = \widetilde{k}^*$) and $\widetilde{C}\lambda_1(\widetilde{k}^*)^{\lambda_1 - 1} = \Sigma$ (differentiability at $k_0 = \widetilde{k}^*$). It is easy to obtain the solutions:

$$\widetilde{k}^* = \frac{\lambda_1\widetilde{\varpi}}{\lambda_1 - 1}, \quad \widetilde{C} = \frac{\Sigma}{\lambda_1}(\widetilde{k}^*)^{1-\lambda_1}. \tag{32}$$

Step 5: It remains to be verified that with these values of $\widetilde{k}^*$ and $\widetilde{C}$ the function ϕ given by (31) satisfies all of the conditions (i)-(xi) of Theorem 3.2 (Integro-variational inequalities for optimal stopping, pp.53–54) of Øksendal and Sulem (2009). To this end, first note that (i) and (ix) hold by construction of ϕ. Moreover, $\phi = \psi$ outside of D. Accordingly, to verify (ii), we only need to prove that $\phi \geq \psi$ on D, i.e., that

$$\widetilde{C}k_0^{\lambda_1} \geq \Sigma\left(k_0 - \widetilde{\varpi}\right) \quad \text{for } 0 < k_0 < \widetilde{k}^*. \tag{33}$$

Define the difference by $\zeta(k_0) \equiv \widetilde{C}k_0^{\lambda_1} - \Sigma\left(k_0 - \widetilde{\varpi}\right)$. By our chosen values of $\widetilde{C}$ and $\widetilde{k}^*$ in (32), we have $\zeta(\widetilde{k}^*) = \zeta'(\widetilde{k}^*) = 0$. Additionally, due to $\lambda_1 > 1$ by (22)-(23), $\zeta''(k_0) = \widetilde{C}\lambda_1(\lambda_1 - 1)k_0^{\lambda_1 - 2} > 0$ for $0 < k_0 < \widetilde{k}^*$. Consequently, $\zeta(k_0) > 0$ for $0 < k_0 < \widetilde{k}^*$ and (33) holds true, and hence (ii) is verified.

Step 6: For (iii), note that the boundary of set D is given by $\partial D = \left\{(t_0, k_0) : k_0 = \widetilde{k}^*\right\}$, we hence have

$$\mathbb{E}^{Z(0)}\left[\int_0^\infty \mathbb{I}_{\partial D}(Z(t))dt\right] = \int_0^\infty P^{k_0}\left[k(t) = \widetilde{k}^*\right]dt = 0,$$

where $\mathbb{I}_{\partial D}(\cdot)$ denotes an indicator function. Also, by our constructions of D and ϕ, it is trivial to see that ∂D is a Lipschitz surface and $\phi \in C^2(\mathbb{R} \times (0, \infty) \setminus \partial D)$ has locally bounded derivatives near ∂D, namely (iv) and (v) always hold true. In addition, it is straightforward to verify that (vii) holds based on our construction of ϕ.

Step 7: For (vi), namely, $\mathcal{A}\phi \leq 0$ on $\mathbb{R} \times (0, \infty) \setminus \partial D$, we know that outside D, we have $\phi(t_0, k_0) = e^{-\rho t_0}\Sigma\left(k_0 - \widetilde{\varpi}\right)$ and therefore $\mathcal{A}\phi = e^{-\rho t_0}\Sigma[(\mu - \rho)k_0 + \rho\widetilde{\varpi}]$, which combined with (23) reveals that $(\mu - \rho)k_0 + \rho\widetilde{\varpi} \leq 0$ for all $k_0 \geq \widetilde{k}^*$ is equivalent to $\widetilde{k}^* \geq \frac{\rho\widetilde{\varpi}}{\rho - \mu}$. This is completely consistent with requirement (30). Hence, combining it with (32) leads us to

$$\frac{\lambda_1\widetilde{\varpi}}{\lambda_1 - 1} \geq \frac{\rho\widetilde{\varpi}}{\rho - \mu} \iff \lambda_1 \leq \frac{\rho}{\mu}. \tag{34}$$

Step 8: To check if (x) holds true, i.e., $\tau_\Delta^* = \tau_D \equiv \inf\{t > 0 : k(t) \notin D\} < \infty$ a.s., we consider the solution of $k(t)$ given by equation (25). By applying the law of iterated logarithm for Brownian motion we conclude that if $\mu > \frac{1}{2}\sigma^2$, then $\lim_{t\to\infty}k(t) = \infty$ a.s., and, in particular, that $\tau_\Delta^* = \tau_D < \infty$ almost surely. Here, for (viii) to hold, it suffices that (xi) holds true. In what follows, we provide conditions under which (xi) holds. Since by applying the Heine-Borel theorem and the Weierstrass theorem, we know that ϕ is bounded on compact set $[0, \widetilde{k}^*]$, it suffices to verify that $\{e^{-\rho\tau_\Delta}k(\tau_\Delta)\}_{\tau_\Delta \in \mathcal{T}_\Delta}$ is uniformly integrable. For this to be true, it suffices that there exists a constant $W > 0$ such that

$$\mathbb{E}\left[e^{-2\rho\tau_\Delta}k^2(\tau_\Delta)\right] \leq W \quad \text{for } \forall \tau_\Delta \in \mathcal{T}_\Delta. \tag{35}$$

Since we have from (25) that $\mathbb{E}\left[e^{-2\rho\tau_\Delta}k^2(\tau_\Delta)\right] = k_0^2\mathbb{E}\left(\exp\left\{\left[2(\mu - \rho) + \sigma^2\right]\tau_\Delta\right\}\right)$, we can conclude that if

$$2(\mu - \rho) + \sigma^2 \leq 0, \tag{36}$$

then (35) holds, and hence (xi) holds as well.

Now, we summarize what we have proven as follows:

Claim 6.3. *Suppose (23), (34), (36), and $\mu > \frac{1}{2}\sigma^2$ hold true for $\mu \equiv r - \delta - \rho(2-\varepsilon) > 0$. Then, with λ_1, $\widetilde{C}$ and $\widetilde{k}^*$ given by (22) and (32), the function ϕ given by (31) coincides with the value function Φ_Δ of our problem, and $\tau_\Delta^* = \tau_D \equiv \inf\left\{t > 0 : k(t) = \widetilde{k}^*\right\}$ is an optimal exit time, where D is the continuation region given by (29).*

Claim 6.4. *Suppose the capital return rate is restricted as in the following Proof, then the conditions used in Claim 6.3 hold true.*

Proof. *First, we have $\mu > \frac{1}{2}\sigma^2 \Leftrightarrow r > \delta + \rho(2-\varepsilon) + \frac{1}{2}\sigma^2 \equiv r_{\min}$. Since it is easy to show that (23) implies (34), we just need to show that $\mu < \rho \Leftrightarrow r < \delta + \rho(2-\varepsilon) + \rho \equiv r_{\max}$. Also, note that (36) yields $\frac{1}{2}\sigma^2 \le \rho - \mu$, we hence have $r_{\min} < r_{\max}$. As a consequence, the required conditions hold true as long as $r \in (r_{\min}, r_{\max})$.* □

Step 9: To complete the Proof, we need the following result.

Therefore, we obtain the optimal exit time by combining these results. **QED**

C. Proof of Lemma 3.6

We shall complete it in three steps.

Step 1: To justify cooperation, we will show that group rationality, individual rationality, and subgame consistency are satisfied. In addition, no one will unilaterally deviate from cooperation under some given Pareto-optimal payoff allocation principles.

Claim 6.5. *Group rationality is satisfied for the cooperative equilibrium.*

Proof. It follows from Lemmas 3.4 and 3.5 that we only need to confirm that $C_6 > C_8 + C_{10}$. In fact, we have

$$C_6 > C_8 + C_{10} \iff [2-(1-\varepsilon)]^{2-(1-\varepsilon)} e^{2(1-\varepsilon)} > 4. \tag{37}$$

Defining $p \equiv 1-\varepsilon$, *then* $0 < p < 1$ *based on our specification. Considering function* $f(p) \equiv (2-p)^{2-p}e^{2p}$, *it is easy to obtain* $\frac{\partial \ln f(p)}{\partial p} = \ln\left(\frac{e}{2-p}\right) > 0$, *which implies that* $\inf_{0<p<1} \ln f(p) = \lim_{p\downarrow 0} \ln f(p) = \ln 4$. *Thus,* $f(p) > 4$ *always holds true for* $0 < p < 1$, *which means that (37) holds and hence group rationality is satisfied.* □

Here, we shall obtain a subgame-consistent PDP.

Claim 6.6. *An instantaneous payment at time $t' \in (0, \tau_0)$ equaling*

$$\xi^i(t') = -v_t^{(t_0)i}\left(t', k_{t'}^{**}\right) - \frac{1}{2}\sigma^2\left(k_{t'}^{**}\right)^2 v_{kk}^{(t_0)i}\left(t', k_{t'}^{**}\right) - v_k^{(t_0)i}\left(t', k_{t'}^{**}\right) k_{t'}^{**}(r-\delta-\rho)$$

for $i \in \{C, G\}$ and $k_{t'}^{**} \in \Gamma_{t'}^{**}$ yields a subgame consistent solution imputation.

Proof. *We omit it as it is similar to the proof of Theorem* 5.8.3 *in* Yeung and Petrosyan (2006). □

Claim 6.7. *Both the Nash bargaining solution/Shapley value and proportional distribution can provide us with a valid imputation.*

Proof. A trivial application of Claim 6.5. □

Claim 6.8. *Both the Nash bargaining solution/Shapley value and proportional distribution principle meet subgame consistency.*

Proof. *Since the equilibrium feedback strategies are Markovian in the sense that they just depend on the current state and current time, one can readily observe that*

$$\begin{pmatrix} c^{**(t_0)}\left(t, k_t^{**}\right) \\ \tau_k^{**(t_0)}\left(t, k_t^{**}\right) \end{pmatrix} = \begin{pmatrix} c^{**(t')}\left(t, k_t^{**}\right) \\ \tau_k^{**(t')}\left(t, k_t^{**}\right) \end{pmatrix}$$

for $t_0 \le t' \le t < \tau_0$ and $k^{**}(t) \equiv k_t^{**} \in \Gamma_t^{**}$. In addition, by using this property, we can get that $J^{(t_0)C}\left(t', k_{t'}^{**}\right) = e^{-\rho(t'-t_0)} J^{(t')C}\left(t', k_{t'}^{**}\right)$, $J^{(t_0)G}\left(t', k_{t'}^{**}\right) = e^{-\rho(t'-t_0)} J^{(t')G}\left(t', k_{t'}^{**}\right)$ and $J^{(t_0)}\left(t', k_{t'}^{**}\right) = e^{-\rho(t'-t_0)} J^{(t')}\left(t', k_{t'}^{**}\right)$, in which the LHS (left-hand side) measures the expected present values of non-cooperative and cooperative payoffs in time interval $[t', \tau_0)$, when $k^{**}(t') = k_{t'}^{**}$, and the game starts from time $t_0 \le t'$.

For the Nash bargaining solution/Shapley value, we then have

$$\begin{aligned} v^{(t_0)i}\left(t', k_{t'}^{**}\right) &= J^{(t_0)i}\left(t', k_{t'}^{**}\right) + \frac{1}{2}\left[J^{(t_0)}\left(t', k_{t'}^{**}\right) - \sum_{j\in\{C,G\}} J^{(t_0)j}\left(t', k_{t'}^{**}\right)\right] \\ &= e^{-\rho(t'-t_0)}\left\{J^{(t')i}\left(t', k_{t'}^{**}\right) + \frac{1}{2}\left[J^{(t')}\left(t', k_{t'}^{**}\right) - \sum_{j\in\{C,G\}} J^{(t')j}\left(t', k_{t'}^{**}\right)\right]\right\} \\ &= e^{-\rho(t'-t_0)} v^{(t')i}\left(t', k_{t'}^{**}\right) \end{aligned}$$

for $i \in \{C, G\}$, $t_0 \le t' < \tau_0$, and $k^{**}_{t'} \in \Gamma^{**}_{t'}$. Similarly, for the proportional distribution principle,

$$\begin{aligned} v^{(t_0)i}\left(t', k^{**}_{t'}\right) &= \frac{J^{(t_0)i}\left(t', k^{**}_{t'}\right)}{\sum_{j\in\{C,G\}} J^{(t_0)j}\left(t', k^{**}_{t'}\right)} J^{(t_0)}\left(t', k^{**}_{t'}\right) \\ &= \frac{e^{-\rho(t'-t_0)} J^{(t')i}\left(t', k^{**}_{t'}\right)}{\sum_{j\in\{C,G\}} e^{-\rho(t'-t_0)} J^{(t')j}\left(t', k^{**}_{t'}\right)} \\ & e^{-\rho(t'-t_0)} J^{(t')}\left(t', k^{**}_{t'}\right) \\ &= e^{-\rho(t'-t_0)}\left[\frac{J^{(t')i}\left(t', k^{**}_{t'}\right)}{\sum_{j\in\{C,G\}} J^{(t')j}\left(t', k^{**}_{t'}\right)} J^{(t')}\left(t', k^{**}_{t'}\right)\right] \\ &= e^{-\rho(t'-t_0)} v^{(t')i}\left(t', k^{**}_{t'}\right) \end{aligned}$$

for $i \in \{C, G\}$, $t_0 \le t' < \tau_0$, *and* $k^{**}_{t'} \in \Gamma^{**}_{t'}$. *Therefore, the required assertion follows.* □

Claim 6.9. *Under the Nash bargaining solution/Shapley value and proportional distribution principle, neither the capitalist nor the government will unilaterally deviate from cooperation.*

Proof. We first consider the case under the Nash bargaining solution/Shapley value. At date $t \ge t_0$, if no one deviates from cooperation, the payoff allocation is

$$v^i\left(t, k^{**}(t)\right) = J^i\left(t, k^{**}(t)\right) + \frac{1}{2}\left[J\left(t, k^{**}(t)\right) - \sum_{j\in\{C,G\}} J^j\left(t, k^{**}(t)\right)\right]$$

for $i \in \{C, G\}$. It follows from Claim 6.5 that $v^i(t, k^{**}(t)) > J^i(t, k^{**}(t))$ for all $i \in \{C, G\}$. If the capitalist unilaterally deviates from cooperation, he gets payoff $J^C\left(t, \widehat{k}(t)\right) = e^{-\rho(t_0+t)}\left[C_7 \ln \widehat{k}(t) + C_8\right]$, with C_7 and C_8 given in Lemma 3.5, and $\widehat{k}(t)$ a solution of

$$d\widehat{k}(t) = \left\{r - \delta - \frac{\rho}{2}\left[(1-\varepsilon) + 2\right]\right\}\widehat{k}(t)dt + \sigma\widehat{k}(t)dB(t),$$

which, compared with (9), shows $k^{**}(t) > \widehat{k}(t)$ for $\forall t$. As we have $J^C(t, k^{**}(t)) = e^{-\rho(t_0+t)}[C_7 \ln k^{**}(t) + C_8]$ with the same C_7 and C_8, it is immediate that $J^C\left(t, \widehat{k}(t)\right) < J^C(t, k^{**}(t)) < v^C(t, k^{**}(t))$. On the other hand, if the government unilaterally deviates from cooperation, it gets payoff $J^G\left(t, \widetilde{k}(t)\right) = e^{-\rho(t_0+t)}\left[C_9 \ln \widetilde{k}(t) + C_{10}\right]$ with C_9 and C_{10} given by Lemma 3.5, and $\widetilde{k}(t)$ a solution of

$$d\widetilde{k}(t) = \left[r - \delta - \rho\left(1 + \frac{1-\varepsilon}{2}\right)\right]\widetilde{k}(t)dt + \sigma\widetilde{k}(t)dB(t),$$

which shows $k^{**}(t) > \widetilde{k}(t) = \widehat{k}(t)$ for $\forall t$. Then, it is easy to obtain $J^G\left(t, \widetilde{k}(t)\right) < J^G(t, k^{**}(t)) < v^G(t, k^{**}(t))$. To sum up, sunilateral deviation always results in lower payoffs; hence neither the capitalist nor the government will unilaterally deviate from cooperation.

For the case under the proportional distribution principle, since by Claim 6.5 the payoff allocation under cooperation satisfies

$$v^i\left(t, k^{**}(t)\right) = \frac{J^i(t, k^{**}(t))}{\sum\limits_{j\in\{C,G\}} J^j(t, k^{**}(t))} J\left(t, k^{**}(t)\right) > J^i(t, k^{**}(t))$$

for $i \in \{C, G\}$, no one will unilaterally deviate from cooperation for the same reason shown above, which is hence omitted to economize on space. □

Step 2: Solving the problem in Stage 1 gives rise to:

Claim 6.10. *Suppose* $r - \delta < 2\rho$, $h_1 \le \frac{\rho}{r-\delta-\rho}$, $r - \delta - \rho - \frac{1}{2}\sigma^2 > 0$ *and* $2(r-\delta) - 4\rho + \sigma^2 \le 0$ *hold true for* $h_1 = \frac{\sigma^2 - 2(r-\delta-\rho) + \sqrt{[2(r-\delta-\rho)-\sigma^2]^2 + 8\rho\sigma^2}}{2\sigma^2}$, $k^{**} = \frac{h_1\varpi}{h_1 - 1}$ *and* $\overline{C} = \frac{1}{h_1}(k^{**})^{1-h_1}$. *Then, we can derive function*

$$\phi(t_0, k_0) = \begin{cases} e^{-\rho t_0}\overline{C}k_0^{h_1} & \text{if } 0 < k_0 < k^{**} \\ e^{-\rho t_0}(k_0 - \varpi) & \text{if } k^{**} \le k_0 \end{cases}$$

such that it coincides with value function Φ_0 *of our problem, and* $\tau_0^{**} = \tau_D \equiv \inf\{t > 0 : k(t) = k^{**}\}$ *is an optimal exit time with continuation region* $D = \{(t_0, k_0) : 0 < k_0 < k^{**}\}$.

Proof. *It is similar to that of* Claim 6.2 *and is therefore omitted.* □

Step 3: To complete the Proof, we need the following result.

Claim 6.11. *Suppose the capital return rate is restricted as shown in the following Proof, then the conditions required in Claim 6.10 hold true.*

Proof. First, we have $r-\delta-\rho-\frac{1}{2}\sigma^2>0 \Leftrightarrow r>\delta+\rho+\frac{1}{2}\sigma^2 \equiv \widetilde{r}_{\min}$. In addition, since $2(r-\delta)-4\rho+\sigma^2 \le 0$ implies $r-\delta<2\rho$, we just show that $2(r-\delta)-4\rho+\sigma^2 \le 0 \Leftrightarrow r \le \delta+2\rho-\frac{1}{2}\sigma^2 \equiv \widetilde{r}_{\max}$. Also, it is easy to show that $h_1 \le \frac{\rho}{r-\delta-\rho}$ is implied by $r-\delta<2\rho$. In consequence, we just need $\rho>\sigma^2$ to ensure that $\widetilde{r}_{\min}<\widetilde{r}_{\max}$. To conclude, the required conditions hold true as long as $r \in (\widetilde{r}_{\min}, \widetilde{r}_{\max}]$. □

The Proof of Lemma 3.6 is therefore completed using Claims 6.5-6.11. **QED**

D. Proof of Theorem 4.1

We shall complete it in four steps.

Step 1: By using Lemmas 3.1-3.2, we have

$$\widetilde{k}^* = k(\tau_\Delta^*) = k_0 \exp\left\{\left[r-\delta-\rho(2-\varepsilon)-\frac{1}{2}\sigma^2\right]\tau_\Delta^* + \sigma B(\tau_\Delta^*)\right\},$$

which yields

$$\ln\left(\frac{\widetilde{k}^*}{k_0}\right) = \left[r-\delta-\rho(2-\varepsilon)-\frac{1}{2}\sigma^2\right]\mathbb{E}(\tau_\Delta^*),$$

i.e.,

$$\mathbb{E}(\tau_\Delta^*) = \frac{\ln\left(\frac{\widetilde{k}^*}{k_0}\right)}{r-\delta-\rho(2-\varepsilon)-\frac{1}{2}\sigma^2}. \tag{38}$$

Similarly, it follows from Lemma 3.3 that

$$\mathbb{E}(\tau_0^*) = \frac{\ln\left(\frac{k^*}{k_0}\right)}{r-\delta-\rho(2-\varepsilon)-\frac{1}{2}\sigma^2}. \tag{39}$$

Since $\widetilde{\varpi}<\varpi$ by (7), it is easy to see that $\widetilde{k}^*<k^*$ and hence $\mathbb{E}(\tau_\Delta^*)<\mathbb{E}(\tau_0^*)$. Moreover, to make $\mathbb{E}(\tau_\Delta^*)>0$ we require that $k_0<\widetilde{k}^*$, which implies that $\frac{\varpi}{k_0}>\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta}$ for $\forall\Delta>0$.

Step 2: We now proceed to show that $k^*<k^{**}$ for $\forall\varpi>0$. Define a function $l(x)\equiv -x+\sqrt{x^2+8\rho\sigma^2}$ for $x>0$, by which we obtain $l'(x)=-1+\frac{x}{\sqrt{x^2+8\rho\sigma^2}}<0$ for $\forall x$, i.e., $l(x)$ is a strictly decreasing function with respect to x. Then, by comparing (6) with (11), we immediately get $\lambda_1>h_1$. As a result, $k^*=\frac{\lambda_1\varpi}{\lambda_1-1}<\frac{h_1\varpi}{h_1-1}=k^{**}$ for $\forall\varpi>0$.

Step 3: It follows from Lemma 3.6 that

$$\mathbb{E}(\tau_0^{**}) = \frac{\ln\left(\frac{k^{**}}{k_0}\right)}{r-\delta-\rho-\frac{1}{2}\sigma^2}. \tag{40}$$

Combining (38) with (40) and using the definition of $\widetilde{k}^*$ shows that

$$\begin{aligned}\frac{\mathbb{E}(\tau_\Delta^*)}{\mathbb{E}(\tau_0^{**})}<1 &\iff \frac{\ln\left(\frac{k^*}{k_0}\right)-\mu\Delta}{\ln\left(\frac{k^{**}}{k_0}\right)} < \frac{\mu-\frac{1}{2}\sigma^2}{r-\delta-\rho-\frac{1}{2}\sigma^2}\\ &\iff \Delta > \frac{\ln\left(\frac{k^*}{k_0}\right)-\ln\left(\frac{k^{**}}{k_0}\right)\left(\frac{\mu-\frac{1}{2}\sigma^2}{r-\delta-\rho-\frac{1}{2}\sigma^2}\right)}{\mu},\end{aligned}$$

in which we also impose the assumption that $\frac{\varpi}{k_0}>\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta}$. First, note that

$$\begin{aligned}&\ln\left(\frac{k^*}{k_0}\right) \le \ln\left(\frac{k^{**}}{k_0}\right)\left(\frac{\mu-\frac{1}{2}\sigma^2}{r-\delta-\rho-\frac{1}{2}\sigma^2}\right)\\ \iff &\frac{\varpi}{k_0} \le \left(\frac{h_1}{h_1-1}\right)^{\frac{\mu-\frac{1}{2}\sigma^2}{\rho(1-\varepsilon)}}\left(\frac{\lambda_1-1}{\lambda_1}\right)^{\frac{r-\delta-\rho-\frac{1}{2}\sigma^2}{\rho(1-\varepsilon)}} \equiv \Xi^*\end{aligned}$$

and

$$\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta}<\Xi^* \iff \Delta < \underbrace{\left(\frac{1}{\mu}\right)\left[\frac{\mu-\frac{1}{2}\sigma^2}{\rho(1-\varepsilon)}\right]\ln\left(\frac{\frac{h_1}{h_1-1}}{\frac{\lambda_1}{\lambda_1-1}}\right)}_{>0} \equiv \Delta_1^*$$

then it is immediate that $\mathbb{E}(\tau_\Delta^*) < \mathbb{E}(\tau_0^{**})$ for any $\Delta < \Delta_1^*$ and $\frac{\varpi}{k_0} \le \Xi^*$, as desired. Otherwise, we consider the case with $\frac{\varpi}{k_0} > \Xi^*$. Noting that

$$\Delta > \frac{\ln\left(\frac{k^*}{k_0}\right) - \ln\left(\frac{k^{**}}{k_0}\right)\left(\frac{\mu-\frac{1}{2}\sigma^2}{r-\delta-\rho-\frac{1}{2}\sigma^2}\right)}{\mu}$$

$$\iff \left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta} > \left(\frac{h_1-1}{h_1}\right)^{\frac{\mu-\frac{1}{2}\sigma^2}{r-\delta-\rho-\frac{1}{2}\sigma^2}}\left(\frac{\varpi}{k_0}\right)^{\frac{\rho(1-\varepsilon)}{r-\delta-\rho-\frac{1}{2}\sigma^2}},$$

$$\frac{\varpi}{k_0} > \left(\frac{h_1-1}{h_1}\right)^{\frac{\mu-\frac{1}{2}\sigma^2}{r-\delta-\rho-\frac{1}{2}\sigma^2}}\left(\frac{\varpi}{k_0}\right)^{\frac{\rho(1-\varepsilon)}{r-\delta-\rho-\frac{1}{2}\sigma^2}} \iff \frac{\varpi}{k_0} > \frac{h_1-1}{h_1},$$

$$\Xi^*/\frac{h_1-1}{h_1} = \left(\frac{h_1}{h_1-1}/\frac{\lambda_1}{\lambda_1-1}\right)^{\frac{r-\delta-\rho-\frac{1}{2}\sigma^2}{\rho(1-\varepsilon)}} > 1,$$

and also

$$\frac{\varpi}{k_0} > \left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta} \iff \Delta < \frac{1}{\mu}\ln\left[\left(\frac{\lambda_1}{\lambda_1-1}\right)\left(\frac{\varpi}{k_0}\right)\right],$$

then we claim that there exists a lower bound, written as

$$\Delta_2^* \equiv \frac{\ln\left(\frac{k^*}{k_0}\right) - \ln\left(\frac{k^{**}}{k_0}\right)\left(\frac{\mu-\frac{1}{2}\sigma^2}{r-\delta-\rho-\frac{1}{2}\sigma^2}\right)}{\mu} > 0,$$

of Δ such that $\mathbb{E}(\tau_\Delta^*) < \mathbb{E}(\tau_0^{**})$ for any $\Delta \in \left(\Delta_2^*, \frac{1}{\mu}\ln\left[\left(\frac{\lambda_1}{\lambda_1-1}\right)\left(\frac{\varpi}{k_0}\right)\right]\right)$. Using the above calculation, we can also have $\mathbb{E}(\tau_\Delta^*) = \mathbb{E}(\tau_0^{**})$ for $\Delta = \Delta_2^*$ and $\mathbb{E}(\tau_\Delta^*) > \mathbb{E}(\tau_0^{**})$ for any $\Delta < \Delta_2^*$, as required.

Step 4: Making use of (39) and (40) reveals that

$$\frac{\mathbb{E}(\tau_0^*)}{\mathbb{E}(\tau_0^{**})} = \frac{\ln\left(\frac{\lambda_1}{\lambda_1-1}\right) + \ln\left(\frac{\varpi}{k_0}\right)}{\ln\left(\frac{h_1}{h_1-1}\right) + \ln\left(\frac{\varpi}{k_0}\right)}\left(\frac{r-\delta-\rho-\frac{1}{2}\sigma^2}{\mu-\frac{1}{2}\sigma^2}\right).$$

Thus, by rearranging the terms, we have

$$\frac{\mathbb{E}(\tau_0^*)}{\mathbb{E}(\tau_0^{**})} \le 1 \iff \ln\left(\frac{\varpi}{k_0}\right) \le \frac{\left(\frac{\mu-\frac{1}{2}\sigma^2}{r-\delta-\rho-\frac{1}{2}\sigma^2}\right)\ln\left(\frac{h_1}{h_1-1}\right) - \ln\left(\frac{\lambda_1}{\lambda_1-1}\right)}{1-\left(\frac{\mu-\frac{1}{2}\sigma^2}{r-\delta-\rho-\frac{1}{2}\sigma^2}\right)},$$

which gives rise to $\mathbb{E}(\tau_0^*) \le \mathbb{E}(\tau_0^{**}) \Leftrightarrow \frac{\varpi}{k_0} \le \Xi^*$. In the meantime, note that we need the constraint $\frac{\varpi}{k_0} > \frac{\lambda_1-1}{\lambda_1}$ to make $\mathbb{E}(\tau_0^*) > 0$. Since it is easy to verify that $\Xi^* > \frac{\lambda_1-1}{\lambda_1}$, we hence have $\mathbb{E}(\tau_0^*) < \mathbb{E}(\tau_0^{**})$ for any $\frac{\varpi}{k_0} \in \left(\frac{\lambda_1-1}{\lambda_1}, \Xi^*\right)$, $\mathbb{E}(\tau_0^*) = \mathbb{E}(\tau_0^{**})$ for $\frac{\varpi}{k_0} = \Xi^*$, and also $\mathbb{E}(\tau_0^*) > \mathbb{E}(\tau_0^{**})$ for any $\frac{\varpi}{k_0} > \Xi^*$, as desired in (v). **QED**

E. Proof of Corollary 4.1

First, $\mathbb{E}(\tau_\Delta^*) > 0$ requires that $\varpi > k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta}$ for $\forall\Delta > 0$. By (38) and (22), we have

$$\frac{\partial\mathbb{E}(\tau_\Delta^*)}{\partial r} = -\left(\frac{1}{\mu-\frac{1}{2}\sigma^2}\right)^2\ln\left[\frac{\lambda_1\widetilde{\varpi}}{(\lambda_1-1)k_0}\right] + \left(\frac{1}{\mu-\frac{1}{2}\sigma^2}\right)\left[\underbrace{\frac{1}{\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial r}}_{>0} - \Delta\right].$$

Using (28), we then get

$$\frac{\partial\mathbb{E}(\tau_\Delta^*)}{\partial r} > 0 \Leftrightarrow \varpi < k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left[\left(\mu-\frac{1}{2}\sigma^2\right)\frac{1}{\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial r} + \frac{1}{2}\sigma^2\Delta\right].$$

We hence need the following condition:

$$k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta} < k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left[\left(\mu-\frac{1}{2}\sigma^2\right)\frac{1}{\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial r} + \frac{1}{2}\sigma^2\Delta\right],$$

which yields that

$$\Delta < \frac{1}{\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial r}.$$

The required assertion in part (i) thus follows. Similarly, we can obtain the assertions in parts (ii) and (iii). **QED**

F. Proof of Corollary 4.2

In view of (38) and (22), we have

$$\frac{\partial \mathbb{E}(\tau_\Delta^*)}{\partial \varepsilon} = -\rho\left(\frac{1}{\mu-\frac{1}{2}\sigma^2}\right)^2 \ln\left[\frac{\lambda_1\widetilde{\varpi}}{(\lambda_1-1)k_0}\right] + \left(\frac{1}{\mu-\frac{1}{2}\sigma^2}\right)\left[\underbrace{\frac{1}{\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial\varepsilon}}_{>0} - \rho\Delta\right].$$

The remaining Proof is quite similar to that of Corollary 4.1 and is hence omitted to economize on space. **QED**

G. Proof of Corollary 4.3

By (38) and (22), we have

$$\frac{\partial \mathbb{E}(\tau_\Delta^*)}{\partial \rho} = (2-\varepsilon)\left(\frac{1}{\mu-\frac{1}{2}\sigma^2}\right)^2 \ln\left[\frac{\lambda_1\widetilde{\varpi}}{(\lambda_1-1)k_0}\right] + \left(\frac{1}{\mu-\frac{1}{2}\sigma^2}\right)\left[\underbrace{\frac{1}{\lambda_1(1-\lambda_1)}\frac{\partial\lambda_1}{\partial\rho}}_{<0} + (2-\varepsilon)\Delta\right],$$

where we have used (6) to show that

$$\frac{\partial\lambda_1}{\partial\rho} = \frac{(2-\varepsilon)\left[\sqrt{(2\mu-\sigma^2)^2+8\rho\sigma^2}-(2\mu-\sigma^2)\right]+2\sigma^2}{\sigma^2\sqrt{(2\mu-\sigma^2)^2+8\rho\sigma^2}} > 0.$$

Repeating the same procedure as before, the required assertion in part (i) follows. Similarly, we can establish the assertions stated in parts (ii) and (iii). **QED**

H. Proof of Proposition 5.1

We shall complete it in two steps.

Step 1: We first prove the claim shown in part (i). Similar to the expected optimal exit time shown in equation (40), by applying Lemma 5.1, we can obtain

$$\mathbb{E}(\tau_\Delta^{**}) = \frac{\ln\left(\frac{\widetilde{k}^{**}}{k_0}\right)}{r-\delta-\rho-\frac{1}{2}\sigma^2}, \tag{41}$$

which is well defined whenever we have $\widetilde{k}^{**} > k_0$, which implies

$$\frac{\varpi}{k_0} > \left(\frac{h_1-1}{h_1}\right)e^{(r-\delta-\rho)\Delta}. \tag{42}$$

Applying equations (39) and (41) and rearranging the algebra, we get that $\mathbb{E}(\tau_0^*) < \mathbb{E}(\tau_\Delta^{**})$ is equivalent to

$$\Delta < \frac{\ln\left(\frac{k^{**}}{k_0}\right) - \ln\left(\frac{k^*}{k_0}\right)\left(\frac{r-\delta-\rho-\frac{1}{2}\sigma^2}{\mu-\frac{1}{2}\sigma^2}\right)}{r-\delta-\rho} \equiv \Delta_3^*,$$

as desired. Noting from Step 3 in the Proof of Theorem 4.1 that

$$\left(\frac{r-\delta-\rho-\frac{1}{2}\sigma^2}{\mu-\frac{1}{2}\sigma^2}\right)\ln\left(\frac{k^*}{k_0}\right) < \ln\left(\frac{k^{**}}{k_0}\right)$$
$$\Longleftrightarrow \frac{\varpi}{k_0} < \left(\frac{h_1}{h_1-1}\right)^{\frac{\mu-\frac{1}{2}\sigma^2}{\rho(1-\varepsilon)}}\left(\frac{\lambda_1-1}{\lambda_1}\right)^{\frac{r-\delta-\rho-\frac{1}{2}\sigma^2}{\rho(1-\varepsilon)}} \equiv \Xi^*,$$

we thus have $\Delta_3^* > 0$ for $\varpi/k_0 < \Xi^*$. Also, as shown in the Proof of Theorem 4.1, $\mathbb{E}(\tau_0^*)$ is well defined for $\varpi/k_0 > (\lambda_1 - 1)/\lambda_1$. It is also easy to verify that $(\lambda_1 - 1)/\lambda_1 < \Xi^*$ always holds true. Moreover, to make condition $\varpi/k_0 < \Xi^*$ compatible with the requirement given by (42), we must have $[(h_1 - 1)/h_1]e^{(r-\delta-\rho)\Delta} < \Xi^*$, which is equivalent to having, after rearranging the algebra, that

$$\Delta < \underbrace{\left(\frac{1}{r-\delta-\rho}\right)\left[\frac{r-\delta-\rho-\frac{1}{2}\sigma^2}{\rho(1-\varepsilon)}\right]\ln\left(\frac{\frac{h_1}{h_1-1}}{\frac{\lambda_1}{\lambda_1-1}}\right)}_{>0} \equiv \Delta_4^*.$$

Using these conditions, the assertion in part (i) immediately follows.

Step 2: We now prove the claim in part (ii). Using equations (41) and (38) and rearranging the algebra, we get that $\mathbb{E}(\tau_\Delta^*) < \mathbb{E}(\tau_\Delta^{**})$ is equivalent to

$$\Delta > \frac{2\left(r-\delta-\rho-\frac{1}{2}\sigma^2\right)}{\rho(1-\varepsilon)\sigma^2}\left[\ln\left(\frac{k^*}{k_0}\right)-\ln\left(\frac{k^{**}}{k_0}\right)\left(\frac{\mu-\frac{1}{2}\sigma^2}{r-\delta-\rho-\frac{1}{2}\sigma^2}\right)\right] \equiv \Delta_5^*.$$

The remaining Proof is omitted due to its similarity with Step 1. **QED**

I. Proof of Lemma 5.2

We shall complete it in two steps.

Step 1: Using backward induction, we first solve the problem in Stage 2. We apply the Itô formula to rewrite the maximization problem as follows:

$$\max_{r_c(t)>0} \mathbb{E}_{t_0}\left[\int_0^{\tau_\Delta} e^{-\rho(t_0+t)}\left(\ln r_c(t)+\ln k(t)\right)dt\right]$$

subject to

$$d\ln k(t) = \left[(1-\tau_k(t))(r-\delta)-r_c(t)-\frac{1}{2}\sigma^2\right]dt+\sigma dB(t).$$

Applying the stochastic maximum principle developed via Malliavin calculus (see, Meyer-Brandis et al., 2012; Di Nunno et al., 2011), we obtain the following Hamiltonian:

$$\begin{aligned} H(t,\ln k(t),r_c(t)) &= e^{-\rho(t_0+t)}\left[\ln r_c(t)+\ln k(t)\right]+\sigma D_t Q(t) \\ &\quad +Q(t)\left[(1-\tau_k(t))(r-\delta)-r_c(t)-\frac{1}{2}\sigma^2+D_{t+}\sigma\right], \end{aligned}$$

where

$$Q(t) \equiv \int_t^{\tau_\Delta} e^{-\rho(t_0+s)}ds = \frac{e^{-\rho(t_0+t)}-e^{-\rho(t_0+\tau_\Delta)}}{\rho}$$

and $D_{t+}\sigma \equiv \lim_{s\to t+} D_s\sigma = 0$. Here, D_s stands for the Malliavin derivative. So, at the optimal $r_c^{***}(t)$ we should have

$$\mathbb{E}\left[\frac{\partial}{\partial r_c}H\left(t,\ln k^{***}(t),r_c^{***}(t)\right)|\mathfrak{F}_{t-\Delta}\right]=0,$$

which implies

$$\mathbb{E}\left[e^{-\rho(t_0+t)}\frac{1}{r_c^{***}(t)}-Q(t)|\mathfrak{F}_{t-\Delta}\right]=0.$$

Since τ_Δ is $\mathfrak{F}_{t-\Delta}$-measurable, $Q(t)$ is $\mathfrak{F}_{t-\Delta}$-measurable. Therefore, $e^{-\rho(t_0+t)} = r_c^{***}(t)Q(t)$, by which we can obtain the required result.

Step 2: Given the result derived in Step 1, the government's Hamiltonian reads as follows:

$$\begin{aligned} H(t,\ln k(t),\tau_k(t)) &= e^{-\rho(t_0+t)}\left\{\varepsilon\ln\left[r_c^{***}(t)k(t)\right]+(1-\varepsilon)\right. \\ &\quad \left.\ln[\tau_k(t)(r-\delta)k(t)]\right\}+\sigma D_t Q(t)+Q(t) \\ &\quad \left[(1-\tau_k(t))(r-\delta)-r_c^{***}(t)-\frac{1}{2}\sigma^2\right], \end{aligned}$$

where

$$Q(t) \equiv \int_t^{\tau_\Delta} e^{-\rho(t_0+s)}[\varepsilon+(1-\varepsilon)]ds = \frac{e^{-\rho(t_0+t)}-e^{-\rho(t_0+\tau_\Delta)}}{\rho}.$$

At optimal $\tau_k^{***}(t)$, we should have

$$\mathbb{E}\left[\frac{\partial}{\partial \tau_k}H\left(t,\ln k^{***}(t),\tau_k^{***}(t)\right)|\mathfrak{F}_t\right]=0,$$

which implies

$$\mathbb{E}\left[e^{-\rho(t_0+t)}\left(\frac{1-\varepsilon}{\tau_k^{***}(t)}\right)-(r-\delta)Q(t)|\mathfrak{F}_t\right]=0.$$

Since τ_Δ is $\mathfrak{F}_{t-\Delta}$-measurable and hence $\mathfrak{F}_t$-measurable, we get that $Q(t)$ is $\mathfrak{F}_t$-measurable. Accordingly, we arrive at $e^{-\rho(t_0+t)}(1-\varepsilon) = (r-\delta)Q(t)\tau_k^{***}(t)$, as required. **QED**

J. Proof of Lemma 5.3

By using (18), (19), and (25), it is easy to verify that

$$\mathbb{E}_{t_0}\left[\int_0^{\mathbb{E}(\tau_\Delta^*)}|V^G(t,k(t))|dt\right]=\int_{\omega\in\Omega}\int_0^{\mathbb{E}(\tau_\Delta^*)}|V^G(t,k(t))|dtdP(\omega)<\infty$$

for any $\mathbb{E}(\tau_\Delta^*)<\infty$; thus, applying Fubini's Theorem implies that

$$V_\Delta^G\equiv\mathbb{E}_{t_0}\left[\int_0^{\mathbb{E}(\tau_\Delta^*)}V^G(t,k(t))dt\right]=\int_0^{\mathbb{E}(\tau_\Delta^*)}\mathbb{E}_{t_0}[V^G(t,k(t))]dt.$$

By applying the formula of integration by parts, we thus have

$$\begin{aligned}V_\Delta^G=&-\left[\left(\frac{\mu-\frac{1}{2}\sigma^2}{\rho^2}\right)\mathbb{E}(\tau_\Delta^*)+\frac{\mu-\frac{1}{2}\sigma^2+\rho\ln k_0+\rho^2C_4}{\rho^3}\right]e^{-\rho\mathbb{E}(\tau_\Delta^*)}\\&+\frac{\mu-\frac{1}{2}\sigma^2+\rho\ln k_0+\rho^2C_4}{\rho^3},\end{aligned}$$

where, without any loss of generality, we normalize t_0 to zero for notational simplicity. Applying the chain rule gives rise to

$$\begin{aligned}\frac{\partial V_\Delta^G}{\partial\Delta}&=\frac{\partial V_\Delta^G}{\partial\mathbb{E}(\tau_\Delta^*)}\frac{\partial\mathbb{E}(\tau_\Delta^*)}{\partial\Delta}\\&=-\left(\frac{\mu}{\mu-\frac{1}{2}\sigma^2}\right)\left[\left(\frac{\mu-\frac{1}{2}\sigma^2}{\rho}\right)\mathbb{E}(\tau_\Delta^*)+\frac{\ln k_0}{\rho}+C_4\right]e^{-\rho\mathbb{E}(\tau_\Delta^*)},\end{aligned}$$

which, combined with (38) and $\Delta_{\min}\equiv\frac{1}{\mu}\ln\left(\frac{\lambda_1\varpi}{\lambda_1-1}\right)+\frac{\rho}{\mu}C_4>0\Leftrightarrow\varpi>\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{-\rho C_4}$, gives rise to the assertion required in part (i). Making use of the chain rule and (38) again, we obtain

$$\left.\frac{\partial^2V_\Delta^G}{\partial\Delta^2}\right|_{\Delta=\Delta_{\min}}=\left.\frac{\partial\left(\partial V_\Delta^G/\partial\Delta\right)}{\partial\mathbb{E}\left(\tau_\Delta^*\right)}\frac{\partial\mathbb{E}\left(\tau_\Delta^*\right)}{\partial\Delta}\right|_{\Delta=\Delta_{\min}}>0,$$

which confirms the uniqueness of the critical value of Δ. So, the Proof of part (i) is complete. For part (ii), we just need to note that

$$\frac{\partial V_\Delta^G}{\partial\Delta}>0\Leftrightarrow\varpi<\left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left(\mu\Delta-\rho C_4\right),$$

$$\mathbb{E}(\tau_{\Delta_{\min}}^*)=\left(\frac{1}{\mu-\frac{1}{2}\sigma^2}\right)\left[\ln\left(\frac{1}{k_0}\right)-\rho C_4\right]>0\Leftrightarrow k_0<e^{-\rho C_4}$$

and also for all $\Delta>0$, $\mathbb{E}(\tau_\Delta^*)>0\Leftrightarrow\varpi>k_0\left(\frac{\lambda_1-1}{\lambda_1}\right)e^{\mu\Delta}$. **QED**

K. Proof of Lemma 5.4

Applying Fubini's Theorem, the formula of integration by parts, the chain rule, and (38), we obtain

$$\frac{\partial V_\Delta^G}{\partial\varpi}=\frac{\partial V_\Delta^G}{\partial\mathbb{E}(\tau_\Delta^*)}\frac{\partial\mathbb{E}(\tau_\Delta^*)}{\partial\varpi}=\frac{1}{\left(\mu-\frac{1}{2}\sigma^2\right)\varpi}\left[\left(\frac{\mu-\frac{1}{2}\sigma^2}{\rho}\right)\mathbb{E}(\tau_\Delta^*)+\frac{\ln k_0}{\rho}+C_4\right]e^{-\rho\mathbb{E}(\tau_\Delta^*)},$$

where we have normalized t_0 to zero for notational simplicity. Assuming that $\partial V_\Delta^G/\partial\varpi=0$, we thus obtain the critical value as $\varpi_{\min}\equiv\left(\frac{\lambda_1-1}{\lambda_1}\right)\exp\left(\mu\Delta-\rho C_4\right)>0$. Note that

$$\frac{\partial\left(\partial V_\Delta^G/\partial\varpi\right)}{\partial\mathbb{E}(\tau_\Delta^*)}=\frac{1}{\varpi}\left[\frac{1}{\rho}-\mathbb{E}(\tau_\Delta^*)-\frac{\ln k_0+\rho C_4}{\mu-\frac{1}{2}\sigma^2}\right]e^{-\rho\mathbb{E}(\tau_\Delta^*)}$$

and

$$\mathbb{E}(\tau_\Delta^*)|_{\varpi=\varpi_{\min}}=-\frac{\ln k_0+\rho C_4}{\mu-\frac{1}{2}\sigma^2}>0\Leftrightarrow k_0<e^{-\rho C_4},$$

we hence have

$$\begin{aligned}\frac{\partial^2V_\Delta^G}{\partial\varpi^2}|_{\varpi=\varpi_{\min}}&=\frac{\partial\left(\partial V_\Delta^G/\partial\varpi\right)}{\partial\mathbb{E}(\tau_\Delta^*)}\frac{\partial\mathbb{E}(\tau_\Delta^*)}{\partial\varpi}|_{\varpi=\varpi_{\min}}\\&=\frac{1}{\rho\left(\mu-\frac{1}{2}\sigma^2\right)\varpi_{\min}{}^2}\exp\left[\frac{\rho(\ln k_0+\rho C_4)}{\mu-\frac{1}{2}\sigma^2}\right]>0,\end{aligned}$$

which, accordingly, leads us to the assertion desired. **QED**

L. Optimal Stopping Rule when the Objective is of Integral Form

To economize on space, we shall take the optimal stopping problem under an authoritarian, transparent government as an example because the optimal stopping problems under other types of governments can be solved analogously. Making use of Lemma 3.1, the optimal stopping problem for $\Delta = 0$ reads as follows:

$$\Phi_0(t_0,k_0) \equiv \sup_{\tau\in\mathcal{T}} \mathbb{E}^{t_0,k_0}\left[\int_0^\tau e^{-\rho(t_0+t)}\ln c(t)dt + e^{-\rho(t_0+\tau)}\ln k(\tau)\right], \tag{43}$$

subject to $c(t) = \rho k(t)$ and the stochastic differential equation of capital accumulation:

$$dk(t) = [r-\delta-\rho(2-\varepsilon)]k(t)dt + \sigma k(t)dB(t). \tag{44}$$

In fact, solving problem (43) leads us to the following lemma.

Lemma 6.1. *Suppose the economy is under an authoritarian regime with a transparent government. If the parameters satisfy $\sigma > 0$ and $\frac{1}{2}\sigma^2 < \mu \le \min\left\{\frac{1}{2}\sigma^2 - \rho^2\ln\rho, \rho - \frac{1}{2}\sigma^2\right\}$, then the optimal exit time is $\tau_0^* = \inf\left\{t>0 : k^*(t) = k^* \equiv \exp\left(\frac{1}{\beta} + \frac{\rho C_{11} - \rho\ln\rho}{\rho-1}\right)\right\}$, in which $k^*(t)$ is a solution to equation (44) and*

$$\begin{aligned}
\beta &= \frac{-\left(\mu-\frac{1}{2}\sigma^2\right) - \sqrt{\left(\mu-\frac{1}{2}\sigma^2\right)^2 + 2\rho\sigma^2}}{\sigma^2} < 0;\\
C_{11} &= \frac{1}{\rho}\left(\frac{\mu}{\rho} - \frac{\sigma^2}{2\rho} + \ln\rho\right);\\
\mu &\equiv r-\delta-\rho(2-\varepsilon).
\end{aligned}$$

Proof. *Let $\Sigma(t) \equiv (t_0+t, k(t))'$, $\Sigma(0) \equiv (t_0,k_0)'$ and $'$ denote a transposed operation, then it is easily seen from (44) that the generator of $\Sigma(t)$ reads as follows:*

$$\mathcal{A}\phi(t_0,k_0) = \frac{\partial\phi}{\partial t_0} + \mu k_0\frac{\partial\phi}{\partial k_0} + \frac{1}{2}\sigma^2 k_0^2\frac{\partial^2\phi}{\partial k_0^2}$$

for $\mu \equiv r - \delta - \rho(2-\varepsilon)$ and $\forall\phi \in C^2(\mathbb{R}^2)$. If we try a function ϕ of the form, $\phi(t_0,k_0) = \exp(-\rho t_0)\varphi(k_0)$ for $\varphi \in C^2(\mathbb{R})$, we then have

$$\begin{aligned}
\mathcal{A}\phi(t_0,k_0) &= \exp(-\rho t_0)\left[-\rho\varphi(k_0) + \mu k_0\varphi'(k_0) + \frac{1}{2}\sigma^2k_0^2\varphi''(k_0)\right]\\
&\equiv \exp(-\rho t_0)\mathcal{A}_0\varphi(k_0).
\end{aligned} \tag{45}$$

Define $g_1(k_0) \equiv \ln k_0$ and $g_2(k_0) \equiv \ln(\rho k_0)$. *Then, it follows from (45) that*

$$\begin{aligned}
&\mathcal{A}_0 g_1(k_0) + g_2(k_0) > 0\\
\Leftrightarrow\ &\rho\ln k_0 - \ln(\rho k_0) < \mu - \frac{1}{2}\sigma^2\\
\Leftrightarrow\ &k_0 > \rho^{\frac{1}{\rho-1}}\exp\left(\frac{\mu-\frac{1}{2}\sigma^2}{\rho-1}\right).
\end{aligned}$$

Hence, *we define*

$$U \equiv \left\{(t_0,k_0) : k_0 > \rho^{\frac{1}{\rho-1}}\exp\left(\frac{\mu-\frac{1}{2}\sigma^2}{\rho-1}\right)\right\}.$$

In view of $U \subseteq D$ *it is natural to guess that the continuation region D has the following form*:

$$D \equiv \left\{(t_0,k_0) : k_0 > k^*\right\}$$

for some k^* satisfying

$$k^* \le \rho^{\frac{1}{\rho-1}}\exp\left(\frac{\mu-\frac{1}{2}\sigma^2}{\rho-1}\right). \tag{46}$$

Now, *in D we try to solve the equation*:

$$\mathcal{A}_0\varphi(k_0) + g_2(k_0) = 0. \tag{47}$$

The homogenous equation $\mathcal{A}_0\varphi_0(k_0) = 0$ *has a solution of* $\varphi_0(k_0) = k_0^\beta$ *if and only if parameter β is the solution to the equation* $\sigma^2\beta^2 + (2\mu - \sigma^2)\beta - 2\rho = 0$, *which has the following roots*:

$$\beta_1 = \frac{-\left(\mu-\frac{1}{2}\sigma^2\right) + \sqrt{\left(\mu-\frac{1}{2}\sigma^2\right)^2 + 2\rho\sigma^2}}{\sigma^2} > 0, \tag{48}$$

$$\beta_2 = \frac{-\left(\mu - \frac{1}{2}\sigma^2\right) - \sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\rho\sigma^2}}{\sigma^2} < 0. \tag{49}$$

To find a particular solution, *denoted* $\varphi_1(k_0)$, *to the non-homogenous equation*:

$$\mathcal{A}_0\varphi_1(k_0) + g_2(k_0) = \mathcal{A}\varphi_1(k_0) + \ln(\rho k_0) = 0, \tag{50}$$

we try

$$\varphi_1(k_0) = C_{11} + C_{12}\ln k_0 \tag{51}$$

for some constants C_{11} and C_{12} to be determined. Substituting (51) into (50) and performing the operator (45) gives rise to

$$-\rho C_{11} - \rho C_{12}\ln k_0 + \mu C_{12} - \frac{1}{2}\sigma^2 C_{12} + \ln\rho + \ln k_0 = 0, \text{ for } \forall k_0 > 0$$

which implies that $C_{12} = \rho^{-1}$. Hence, we have

$$C_{11} = \frac{1}{\rho}\left(\mu\rho^{-1} - \frac{1}{2}\sigma^2\rho^{-1} + \ln\rho\right). \tag{52}$$

Consequently, for any constant C, the function $\varphi(k_0) = Ck_0^\beta + \rho^{-1}\ln k_0 + C_{11}$ is a solution to the equation defined in (47) with C_{11} given by (52). Therefore, one can try to put

$$\varphi(k_0) = \begin{cases} \ln(\rho k_0) \equiv g_2(k_0), & \bar{k} \le k_0 \le k^* \\ Ck_0^\beta + \rho^{-1}\ln k_0 + C_{11}, & k_0 > k^* \end{cases} \tag{53}$$

where β is given by (48) or (49), while $C > 0$, $\bar{k}$, and k^* are to be determined. The continuity and differentiability of φ at $k_0 = k^*$ provide us with the following equations:

$$\ln(\rho k^*) = C(k^*)^\beta + \rho^{-1}\ln k^* + C_{11} \tag{54}$$

and

$$(k^*)^{-1} = C\beta(k^*)^{\beta-1} + \rho^{-1}(k^*)^{-1}. \tag{55}$$

By (55), *we obtain*

$$(k^*)^\beta = \frac{1-\rho^{-1}}{C\beta} \tag{56}$$

Inserting (56) *into* (54) *produces*

$$k^* = \exp\left(\frac{1}{\beta} + \frac{\rho C_{11} - \rho\ln\rho}{\rho - 1}\right). \tag{57}$$

By (56), *we get that*

$$C = \frac{\rho - 1}{\rho\beta(k^*)^\beta} > 0 \tag{58}$$

which implies that we should choose $\beta < 0$, i.e., $\beta = \beta_2 < 0$ given by equation (49). We also obtain the maximized level of the objective as

$$\Phi_0^*(t_0, k_0) = \exp(-\rho t_0)\left[\frac{\rho - 1}{\rho\beta(k^*)^\beta}k_0^\beta + \rho^{-1}\ln k_0 + C_{11}\right].$$

Now, it follows from the "Integro-variational inequalities for optimal stopping"(see, Øksendal and Sulem (2009)) that we need to verify whether the following requirements are fulfilled.

(i) We show that $\phi \ge g_2$ on the continuation region D, i.e.,

$$Ck_0^\beta + \rho^{-1}\ln k_0 + C_{11} \ge \ln(\rho k_0) \quad \text{for} \quad k_0 > k^*. \tag{59}$$

Defining $h(k_0) \equiv Ck_0^\beta + \rho^{-1}\ln k_0 + C_{11} - \ln(\rho k_0)$. By our chosen values of C and k^*, we see that $h(k^*) = h'(k^*) = 0$. We also have

$$h''(k_0) = \underbrace{C\beta(\beta-1)k_0^{\beta-2}}_{>0} + \underbrace{\left(1-\rho^{-1}\right)k_0^{-2}}_{<0}$$

by (58) and (49). Using (58) again, we obtain

$$h''(k_0) > 0 \Longleftrightarrow \frac{\rho-1}{\rho(k^*)^\beta}(\beta-1)k_0^\beta > \rho^{-1} - 1 \Longleftrightarrow (1-\beta)k_0^\beta > (k^*)^\beta. \tag{60}$$

Thus, if (60) holds, then we have $h''(k_0) > 0$ for all $k_0 > k^*$. As such, (59) follows as long as (60) is satisfied.

(ii) Outside of the continuation region D, we have $\varphi(k_0) = \ln(\rho k_0)$ from (53), and hence, by using (45), we obtain

$$\begin{aligned}
&\mathcal{A}_0\varphi(k_0) = -\ln(\rho k_0) + \left(\mu - \frac{1}{2}\sigma^2\right) \le 0, \quad \forall \bar{k} \le k_0 \le k^* \\
&\iff \ln k_0 \ge -\ln\rho + \frac{\mu - \frac{1}{2}\sigma^2}{\rho}, \quad \forall \bar{k} \le k_0 \le k^* \\
&\iff k_0 > \frac{1}{\rho}\exp\left(\frac{\mu - \frac{1}{2}\sigma^2}{\rho}\right), \quad \forall \bar{k} \le k_0 \le k^*.
\end{aligned} \tag{61}$$

Thus, we directly put

$$\bar{k} \equiv \frac{1}{\rho}\exp\left(\frac{\mu - \frac{1}{2}\sigma^2}{\rho}\right). \tag{62}$$

Combining (61) with (46) gives rise to

$$\frac{1}{\rho}\exp\left(\frac{\mu - \frac{1}{2}\sigma^2}{\rho}\right) \le k_0 \le k^* \le \rho^{\frac{1}{\rho-1}}\exp\left(\frac{\mu - \frac{1}{2}\sigma^2}{\rho - 1}\right)$$

which yields

$$\rho^2 \ln\left(\frac{1}{\rho}\right) \ge \mu - \frac{1}{2}\sigma^2. \tag{63}$$

Thus, provided the definition of $\bar{k}$ given in (62), (61) holds when (63) is satisfied.

(iii) To check if $\tau^* < \infty$ almost surely. We first need to find a (strong) solution to the following stochastic differential equation:

$$dk(t) = \mu k(t)dt + \sigma k(t)dB(t). \tag{64}$$

By using (64) and applying Itô's rule, we get that

$$k(t) = k_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B(t)\right] \tag{65}$$

We see that if

$$\mu > \frac{1}{2}\sigma^2 \tag{66}$$

and $\sigma > 0$, then, by the law of the iterated logarithm of Brownian motion, we get that $\lim_{t\to\infty} k(t) = \infty$, almost surely. Particularly, we get $\tau^* < \infty$ almost surely.

(iv) Now, we are about to show that ϕ is uniformly integrable for $\tau^* \in \mathcal{T}$. First, conditional on (62), we argue that (53) can be expressed as follows:

$$\varphi(k_0) = \begin{cases} \ln(\rho k_0) \equiv g_2(k_0), & \bar{k} \le k_0 \le k^* \\ Ck_0^{\beta} + \rho^{-1}\ln k_0 + C_{11}, & k_0 > k^* \end{cases} \tag{67}$$

where β is given by (49) while $C > 0$, C_{11}, and k^* are given by (58), (52), and (57), respectively. The minimum value of the initial level of capital stock is given by

$$\bar{k} = \frac{1}{\rho}\exp\left(\frac{\mu - \frac{1}{2}\sigma^2}{\rho}\right) > 0. \tag{68}$$

Noting from (57) that $k^* < \infty$, thus $[\bar{k}, k^*]$ is compact by using the Heine-Borel theorem. Accordingly, ϕ is uniformly bounded on $[\bar{k}, k^*]$ by applying the fact that $\phi \in C^2(\mathbb{R}^2)$ and the well-known Weierstrass theorem. So, it suffices to check that $\left\{\exp(-\rho\tau)[Ck^{\beta}(\tau) + \rho^{-1}\ln k(\tau) + C_{11}]\right\}_{\tau\in\mathcal{T}}$ is uniformly integrable on (k^*, ∞), where the uniform topology is naturally induced by the norm, which is induced by the inner product of Hilbert space $L^2(\Omega, P)$. However, noting that we have chosen $\beta = \beta_2 < 0$ in (49), thus $k_0^{\beta} \to 0$ as $k_0 \to \infty$. In consequence, ignoring the constants, it suffices to demonstrate that there exists an upper bound $M < \infty$ such that

$$\mathbb{E}\left\{\exp(-2\rho\tau)[\ln k(\tau)]^2\right\} \le M \quad \text{for all} \quad \tau \in \mathcal{T} \quad \text{and} \quad k(\tau) > k^*. \tag{69}$$

Since we have $0 < \ln k(t) \le k(t)$ on (k^*, ∞). Hence, by (65), we have

$$\mathbb{E}\left\{\exp(-2\rho\tau)[\ln k(\tau)]^2\right\} \le \mathbb{E}\left\{\exp(-2\rho\tau)[k(\tau)]^2\right\} = k_0^2\exp[(2\mu + \sigma^2 - 2\rho)\tau].$$

We hence conclude that if

$$\mu + \frac{1}{2}\sigma^2 < \rho, \tag{70}$$

then (69) holds true and so does (iv). The Proof is completed by combining (70) with (66) and (63) to establish the range of the relevant parameters. □

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