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Two further impossibility results on responsive affirmative action in school choice

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HIGHLIGHTS

- This paper investigates the responsiveness of the priority-based affirmative action in school choice.
- We consider two efficiency-improved mechanisms—Kesten's EADAM and the DA-TTC mechanism.
- We show that neither EADAM nor DA-TTC is minimally responsive to the priority-based affirmative action policy.

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ABSTRACT

This paper investigates the responsiveness of priority-based affirmative action in school choice. We consider two efficiency-improved mechanisms—Kesten's EADAM (efficiency-adjusted deferred acceptance mechanism) and DA-TTC mechanism (running deferred acceptance and then running top trading cycles algorithm). We show that neither EADAM nor DA-TTC is minimally responsive to the priority-based affirmative action policy. That is, under EADAM or DA-TTC, there are market situations in which a stronger affirmative action policy may result in a Pareto inferior assignment for the minority students.

1. Introduction

School choice programs aim to give students the option to choose their school. At the same time, underrepresented students should be favored to close the opportunity gap. Affirmative action policies that give minority students higher chances to attend their desired schools have been playing an important role in achieving this goal in the United States and many other countries.

There are three popular affirmative action policies in school choice: *quota-based, reserve-based, and priority-based* affirmative actions. The *quota-based* (or *majority-quotas* type) affirmative action policy in school choice gives minority students higher chances to attend more preferred schools by *limiting the number of admitted majority students* at some schools. There are many examples of majority-quotas type public school admission policies in the United States.[1] The *reserve-based* affirmative action policy proposed by Hafalir et al. (2013) is to *reserve some seats at each school for the minority students*, and to require that a reserved seat at a school can be assigned to a majority student only if no minority student prefers that school to her assignment. The *priority-based* affirmative action favors minority students by means of *promoting their priorities at schools*. In Chinese college admissions, the minority students are favored by a priority-based affirmative action policy that awards bonus points to the minority students at the national college entrance exam.

Since the affirmative action policies in school choice aim to improve the welfare of the minority students, the *minimal* requirement for a satisfactory assignment mechanism should be: running such an mechanism, a stronger affirmative action makes *at least one* minority student better off in case there is a minority student who becomes worse off. In other words, a stronger affirmative

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[1] See, for instance, Hafalir et al. (2013), Ehlers et al. (2014) and Doğan (2016) for detailed discussions, including the disadvantages of the policy, such as avoidable inefficiency.

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action should not hurt a minority student without benefiting anyone of the minority students. If an assignment mechanism satisfies this property, then we say that it is *minimally responsive to the affirmative action*, or simply *minimally responsive*.[2]

According to Kojima (2012) and Hafalir et al. (2013), we know that, on the full domain of school choice problems, there exists no stable mechanism that is minimally responsive to any type of the three popular affirmative action policies. A natural question is: If we relax the stability requirement, can we obtain a desirable result. Afacan and Salman (2016) show that the Boston mechanism (which is not stable) is minimally responsive to the priority-based and reserve-based affirmative action policies. For another unstable mechanism—the top trading cycles mechanism (TTC), Kojima (2012) shows that it is not minimally responsive to priority-based or quota-based affirmative action policies. Following this direction, this paper considers two unstable mechanisms—Kesten's EADAM and DA-TTC mechanism. The prominent advantage of these two mechanisms is that they can improve the efficiency based on the DA matching. We show that neither EADAM nor DA-TTC mechanism is minimally responsive to the priority-based affirmative action (Theorems 1 and 2).

2. Model

Let S and C be finite and disjoint sets of **students** and **schools**. There are two types of students: **minority students** and **majority students**. Let S^m and S^M denote the sets of minority and majority students, respectively. They are nonempty sets such that $S^m \cup S^M = S$ and $S^m \cap S^M = \emptyset$. Suppose that $|C|, |S| \geq 2$.

For each student $s \in S$, $\succ_s$ is a strict (i.e., complete, transitive, and anti-symmetric) preference relation over $C \cup \{s\}$, where s denotes the outside option, which can be attending a private school or being home-schooled. School c is acceptable to student s if $c \succ_s s$. The list of preferences for a group of students S' is denoted by $\succ_{S'} = (\succ_s)_{s \in S'}$. For each school $c \in C$, $\succ_c$ is a strict priority order over S and being unmatched (being unmatched is denoted by $\varnothing$). Student s is acceptable to school c if $s \succ_c \varnothing$. The list of priorities for a group of schools C' is denoted by $\succ_{C'} = (\succ_c)_{c \in C'}$. For agents i, j in C (resp. S) and k in S (resp. C), $i \succeq_k j$ denotes either $i \succ_k j$ or $i = j$.

For each $c \in C$, q_c is the capacity of c or the number of seats at school c. We assume that there are enough seats for all students, so $\sum_{c \in C} q_c \geq |S|$. Let $q = (q_c)_{c \in C}$ be the capacity profile.

A *school choice problem* or simply a **problem** is a tuple $G = (S^m, S^M, C, \succ_S, \succ_C, q)$. Since S^m, S^M, C and q will be fixed, we also denote a problem by $(\succ_S, \succ_C)$.

For **priority-based affirmative action** problems, we say that $G' = (\succ_S, \succ'_C)$ **has a stronger priority-based affirmative action policy** than $G = (\succ_S, \succ_C)$ if, for every $c \in C$ and any $s, s' \in S$, (1) if $s \succeq_c s'$ and $s \in S^m$ then $s \succeq'_c s'$, (2) if $s, s' \in S^M$ and $s \succeq_c s'$, then $s \succeq'_c s'$. In other words, a stronger priority-based affirmative action policy promotes the ranking of minority students at schools relative to majority students while keeping the relative ranking of each student within her own group fixed.

A matching is an assignment of students to schools such that each student is assigned to a school or to her outside option, and no more students are assigned to a school than its capacity. Formally, a **matching** μ is a mapping from $C \cup S$ to the subsets of $C \cup S$ such that

(1) $\mu(s) \in C \cup \{s\}$ for every $s \in S$,
(2) $\mu(c) \subseteq S$ and $|\mu(c)| \leq q_c$ for every $c \in C$,
(3) $\mu(s) = c$ if and only if $s \in \mu(c)$ for every $c \in C$ and $s \in S$.

A matching μ' is **Pareto inferior to μ for the minority** if (i) $\mu(s) \succeq_s \mu'(s)$ for every $s \in S^m$; and (ii) $\mu(s) \succ_s \mu'(s)$ for some $s \in S^m$. A **mechanism** ϕ chooses a matching $\phi(G)$ for each problem G.

Definition 1. A matching mechanism ϕ is said to **be minimally responsive to the priority-based affirmative action** if there are no problems G and $\tilde{G}$ such that $\tilde{G}$ has a stronger priority-based affirmative action policy than G and $\phi(\tilde{G})$ is Pareto inferior to $\phi(G)$ for the minority.

3. Results

In this section, we study the minimal responsiveness of two efficiency-improved mechanisms—Kesten's EADAM and DA-TTC mechanism. The appealing advantage of these two mechanisms is that they are Pareto efficient (see Kesten (2010) and Cantala and Pápai (2014)).

For completeness, we first present the EADAM with all students consenting. For any school choice problem G, Kesten's EADAM with all students consenting operates as follows:

Round 0 Run DA for the problem G.[3]

Round k, $k \geq 1$ In the DA procedure of a school choice problem, if student s is tentatively accepted by school c at some Step t and is later rejected by school c at some Step $t' > t$, and there is at least one other student who is rejected by school c at some Step r such that $t \leq r < t'$, then we call student s an **interrupter** at school c, and the pair (s, c) an **interrupting pair** of Step t'. Identify the last step of the round-$(k-1)$ DA procedure in which there is (are) some interrupter(s) being rejected, and then identify all interrupting pairs of this step and, for each pair, remove the respective school from the interrupter's preference. After that, rerun DA (round-k DA) with the new preference profile.

Stop when there are no more interrupters.

According to the procedure of EADAM, one can see that EADAM iteratively removes the interrupters in DA. Recently, Ju et al. (2018) study the minimal responsiveness of the reserve-based affirmative action policy. They propose a new (unstable) mechanism called the efficiency-improved DA with minority reserves (EIDA^m), which operates through iteratively removing the under-demanded schools,[4] instead of removing the interrupters in the deferred acceptance with minority reserves. Tang and Yu (2014) show that removing the under-demanded schools and removing the interrupters in the deferred acceptance produce the same matching outcome. Ju et al. (2018) obtain that EIDA^m is Pareto efficient and is minimally responsive to the reserve-based affirmative action policy. It is known that EADAM is Pareto efficient, for the priority-based affirmative action it would be interesting to check whether EADAM is minimally responsive. In this paper we first have the following result.

Theorem 1. *EADAM (with all students consenting) is not minimally responsive to the priority-based affirmative action policy.*

Proof. We consider the following example. Let $C = \{c_1, c_2, c_3, c_4\}$, $S^M = \{s_1, s_3, s_4\}$ and $S^m = \{s_2\}$. All schools have a capacity of 1: $q = (1, 1, 1, 1)$. Students' preferences and schools' priorities are given by the following table:

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$	$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$	$\succ_{c_4}$
c_1	c_1	c_2	c_3	s_4	s_3	s_2	s_1
c_4	c_2	c_4	c_1	s_1	s_2	s_4	s_3
$\vdots$	c_3	$\vdots$	$\vdots$	s_2	$\vdots$	$\vdots$	$\varnothing$
	c_4			s_3			
	s_2			$\varnothing$			

[2] Kojima (2012) first introduces this concept. He calls it "respecting the spirit of affirmative action". Recently, Doğan (2016) starts to use the notion of "minimal responsiveness".

[3] The DA (deferred acceptance) algorithm was originally introduced by Gale and Shapley (1962). It has been extensively studied for its stability and other desirable properties.

[4] See Kesten and Kurino (2016). Given a matching μ for a problem G, a school c is said to be *under-demanded* in matching μ if all the students (in G) weakly prefer their assignments under μ to school c, that is, $\mu(s) \succeq_s c$ for each student s.

Then the procedure of the DA mechanism under $(\succ_C, \succ_S)$ is as follows:

Step	c_1	c_2	c_3	c_4
1	$\boxed{s_1}$, s_2	$\boxed{s_3}$	$\boxed{s_4}$	
2		s_2, $\boxed{s_3}$		
3			$\boxed{s_2}$, s_4	
4	s_1, $\boxed{s_4}$			
5	$\boxed{s_4}$	$\boxed{s_3}$	$\boxed{s_2}$	$\boxed{s_1}$

One can see that s_1 is the only interrupter (at c_1) in the procedure of DA algorithm. We remove c_1 from s_1's preference and rerun DA with the new preference profile. Then the matching produced by EADAM is

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_2 & s_3 & s_4 & s_1 \end{pmatrix}.$$

If we change the schools' priorities as follows: $\succ'_{c_2}$: $s_2, s_3, \ldots$, and $\succ'_{c_i} = \succ_{c_i}$ for other schools. Then the procedure of the DA mechanism under $(\succ'_C, \succ_S)$ is as follows:

Step	c_1	c_2	c_3	c_4
1	$\boxed{s_1}$, s_2	$\boxed{s_3}$	$\boxed{s_4}$	
2		$\boxed{s_2}$, s_3		
3	$\boxed{s_1}$	$\boxed{s_2}$	$\boxed{s_4}$	$\boxed{s_3}$

One can see that there exists no interrupter in the procedure of DA algorithm. Then DA and EADAM produce the same matching as follows:

$$\mu' = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_1 & s_2 & s_4 & s_3 \end{pmatrix}.$$

One can check that $(\succ'_C, \succ_S)$ has a stronger priority-based affirmative action policy than$(\succ_C, \succ_S)$. However, running EADAM, student s_2 under $(\succ'_C, \succ_S)$ is worse than under $(\succ_C, \succ_S)$. Then, under EADAM, the stronger affirmative action policy makes all minority student (only one- s_2) worse off. Thus, EADAM is not minimally responsive to the priority-based affirmative action policy. □

Next we consider another efficient mechanism—DA-TTC mechanism (deferred acceptance then top trading cycles mechanism). We specify the procedure of the DA-TTC mechanism as follows:

Round 1. For any school choice problem G, run DA algorithm, denote the matching by $\mu = DA(G)$.

Round 2. Allow students to trade by running TTC. For each student $s \in S$, using the DA assignment $\mu(s)$ as her ownership right in the TTC mechanism.[5]

It is easy to show that DA-TTC mechanism will guarantee that students always receive an assignment that is no worse than their DA matching, which is not guaranteed by using TTC alone. Cantala and Pápai (2014) show that the DA-TTC mechanism is Pareto efficient for students. However, the DA-TTC mechanism is not minimally responsive to the priority-based affirmative action policy. Specifically, we consider the example in the proof of Theorem 1 above. It is easy to check that, with respect to $(\succ_C, \succ_S)$, the assignment produced by DA-TTC is

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_2 & s_3 & s_4 & s_1 \end{pmatrix}.$$

And for $(\succ'_C, \succ_S)$, the assignment produced by DA-TTC is

$$\mu' = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_1 & s_2 & s_4 & s_3 \end{pmatrix}.$$

We know that $(\succ'_C, \succ_S)$ has a stronger priority-based affirmative action policy than$(\succ_C, \succ_S)$. However, running DA-TTC, student s_2 under $(\succ'_C, \succ_S)$ is worse than under $(\succ_C, \succ_S)$. Under DA-TTC, the stronger affirmative action policy makes all minority student (only one- s_2) worse off. Thus, DA-TTC is not minimally responsive to the priority-based affirmative action. We conclude this result as follows:

Theorem 2. *DA-TTC mechanism is not minimally responsive to the priority-based affirmative action policy.*

4. Discussion

Kojima (2012) obtains that both DA and TTC mechanism are not minimally responsive to the priority-based affirmative action policy. In this paper, we considered the minimal responsiveness of two Pareto efficient (for students) mechanisms—EADAM and DA-TTC, and obtained impossibility results. Our results complement those in Kojima (2012). From the counterexample of the present paper, one can also see that neither EADAM nor DA-TTC respects improvements (in the sense of Balinski and Sönmez (1999)). We know that both EADAM and DA-TTC are not stable or fair mechanisms. Then such a by-product of our counterexample complements results (Theorem 5 and 6) in Balinski and Sönmez (1999).

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[5] The original top trading cycles algorithm was proposed for housing markets and is attributed to David Gale by Shapley and Scarf (1974). It attracted extensive attention for its strategy-proofness and efficiency property.