

On the Existence of Optimal Truth-Dominant Mechanisms

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Abstract

The existence of optimal dominant strategy mechanisms has been shown only for quadratic valuation functions. This paper shows that non-trivial Pareto-efficient and truth-dominant mechanisms exist for a slightly larger set of valuation functions.

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JEL classification: C72; D71; H21

1. Introduction

In the earlier literature on mechanism design, a central question asked was: Is there a mechanism that yields Pareto-efficient allocations when individuals' self-interested behavior is described by dominant strategy equilibrium? That is, is there a mechanism that simultaneously yields Pareto-efficient allocations and provides individual agents with incentives to report their true preferences honestly? Most results in the literature, however, are negative. Hurwicz (1975), Green and Laffont (1979), Walker (1980), Hurwicz and Walker (1990), and others have shown that no mechanism can always yield Pareto-optimal outcomes if truthful behavior is always a dominant strategy for each of the mechanism's participants when the class of economic environments is sufficiently 'thick'. Because of these impossibility results on dominant/Bayesian incentive compatibility, economists turn to other equilibrium solution concepts such as Nash equilibrium and his refinements, and get positive results such as those in Groves and Ledyard (1977), Maskin (1977), Hurwicz (1979), Walker (1981), Moore and Repullo (1988), and Tian (1989), among many others.

However, when the set of economic environments is 'thin', an optimal incentive-compatible mechanism under complete/incomplete information may exist. Groves and Loeb (1975) were the first to give an optimal truth-dominant mechanism for the class of quadratic valuation functions when they studied optimal taxation issues. Green and Laffont (1979) and Laffont

and Maskin (1980) then provided the necessary and sufficient conditions for checking the existence of an optimal dominant mechanism. The positive result on the existence of an optimal dominant mechanism, however, has been given only for the class of quadratic valuation functions. In fact, there is a conjecture made by Laffont and Maskin (1980) and believed by many others that the quadratic case is the only non-trivial case which permits Pareto-efficient and truth-dominant mechanisms for public goods economic environments.

This paper deals with characterizing a class of transferable utility functions $u_i(x_i, y) = x_i + v_i(y, \theta)$ in public goods economies, where the valuation functions v_i come from a wider class than that of quadratic ones, such that there exists a Pareto-optimal and truth-dominant mechanism. Thus our results generalize the results of Green and Laffont (1979) and Laffont and Maskin (1980).

2. The model and truth-dominant mechanisms

In an economy with public goods, there are n agents who consume one private good and one public good, x being private and y public. The private good x can be thought of as money, and the public good y can be thought of as a public project. Denote by $N = \{1, 2, \dots, n\}$ the set of agents. The utility function of agent i , $u_i(x_i, y)$, is transferable or quasi-linear: $u_i(x_i, y) = x_i + v_i(y, \theta_i)$, where v_i is a real-valued function and is called agent i 's valuation function, θ_i lying in a space Θ_i is a parameter of the valuation function, and where for simplicity we assume that $y \in R_+$. Since the endowments of the private good play no role in the paper, we will always interpret x_i as the increment (or transfer) of the private good accruing to agent i ; thus, we define the feasible states to be whatever satisfies the condition¹

$$\sum_{i=1}^n x_i \leq 0. \quad (1)$$

Then Pareto-optimal states can be characterized as those which, at the same time, maximize the aggregate valuation $\sum_{i=1}^n v_i: R_+ \times \Theta_i \rightarrow R$, and also satisfy the feasibility constraint (1) with equality.

The decision-maker is assumed to know the functional forms $v_i(\cdot, \cdot)$ but is ignorant of the true value $\hat{\theta}_i$ of the parameter θ_i which identifies agent i 's tastes for the public good. The purpose of a mechanism is to choose an optimal level of the public good in this framework of imperfect information. The decision rule, or mechanism, for choosing an outcome (more precisely, a direct revelation mechanism) is a mapping, $(n+1)$ -tuple $h = (h_y, h_{x_1}, \dots, h_{x_n})$ of functions on $\Theta = \prod_{i=1}^n \Theta_i$, with the outcome determined by $y = h_y(\theta)$ and $x_i = h_{x_i}(\theta)$. A mechanism $h: \Theta \rightarrow R_+ \times R^n$ is said to be *continuously differentiable* or C^1 if h is continuously differentiable. A mechanism $h: \Theta \rightarrow R_+ \times R^n$ is said to be *balanced* if for all $\theta \in \Theta$, $\sum_{i=1}^n h_{x_i}(\theta) = 0$. A mechanism $h: \Theta \rightarrow R_+ \times R^n$ is said to be *successful* if $h_y(\theta)$ maximizes the aggregate valuation $\sum_{i=1}^n v_i$, i.e. if

¹ Here, like Green and Laffont (1979) and Laffont and Maskin (1980), we have ignored the lower bound restrictions of individuals' transfers.

$$\sum_{i=1}^n v_i(h_y(\theta), \theta_i) \geq \sum_{i=1}^n v_i(y, \theta_i) \quad \text{for all } y \in R_+ \text{ and } \theta \in \Theta. \quad (2)$$

A mechanism $h: \Theta \rightarrow R_+ \times R^n$ is truth dominant (or strongly individually incentive compatible) if the truth is a dominant strategy for each individual, i.e. if for any $i \in N$, any $\theta \in \Theta$

$$v_i(h_y(\hat{\theta}_i, \theta_{-i}), \hat{\theta}_i) + h_{xi}(\hat{\theta}_i, \theta_{-i}) \geq v_i(h_y(\theta_i, \theta_{-i}), \theta_i) + h_{xi}(\theta_i, \theta_{-i}),$$

where $\theta_{-i} \in \Theta_{-i} = \prod_{j \neq i} \Theta_j$.

A mechanism $h: \Theta \rightarrow R_+ \times R^n$ is said to be a *Groves mechanism* (see Groves (1973)) if it is successful and the transfer outcomes are given by

$$h_{xi}(\theta) = \sum_{j \neq i} v_j(h_y(\theta), \theta_j) + g_i(\theta_{-i}), \quad (3)$$

where $g_i(\cdot)$ is an arbitrary function from Θ_{-i} to R . Thus a Groves mechanism is Pareto efficient if and only if it is balanced. Green and Laffont (1979), Walker (1978), Laffont and Maskin (1980), and others have shown that, under certain fairly general assumptions about the admissible set $V = \{v_1(\cdot, \theta_1), \dots, v_n(\cdot, \theta_n)\}$, a mechanism is truth dominant and successful if and only if it is a Groves mechanism. It follows that the only mechanisms which are both Pareto efficient and truth dominant are balanced Groves mechanisms.

When valuation functions are differentiable, Green and Laffont (1979), and Laffont and Maskin (1980) gave a necessary and sufficient condition for the existence of Pareto-optimal and truth-dominant mechanisms (balanced Groves mechanisms) by making the following assumption:

Assumption 1. For $i \in N$, let Θ_i be an open interval in R and $v_i: R_+ \times \Theta_i \rightarrow R$ be a continuously differentiable function such that for any $\theta \in \Theta$, there exists $y^*(\theta) \in R_+$ for which: (i) $\sum_{i=1}^n v_i(y^*(\theta), \theta_i) = \max_{y>0} \sum_{i=1}^n v_i(y, \theta_i)$, and (ii) $y^*(\theta)$ is continuously differentiable.

Note that by Eq. (3) and Assumption 1, we have

$$\frac{\partial h_{xi}}{\partial \theta_i}(\theta_i, \theta_{-i}) \equiv -\frac{\partial v_i}{\partial h_y(\theta)} \cdot \frac{\partial h_y(\theta)}{\partial \theta_i} \quad (4)$$

for $h_y(\theta) = y^*(\theta)$.

They then prove that under Assumption 1 there exists a Pareto-optimal and truth-dominant mechanism for the class of admissible C^1 -functions $V = \{v_1(\cdot, \theta_1), \dots, v_n(\cdot, \theta_n)\}$ if and only if

$$\sum_{i=1}^n \frac{\partial^{n-1}}{\partial \theta_{-i}} \left[\frac{\partial v_i}{\partial y} \cdot \frac{\partial h_y}{\partial \theta_i} \right] \equiv 0.$$

Here the notation $\partial^{n-1}/\partial \theta_{-i}$ stands for $\partial^{n-1}/\partial \theta_1, \dots, \theta_{i-1} \theta_{i+1}, \dots, \theta_n$. They also prove that there exists no Pareto-efficient and truth-dominant C^1 -mechanism for $n = 2$. For $n \geq 3$, the possibility results for the existence of balanced Groves mechanisms for specific functional forms of valuation functions are obtained in the literature (cf. Groves and Loeb, 1975; Green and Laffont, 1979; and Laffont and Maskin, 1980), but only for the family of quadratic functions, and Laffont and Maskin (1980) conjectured that the quadratic case is the only non-trivial case which permits balance.

3. Optimal truth-dominant mechanisms

In this section we investigate the existence of Pareto-efficient and truth-dominant mechanisms for a family of admissible valuation functions which have specific functional forms, and which contain the quadratic functions as special cases. The family of admissible valuation functions we consider satisfies Assumption 1 and has the form of

$$V = \left\{ v_i(y, \theta_i) = f(y)\psi_i(\theta_i) - \frac{1}{\alpha c} (cf(y) + d)^\alpha, \theta_i \in \Theta_i, i \in N \right\}, \quad (5)$$

where $\alpha > 1$, $c > 0$, $d \geq 0$, $f(y) \geq 0$ for all y , and f and ψ_i are both continuously differentiable and their derivatives are non-zero. Note that the valuation functions defined in (5) reduce to the quadratic function when $f(y) = y$, $\psi_i(\theta_i) = \theta_i$, $\alpha = 2$, $c = 1$, and $d = 0$.

Let y^* be an extreme point of $\sum_{i=1}^n v_i(y, \theta_i)$, we then have $\sum_{i=1}^n \partial v_i(y^*, \theta_i) / \partial y = 0$, and thus $(cf(y^*) + d)^{\alpha-1} = \bar{\psi}(\theta)$, where $\bar{\psi}(\theta) = (1/n) \sum_{i=1}^n \psi_i(\theta_i)$. Then

$$f(y^*) = \frac{1}{c} [\bar{\psi}(\theta)^{1/(\alpha-1)} - d]. \quad (6)$$

Since $\sum_{i=1}^n \partial^2 v_i(y^*, \theta_i) / \partial y^2 = -nc(\alpha-1)f'(y^*)^2(cf(y^*) + d)^{\alpha-2} < 0$, we know that y^* maximizes $\sum_{i=1}^n v_i(y, \theta_i)$. Denote $h_y(\theta) = y^*(\theta)$. Implicitly differentiating (6) yields

$$\frac{\partial h_y(\theta)}{\partial \theta_i} = \frac{\bar{\psi}(\theta)^{1/(\alpha-1)-1} \psi'_i(\theta_i)}{cn(\alpha-1)f'(y^*)}, \quad (7)$$

which is not equal to zero for any $\theta_i \in \Theta_i$ since $\psi'_i \neq 0$ and $f' \neq 0$. The mechanism is then non-trivial. Thus,

$$\frac{\partial h_{x_i}}{\partial \theta_i}(\theta_i, \theta_{-i}) = -\frac{1}{(\alpha-1)cn} \psi'_i(\theta_i) \psi_i(\theta_i) \bar{\psi}(\theta)^{1/(\alpha-1)-1} + \frac{1}{(\alpha-1)cn} \psi'_i(\theta_i) \bar{\psi}(\theta)^{1/(\alpha-1)}. \quad (8)$$

By the results of Green and Laffont (1979) and Laffont and Maskin (1980) we know that a balanced Groves mechanism exists if and only if

$$\sum_{i=1}^n \frac{\partial^{n-1}}{\partial \theta_{-i}} \left[\frac{\partial v_i}{\partial y} \cdot \frac{\partial h_y}{\partial \theta_i} \right] \equiv 0.$$

Let $m = \alpha/(\alpha-1)$. Then $\alpha = m/(m-1)$ ($m \neq 1$) and, by (8), we have

$$\frac{\partial v_i}{\partial h_y(\theta)} \cdot \frac{\partial h_y(\theta)}{\partial \theta_i} = \frac{m-1}{cn} \psi'_i(\theta_i) \psi_i(\theta_i) \bar{\psi}(\theta)^{m-2} - \frac{m-1}{cn} \psi'_i(\theta_i) \bar{\psi}(\theta)^{m-1} \quad (9)$$

for $m \neq 1$. Utilizing (9), we have

$$\sum_{i=1}^n \frac{\partial^{n-1}}{\partial \theta_{-i}} \left[\frac{\partial v_i}{\partial y} \cdot \frac{\partial h_y}{\partial \theta_i} \right] = \frac{n-1}{cn^{n-1}} (m-1)(m-2), \dots, (m-n+1) \bar{\psi}(\theta)^{m-n} \prod_{j=1}^n \psi'_j(\theta_j) \quad (10)$$

for $m \neq 1$. Thus,

$$\sum_{i=1}^n \frac{\partial^{n-1}}{\partial \theta_{-i}} \left[\frac{\partial v_i}{\partial y} \cdot \frac{\partial h_y}{\partial \theta_i} \right] \equiv 0$$

if and only if m is an integer satisfying $2 \leq m < n$. Hence a balanced Groves mechanism exists if and only if $\alpha/(\alpha - 1)$ is an integer greater than one and less than n , i.e. if and only if $\alpha \in \{2, \frac{3}{2}, \frac{4}{3}, \dots, (n - 1)/(n - 2)\}$.

Then, we have the following theorem:

Theorem 1. For a given class of admissible valuation functions which satisfies Assumption 1 and is given by $V = \{v_i(y, \theta_i) = f(y)\psi(\theta_i) - (1/\alpha c) (cf(y) + d)^\alpha, \theta_i \in \Theta_i, i \in N\}$, where $\alpha > 1$, $c > 0$, $d \geq 0$, $f(y) \geq 0$ for all y , and where f and ψ_i are both continuously differentiable with $f' \neq 0$ and $\psi'_i \neq 0$, there exists a Pareto-efficient and truth-dominant C^1 -mechanism if and only if $n \geq 3$ and $\alpha \in \{2, \frac{3}{2}, \frac{4}{3}, \dots, (n - 1)/(n - 2)\}$.

Thus as long as the family of valuation functions has the form

$$v_i(y, \theta_i) = f(y)\psi_i(\theta_i) - \frac{(m - 1)}{mc} (cf(y) + d)^{m/(m-1)}$$

for $2 \leq m < n$, a balanced Groves C^1 -mechanisms exist. Note that $\alpha \leq 2$. And, when $n = 3$, only $\alpha = 2$ permits a Pareto-optimal and truth-dominant mechanism.

Remark 1. Even though, for the family of admissible valuation functions specified in (5), a balanced Groves mechanisms exist only for $\alpha \in \{1, \frac{3}{2}, \frac{4}{3}, \dots, (n - 1)/(n - 2)\}$ which is a finite set, the set of admissible valuation functions is an uncountable infinite set since d and c can be arbitrary numbers and f and ψ_i can be any continuously differentiable functions with $f' \neq 0$ and $\psi'_i \neq 0$. I do not know whether or not the family of valuation functions specified by (5) is the only family of valuation functions which permits the possible Pareto-efficient and truth-dominant mechanisms.

In what follows we compute the explicit formulas for the transfer outcomes $h_{x_i}(\theta)$ of the mechanism. Integrating (8), we have

$$h_{x_i}(\theta) = -\frac{1}{c} \psi_i(\theta_i) \bar{\psi}(\theta)^{1/(\alpha-1)} + \frac{n\alpha - n + 1}{c\alpha} \bar{\psi}(\theta)^{\alpha/(\alpha-1)} + g_i^1(\theta_{-i}), \tag{11}$$

where $g_i^1(\cdot)$ and $g_i^2(\cdot)$ are arbitrary functions from Θ_{-i} to R . As a matter of fact, by using (3), we can also have

$$h_{x_i}(\theta) = -\frac{1}{c} \psi_i(\theta_i) \bar{\psi}(\theta)^{1/(\alpha-1)} + \frac{n\alpha - n + 1}{c\alpha} \bar{\psi}(\theta)^{\alpha/(\alpha-1)} - \frac{d}{c} \sum_{j \neq i} \psi_j(\theta_j) + g_i(\theta_{-i}). \tag{12}$$

Thus, $g_i(\theta_{-i})$ differs from $g_i^1(\theta_{-i})$ in (11) by a function of θ_{-i} , namely by a term of $-d/c \sum_{j \neq i} \psi_j(\theta_j)$.

In order for a Groves mechanism to be balanced, we need

$$\sum_{i=1}^n h_{xi}(\theta) = n(n-1) \frac{\alpha-1}{c\alpha} \bar{\psi}(\theta)^{\alpha/(\alpha-1)} - \frac{d}{c} n(n-1) \bar{\psi}(\theta) + \sum_{i=1}^n g_i(\theta_{-i}) \equiv 0.$$

When $\alpha = m/(m-1)$, we have

$$\sum_{i=1}^n h_{xi}(\theta) = \frac{(n-1)}{cmn^{m-1}} \left[\sum_{i=1}^n \psi_i(\theta_i) \right]^m - \frac{d}{c} (n-1) \sum_{i=1}^n \psi_i(\theta_i) + \sum_{i=1}^n g_i(\theta_{-i}) \equiv 0. \quad (13)$$

For each given value of α , e.g. for each given $2 \leq m < n$, there are many ways to construct the transfers of individuals which permit a balanced mechanism. However, if we want functional forms of the transfer functions to be the same among individuals, i.e. if $g_i(\theta_{-i}) = g(\theta_{-i})$, then a unique balanced Groves mechanism exists.

Example 1. Suppose $m = 2$. In this case, $\alpha = 2$. One can easily check

$$\left[\sum_{i=1}^n \psi_i(\theta_i) \right]^2 = \sum_{i=1}^n \left[\frac{1}{n-1} \sum_{j \neq i} \psi_j(\theta_j)^2 + \frac{1}{n-2} \sum_{j \neq i} \sum_{k \neq i, j} \psi_j(\theta_j) \psi_k(\theta_k) \right].$$

Thus, if we define

$$g_i^1(\theta_{-i}) = -\frac{1}{2nc} \sum_{j \neq i} \psi_j(\theta_j)^2 - \frac{(n-1)}{2nc(n-2)} \sum_{j \neq i} \sum_{k \neq i, j} \psi_j(\theta_j) \psi_k(\theta_k),$$

then we have

$$h_{xi}(\theta) = -\frac{1}{c} \psi_i(\theta_i) \bar{\psi}(\theta) + \frac{n+1}{2c} \bar{\psi}(\theta)^2 - \frac{1}{2nc} \sum_{j \neq i} \psi_j(\theta_j)^2 - \frac{(n-1)}{2c(n-2)} \sum_{j \neq i} \sum_{k \neq i, j} \psi_j(\theta_j) \psi_k(\theta_k)$$

and thus

$$\sum_{i=1}^n h_{xi}(\theta) = 0$$

for all $\theta \in \Theta$, which means that the Groves mechanism defined in the above is balanced. Furthermore, when $f(y) = y$, $\psi_i(\theta_i) = \theta_i$, $d = 0$, and $c = 1$, the above mechanism reduces to the one given in Groves and Loeb (1975) and Green and Laffont (1979).²

Example 2. Now suppose $m = 3$ and $n > 3$. In this case, $\alpha = \frac{3}{2}$. So the valuation function becomes

$$v_i(y, \theta_i) = f(y) \psi_i(\theta_i) - \frac{2}{3c} (cf(y) + d)^{3/2}$$

and the aggregate transfer outcomes become

² The coefficients of Formula (5.56) in Green and Laffont (1979, p. 94) are incorrect. The coefficients $1/N^2$ and $1/N^2(N-2)$ should be $1/N$ and $(N-1)/N(N-2)$, respectively.

$$\sum_{i=1}^n h_{xi}(\theta) = \frac{(n-1)}{3cn^2} \left[\sum_{i=1}^n \psi_i(\theta_i) \right]^3 + \sum_{i=1}^n g'_i(\theta_{-i}).$$

It can be checked that

$$\begin{aligned} \left[\sum_{i=1}^n \psi_i(\theta_i) \right]^3 &= \frac{1}{n-1} \sum_{i=1}^n \sum_{j \neq i} \psi_i(\theta_j)^3 + \frac{3}{n-2} \sum_{i=1}^n \sum_{j \neq i} \sum_{k \neq i,j} \psi_i(\theta_j)^2 \psi_k(\theta_k) \\ &+ \frac{1}{n-3} \sum_{i=1}^n \sum_{j \neq i} \sum_{k \neq i,j} \sum_{l \neq i,j,k} \psi_j(\theta_j) \psi_k(\theta_k) \psi_l(\theta_l). \end{aligned} \tag{14}$$

Thus, if we define

$$\begin{aligned} g^1_i(\theta_{-i}) &= -\frac{1}{3cn^2} \sum_{j \neq i} \psi_j(\theta_j)^3 - \frac{(n-1)}{cn^2(n-2)} \sum_{j \neq i} \sum_{k \neq i,j} \psi_j(\theta_j)^2 \psi_k(\theta_k) \\ &- \frac{(n-1)}{3cn^2(n-3)} \sum_{j \neq i} \sum_{k \neq i,j} \sum_{l \neq i,j,k} \psi_j(\theta_j) \psi_k(\theta_k) \psi_l(\theta_l), \end{aligned} \tag{15}$$

then we have

$$\begin{aligned} h_{xi}(\theta) &= -\frac{1}{c} \psi_i(\theta_i) \bar{\psi}(\theta) + \frac{2n+1}{3c} \bar{\psi}(\theta)^3 - \frac{1}{3cn^2} \sum_{j \neq i} \psi_j(\theta_j)^3 \\ &- \frac{(n-1)}{cn^2(n-2)} \sum_{j \neq i} \sum_{k \neq i,j} \psi_j(\theta_j)^2 \psi_k(\theta_k) \\ &- \frac{(n-1)}{3cn^2(n-3)} \sum_{j \neq i} \sum_{k \neq i,j} \sum_{l \neq i,j,k} \psi_j(\theta_j) \psi_k(\theta_k) \psi_l(\theta_l), \end{aligned} \tag{16}$$

and thus

$$\sum_{i=1}^n h_{xi}(\theta) = 0$$

for all $\theta \in \Theta$, which means that the Groves mechanism is balanced.

4. Conclusion

In this paper we have characterized the existence of an optimal dominant mechanism for a class of transferable utility functions in public goods economies with one private good and one public good. We extend our knowledge about the class of admissible utility functions for which there exist non-trivial Pareto-efficient and truth-dominant mechanisms. We show that, even for a family of admissible utility functions wider than the quadratic case, one can have a mechanism that simultaneously yields Pareto-efficient allocations and provides individual agents with incentives to report their true preferences honestly.

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