Estimating Price Responses of German Imports and Exports*

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Abstract

This paper estimates trade-demand functions for Germany from monthly data covering the period 1959–1988. It is assumed that these trade-demand functions have the form of the Linear Expenditure System, generated by a shifted Cobb-Douglas trade-utility function in which the shift parameter is postulated to be a function of time (including trend and seasonal components) and to have a stochastic term with a lognormal distribution. A procedure called generalized maximum likelihood is used, and the results are compared with those of nonlinear least squares as a benchmark. The approach is applied to two models: (1) a six-commodity model in which the dependent variables are net imports in six categories and the independent variables are six weighted averages of the import- and export-price indices for these categories as well as the trade deficit; (2) a twelve-commodity model in which the dependent variables are the gross imports and gross exports (the latter measured negatively) in the six categories and the independent variables are the twelve import- and export-price indices and the trade deficit. The latter model thus handles the case of "intra-industry trade".

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1 Introduction

Estimation of elasticities of demand for a country's imports and elasticities of supply for its exports is one of the oldest endeavors of applied econometrics, going back at least to Tinbergen (1946) and Chang (1951). The huge amount of activity that has been pursued in this field is evident from the extensive review undertaken by Stern, Francis, and Schumacher (1976). Still the best general treatment of this subject is to be found in Leamer and Stern (1970).

The models that have been used to measure the effects of world price changes on a country's imports and exports have for the most part been based on an analogy between a country and an individual; thus, demand for imports (usually taken in the aggregate) is specified to be a function of a price index of imports, a price index of related goods, and national income (cf., e.g., Leamer and Stern 1970, p. 9). However, it has been recognized since Samuelson's seminal work (1953) that a country's national income, being composed of factor incomes, depends upon factor rentals which—in the general equilibrium of a competitive open economy—in turn depend upon world prices. This suggests that it would be more appropriate to estimate a Marshallian offer function which relates imports and exports to world prices alone—or, if imbalances of trade are taken into account—to world prices and that part of national expenditure which does not depend on world prices, namely the deficit (or surplus) in the country's balance of trade. This approach was pursued in Chipman (1985b).

In Chipman (1985b) it was assumed that the trade-demand functions, which represent "trades" (imports if positive, exports if negative) as functions of external prices and the trade deficit, were of the LES (linear expenditure system) form introduced by Klein and Rubin (1948) and first applied to data by Stone (1954). Stone's algorithm for estimating the parameters was also used. Further, the dependent variables were net imports (imports less exports) in each statistical category, and the independent variables were weighted averages of the corresponding import and export price indices, as well as the trade balance. In the present paper a number of modifications to this approach have been introduced. First of all, in place of Stone's (1954) algorithm a variant of a method of estimation introduced by Chipman and Tian (1990) has been followed; this will be described in the following section. Secondly, import goods and export goods in the same statistical category, in addition to being aggregated as before, have also been treated as distinct commodity groups, allowing for the treatment of so-called "intra-industry trade"; in this case the dependent variables are gross imports and gross exports (the latter measured negatively), and the independent variables are the import and export price indices for these commodity groups, as well as the trade balance.

While we consider these refinements to constitute an improvement over past procedures, we are well aware that they still suffer from severe limitations. As will be explained in the next section, use of the Linear Expenditure System involves somewhat restrictive assumptions concerning the degree of price responsiveness of production. A more general procedure for allowing for price responses in production was proposed in Chipman (1990) requiring use of data on production and factor endowments; limitations of time and financial resources have prevented us from pursuing this approach in the present paper. A second limitation of the present model is that it is static; thus, investment is not taken into account, and intertemporal effects of price changes on the balance of payments are not allowed for. This limitation also holds, it may be remarked, for the traditional approaches surveyed in Stern, Francis, and Schumacher (1976). A multicommodity intertemporal model of an open economy has been developed by Chipman and Tian (1991), but the appropriate econometric procedures for this model have yet to be worked out. The present paper is therefore offered as providing what we believe to be some methodological improvements over previous approaches, but we regard it as constituting only a further step towards a more fully satisfactory approach.

For this type of work, Germany constitutes an ideal laboratory. Few countries have data on import and export price indices as opposed to unit values,[1] and all the studies surveyed in Stern et al. (1976) have employed the latter. Also, few countries possess data organized in so systematic and consistent a manner. Whether one can expect the data sets to continue after 1992 must be considered as uncertain, and thus the present time offers a unique opportunity to exploit this rich source of data.

[1]These countries are Germany, Japan, and Sweden, and to a limited extent Finland. Recently, South Korea, the Netherlands, and the United States have joined the list, but the data series are not yet long and frequent enough to be usable.

2 The Basic Model

2.1 Stochastic Formulation

We start with the assumption that the country acts as if it maximizes a stochastic, state-dependent, trade-utility function

$$\hat{U}(z_t;\alpha,\gamma_t,\varepsilon_t)=\sum_{j=1}^{m}\alpha_j\log(z_{tj}+\gamma_{tj}+\varepsilon_{tj})\quad(\alpha_j\geq 0,\sum_{j=1}^{m}\alpha_j=1)\tag{2.1}$$

subject to the constraint

$$\sum_{j=1}^{m}p_{tj}z_{tj}\leq D_t,\tag{2.2}$$

where $z_t=(z_{t1},z_{t2},\ldots,z_{tm})'$ is a vector of *trades* (quantities imported if positive, quantities exported if negative), $p_t=(p_{t1},p_{t2},\ldots,p_{tm})'$ is a vector of positive prices of traded goods, and D_t is the deficit in the country's balance of trade, where the t subscript denotes time. The term $\varepsilon_t=(\varepsilon_{t1},\varepsilon_{t2},\ldots,\varepsilon_{tm})'$ is a random error which is assumed to have a multivariate two-parameter lognormal distribution with 0 as lower bound.[2] Accordingly, its density function is given by

$$f(\varepsilon_t)=\frac{\prod_{j=1}^{m}\varepsilon_{tj}^{-1}}{(2\pi)^{m/2}|V|^{1/2}}\exp\left\{-\tfrac{1}{2}[\log\varepsilon_t-a]'V^{-1}[\log\varepsilon_t-a]\right\},\tag{2.3}$$

where $\log\varepsilon_t=(\log\varepsilon_{t1},\ldots,\log\varepsilon_{tm})'$ and $a=(a_1,\ldots,a_m)'$ and $V=[v_{ij}]_{i,j=1,\ldots,m}$ are respectively the mean and the covariance matrix of the random vector $\log\varepsilon_t$, assumed constant over time. Alternatively, the random $m\times 1$ vector $\gamma_t+\varepsilon_t$ has a multivariate three-parameter lognormal distribution with the unknown parameter γ_t as lower bound.[3] The form of γ_t as a function of time will be specified in (2.9) below. This maximization when carried out leads to a trade-demand function $\hat{h}$ of the Klein-Rubin (1948) form[4]

$$P_t z_t=P_t\hat{h}(p_t,D_t;\alpha_t,\gamma_t,\varepsilon_t)=D_t\alpha-[I-\alpha\iota']P_t\gamma_t+e_t\tag{2.4}$$

[2]A specification of the type (2.1) was introduced by Pollak and Wales (1969) who assumed, however, that the ε_{tj} were jointly normally distributed. This would imply that (2.1) is undefined for certain realizations of ε_t. For this reason we choose a one-sided distribution such as the lognormal.

[3]For information about the lognormal distribution see Hill (1963), Aitchison and Brown (1957), Johnson and Kotz (1970), and Cohen (1988).

[4]It was first noted by Samuelson (1948), and later by Geary (1950), that if the error term e_t in (2.4) is ignored, a demand function of this form is generated by a utility function of the form (2.1) with the error term ε_t ignored. This seems to be the basis for the inaccurate terminology "Stone-Geary model" frequently applied to the linear expenditure system (2.4).

known as the Linear Expenditure System, where $P_t = \text{diag}\{p_{tj}\}$ and ι is a column vector of ones, and

$$e_t = -[I - \alpha\iota']P_t\varepsilon_t. \tag{2.5}$$

The dependency of the error term on the prices should be noted. The system (2.4) may be given a precise interpretation within the context of a Heckscher-Ohlin-Lerner-Samuelson (HOLS) model. Such a model has been set forth in Chipman (1985b) and the details will not be repeated here. In terms of such a model the $m \times 1$ vector-valued trade-demand function $\hat{h}$ of (2.4) may be defined by

$$\begin{aligned}&\hat{h}(p_t, D_t; l_t, t, \varepsilon_t) = \\ &\quad h\Big(p_t, \tilde{p}^3(p_t, D_t, l_t, t,), \Pi(p_t^1, \tilde{p}^3(p_t, D_t, l_t, t), l_t) + D_t, \varepsilon_t\Big) - \tilde{y}(p_t, D_t, l_t, t)\end{aligned} \tag{2.6}$$

where $h(p_t^1, p_t^2, p_t^3, E_t)$ is the consumer demand function for tradable goods expressed as a function of the price vectors $p_t = (p_t^{1\prime}, p_t^{2\prime})'$ of tradables (p_t^1 and p_t^2 referring to price vectors of tradables produced and not produced at home, respectively) and p_t^3 of nontradables, as well as of disposable national income (expenditure) E_t; l_t is a vector of the country's factor endowments, and $\tilde{p}^3$ and $\tilde{y}$ denote vectors of prices of nontradables and vectors of net outputs of tradables expressed as functions of the remaining variables; these functions are obtained by solving a system of equations of general equilibrium as shown in Chipman (1985b). Now let us suppose that the $m^* \times 1$ consumer demand functions h^* (where $m^* = m + m_3 = m_1 + m_2 + m_3$, m being the number of tradables and m_3 the number of nontradables) are themselves of the Klein-Rubin LES form

$$P_t^* h^*(p_t^*, E_t, t) = E_t\alpha^* - [I - \alpha^*\iota']P_t^*\gamma_t^* + e_t^* \tag{2.7}$$

where $p_t^* = (p_t^1, p_t^2, p_t^3)' = (p_t, p_t^3)'$ and $P_t^* = \text{diag}\{p_t^*\}$, and likewise $\alpha^* = (\alpha^{*1}, \alpha^{*2}, \alpha^{*3})'$, etc. Then consumer expenditures on tradables have the form

$$P_t x_t = (E_t + \tilde{p}_t^{3\prime}\gamma_t^3)\alpha - [I - \alpha\iota']P_t\gamma_t + e_t. \tag{2.8}$$

(cf. Chipman 1990, p. 7). In the absence of nontradable goods this function is linear in the prices of tradables. If the function $P_t\tilde{y}(p_t, D_t, l_t, t)$ is also linear in these prices, then so is the function $P_t\hat{h}$ of (2.6). A sufficient condition for this may be stated. If there are at least as many factors as commodities and no nontradables are produced, then the function $\tilde{y}$ coincides with the Rybczynski function $\hat{y}(p_t^1, l_t, t)$. If all tradable commodities are produced at home and there are exactly as many products as factors, this function has the form $y_t = B^{-1}(p_t)l_t$

where B^{-1} is the inverse of the matrix of cost-minimizing factor-output ratios. If the elements of B^{-1} have the form $b^{ij}(p_t) = \sum_{k=1}^m b_k^{ij} p_k/p_i$, where the b_k^{ij} are constants, then $P_t\hat{y}(p_t^1, l_t, t)$ is linear in the prices. This includes the special case $b_k^{ij} = 0$ for $i \neq k$ in which $B^{ij}(p_t) =$ constant and the model reduces to one of pure exchange; in this case one can identify the parameter γ_t with y_t.

This model does not allow as much freedom as one would wish for output—as opposed to consumption—to vary in response to price changes; too much of a burden is placed on consumption to explain variations in imports and exports in response to price changes. It is therefore desirable to allow for considerable time variation in output. In particular one wants to allow for the effects of technical change, increases in factor endowments, and seasonal variation. For monthly data the γ_t term in (2.1) has therefore been postulated to have the form

$$\gamma_{tj} = \kappa_j(t) + \xi_j \cos\left(\frac{\pi}{6}t\right) + \zeta_j \sin\left(\frac{\pi}{6}t\right) \tag{2.9}$$

for $j = 1, 2, \ldots, m; t = 1, 2, \ldots, T$, where

$$\kappa_j(t) = \sum_{k=1}^N \phi_{jk}(t)\kappa_{jk} \quad (j = 1, 2, \ldots, m) \tag{2.10}$$

is a cubic spline with N knots $t_1, t_2, \ldots, t_N$, the $\phi_{jk}(t)$ being well-defined numbers (cf. Chipman 1985a, p. 411).[5] The $\kappa_{jk} = \kappa_j(t_k)$ ($j = 1, \ldots, m; k = 1, \ldots, N$) are mN parameters to be estimated. In the special case $N = 2$, (2.10) reduces to a linear trend $a_j + b_j t$.

2.2 Derivation of the Likelihood Function

From (2.4) and (2.5) we may write the relationship between the trade-values and the prices, trade deficit, and error terms as

$$[I - \alpha\iota']P_t(\gamma_t + \varepsilon_t) = D_t\alpha - P_t z_t. \tag{2.11}$$

To set up the likelihood function, we wish to solve this equation for the error term ε_t; however, this cannot be done in the usual way, since the matrix $I - \alpha\iota'$ is singular. Extending an approach discussed in Chipman and Tian (1990), we may proceed as follows. From the results of Penrose (1955) we know that an equation

$$Ax = b \tag{2.12}$$

[5] For a treatment of spline functions see Greville (1969).

(where A is an $m \times n$ matrix) has a solution, x, for given A and conformable b, if and only if

$$AA^{-}b = b, \tag{2.13}$$

where A^{-} is any generalized inverse of A in the sense of Rao (1966), that is, any $n \times m$ matrix A^{-} such that $AA^{-}A = A$ (such a matrix always exists). The general solution of (2.12) is then given by (cf. Penrose, 1955)

$$x = A^{-}b + (I - A^{-}A)c, \tag{2.14}$$

where c is an arbitrary $n \times 1$ vector. Applying this result to (2.11), and noting that the matrix $I - \alpha\iota'$ is idempotent (of rank $m-1$), hence is its own generalized inverse (in Rao's sense),[6] the solvability condition (2.13) becomes

$$[I - \alpha\iota'](D_t\alpha - P_t z_t) = D_t\alpha - P_t z_t. \tag{2.15}$$

However, since $[I - \alpha\iota']\alpha = 0$, this condition reduces to

$$\alpha\iota' P_t z_t = D_t\alpha, \tag{2.16}$$

which necessarily holds on account of the budget identity $\iota' P_t z_t = D_t$. Using this budget constraint, as well as the relations $[I - \alpha\iota']\alpha = 0$ and $[I - \alpha\iota'] = [I - \alpha\iota']^{-}$, from (2.14) the general solution of (2.11) is then given by

$$\left.\begin{aligned} \gamma_t + \varepsilon_t &= P_t^{-1}\{[I - \alpha\iota'](D_t\alpha - P_t z_t) + \alpha\iota' c_t\} \\ &= P_t^{-1}\{\alpha\iota' c_t - [I - \alpha\iota'] P_t z_t\} \\ &= [D_t + \iota' c_t] P_t^{-1}\alpha - z_t \end{aligned}\right\} \text{ for } c_t \in \Re^m, \tag{2.17}$$

where $\Re^m$ denotes the m-dimensional Euclidean space. Equivalently, the set of solutions, ε_t, is given by

$$\begin{aligned} \{\varepsilon_t \mid [I - \alpha\iota'] P_t(\gamma_t + \varepsilon_t) = D_t\alpha - P_t z_t\} &= P_t^{-1}\{\alpha\iota'\Re^m - [I - \alpha\iota'] P_t z_t\} \\ &= [D_t + \iota'\Re^m] P_t^{-1}\alpha - \gamma_t - z_t. \end{aligned} \tag{2.18}$$

Letting c_t be some well-defined parametric function of time (whose parameters are to be estimated), and defining

$$\eta_t = [D_t + \iota' c_t] P_t^{-1}\alpha - \gamma_t - z_t \quad \text{for } c_t \in \Re^m, \tag{2.19}$$

[6] It also satisfies the second of Penrose's conditions, $A^{-}AA^{-} = A^{-}$, but not the remaining two conditions that $A^{-}A$ and AA^{-} be symmetric. Thus it is not a full generalized inverse (or Moore-Penrose "pseudoinverse") in Penrose's sense.

we see from (2.17) that η_t belongs to the solution set (2.18) hence is a particular solution. Accordingly, from (2.19) and the budget equation $\iota' P_t z_t = p_t' z_t = D_t$ we have

$$p_t'(\gamma_t + \eta_t) = [D_t + \iota' c_t]\iota' P_t P_t^{-1}\alpha - p_t' z_t = \iota' c_t \equiv \mu_t, \tag{2.20}$$

(defining μ_t), from which it follows that the components of η_t are linearly dependent, hence the distribution of η_t is singular.

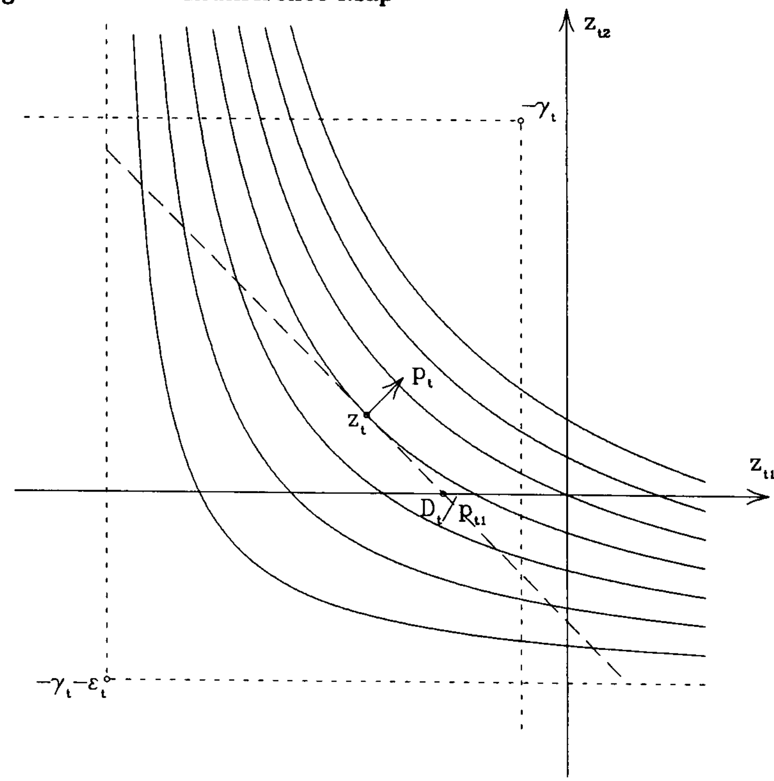
Notice from (2.1) and the choice of solution (2.19) for ε_t that the vector

$$z_t + \gamma_t + \eta_t = [D_t + \iota' c_t] P_t^{-1}\alpha = [D_t + \mu_t] P_t^{-1}\alpha \tag{2.21}$$

must have all its components positive. Thus the arbitrary vector c_t must be chosen so that

$$\mu_t \equiv \iota' c_t > -D_t. \tag{2.22}$$

Figure 1: Trade-Indifference Map



An illustration is shown in Figure 1 of the trade-indifference map corresponding to the trade-utility function (2.1). The positivity of the vector (2.21) is shown by the fact that $z_t > -\gamma_t - \varepsilon_t$, i.e., the observed trade vector lies northwest of the shifted and moving random origin $-\gamma_t - \varepsilon_t$ of the system of Cobb-Douglas trade-indifference curves. The diagram also places $z_t < -\gamma_t$, where $-\gamma_t$ is the upper bound of the negative of the lognormal distribution—the latter being concentrated in the negative quadrant emanating from $-\gamma_t$, and of course including the realization $-\gamma_t - \varepsilon_t$. Since from (2.21),

$$p_t'(z_t + \gamma_t) = D_t + \mu_t - p_t'\eta_t, \tag{2.23}$$

and both $D_t + \mu_t > 0$ and $p_t'\eta_t > 0$, this expression will be negative if μ_t is such that $D_t + \mu_t$ is sufficiently small; hence $p_t'z_t < -p_t'\gamma_t$ in Figure 1. However, if $D_t + \mu_t$ is sufficiently large, (2.23) could be positive in which case $-\gamma_t$ would lie below the budget line in Figure 1.

Substituting the particular solution η_t for ε_t in the second and third expressions on the right in (2.17), we have

$$P_t(\gamma_t + \eta_t) = -[I - \alpha\iota']P_t z_t + \alpha\iota' c_t = [D_t + \mu_t]\alpha - P_t z_t. \tag{2.24}$$

This expresses the product of P_t with the random variable $\gamma_t + \eta_t$ as an affine projection onto an $(m-1)$-dimensional coset of the $(m-1)$-dimensional subspace $-[I - \alpha\iota']P_t\Re^m$. Thus for fixed c_t the support of the (three-parameter) lognormal distribution of $\gamma_t + \eta_t$ is contained in an $(m-1)$-dimensional affine subspace. As c_t varies, however, the distribution of $\gamma_t + \varepsilon_t$ spans an orthant (from γ_t) of m-dimensional space. Premultiplying both sides of (2.24) by the idempotent matrix $I - \alpha\iota'$ we obtain

$$[I - \alpha\iota']P_t(\gamma_t + \eta_t) = -[I - \alpha\iota']P_t z_t = D_t\alpha - P_t z_t, \tag{2.25}$$

showing that the singular random variable η_t satisfies (2.11). So, however, does the nonsingular random variable ε_t since (2.11) defines a projection of $P_t(\gamma_t + \varepsilon_t)$ into an $(m-1)$-dimensional subspace of the m-dimensional space.

Now substituting (2.4) in (2.19) we obtain

$$\eta_t = P_t^{-1}[\iota'(c_t - P_t\gamma_t)\alpha - e_t] = P_t^{-1}[(\mu_t - p_t'\gamma_t)\alpha - e_t], \tag{2.26}$$

so that once again we see that the distribution of η_t is singular, since that of e_t is (as is clear from (2.5)). However, in the following section we proceed as though

it were distributed as ε_t. In order to proceed, however, we need to have a way to handle the parameter μ_t of (2.20).

1. One procedure, which might be suitable in the case of the analysis of consumer-expenditure data—where D_t would be interpreted as total expenditure, which must be positive—would be to set $c_t = 0$ in (2.17) and thus $\mu_t = \iota' c_t = 0$.

2. In the present application, however, where D_t represents the trade deficit, the above choice is clearly inappropriate. However, one may specify c_t in (2.17) to be some constant vector, c, and thus $\mu_t = \mu = \iota' c$ for all t. In the case of Germany, which except for a few months in the period 1959–88 has persistently had a trade surplus (i.e., $D_t < 0$), this means that μ must be chosen larger than the largest trade surplus observed during the sample period, i.e.,

$$\mu > \max_t\{-D_t\} = -\min_t\{D_t\} \equiv -D_{\min}. \tag{2.27}$$

Note that, even though the arbitrary vector c is unidentifiable, μ is identifiable and can be estimated with the other parameters in the model, subject to the restriction (2.27). In fact, the procedure we have chosen is to set

$$\mu_t = \mu = -D_{\min} + \exp(\nu) \tag{2.28}$$

where ν is a parameter to be estimated. Accordingly, substituting (2.28) in (2.24) we obtain

$$\eta_t = [D_t + \mu_t]P_t^{-1}\alpha - \gamma_t - z_t = [D_t - D_{\min} + \exp(\nu)]P_t^{-1}\alpha - \gamma_t - z_t. \tag{2.29}$$

Although as pointed out above this random variable necessarily has a singular distribution, we proceed as if it was distributed as ε_t. Accordingly, from (2.3) we then have—assuming the η_t's to be independently and identically distributed for $t = 1, 2, \ldots, T$—the following loglikelihood function:

$$\begin{aligned} L(a, V, \alpha, \nu, K, \xi, \zeta) &= -\frac{Tm}{2}\log(2\pi) - \frac{T}{2}\log|V| \\ &- \sum_{t=1}^{T}\sum_{j=1}^{m}\log\eta_{tj} - \frac{1}{2}\sum_{t=1}^{T}[\log\eta_t - a]'V^{-1}[\log\eta_t - a], \end{aligned} \tag{2.30}$$

where η_t is given by the third expression of (2.29) and

$$K = [\kappa_{jl}]_{j=1,\ldots,m;l=1,\ldots,N}, \quad \xi = (\xi_1, \ldots, \xi_m)', \quad \zeta = (\zeta_1, \ldots, \zeta_m)' \tag{2.31}$$

are defined as in (2.9) and (2.10). The estimators $\hat{a}, \hat{V}, \hat{\alpha}, \hat{\nu}, \hat{K}, \hat{\xi}, \hat{\zeta}$ obtained by maximizing (2.30) will be termed *generalized maximum-likelihood* (GML) estimators.

3. In the above paragraphs we have specified GML estimation for the cases $\mu_t = 0$ and μ_t equal to a parameter μ to be estimated. In both cases we assume μ_t to be constant over time t. However, this is certainly not a satisfactory assumption, since from (2.20) we have the following identity:

$$\mu_t = p_t'(\gamma_t + \eta_t). \tag{2.32}$$

Thus, μ_t depends on time t. Interpreting μ_t to be a singular random variable having the value μ_t at each time period t, and taking expectations of both sides of the above equation and noting that $\mathrm{E}\varepsilon_{ti} = \exp(a_i + v_{ii}/2)$,[7] we have

$$\mu_t = \mathrm{E}\mu_t = p_t'(\gamma_t + \mathrm{E}\varepsilon_t) = p_t'(\gamma_t + \exp(a + v/2)), \tag{2.33}$$

where v is a vector whose i-th component is v_{ii} (the i-th diagonal element of V) and thus $\exp(a + v/2)$ is a vector whose i-th component is $\exp(a_i + v_{ii}/2)$. Thus, substituting $\mu_t = p_t'(\gamma_t + \exp(a + v/2))$ into (2.21), we have

$$z_t + \gamma_t + \eta_t = [D_t + p_t'(\gamma_t + \exp(a + v/2))]P_t^{-1}\alpha \tag{2.34}$$

from which the GML estimators can be obtained.

We have not pursued this approach in the present paper, however, since the matrix of second-order partial derivatives of the loglikelihood function is extremely complicated, and application of Newton's method is not practicable. However, we plan to implement this third approach in a future paper.

3 Empirical Implementation

The model was applied to data on six main categories of goods: agriculture, mining products, basic materials, capital goods, consumer goods, and foodstuffs. In one model, the goods contained in the export basket were considered to be distinct from the goods in the import basket; thus, the gross imports and the negatives of the gross exports constituted 12 dependent variables; the 13 independent variables were the corresponding 12 import- and export-price indices and the deficit in the

[7] The reason why this formula is true for any multivariate lognormal distribution is the following. If the random vector $x_t = \log \varepsilon_t$ has a normal distribution with the mean a and variance matrix V, we know that $x_{ti} = \log \varepsilon_{ti}$ also has a normal distribution with mean and variance a_i and v_{ii}. And consequently we have $\mathrm{E}\varepsilon_{ti} = \exp(a_i + v_{ii}/2)$. Cf. Aitchison and Brown (1957), p. 8.

balance of trade. The data (furnished by the Statistisches Bundesamt, Wiesbaden) consisted of 360 monthly observations from January 1959 to December 1988: the import and export values (the latter converted to negative amounts) and thus the trade deficit (the sum of these) are expressed in thousands of current D-marks; the import- and export-price indices are Laspeyres series with bases 1958, 1962, 1970, 1976, 1980, and 1985, linked at each January of the respective base year and expressed in terms of 1985 = 100. In the other model, the dependent variables were the six net imports (gross imports less gross exports) and the independent variables were weighted averages of the import and export price indices for each of the six categories, and the trade deficit. In each case, a spline function with three evenly spaced knots was used, the second knot being placed at January 1974. The results are exhibited in the accompanying tables. Plots are shown of the actual data series as well as of the difference between predicted and observed series. Own-price, cross-price, and net-expenditure elasticities as well as trade-Slutsky terms of imports, exports, and net imports were computed according to the methods indicated in subsection 3.3, and evaluated at all 360 data points; the medians of the elasticities and Slutsky terms are shown in the accompanying tables. The results are discussed in Section 4.

3.1 Computation of GML Estimates

In addition to the estimates for the parameters of the trade-utility function, those for the parameters of the lognormal distribution have to be determined by GML estimation of the model described above. Accordingly, three sets of restrictions on the parameters have to be taken into account: (1) all components of the α-vector have to be nonnegative and sum to one; (2) the parameter μ must satisfy (2.27); and (3) the vector η_t must be positive.

Both the adding-up and nonnegativity constraints are enforced by substituting α by the $m \times 1$-vector $\tilde{\alpha}(\beta)$ obtained from a multinomial logit-type transformation of an unconstrained parameter vector β

$$\tilde{\alpha}_i(\beta) = \frac{\exp(\beta_i)}{\sum_{k=1}^m \exp(\beta_k)}, \tag{3.1}$$

which enters the loglikelihood function (2.30).[8]

[8] Previous estimation results showed that the nonnegativity constraint is not always fulfilled if α rather than β enters the loglikelihood function.

Identifiability is ensured by arbitrarily fixing one of the β-components at the zero level. In order to fulfill restriction (2.26), μ_t is set equal to $-D_{\min}+\exp(\nu)$ in accordance with (2.28), ν being determined by GML estimation. The loglikelihood function L that has been previously defined in (2.30) is consequently redefined as follows:

$$\begin{aligned} L^*(a,V,\bar\beta,\nu,K,\xi,\zeta) &= L(a,V,\tilde\alpha(\beta),\nu,K,\xi,\zeta) \\ &= -\frac{Tm}{2}\log(2\pi)-\frac{T}{2}\log\mid V\mid-\sum_{t=1}^{T}\sum_{j=1}^{m}\log\eta^*_{tj} \\ &\quad -\frac{1}{2}\sum_{t=1}^{T}[\log\eta^*_t-a]'V^{-1}[\log\eta^*_t-a], \end{aligned} \tag{3.2}$$

where $\bar\beta$ denotes

$$\bar\beta=(\beta_1,\ldots,\beta_{r-1},\beta_{r+1},\ldots,\beta_m)' \tag{3.3}$$

and β_r is set equal to zero, r being some integer chosen in advance.[9] In (3.2) and the following, it is understood that the r-th component of β is set equal to zero. The vector η^*_t is obtained from

$$\eta^*_t=[D_t-D_{\min}+\exp(\nu)]P_t^{-1}\tilde\alpha(\beta)-z_t-\gamma_t, \tag{3.4}$$

where γ_t is determined from (2.9).

The $(N+3)m$ GML estimators for the parameters of the trade-utility function and the parameter ν, all comprised in the parameter vector θ,

$$\theta=(\bar\beta',\nu,\kappa_1',\ldots,\kappa_N',\xi',\zeta')', \tag{3.5}$$

where κ_l denotes the l-th column of the matrix K of (2.31), i.e.,

$$K=(\kappa_1,\ldots,\kappa_N),\qquad \kappa_l=(\kappa_{1l},\ldots,\kappa_{ml})', \tag{3.6}$$

and ξ, ζ are given by (2.31), as well as the GML estimators for the mean vector a and covariance matrix V of the lognormal distribution, are obtained by solving the following system of equations:

$$a = \frac{1}{T}\sum_{t=1}^{T}\log\eta^*_t, \tag{3.7}$$

$$V = \frac{1}{T}\sum_{t=1}^{T}(\log\eta^*_t-a)(\log\eta^*_t-a)', \tag{3.8}$$

$$\sum_{t=1}^{T}G_tH_t^{-1}\left(\iota+V^{-1}(\log\eta^*_t-a)\right)=0. \tag{3.9}$$

In (3.9), G_t denotes the $(N+3)m\times m$ matrix

$$\begin{aligned} G_t = \Big(&\left[(D_t-D_{\min}+\exp(\nu))\,J_r\left(\tilde\alpha(\beta)\tilde\alpha(\beta)'-A\right)P_t^{-1}\right]', \\ &\left[-\exp(\nu)\tilde\alpha(\beta)'P_t^{-1}\right]',\Phi_1(t),\ldots,\Phi_N(t),\cos\left(\frac{\pi}{6}t\right)I,\sin\left(\frac{\pi}{6}t\right)I\Big)', \end{aligned} \tag{3.10}$$

where the $(m-1)\times m$ matrix J is defined as

$$J_r=\begin{pmatrix} I_{r-1} & 0 & 0 \\ 0 & 0 & I_{m-r}\end{pmatrix}, \tag{3.11}$$

A denotes

$$A=\operatorname{diag}(\tilde\alpha(\beta)), \tag{3.12}$$

$\Phi_l(t)$ denotes

$$\Phi_l(t)=\operatorname{diag}(\phi_{1l}(t),\ldots,\phi_{ml}(t))\qquad(l=1,2,\ldots,N), \tag{3.13}$$

and H_t^{-1} is given as

$$H_t^{-1}=\operatorname{diag}\left(\eta_{t1}^{-1},\ldots,\eta_{tm}^{-1}\right)\qquad(t=1,2,\ldots,T). \tag{3.14}$$

The Newton-Raphson algorithm has been chosen as maximization algorithm

$$\theta_{n+1}=\theta_n-\lambda M_n^{-1}\Delta_n, \tag{3.15}$$

where Δ and M correspond to the gradient and the Hessian matrix of the log-likelihood function L^* at the n-th iteration step. Whereas the gradient is given by

$$\Delta=\frac{\partial L^*}{\partial\theta}=\sum_{t=1}^{T}G_tH_t^{-1}\left(\iota+V^{-1}(\log\eta^*_t-a)\right), \tag{3.16}$$

[9] Whereas β_6, the coefficient corresponding to net imports of foodstuffs, was set equal to zero for the six-commodity model, β_7, i.e. the coefficient for exports of agricultural goods, was fixed to the same value in the case of the twelve-commodity model. Estimation results for the twelve-commodity model proved to be numerically very unstable if exports of foodstuffs were chosen as the base category ($\beta_{12}=0$).

the Hessian, i.e., the matrix of second-order partial derivatives $\partial^2 L^*/\partial\theta\partial\theta' = M$, is given as follows:

$$M = \sum_{t=1}^{T} G_t H_t^{-1}\left[I + B_t - V^{-1}\right] H_t^{-1} G_t' + \sum_{t=1}^{T} \Lambda_t, \tag{3.17}$$

where B_t is an $m \times m$ diagonal matrix of the form

$$B_t = \begin{pmatrix} \delta_1' V^{-1}(\log \eta_t^* - a) & & & \\ & \delta_2' V^{-1}(\log \eta_t^* - a) & & \\ & & \ddots & \\ & & & \delta_m' V^{-1}(\log \eta_t^* - a) \end{pmatrix} \tag{3.18}$$

and δ_i denotes a vector that takes the value one at the i-th place and zeros elsewhere. The block-diagonal matrix Λ_t is defined as

$$\Lambda_t = \begin{pmatrix} N & Q & 0 \\ Q' & S & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{3.19}$$

where the matrices N, Q, and S are given as follows:

$$\begin{aligned} N &= -\sum_{t=1}^{T}(D_t - D_{\min} + \exp(\nu)) \\ &\quad \cdot \left\{\tilde{\alpha}(\beta)' P_t^{-1} H_t^{-1}\left(\iota + V^{-1}(\log \eta_t^* - a)\right) J_r\left[2\tilde{\alpha}(\beta)\tilde{\alpha}(\beta)' - A\right] J_r' \right. \\ &\quad - J_r\left[\operatorname{diag}\left(P_t^{-1} H_t^{-1}\left(\iota + V^{-1}(\log \eta_t^* - a)\right)\right)\left(\tilde{\alpha}(\beta)\tilde{\alpha}(\beta)' - A\right)\right] J_r' \\ &\quad \left. - J_r\left[\tilde{\alpha}(\beta)\tilde{\alpha}(\beta)' \operatorname{diag}\left(H_t^{-1} P_t^{-1}\left(\iota + V^{-1}(\log \eta_t^* - a)\right)\right)\right] J_r'\right\}, \end{aligned} \tag{3.20}$$

$$Q = \sum_{t=1}^{T} \exp(\nu) J_r\left[\tilde{\alpha}(\beta)\tilde{\alpha}(\beta)' - A\right] H_t^{-1} P_t^{-1}\left(\iota + V^{-1}(\log \eta_t^* - a)\right), \tag{3.21}$$

$$S = -\sum_{t=1}^{T} \exp(\nu)\tilde{\alpha}(\beta)' H_t^{-1} P_t^{-1}\left(\iota + V^{-1}(\log \eta_t^* - a)\right). \tag{3.22}$$

The scalar λ in (3.15) allows for gradual reduction of stepsize for given values of θ_n, Δ_n, and M_n: Initially taking the value of one, λ is repeatedly reduced, in case the newly determined vector θ_{n+1} leads to either a decrease of the respective loglikelihood value or negativity of any of the components of the vector η_t^*. As soon as the stepsize is small enough to yield a vector θ_{n+1} for which neither of the two conditions (decrease of the loglikelihood value or negativity of any η_t^*-component) is fulfilled, λ is reset to its initial value, whereas θ_{n+2} is determined

on the basis of the newly determined value of θ_{n+1} and the respective values of the gradient and the Hessian matrix.[10]

Two different routines have been used for the inversion of matrices: (1) Owing to the method's numerical stability and speed of computation, the combined method of LU-decomposition and subsequent backsubstitution has been chosen for the inversion of the covariance matrix V. The matrix is decomposed into a lower-triangular and an upper-triangular matrix by means of Crout's method with partial pivoting. Backsubstitution is then repeated for each column of the inverse matrix.[11] (2) The method of singular-value decomposition has been chosen for the inversion of the Hessian matrix M of second-order partial derivatives, as the matrices of both the six- and twelve-commodity models proved to be fairly ill-conditioned for θ-values that were not close to the maximum. The matrix M is decomposed into the product of two orthogonal matrices U and V of order $(N+3)m$ and the $(N+3)m$-order diagonal matrix of singular values W:

$$M = UWV'. \tag{3.23}$$

The inverse of M is easily determined as

$$M^{-1} = VW^{-1}U'. \tag{3.24}$$

However, the reciprocals of those singular values that are smaller than 10^{-11} times the largest singular value[12] are set equal to zero to avoid overshooting effects.[13] Denoting by $\overline{W}$ the matrix obtained from W by replacing the "small" singular values (as defined above) by zeros, what is actually computed, then, is the generalized inverse,

$$\overline{M}^{\dagger} = V\overline{W}^{\dagger}U'$$

of the matrix $\overline{M} = U\overline{W}V'$, which is the best approximation of M by a matrix whose rank is equal to that of $\overline{W}$, i.e., to the number of "large" singular values.

The negative generalized inverse of the Hessian matrix (the generalized information matrix) has been used as the asymptotic covariance matrix of the estimators for $\bar{\beta}$, ν, K, ξ, and ζ. Transformation of the covariance matrix with respect to $\bar{\beta}$ allowed to construct the t-ratios corresponding to $\tilde{\alpha}(\beta)$.[14]

[10] For a detailed description of the method of backtracking see Dennis and Schnabel (1983), pp. 126–129.

[11] See Press et al. (1988), pp. 31–39.

[12] For the case of the twelve-commodity model the respective factor had to be set to 10^{-10}.

[13] See Press et al. (1988), pp. 52–64.

[14] See, e.g., Fomby, Hill, and Johnson (1988), p. 58.

For the particular solution η_t for ε_t, (2.24) implies that the predicted net expenditure as depicted in Figures 2–3, which we denote $P_t\hat{z}_t^D$, can be determined as follows:

$$P_t\hat{z}_t^D = [D_t - D_{\min} + \exp(\hat{\nu})]\tilde{\alpha}(\hat{\beta}) - P_t\left(\hat{\gamma}_t + \hat{\mathrm{E}}(\varepsilon_t)\right), \tag{3.25}$$

where the circumflex indicates that estimates of the respective parameters are inserted. $\hat{\beta}$ denotes that the estimates of the $\bar{\beta}$-components are inserted into the parameter vector β, i.e.,

$$\hat{\beta} = (\hat{\beta}_1, \ldots, \hat{\beta}_{r-1}, 0, \hat{\beta}_{r+1}, \ldots, \hat{\beta}_m)'.$$

Define v as the vector of diagonal elements $v_{11}, \ldots, v_{mm}$ of the covariance matrix V. The estimate of the expected value of ε_t is then obtained from

$$\hat{\mathrm{E}}(\varepsilon_t) = \exp\left(\hat{a} + \frac{\hat{v}}{2}\right). \tag{3.26}$$

Note that if we instead compute predicted net expenditure directly from (2.4) and (2.5), i.e., by

$$P_t z_t = D_t\alpha - [I - \alpha\iota']P_t(\gamma_t + \varepsilon_t) = [D_t + p_t'(\gamma_t + \varepsilon_t)]\alpha - P_t(\gamma + \varepsilon_t), \tag{3.27}$$

the appropriate formula for predicted net expenditure is

$$\begin{aligned} P_t z_t^G &= D_t\tilde{\alpha}(\hat{\beta}) - [I - \tilde{\alpha}(\hat{\beta})\iota']P_t\left(\hat{\gamma}_t + \hat{\mathrm{E}}(\varepsilon_t)\right) \\ &= \left[D_t + p_t'\left(\hat{\gamma}_t + \hat{\mathrm{E}}(\varepsilon_t)\right)\right]\tilde{\alpha}(\hat{\beta}) - P_t\left(\hat{\gamma}_t + \hat{\mathrm{E}}(\varepsilon_t)\right). \end{aligned} \tag{3.28}$$

Formulas (3.25) and (3.28) do not coincide unless the equation

$$\sum_{j=1}^{m} p_{tj}\left(\hat{\gamma}_{tj} + \hat{\mathrm{E}}(\varepsilon_{tj})\right) = -D_{\min} + \exp(\hat{\nu}) \tag{3.29}$$

holds. Figures 2–3 use (3.25) for reasons to be given presently.

Prediction quality of the respective models is reflected by goodness-of-fit and goodness-of-forecast measures as shown in Tables 1a and 1b.[15] Goodness of fit, depicted in the first column of each table, is computed for each commodity as the Euclidean distance between predicted and observed trade values:

$$\sqrt{\sum_{t=1}^{T}(p_{ti}\hat{z}_{ti} - p_{ti}z_{ti})^2} \qquad (i = 1, 2, \ldots, m), \tag{3.30}$$

[15]The same measures have been used to reflect prediction quality for the nonlinear least-squares estimation.

whereas goodness of forecast, shown in the second column of the respective tables, is determined from:

$$\sqrt{\sum_{t=1}^{T}(p_{ti}\hat{z}_{ti} - p_{ti}z_{ti})^2} + \sqrt{\sum_{t=1}^{T-1}[p_{t+1,i}\hat{z}_{t+1,i} - p_{ti}\hat{z}_{ti} - (p_{t+1,i}z_{t+1,i} - p_{ti}z_{ti})]^2} \tag{3.31}$$

for $i = 1, 2, \ldots, m$. It was found—not surprisingly, in view of the fact that the GML estimation is based on (2.29)—that (3.25) gives the better fit and forecast according to these metrics, hence this is the one shown in Tables 1a and 1b and in Figures 2–3.

3.2 Computation of NLLS Estimates

Neglecting the nonnegativity constraint for $\gamma_{tj} + z_{tj} + \varepsilon_{tj}$ in (2.1), the parameters of the trade-utility function can be obtained from the following regression equations by the method of *nonlinear least-squares estimation* (NLLS):

$$w_{tj} = p_{tj}z_{tj} = D_t\alpha_j + \sum_{i=1}^{m} p_{ti}\gamma_{ti}(\alpha_j - \delta_{ij}) + \varepsilon_{tj}, \tag{3.32}$$

where w_{tj} denotes the observed imports and exports or net imports of commodity j in period t. The expected value of ε_t is assumed to be equal to zero and the corresponding covariance matrix Σ.

Estimation results showed that no multinomial logit-type transformation is needed to enforce nonnegativity of the α-components. To fulfill the adding-up constraint for the elements of vector α, the estimate of the vector's (arbitrarily chosen) m-th component is determined from the estimates of the remaining $m-1$ elements[16]

$$\alpha_m = 1 - \sum_{i=1}^{m-1}\alpha_i. \tag{3.33}$$

To take account of the (weaker) restriction that the expected value of $\gamma_{tj} + z_{tj} + \varepsilon_{tj}$ must be positive, the parameters of the spline function defined in (2.10) have been transformed to

$$g_j(t_k) = \kappa_j(t_k)^2 - b_{jk}, \tag{3.34}$$

where b_{jk} takes the role of a lower bound for z_{tj}. b_{jk} is determined as the minimum of annual averages for import, export or net import quantities per month in the

[16]The covariance matrix corresponding to α has been obtained by transformation of the covariance matrix with respect to $(\alpha_1, \ldots, \alpha_{m-1})'$ in accordance with the formula given, e.g., in Fomby, Hill, and Johnson (1988), p. 58.

period between knot k of the spline function and its successor. Using average values and disregarding seasonality terms in (2.9), the transformation of $\kappa_j(t_k)$ cannot ensure, however, that the nonnegativity constraint for $\gamma_t + z_t$ is fulfilled for observations where seasonal fluctuations have been particularly strong.[17]

Taking account of the adding-up constraint for the α-components, the covariance matrix of the residuals is singular. The logarithm of the determinant of this matrix, reduced by one dimension, has therefore been chosen as target function. Minimization of this target function was carried out by utilization of the Broydon-Fletcher-Goldfarb-Shanno-algorithm,[18] a quasi-Newton method, where the inverse of the information matrix is substituted by a sequence of matrices H_i, determined during each iteration step i:

$$\begin{aligned} H_{n+1} &= H_n + \frac{(\theta_{n+1}-\theta_n)(\theta_{n+1}-\theta_n)'}{(\theta_{n+1}-\theta_n)'(\nabla f_{n+1}-\nabla f_n)} \\ &\quad - \frac{[H_n(\nabla f_{n+1}-\nabla f_n)]\,[H_n(\nabla f_{n+1}-\nabla f_n)]'}{(\nabla f_{n+1}-\nabla f_n)'H_n(\nabla f_{n+1}-\nabla f_n)} \\ &\quad + [(\nabla f_{n+1}-\nabla f_n)'H_n(\nabla f_{n+1}-\nabla f_n)]\,uu', \end{aligned} \tag{3.35}$$

where ∇f_n denotes the gradient of the target function at iteration n, θ_n denotes the respective parameter vector, and u is defined as follows:

$$\begin{aligned} u &= \frac{\theta_{n+1}-\theta_n}{(\theta_{n+1}-\theta_n)'(\nabla f_{n+1}-\nabla f_n)} \\ &\quad - \frac{H_n(\nabla f_{n+1}-\nabla f_n)}{(\nabla f_{n+1}-\nabla f_n)'H_n(\nabla f_{n+1}-\nabla f_n)}. \end{aligned} \tag{3.36}$$

Speed of convergence was further increased by gradual bracketing of the minimum and inverse parabolic interpolation of the function between the respective brackets. The minimum value for each component of the parameter vector is determined successively: Two values are searched for each component of the parameter vector, for which the partial derivative of the target function takes opposite sign. The target function is then interpolated to find new values of smaller distance.[19]

Given the assumption that the expected value of ε_t is equal to zero and the respective covariance matrix is Σ, (3.32) leads to the following formula for the predicted net expenditure for each commodity j :

$$p_{tj}\hat{z}_{tj}^N = D_t\hat{\alpha}_j + \sum_{i=1}^{m} p_{ti}\hat{\gamma}_{ti}(\hat{\alpha}_j - \delta_{ij}) \tag{3.37}$$

The same goodness-of-fit and goodness-of-forecast measures (3.30) and (3.31) as for the GML estimates are used to convey an impression of the prediction quality of both models; the two measures for the NLLS estimates are depicted in the last two columns of Tables 1a and 1b.

3.3 Computation of Elasticities and Slutsky Terms

Compensated and uncompensated elasticities of predicted trade-demand quantities with respect to prices and the trade balance clearly depend on the assumption as to which distribution the random term ε_t follows. In the case of the lognormal specification they also depend on which of the specifications (3.28) and (3.25) is used. We first employ the specification (3.28).

Rewriting (2.4) and (2.5) as

$$p_{ti}z_{ti} = D_t\alpha_i + \sum_{k=1}^{m} p_{tk}\gamma_{tk}(\alpha_i - \delta_{ik}) + e_{ti} \quad (i = 1, 2, \ldots, m) \tag{3.38}$$

where

$$e_{ti} = \sum_{k=1}^{m}(\alpha_i - \delta_{ik})p_{tk}\varepsilon_{tk} \quad (i = 1, 2, \ldots, m), \tag{3.39}$$

we construct the predicted trade-demand quantities $\hat{z}_{ti}^G$ based on the GML estimates as follows:

$$\hat{z}_{ti}^G = \frac{D_t\tilde{\alpha}_i(\hat{\beta})}{p_{ti}} + \frac{1}{p_{ti}}\sum_{k=1}^{m} p_{tk}\hat{\gamma}_{tk}\left(\tilde{\alpha}_i(\hat{\beta}) - \delta_{ik}\right) + \frac{1}{p_{ti}}\hat{\mathrm{E}}(\hat{e}_{ti}), \tag{3.40}$$

where

$$\hat{e}_{ti} = \sum_{k=1}^{m}\left(\tilde{\alpha}_i(\hat{\beta}) - \delta_{ik}\right)p_{tk}\varepsilon_{tk} \tag{3.41}$$

and δ_{ik} denotes Kronecker's δ, and thus $\hat{\mathrm{E}}(\hat{e}_{ti})$ is determined from

$$\hat{\mathrm{E}}(\hat{e}_{ti}) = \sum_{k=1}^{m}\left(\tilde{\alpha}_i(\hat{\beta}) - \delta_{ik}\right)p_{tk}\exp\left(\hat{a}_k + \frac{\hat{v}_{kk}}{2}\right). \tag{3.42}$$

Hence (3.40) becomes

$$\hat{z}_{ti}^G = \frac{D_t\tilde{\alpha}_i(\hat{\beta})}{p_{ti}} + \frac{1}{p_{ti}}\sum_{k=1}^{m}\left(\tilde{\alpha}_i(\hat{\beta}) - \delta_{ik}\right)p_{tk}\left[\hat{\gamma}_{tk} + \exp\left(\hat{a}_k + \frac{\hat{v}_{kk}}{2}\right)\right]. \tag{3.43}$$

[17]For agricultural products, basic materials, and capital goods the constraint is not always fulfilled.

[18]For the implementation of this routine see Press et al. (1988), pp. 307–311.

[19]For further details see Press et al. (1988), pp. 294–297.

Elasticities of predicted trade-demand quantities with respect to prices of the same commodity i or other commodities j are therefore determined from

$$\frac{\partial \hat{z}^G_{ti}}{\partial p_{tj}}\frac{p_{tj}}{\hat{z}^G_{ti}} = -\delta_{ij} + \left(\tilde{\alpha}_i(\hat{\beta}) - \delta_{ij}\right) R_{tj}, \tag{3.44}$$

where

$$R_{tj} = \frac{p_{tj}\left[\hat{\gamma}_{tj} + \exp(\hat{a}_j + \frac{\hat{v}_{jj}}{2})\right]}{D_t\tilde{\alpha}_i(\hat{\beta}) + \sum_{k=1}^m p_{tk}\left[\hat{\gamma}_{tk} + \exp(\hat{a}_k + \frac{\hat{v}_{kk}}{2})\right]\left(\tilde{\alpha}_i(\hat{\beta}) - \delta_{ik}\right)}, \tag{3.45}$$

whereas the net-expenditure elasticities for each period are obtained from

$$\frac{\partial \hat{z}^G_{ti}}{\partial D_t}\frac{D_t}{\hat{z}^G_{ti}} = \frac{D_t\tilde{\alpha}_i(\hat{\beta})}{D_t\tilde{\alpha}_i(\hat{\beta}) + \sum_{k=1}^m p_{tk}\left[\hat{\gamma}_{tk} + \exp(\hat{a}_k + \frac{\hat{v}_{kk}}{2})\right]\left(\tilde{\alpha}_i(\hat{\beta}) - \delta_{ik}\right)}. \tag{3.46}$$

Finally,

$$s^G_{t,ij} = \frac{\tilde{\alpha}_j(\hat{\beta})(\tilde{\alpha}_i(\hat{\beta}) - \delta_{ij})}{p_{ti}p_{tj}}\left\{D_t + \sum_{k=1}^m p_{tk}\left[\hat{\gamma}_{tk} + \exp\left(\hat{a}_k + \frac{\hat{v}_{kk}}{2}\right)\right]\right\} \tag{3.47}$$

yield the trade-Slutsky terms.

Construction of the elasticity and trade-Slutsky terms on the basis of formula (3.25), where $-D_{\min} + \exp(\hat{\nu})$ substitutes for $\sum_{j=1}^m p_{tj}\left(\hat{\gamma}_{tj} + \hat{\mathrm{E}}(\varepsilon_{tj})\right)$, leads to the striking result that all cross-price elasticities vanish. Own-price elasticities of the corresponding predicted trade demand quantities

$$\hat{z}^D_{ti} = \frac{[D_t - D_{\min} + \exp(\hat{\nu})]\tilde{\alpha}_i(\hat{\beta})}{p_{ti}} - \left[\hat{\gamma}_{ti} + \exp\left(\hat{a}_i + \frac{\hat{v}_{ii}}{2}\right)\right]$$

can be determined from

$$\frac{\partial \hat{z}^D_{ti}}{\partial p_{ti}}\frac{p_{ti}}{\hat{z}^D_{ti}} = -1 - \frac{p_{ti}\left[\hat{\gamma}_{ti} + \exp\left(\hat{a}_i + \frac{\hat{v}_{ii}}{2}\right)\right]}{\left[D_t - D_{\min} + \exp(\hat{\nu})\right]\tilde{\alpha}_i(\hat{\beta}) - p_{ti}\left[\hat{\gamma}_{ti} + \exp\left(\hat{a}_i + \frac{\hat{v}_{ii}}{2}\right)\right]}, \tag{3.48}$$

whereas the net-expenditure elasticities are yielded by

$$\frac{\partial \hat{z}^D_{ti}}{\partial D_t}\frac{D_t}{\hat{z}^D_{ti}} = \frac{\tilde{\alpha}_i(\hat{\beta})D_t}{\left[D_t - D_{\min} + \exp(\hat{\nu})\right]\tilde{\alpha}_i(\hat{\beta}) - p_{ti}\left[\hat{\gamma}_{ti} + \exp\left(\hat{a}_i + \frac{\hat{v}_{ii}}{2}\right)\right]}. \tag{3.49}$$

The trade-Slutsky terms are finally given by

$$s^D_{t,ij} = \frac{\tilde{\alpha}_j(\hat{\beta})}{p_{ti}p_{tj}}\left\{\left[D_t - D_{\min} + \exp(\hat{\nu})\right]\left(\tilde{\alpha}_i(\hat{\beta}) - \delta_{ij}\right) - p_{tj}\left[\hat{\gamma}_{tj} + \exp\left(\hat{a}_j + \frac{\hat{v}_{jj}}{2}\right)\right]\right\}. \tag{3.50}$$

We now turn to the corresponding expressions for the case of the nonlinear least-squares estimates. Predicted trade demand quantities z^N_{ti} being derived from (3.37), the respective price elasticities are obtained from

$$\frac{\partial \hat{z}^N_{ti}}{\partial p_{tj}}\frac{p_{tj}}{\hat{z}^N_{ti}} = -\delta_{ij} + (\hat{\alpha}_i - \delta_{ij})\frac{p_{tj}\hat{\gamma}_{tj}}{D_t\hat{\alpha}_i + \sum_{k=1}^m p_{tk}\hat{\gamma}_{tk}(\hat{\alpha}_i - \delta_{ik})}, \tag{3.51}$$

whereas the net-expenditure elasticities are constructed as follows:

$$\frac{\partial \hat{z}^N_{ti}}{\partial D_t}\frac{D_t}{\hat{z}^N_{ti}} = \frac{D_t\hat{\alpha}_i}{D_t\hat{\alpha}_i + \sum_{k=1}^m p_{tk}\hat{\gamma}_{tk}(\hat{\alpha}_i - \delta_{ik})}. \tag{3.52}$$

Finally, the trade-Slutsky terms are given by

$$s^N_{t,ij} = \frac{\hat{\alpha}_j(\hat{\alpha}_i - \delta_{ij})}{p_{ti}p_{tj}}\left\{D_t + \sum_{k=1}^m p_{tk}\hat{\gamma}_{tk}\right\}. \tag{3.53}$$

It should be noted that the elasticities and Slutsky terms derived from (3.27) differ from those derived from the deterministic LES model only with respect to the expected value of ε_t. Given that this expected value takes the value zero in the NLLS case, the elasticities and Slutsky terms corresponding to the NLLS estimates do not differ from their deterministic counterparts.[20]

Tables 6a, 6b, 7, 8a, 8b, and 9 show the medians of own- and (if unequal to zero) cross-price elasticities computed from formulas (3.44), (3.48), and (3.51), respectively; the first three tables show results corresponding to the six-commodity case, results for the twelve-commodity models are depicted in the latter three tables. The medians of net-expenditure elasticities as determined from (3.46), (3.49), and (3.52) are presented in Tables 10a, 10b, 11, 12a, 12b, and 13, whereas the medians of trade-Slutsky terms obtained from (3.47), (3.50), and (3.53) are shown in Tables 14a, 14b, 15, 16a, 16b, and 17. Additionally, own-price elasticities are depicted in Figures 8–9. For their interpretation, it should be noted that, whereas the trade deficits D_t are negative for all but five months[21] of the observation period, net imports of the six commodities in question frequently change signs.

4 Discussion of the Results

The six panels of Figures 2a and 2b depict the predicted and actual values of the six net-import categories in the six-commodity model according to the GML

[20] Cf. Philips (1983), p. 128.

[21] With the exceptions of June 1965, August 1965, August 1980, January 1981, and March 1981, the German trade balance has continuously been characterized by a surplus.

method, the predicted values being computed from formula (3.25); the twelve panels of Figures 3a–3d do the same for the six gross imports and six gross exports of the twelve-commodity model. As a benchmark for goodness of fit and forecast, the corresponding plots of predicted and actual values are shown in Figures 4a–4b and 5a–5d for the NLLS procedure, the predicted values being computed from (3.37). The predictive quality of the GML estimation procedure stands up well in this comparison, as far as the eye can tell. This is confirmed by Tables 1a and 1b where the goodness-of-fit and goodness-of-forecast measures (3.30) and (3.31) show these to be on average only slightly worse (larger) for GML estimation than for NLLS estimation, and in several cases better.

Figures 6a–6b and 7a–7d display the residuals from GML estimation. These do not show any particular trend in the means but do show an increasing trend in the variance. This is not surprising, since the import and export values, as well as the price indices, are measured in absolute terms and reflect the steady inflation over the period. Moreover, the true residuals as given by (2.5) and (3.39) depend on the prices (see also (3.41)). What is most striking is the apparent structural change beginning at the end of 1973. The period 1974–88 is much more volatile than the period 1959–73, reflecting the successive oil shocks as well as the change from fixed to flexible exchange rates. Again, however, this is to be expected from the structure of the error term and its dependence on prices.[22]

Tables 2 and 3 furnish the parameter estimates from GML and NLLS estimation respectively, for the six-commodity model, and Tables 4 and 5 do the same for the twelve-commodity model. It is of interest that the estimates of the α_i's are quite close for the two methods of estimation, the principal difference being that the GML method yields very small values (less than 10^{-5}) for net imports and gross exports of mining products as well as for gross imports of capital and consumer goods. One could argue that this is because mining products (consisting largely of crude petroleum), as well as capital goods, do not directly enter preferences; but this could not explain the low α_i for consumer-goods gross imports or the high α_i for the capital-goods gross exports.[23]

[22]We have not attempted to ascertain formally whether the dependent variables of the model (the trades) and the independent variables (the prices and trade deficit) are "cointegrated" (cf., e.g., Hendry, 1986; Granger, 1986; Stock and Watson, 1988), but this seems to be a plausible hypothesis.

[23]The model in Section 2 assumes that all goods enter consumer preferences, and does not explicitly take account of trade in intermediate products. Both these assumptions are in need of modification in future work.

The most striking thing about the α_i estimates is that 80% are accounted for by the two sectors basic materials (largely chemicals) and capital goods (largely machinery and transport equipment). Thus, in terms of the six-commodity model one would predict that if Germany borrows heavily from abroad to finance development in the East, out of every mark borrowed, net imports of capital goods will rise by 71 pfennigs and net imports of basic materials will rise by 14 pfennigs. In terms of the twelve-commodity model one would conclude that there would be no change in the gross imports of capital goods but a fall in gross exports of capital goods to the extent of 65 pfennigs.[24] Likewise, in terms of the twelve-commodity model one would predict a rise in imports of basic materials of 5 pfennigs and a fall in exports of 13 pfennigs. These two industries would bear the brunt of adjustment if the German trade balance goes into deficit. One might start to hear clamors about the "loss of competitiveness" in these two sectors.

While the estimates of the α_i parameters are reasonably close as between the GML and NLLS procedures, it will be noted from Tables 4 and 5 that the estimates of the spline parameters are very far apart. This partly reflects the fact that the GML procedure imposes positivity constraints on the $z_{ti} + \gamma_{ti} + \varepsilon_{ti}$ whereas the NLLS procedure imposes them only on their expected values; and partly the fact that the likelihood function of the lognormal distribution is very flat with respect to the γ_{ti} and as a result these terms are very difficult to estimate (cf. Johnson and Kotz, 1970; Cohen, 1951; Hill, 1963).

Figures 8a–8b and 9a–9c plot the own-price elasticities of the trade-demand functions for the six-commodity and twelve-commodity models respectively[25] computed from formula (3.44) using GML estimation, and the medians of these and of the cross-price elasticities are tabulated in Tables 6a and 8a respectively. Table 6b shows the medians of the own-price elasticities from GML estimation using formula (3.48) (plots are not shown, but they are similar to those of Figures 8–9) and Table 7 gives the medians of the own- and cross-price elasticities from NLLS estimation using formula (3.51). Plots of own-price elasticities from NLLS estimation are not shown here, but they are similar to the second panels of Figures

[24]Theoretically, the sums of the α_i columns for imports and exports in Table 4 should be the same as the α_i column in Table 2, but of course they do not correspond exactly because they are different models.

[25]Plots of elasticities for net imports of mining products, gross imports of capital goods and consumer goods, and gross exports of mining products, are omitted since from Tables 2 and 4 these have effectively zero α coefficients, hence the elasticities are also effectively zero as is seen from (3.44) and (3.45).

8a and 8b, exhibiting great spikes which result from denominators approaching zero. Two additional features of these plots should be noted. One is that the elasticities for agriculture and foodstuffs both exhibit seasonal fluctuation; this seems quite reasonable. The second feature is not so reasonable: the tendency of the elasticities to approach zero as time progresses. We believe this result to be an unfortunate consequence of the implicit assumption (3.29) implied (although not imposed) by the GML estimation procedure. Figure 10 displays the two terms of (3.29) for the six- and twelve-commodity models. The NLLS estimates do not exhibit the same kind of trend as the GML estimates. Note that since exports are measured negatively, one expects own-price elasticities to be positive for export goods.

Tables 10a and 10b give the medians of the net-expenditure elasticities from GML estimation in the six-commodity model using formulas (3.46) and (3.49) respectively, while Table 11 gives the medians of the corresponding elasticities from NLLS estimation using formula (3.52). Tables 12a, 12b, and 13 do the same for the twelve-commodity model.

Finally, Tables 14a and 14b furnish medians of the estimates of the trade-Slutsky terms from GML estimation using formulas (3.47) and (3.50) respectively, and Table 15 does the same for NLLS estimation using formula (3.53). Tables 16a, 16b, and 17 do the same for the twelve-commodity model. It will be observed that the GML estimates are in conformity with theory, which requires the own-trade-Slutsky terms to be nonpositive, whereas the NLLS estimates are not.

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Table 1a: Goodness of Fit and Forecast
Generalized Maximum-Likelihood vs. Nonlinear Least-Squares Estimates
(6 Commodities, Period 1959–1988)

Category	GML Estimation		NLLS Estimation	
NET IMPORTS				
Agriculture, etc.	3007601.57	6510996.68	2957929.26	6445418.58
Mining products	5456533.50	9516878.25	4808881.51	8696996.92
Basic materials	7698120.85	12937149.99	7637499.21	13014415.88
Capital goods	15108902.36	25317654.78	10699014.54	21491760.22
Consumer goods	4320137.45	7824008.78	3649435.76	7140894.67
Foodstuffs	2328476.37	4676985.94	2388433.27	4815011.32

Table 1b: Goodness of Fit and Forecast
Generalized Maximum-Likelihood vs. Nonlinear Least-Squares Estimates
(12 Commodities, Period 1959–1988)

Category	GML Estimation		NLLS Estimation	
IMPORTS				
Agriculture, etc.	3295550.56	7249108.95	3002642.37	6830445.84
Mining products	5390793.71	9402640.38	5011106.10	8870155.62
Basic materials	9635717.12	20728060.85	8867076.29	19816801.11
Capital goods	11600516.79	26599949.31	11596087.53	26611553.51
Consumer goods	6714710.47	14553548.78	6423793.44	14602062.72
Foodstuffs	2045627.20	4820293.04	1999682.10	4685410.34
EXPORTS				
Agriculture, etc.	652987.17	1288524.11	645866.89	1275731.10
Mining products	1704124.86	2579133.06	1581607.73	2416502.79
Basic materials	8237902.52	18507390.67	8343121.70	18758154.64
Capital goods	22537140.58	45587246.90	17769514.35	40746116.03
Consumer goods	4648558.34	10203724.76	4770075.62	10495873.19
Foodstuffs	1923753.55	3900589.36	1958607.13	3925062.30

Table 2: Results from Generalized Maximum-Likelihood Estimation
Estimated Coefficients (6 commodities, period: 1959–1988)

Category i	α_i	Spline (1)	Spline (2)	Spline (3)	Cosine	Sine
NET IMPORTS						
Agriculture, etc.	0.02176	−313042.58	−313847.17	−314512.62	11.024	−86.068
	(4.50)	(−2502.44)	(−3427.30)	(−8527.43)	(0.72)	(−5.59)
Mining products	0.00000	−310685.09	−315763.57	−314976.00	−104.572	−40.119
	(6.77)	(−3728.29)	(−6818.00)	(−3776.14)	(−2.51)	(−0.96)
Basic materials	0.13582	−310555.57	−311679.46	−313736.87	101.490	90.963
	(14.08)	(−927.15)	(−1414.77)	(−3695.67)	(2.52)	(2.26)
Capital goods	0.70940	−303908.21	−305465.48	−310952.92	187.905	−10.091
	(56.05)	(−747.51)	(−1182.85)	(−2078.62)	(2.66)	(−0.14)
Consumer goods	0.10815	−309897.97	−311724.92	−313414.47	−75.661	−7.806
	(16.25)	(−1531.65)	(−2364.50)	(−6195.62)	(−3.43)	(−0.35)
Foodstuffs	0.02486	−312892.19	−313387.12	−313227.76	−62.739	37.240
	(6.77)	(−3223.29)	(−4807.18)	(−11560.64)	(−5.79)	(3.45)
ν	−6.71071					
	(−4654.16)					

t-ratios are given in parentheses.

Table 3: Results from Nonlinear Least-Squares Estimation
Estimated Coefficients (6 commodities, period: 1959–1988)

Category i	α_i	Spline (1)	Spline (2)	Spline (3)	Cosine	Sine
NET IMPORTS						
Agriculture, etc.	0.01950	27.274	9.245	−12.970	1.224	−107.924
	(0.25)	(0.40)	(0.19)	(−0.68)	(0.01)	(−0.59)
Mining products	0.04106	87.501	0.001	−20.590	−418.189	−199.529
	(0.75)	(1.16)	(0.00)	(−0.73)	(−0.83)	(−0.83)
Basic materials	0.21072	98.798	32.174	−0.001	−9.533	83.199
	(0.91)	(0.72)	(0.54)	(0.00)	(−0.01)	(0.36)
Capital goods	0.58259	−137.441	49.261	49.854	71.646	2.272
	(1.99)	(−0.50)	(0.44)	(1.09)	(0.36)	(0.05)
Consumer goods	0.10751	69.677	30.029	21.237	−155.454	−27.118
	(1.03)	(0.72)	(0.85)	(1.00)	(−0.52)	(−0.19)
Foodstuffs	0.03862	37.079	−0.001	10.989	−71.160	36.464
	(0.87)	(0.65)	(0.00)	(0.63)	(−0.42)	(0.23)

t-ratios are given in parentheses.

Table 4: Results from Generalized Maximum-Likelihood Estimation Estimated Coefficients (12 commodities, period: 1959–1988)

Category i	α_i	Spline (1)	Spline (2)	Spline (3)	Cosine	Sine
IMPORTS						
Agriculture	0.03468	−306444.77	−307563.59	−308827.75	−11.500	−85.368
	(9.20)	(−3127.54)	(−4243.66)	(−8393.48)	(−0.70)	(−5.15)
Mining products	0.00830	−300775.97	−306449.08	−305029.96	−141.328	−1.522
	(5.32)	(−1468.50)	(−2896.62)	(−3577.27)	(−3.39)	(−0.04)
Basic materials	0.04860	−297546.83	−303472.89	−309805.86	180.213	26.743
	(5.84)	(−897.26)	(−1458.34)	(−2995.52)	(3.56)	(0.53)
Capital goods	0.00000	−298950.57	−302806.84	−311689.96	−43.450	−125.887
	(4.81)	(−2917.28)	(−5320.99)	(−3043.27)	(−0.85)	(−2.46)
Consumer goods	0.00000	−303298.52	−306230.22	−309458.83	−67.932	22.330
	(4.82)	(−4762.96)	(−8657.56)	(−4860.32)	(−2.14)	(0.70)
Foodstuffs	0.01872	−305397.03	−306537.46	−308040.03	−57.552	29.285
	(7.60)	(−4813.07)	(−7111.12)	(−13890.15)	(−6.02)	(3.08)
EXPORTS						
Agriculture	0.00481	−307621.03	−307450.67	−307208.60	14.873	7.255
	(4.90)	(−10066.82)	(−16184.55)	(−39802.62)	(4.59)	(2.25)
Mining products	0.00000	−304854.41	−304829.90	−305130.65	13.394	−6.971
	(4.86)	(−14219.65)	(−25609.71)	(−14248.25)	(1.25)	(−0.65)
Basic materials	0.13015	−315725.00	−311560.55	−307900.16	−58.692	84.101
	(17.66)	(−1346.67)	(−1958.38)	(−3747.53)	(−1.49)	(2.14)
Capital goods	0.64709	−313112.26	−313482.30	−311864.37	250.364	100.563
	(60.73)	(−713.72)	(−1268.40)	(−1443.79)	(2.36)	(0.95)
Consumer goods	0.07988	−310233.04	−308890.19	−307069.08	−2.951	−29.296
	(20.94)	(−2495.95)	(−3956.55)	(−6751.91)	(−0.14)	(−1.36)
Foodstuffs	0.02776	−308928.30	−308393.45	−307174.72	−1.388	7.850
	(11.80)	(−4578.73)	(−7022.37)	(−15046.23)	(−0.16)	(0.88)
ν	−3.48490					
	(−102.52)					

t-ratios are given in parentheses.

Table 5: Results from Nonlinear Least-Squares Estimation Estimated Coefficients (12 commodities, period: 1959–1988)

Category i	α_i	Spline (1)	Spline (2)	Spline (3)	Cosine	Sine
IMPORTS						
Agriculture, etc.	0.02238	29.321	29.900	22.988	−22.497	−119.782
	(0.63)	(1.18)	(1.16)	(−0.84)	(−0.12)	(−0.54)
Mining products	0.01603	66.703	0.005	−30.017	−391.263	−211.786
	(1.04)	(2.24)	(0.00)	(−1.30)	(−0.89)	(−0.83)
Basic materials	0.03762	48.490	46.022	31.587	223.103	−1.628
	(0.52)	(0.86)	(0.99)	(0.81)	(0.73)	(0.00)
Capital goods	0.00145	−16.191	−0.022	−13.139	−93.481	−150.921
	(0.07)	(−0.25)	(0.00)	(−0.23)	(−0.28)	(−0.23)
Consumer goods	0.04119	47.530	−44.356	30.362	−121.133	16.402
	(0.42)	(0.86)	(−0.73)	(0.66)	(−0.46)	(0.04)
Foodstuffs	0.01011	20.624	16.745	−10.144	−59.391	22.852
	(0.38)	(0.80)	(0.52)	(−0.30)	(−0.50)	(0.17)
EXPORTS						
Agriculture, etc.	0.00662	6.287	−0.003	14.245	9.026	2.877
	(1.23)	(0.48)	(0.00)	(1.37)	(0.20)	(0.07)
Mining products	0.00667	23.800	19.434	14.033	−9.952	8.855
	(0.48)	(0.65)	(0.78)	(0.83)	(−0.07)	(0.07)
Basic materials	0.11777	−0.000	−0.014	46.877	−162.527	82.123
	(1.25)	(0.00)	(0.00)	(0.83)	(−0.47)	(0.22)
Capital goods	0.64407	161.913	144.190	108.433	75.772	10.035
	(2.83)	(1.28)	(1.15)	(0.75)	(0.68)	(0.14)
Consumer goods	0.06343	24.637	8.184	34.059	−42.178	−53.555
	(0.73)	(0.35)	(0.04)	(0.77)	(−0.18)	(−0.22)
Foodstuffs	0.03267	23.891	0.001	26.637	−13.715	7.899
	(5.00)	(2.17)	(0.00)	(0.96)	(−0.11)	(0.07)

t-ratios are given in parentheses.

Table 6a: Results from GML Estimation
Medians of Own- and Cross-Price Elasticities[a] (6 commodities, period: 1959–1988)

	External Prices					
Net-Import Category	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
Agriculture, etc.	−0.19536	−0.01847	0.03341	0.22355	0.01581	−0.00191
Mining products	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Basic materials	0.36000	0.41532	3.56284	−4.64285	−0.30808	0.03102
Capital goods	0.15359	0.12647	−0.26593	−0.25251	−0.11786	0.01557
Consumer goods	−0.39188	−0.46684	0.58807	4.39034	−2.70797	−0.04798
Foodstuffs	−0.06194	−0.07114	0.12697	0.84112	0.05148	−0.66442

[a]Computed from formula (3.44) of the text.

Table 6b: Results from GML Estimation
Medians of Own-Price Elasticities[b] (6 commodities, period: 1959–1988)

	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
NET IMPORTS	−0.17226	0.00000	3.29635	1.50734	−3.45004	−0.64807

[b]Computed from formula (3.48) of the text.

Table 7: Results from NLLS Estimation
Medians of Own- and Cross-Price Elasticities[c] (6 commodities, period: 1959–1988)

	External Prices					
Net-Import Category	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
Agriculture, etc.	−0.41195	−0.00884	0.00244	−0.00255	−0.00314	0.00398
Mining products	−0.00677	−0.52468	−0.00051	0.00037	−0.01790	−0.00615
Basic materials	−0.00133	−0.00089	−0.95683	−0.00452	−0.00737	−0.00173
Capital goods	−0.00114	−0.00031	−0.01177	−0.99632	−0.00691	−0.00147
Consumer goods	−0.00323	−0.00450	−0.00733	−0.00238	−0.90057	−0.00375
Foodstuffs	−0.00047	−0.00350	−0.00831	−0.00312	−0.01176	−0.85955

[c]Computed from formula (3.51) of the text.

Table 8a: Results from GML Estimation
Medians of Own- and Cross-Price Elasticities[a] (12 commodities, period: 1959–1988)

Category	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
IMPORTS	External Prices: Imports					
Agriculture, etc.	−0.28843	−0.02583	−0.06510	−0.06000	−0.04377	−0.01812
Mining products	−0.00753	−0.06223	−0.01439	−0.01395	−0.00953	−0.00361
Basic materials	−0.01541	−0.01570	−0.18846	−0.03762	−0.02503	−0.00978
Capital goods	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Consumer goods	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Foodstuffs	−0.02085	−0.01992	−0.05285	−0.04980	−0.03559	−0.22134
EXPORTS	External Prices: Imports					
Agriculture, etc.	0.03139	0.03556	0.08063	0.07936	0.05385	0.02047
Mining products	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Basic materials	0.03616	0.03793	0.09215	0.09102	0.06209	0.02415
Capital goods	0.08917	0.09367	0.22734	0.22489	0.15257	0.05866
Consumer goods	0.05352	0.05896	0.13317	0.13726	0.09020	0.03456
Foodstuffs	0.05666	0.07809	0.15353	0.14982	0.09817	0.03748
IMPORTS	External Prices: Exports					
Agriculture, etc.	0.00518	0.00733	0.12647	0.37063	0.06003	0.01938
Mining products	0.00168	0.00180	0.03993	0.12131	0.02032	0.00633
Basic materials	0.00321	0.00467	0.08051	0.20772	0.03499	0.01219
Capital goods	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Consumer goods	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Foodstuffs	0.00446	0.00631	0.10559	0.28457	0.04947	0.01735
EXPORTS	External Prices: Exports					
Agriculture, etc.	0.30890	−0.00954	−0.15744	−0.42409	−0.07226	−0.02459
Mining products	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Basic materials	−0.00746	−0.01086	0.16987	−0.48383	−0.08253	−0.02869
Capital goods	−0.01876	−0.02747	−0.45634	−0.33769	−0.20783	−0.07113
Consumer goods	−0.01135	−0.01641	−0.27480	−0.72788	0.39876	−0.04185
Foodstuffs	−0.01174	−0.01824	−0.29200	−0.76169	−0.12817	0.50102

[a]Computed from formula (3.44) of the text.

Table 8b: Results from GML Estimation
Medians of Own-Price Elasticities[b] (12 commodities, period: 1959–1988)

	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
IMPORTS	−0.24699	−0.05168	−0.14013	0.00000	0.00000	−0.18950
EXPORTS	0.27403	0.00000	0.31881	0.78521	0.46866	0.51055

[b]Computed from formula (3.48) of the text.

Table 9: Results from NLLS Estimation
Medians of Own- and Cross-Price Elasticities[c] (12 commodities, period: 1959–1988)

Category	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
IMPORTS	External Prices: Imports					
Agriculture, etc.	−0.40736	−0.00861	0.00116	−0.01242	−0.00022	0.00098
Mining products	−0.00673	−0.12177	0.01207	−0.01258	−0.00722	−0.00368
Basic materials	0.00339	0.01379	−0.48056	0.01222	0.01229	0.00411
Capital goods	−0.00057	−0.00079	0.00060	0.00262	−0.00035	−0.00012
Consumer goods	−0.00500	−0.01721	0.01490	−0.00741	−0.75999	−0.01212
Foodstuffs	0.00103	−0.01040	0.00995	−0.00483	−0.00756	−0.31339
EXPORTS	External Prices: Imports					
Agriculture, etc.	−0.00025	0.00711	−0.01615	0.00893	0.00695	0.00134
Mining products	−0.00364	−0.00011	−0.00726	0.00135	0.00243	−0.00260
Basic materials	−0.00188	−0.00015	−0.00528	0.00181	−0.00114	−0.00584
Capital goods	−0.00246	0.00098	−0.01174	0.00251	0.00475	−0.00245
Consumer goods	−0.00804	−0.00129	−0.00856	0.00055	0.00040	−0.00408
Foodstuffs	−0.00357	0.00037	−0.01022	0.00172	0.00357	−0.00303
IMPORTS	External Prices: Exports					
Agriculture, etc.	−0.00007	−0.00103	0.00046	−0.01401	−0.00640	−0.00041
Mining products	0.00060	−0.00074	−0.00760	−0.00505	−0.00535	−0.00090
Basic materials	−0.00112	−0.00095	0.01310	−0.02120	0.00366	0.00001
Capital goods	0.00004	0.00004	−0.00017	0.00072	−0.00034	−0.00001
Consumer goods	0.00012	−0.00385	−0.02070	−0.02254	−0.00627	−0.00292
Foodstuffs	0.00031	−0.00169	−0.01314	−0.01001	−0.00292	−0.00200
EXPORTS	External Prices: Exports					
Agriculture, etc.	−0.82400	−0.00278	0.00381	−0.03627	0.00021	−0.00147
Mining products	−0.00065	−0.61974	−0.00219	−0.02827	−0.00320	−0.00195
Basic materials	−0.00073	−0.00381	−0.91309	−0.03390	−0.00230	−0.00237
Capital goods	−0.00107	−0.00379	−0.00239	−0.97745	−0.00253	−0.00248
Consumer goods	−0.00073	−0.00409	−0.00375	−0.04134	−0.92314	−0.00264
Foodstuffs	−0.00089	−0.00391	−0.00318	−0.04246	−0.00378	−0.92687

[c]Computed from formula (3.51) of the text.

Table 10a: Results from GML Estimation
Medians of Net-Expenditure Elasticities[a] (6 commodities, period: 1959–1988)

	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
NET IMPORTS	−0.02969	0.00000	0.66387	0.28332	−0.74512	−0.10278

[a]Computed from formula (3.46) of the text.

Table 10b: Results from GML Estimation
Medians of Net-Expenditure Elasticities[b] (6 commodities, period: 1959–1988)

	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
NET IMPORTS	−0.02977	0.00000	0.65313	0.28534	−0.78199	−0.10107

[b]Computed from formula (3.49) of the text.

Table 11: Results from NLLS Estimation
Medians of Net-Expenditure Elasticities[c] (6 commodities, period: 1959–1988)

	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
NET IMPORTS	0.44619	0.56734	0.97827	1.02316	0.92264	0.87385

[c]Computed from formula (3.52) of the text.

Table 12a: Results from GML Estimation
Medians of Net-Expenditure Elasticities[a] (12 commodities, period: 1959–1988)

	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
IMPORTS	−0.04302	−0.01212	−0.02907	0.00000	0.00000	−0.03919
EXPORTS	0.06343	0.00000	0.06978	0.17421	0.10960	0.12570

[a]Computed from formula (3.46) of the text.

Table 12b: Results from GML Estimation
Medians of Net-Expenditure Elasticities[b] (12 commodities, period: 1959–1988)

	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
IMPORTS	−0.04236	−0.01219	−0.02905	0.00000	0.00000	−0.03904
EXPORTS	0.06349	0.00000	0.07040	0.17365	0.10901	0.12635

[b]Computed from formula (3.49) of the text.

Table 13: Results from NLLS Estimation
Medians of Net-Expenditure Elasticities[c] (12 commodities, period: 1959–1988)

	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
IMPORTS	0.46482	0.17862	0.45580	−0.00182	0.83452	0.35498
EXPORTS	0.83727	0.65006	0.96417	1.00253	0.98800	0.98319

[c]Computed from formula (3.52) of the text.

Table 14a: Results from GML Estimation
Medians of Trade-Slutsky Terms[a] (6 commodities, period: 1959–1988)

Net-Import Category	External Prices: Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
Agriculture, etc.	−0.51588	0.00000	0.08299	0.39319	0.05955	0.01184
Mining products	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Basic materials	0.08299	0.00000	−3.76535	2.86265	0.43321	0.08536
Capital goods	0.39319	0.00000	2.86265	−5.77586	2.07873	0.41077
Consumer goods	0.05955	0.00000	0.43321	2.07873	−2.59898	0.06178
Foodstuffs	0.01184	0.00000	0.08536	0.41077	0.06178	−0.49918

[a]Computed from formula (3.47) of the text.

Table 14b: Results from GML Estimation
Medians of Trade-Slutsky Terms[b] (6 commodities, period: 1959–1988)

Net-Import Category	External Prices: Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
Agriculture, etc.	−0.44223	0.12473	−0.02211	−0.24791	0.01136	0.01809
Mining products	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Basic materials	0.44423	0.84848	−4.42510	−1.74465	0.08245	0.13264
Capital goods	2.12330	4.27960	−0.73935	−31.32067	0.38935	0.63581
Consumer goods	0.32402	0.62017	−0.11138	−1.27109	−2.77490	0.09651
Foodstuffs	0.06749	0.13621	−0.02431	−0.27062	0.01264	−0.45743

[b]Computed from formula (3.50) of the text.

Table 15: Results from NLLS Estimation
Medians of Trade-Slutsky Terms[c] (6 commodities, period: 1959–1988)

Net-Import Category	External Prices: Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
Agriculture, etc.	0.07621	−0.00761	−0.01942	−0.04801	−0.00882	−0.00293
Mining products	−0.00761	0.99642	−0.09914	−0.24408	−0.04504	−0.01453
Basic materials	−0.01942	−0.09914	0.91267	−0.60895	−0.11209	−0.03720
Capital goods	−0.04801	−0.24408	−0.60895	1.05112	−0.27326	−0.09103
Consumer goods	−0.00882	−0.04504	−0.11209	−0.27326	0.42223	−0.01688
Foodstuffs	−0.00293	−0.01453	−0.03720	−0.09103	−0.01688	0.13617

[c]Computed from formula (3.53) of the text.

Table 16a: Results from GML Estimation
Medians of Trade-Slutsky Terms[a] (12 commodities, period: 1959–1988)

Category	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
IMPORTS	External Prices: Imports					
Agriculture, etc.	−0.86050	0.01771	0.05439	0.00000	0.00000	0.0145
Mining products	0.01771	−1.38395	0.03115	0.00000	0.00000	0.0083
Basic materials	0.05439	0.03115	−1.84036	0.00000	0.00000	0.0260
Capital goods	0.00000	0.00000	0.00000	0.00000	0.00000	0.0000
Consumer goods	0.00000	0.00000	0.00000	0.00000	0.00000	0.0000
Foodstuffs	0.01459	0.00835	0.02600	0.00000	0.00000	−0.3742
EXPORTS	External Prices: Imports					
Agriculture, etc.	0.00431	0.00247	0.00749	0.00000	0.00000	0.0020
Mining products	0.00000	0.00000	0.00000	0.00000	0.00000	0.0000
Basic materials	0.12345	0.07344	0.21476	0.00000	0.00000	0.0586
Capital goods	0.62406	0.37700	1.04703	0.00000	0.00000	0.2909
Consumer goods	0.07444	0.04437	0.12478	0.00000	0.00000	0.0345
Foodstuffs	0.02395	0.01420	0.04065	0.00000	0.00000	0.0113
IMPORTS	External Prices: Exports					
Agriculture, etc.	0.00431	0.00000	0.12345	0.62406	0.07444	0.0239
Mining products	0.00247	0.00000	0.07344	0.37700	0.04437	0.0142
Basic materials	0.00749	0.00000	0.21476	1.04703	0.12478	0.0406
Capital goods	0.00000	0.00000	0.00000	0.00000	0.00000	0.0000
Consumer goods	0.00000	0.00000	0.00000	0.00000	0.00000	0.0000
Foodstuffs	0.00206	0.00000	0.05868	0.29091	0.03453	0.0113
EXPORTS	External Prices: Exports					
Agriculture, etc.	−0.12319	0.00000	0.01699	0.08610	0.01034	0.0034
Mining products	0.00000	0.00000	0.00000	0.00000	0.00000	0.0000
Basic materials	0.01699	0.00000	−3.25719	2.47964	0.29407	0.0943
Capital goods	0.08610	0.00000	2.47964	−7.12115	1.50846	0.4904
Consumer goods	0.01034	0.00000	0.29407	1.50846	−2.04604	0.0575
Foodstuffs	0.00340	0.00000	0.09438	0.49040	0.05752	−0.6491

[a]Computed from formula (3.47) of the text.

Table 16b: Results from GML Estimation
Medians of Trade-Slutsky Terms[b] (12 commodities, period: 1959–1988)

Category	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
IMPORTS			External Prices: Imports			
Agriculture, etc.	−0.68818	0.26147	0.34526	0.20998	0.15949	0.07050
Mining products	0.07392	−1.12709	0.19352	0.10795	0.08004	0.03953
Basic materials	0.19451	0.49300	−1.09482	0.39011	0.26142	0.11544
Capital goods	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Consumer goods	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Foodstuffs	0.05541	0.13737	0.17141	0.09538	0.07893	−0.30691
EXPORTS			External Prices: Imports			
Agriculture, etc.	0.01578	0.03842	0.04924	0.02787	0.02210	0.00999
Mining products	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Basic materials	0.44490	1.09703	1.39035	0.88089	0.65666	0.29092
Capital goods	2.25962	5.60680	7.06684	4.31527	3.24749	1.44167
Consumer goods	0.26689	0.66648	0.84710	0.50254	0.38872	0.17226
Foodstuffs	0.08746	0.21463	0.27720	0.16389	0.12789	0.05642
IMPORTS			External Prices: Exports			
Agriculture, etc.	−0.01392	−0.04269	−0.35108	−0.68828	−0.13990	−0.04102
Mining products	−0.00731	−0.02679	−0.18386	−0.38751	−0.07367	−0.02155
Basic materials	−0.02251	−0.07300	−0.54299	−1.05629	−0.22370	−0.07499
Capital goods	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Consumer goods	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Foodstuffs	−0.00669	−0.01999	−0.16917	−0.33833	−0.06649	−0.01931
EXPORTS			External Prices: Exports			
Agriculture, etc.	−0.11410	−0.00597	−0.04877	−0.09439	−0.01926	−0.00588
Mining products	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Basic materials	−0.05654	−0.17547	−5.01723	−2.68244	−0.56103	−0.17148
Capital goods	−0.28077	−0.93847	−7.01360	−35.52066	−2.80731	−0.89763
Consumer goods	−0.03378	−0.10874	−0.85008	−1.61008	−2.41240	−0.10661
Foodstuffs	−0.01100	−0.03365	−0.27745	−0.53525	−0.11121	−0.66149

[b]Computed from formula (3.50) of the text.

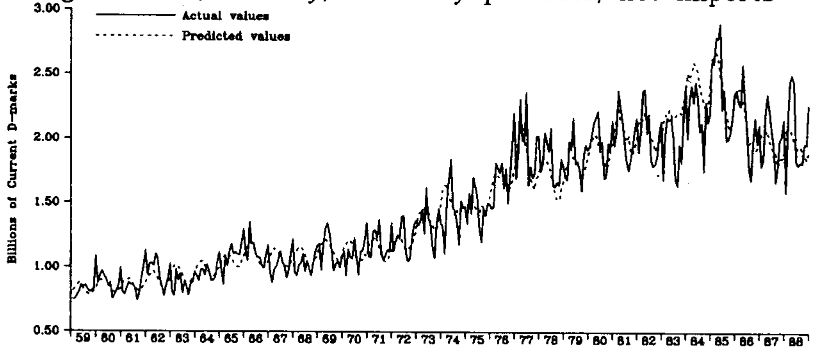
Table 17: Results from NLLS Estimation
Medians of Trade-Slutsky Terms[c] (12 commodities, period: 1959–1988)

Category	Agricul-ture, etc.	Mining products	Basic materials	Capital goods	Consumer goods	Food-stuffs
IMPORTS			External Prices: Imports			
Agriculture, etc.	0.08138	−0.00341	−0.00396	−0.00008	−0.00362	−0.00079
Mining products	−0.00341	0.42308	−0.00773	−0.00015	−0.00694	−0.00145
Basic materials	−0.00396	−0.00773	0.21652	−0.00017	−0.00796	−0.00170
Capital goods	−0.00008	−0.00015	−0.00017	0.00338	−0.00016	−0.00003
Consumer goods	−0.00362	−0.00694	−0.00796	−0.00016	0.16822	−0.00156
Foodstuffs	−0.00079	−0.00145	−0.00170	−0.00003	−0.00156	0.03253
EXPORTS			External Prices: Imports			
Agriculture, etc.	−0.00059	−0.00115	−0.00133	−0.00003	−0.00120	−0.00026
Mining products	−0.00095	−0.00182	−0.00209	−0.00004	−0.00193	−0.00040
Basic materials	−0.01080	−0.01967	−0.02316	−0.00047	−0.02217	−0.00472
Capital goods	−0.05975	−0.11301	−0.12824	−0.00259	−0.11951	−0.02530
Consumer goods	−0.00549	−0.01064	−0.01207	−0.00024	−0.01113	−0.00237
Foodstuffs	−0.00275	−0.00514	−0.00590	−0.00012	−0.00547	−0.00117
IMPORTS			External Prices: Exports			
Agriculture, etc.	−0.00059	−0.00095	−0.01080	−0.05975	−0.00549	−0.00275
Mining products	−0.00115	−0.00182	−0.01967	−0.11301	−0.01064	−0.00514
Basic materials	−0.00133	−0.00209	−0.02316	−0.12824	−0.01207	−0.00590
Capital goods	−0.00003	−0.00004	−0.00047	−0.00259	−0.00024	−0.00012
Consumer goods	−0.00120	−0.00193	−0.02217	−0.11951	−0.01113	−0.00547
Foodstuffs	−0.00026	−0.00040	−0.00472	−0.02530	−0.00237	−0.00117
EXPORTS			External Prices: Exports			
Agriculture, etc.	0.02816	−0.00032	−0.00358	−0.01943	−0.00182	−0.00090
Mining products	−0.00032	0.06862	−0.00560	−0.03130	−0.00293	−0.00140
Basic materials	−0.00358	−0.00560	0.48541	−0.34848	−0.03309	−0.01638
Capital goods	−0.01943	−0.03130	−0.34848	1.07013	−0.18147	−0.08790
Consumer goods	−0.00182	−0.00293	−0.03309	−0.18147	0.24987	−0.00828
Foodstuffs	−0.00090	−0.00140	−0.01638	−0.08790	−0.00828	0.11959

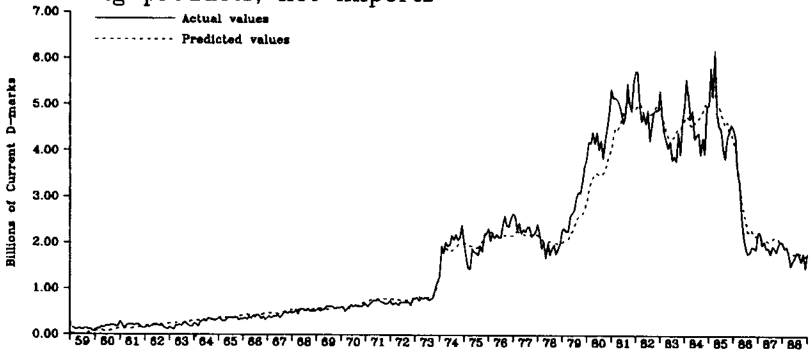
[c]Computed from formula (3.53) of the text.

Figure 2a: Predicted and Actual Values from GML Estimation

Agricultural, forestry, & fishery products, net imports



Mining products, net imports



Basic materials, net imports

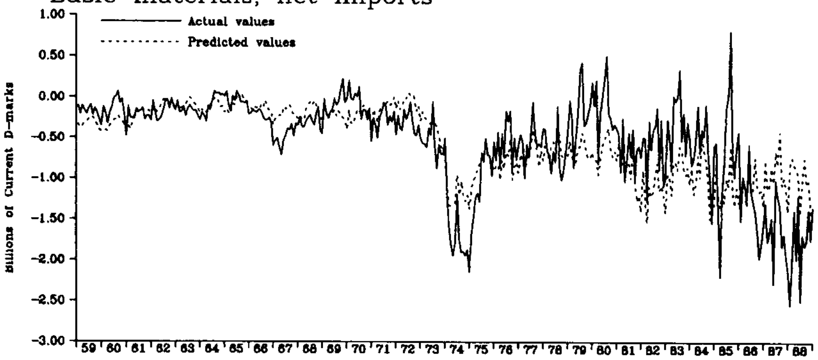
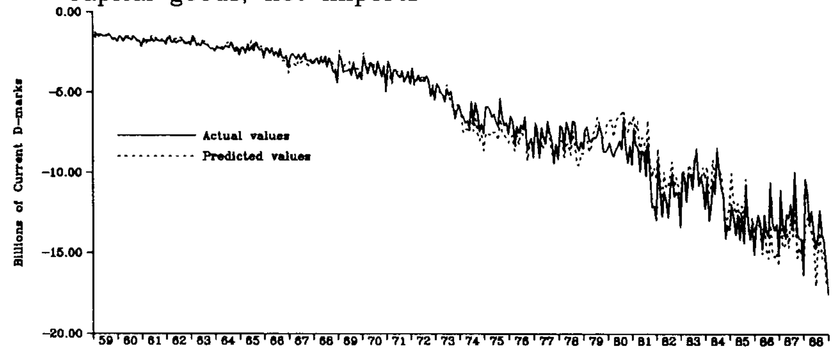
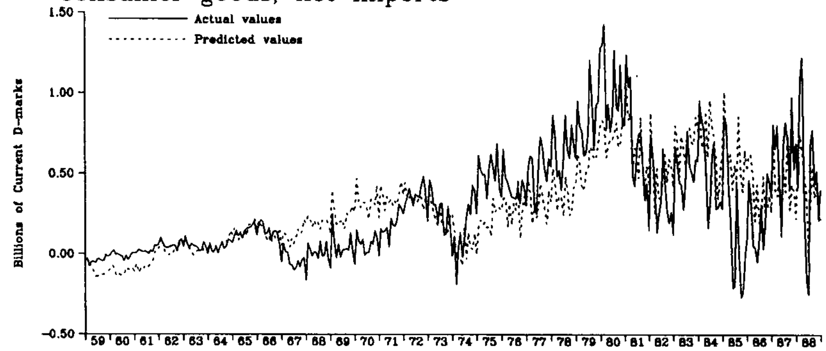


Figure 2b: Predicted and Actual Values from GML Estimation

Capital goods, net imports



Consumer goods, net imports



Food, beverages, & tobacco, net imports

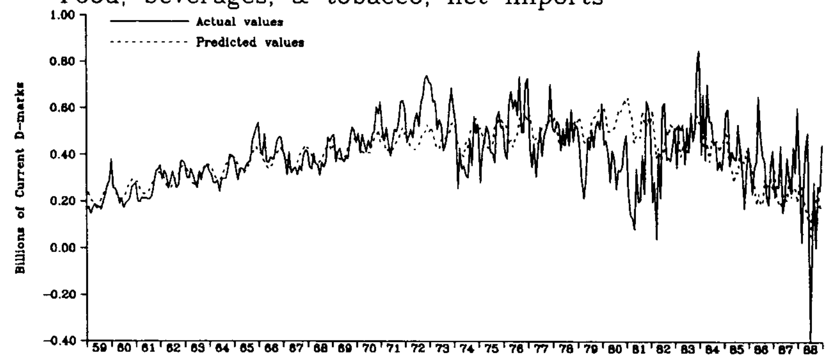
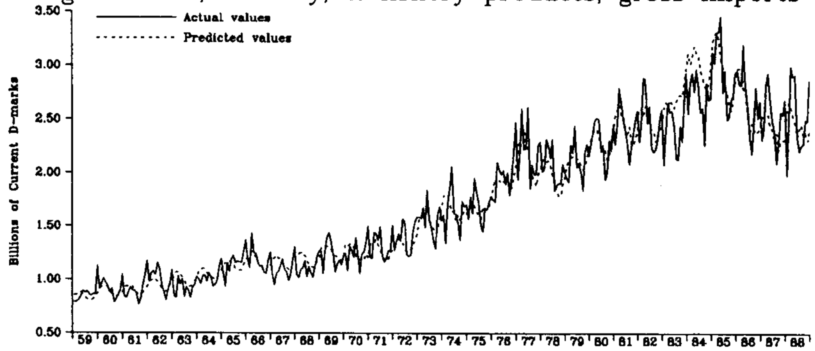
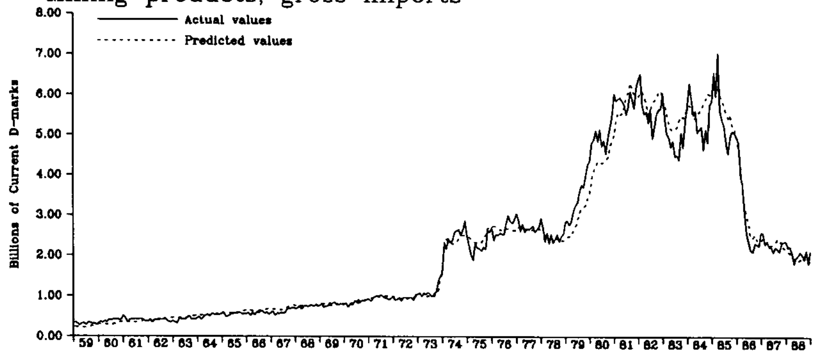


Figure 3a: Predicted and Actual Values from GML Estimation

Agricultural, forestry, & fishery products, gross imports



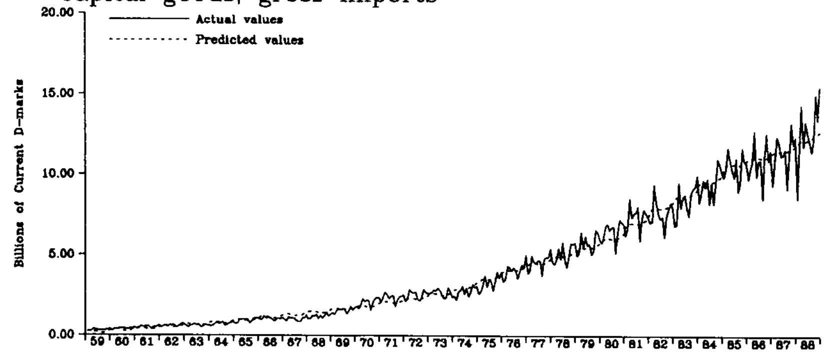
Mining products, gross imports



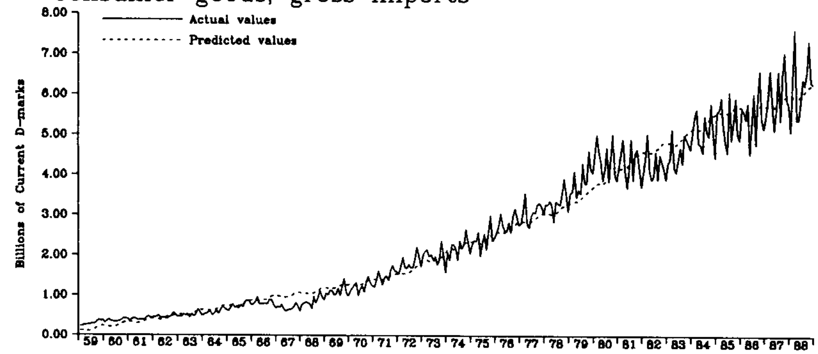
Basic materials, gross imports

Figure 3b: Predicted and Actual Values from GML Estimation

Capital goods, gross imports



Consumer goods, gross imports



Food, beverages, & tobacco, gross imports

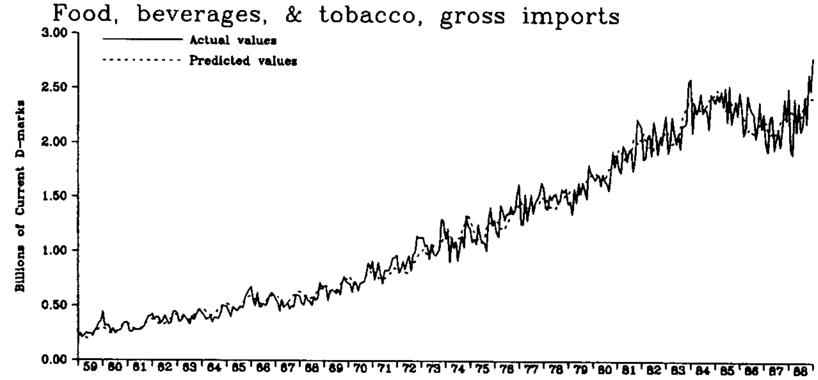
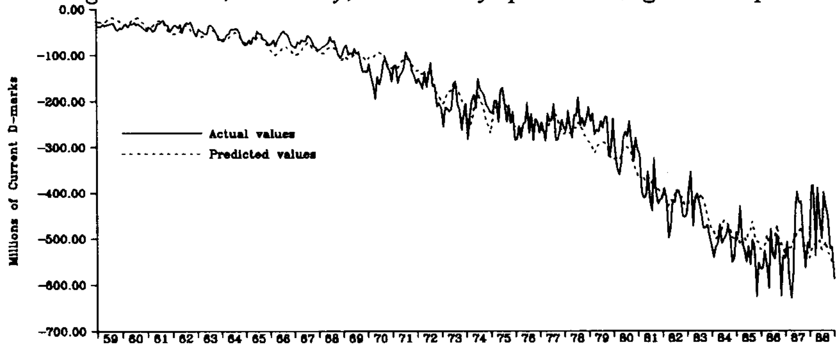
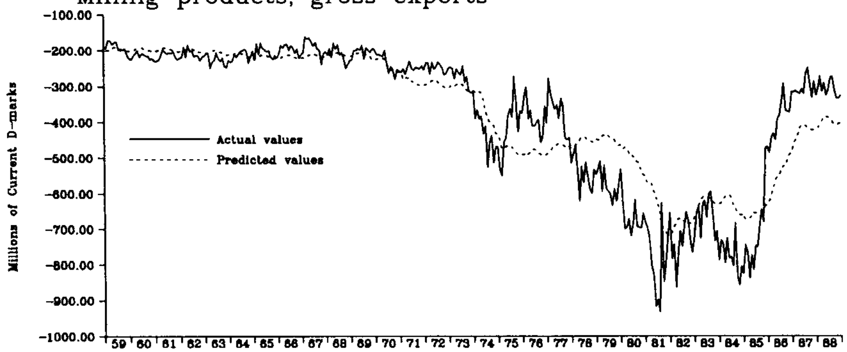


Figure 3c: Predicted and Actual Values from GML Estimation

Agricultural, forestry, & fishery products, gross exports



Mining products, gross exports



Basic materials, gross exports

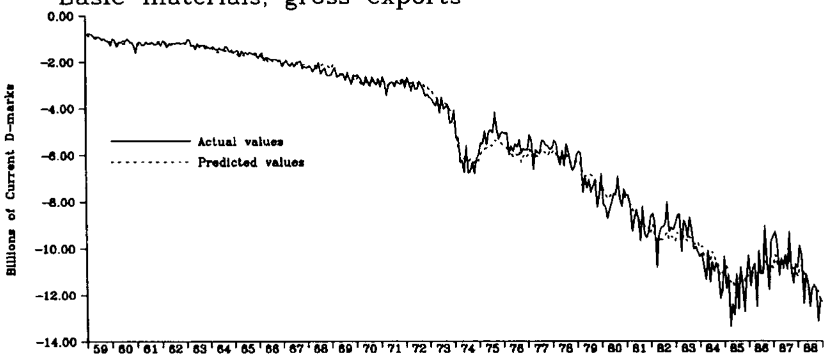
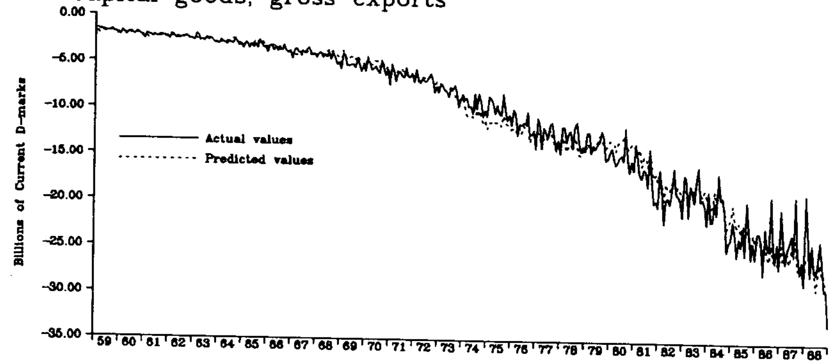
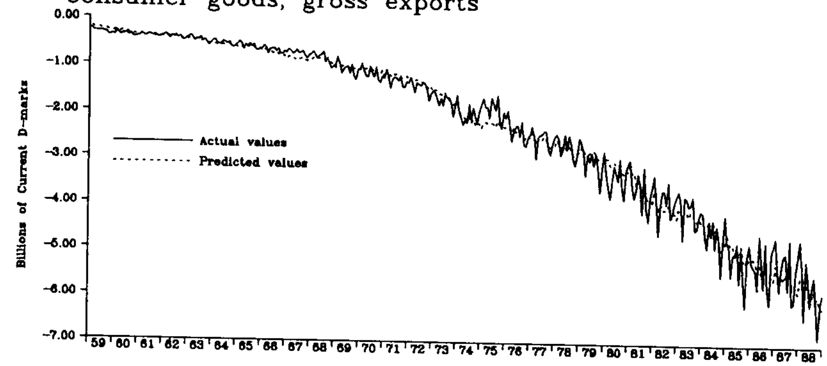


Figure 3d: Predicted and Actual Values from GML Estimation

Capital goods, gross exports



Consumer goods, gross exports



Food, beverages, & tobacco, gross exports

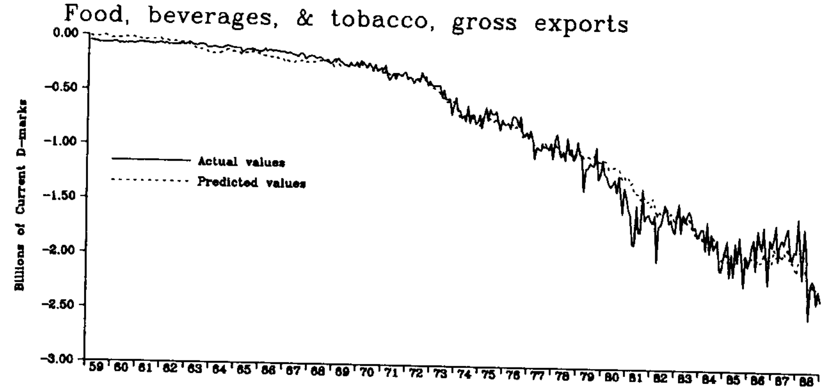


Figure 4a: Predicted and Actual Values from NLLS Estimation

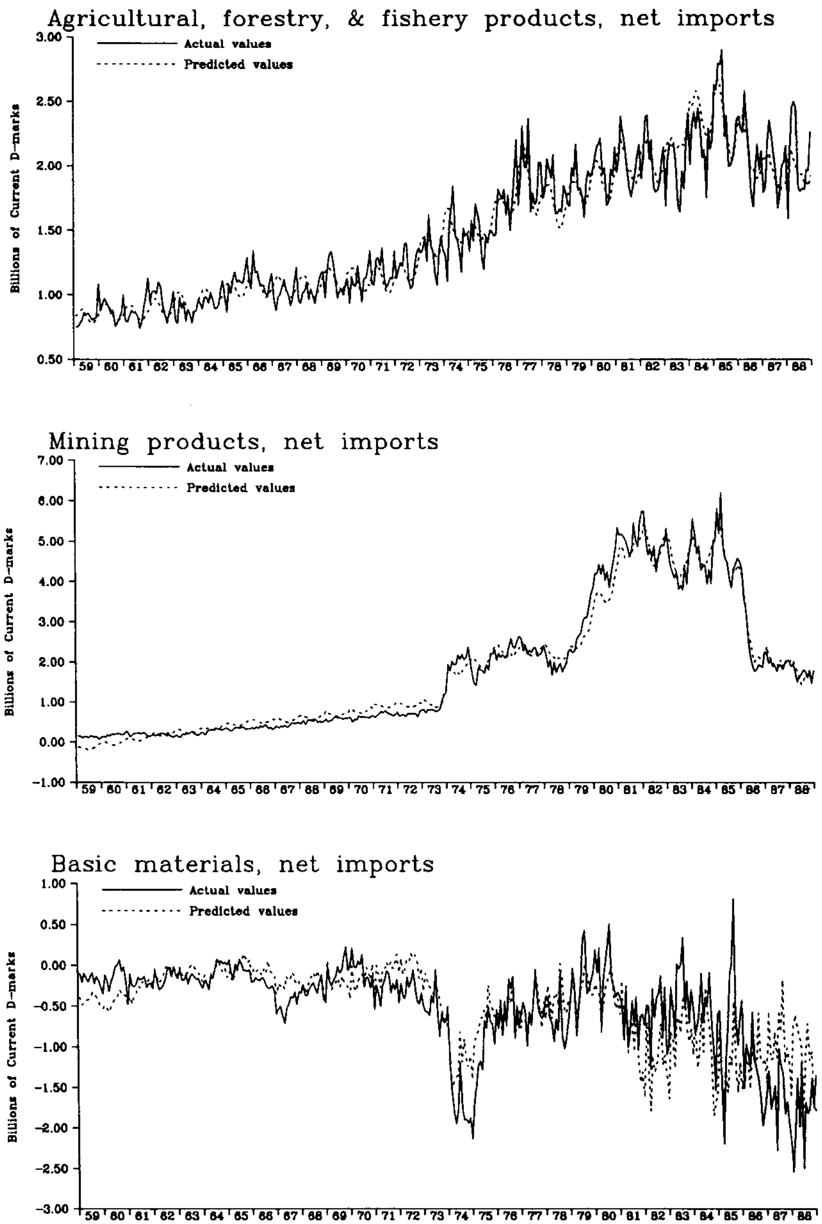


Figure 4b: Predicted and Actual Values from NLLS Estimation

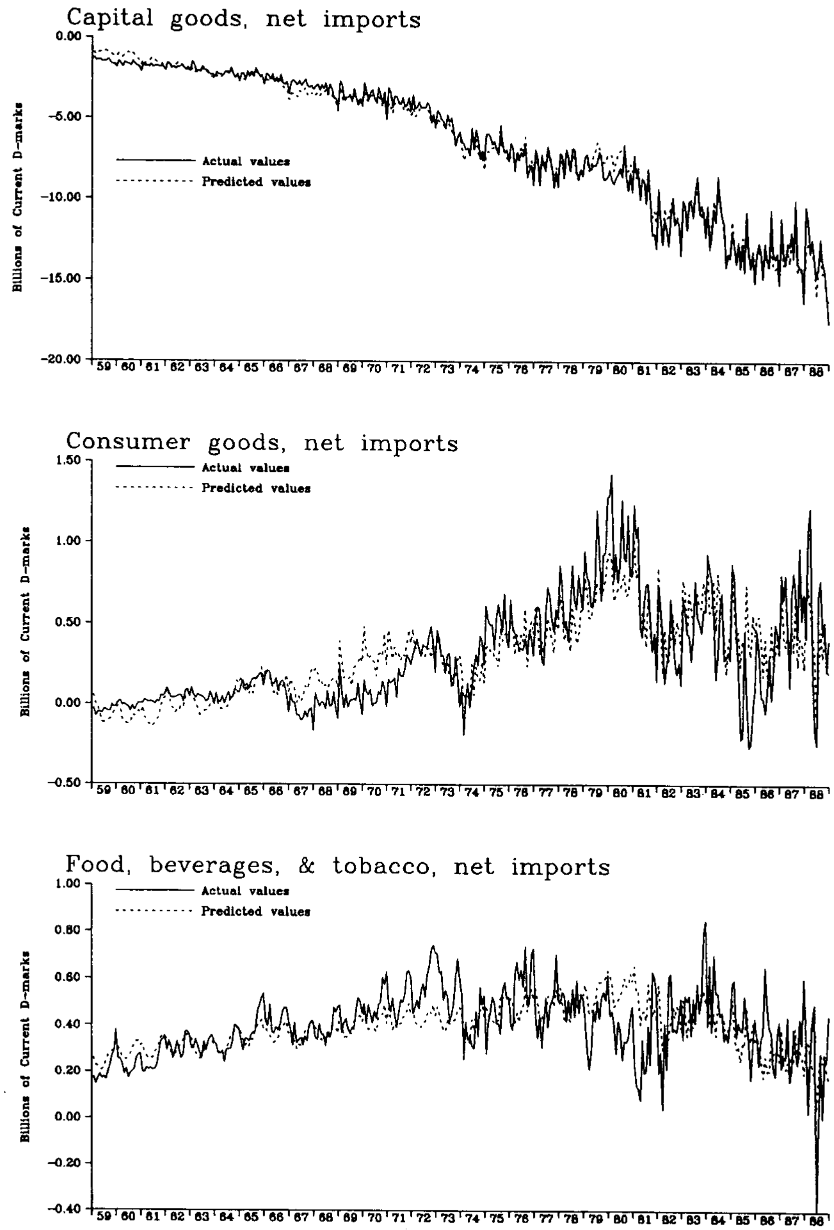
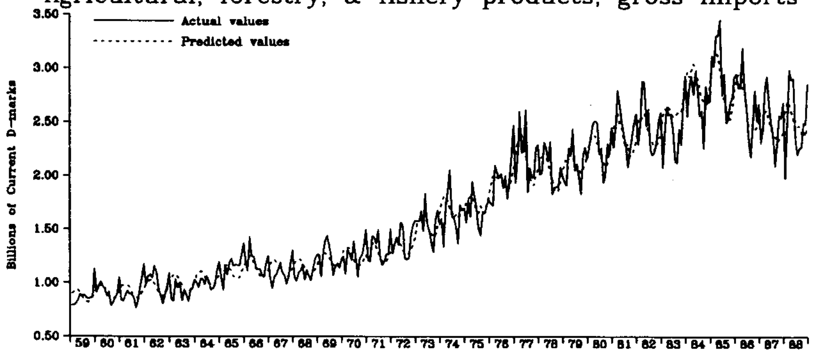
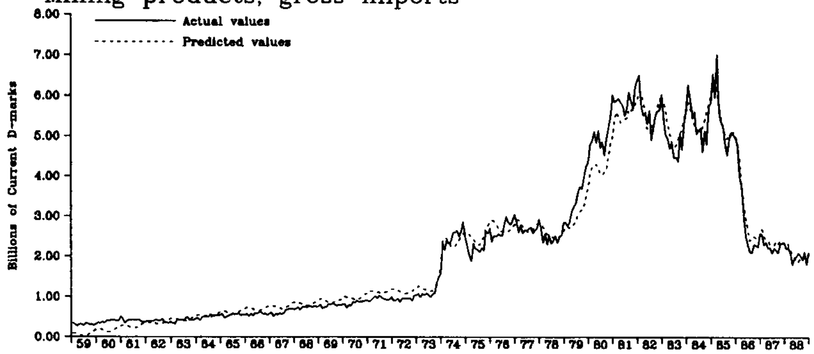


Figure 5a: Predicted and Actual Values from NLLS Estimation

Agricultural, forestry, & fishery products, gross imports



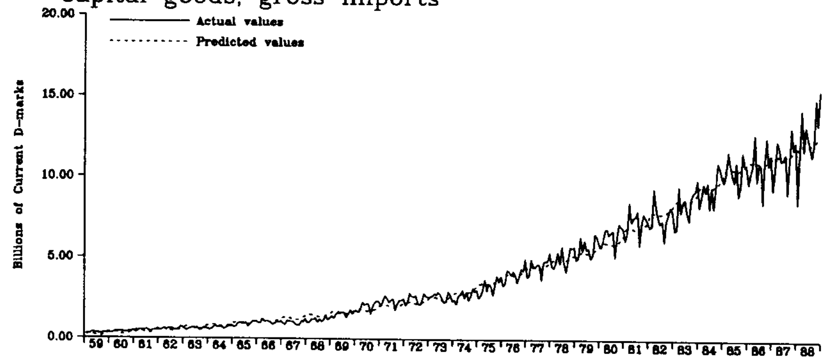
Mining products, gross imports



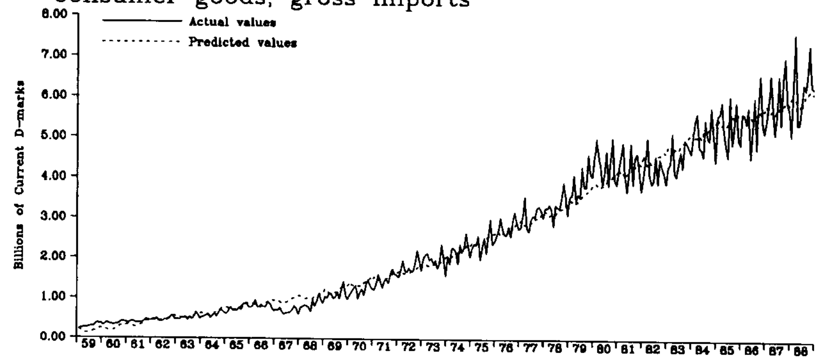
Basic materials, gross imports

Figure 5b: Predicted and Actual Values from NLLS Estimation

Capital goods, gross imports



Consumer goods, gross imports



Food, beverages, & tobacco, gross imports

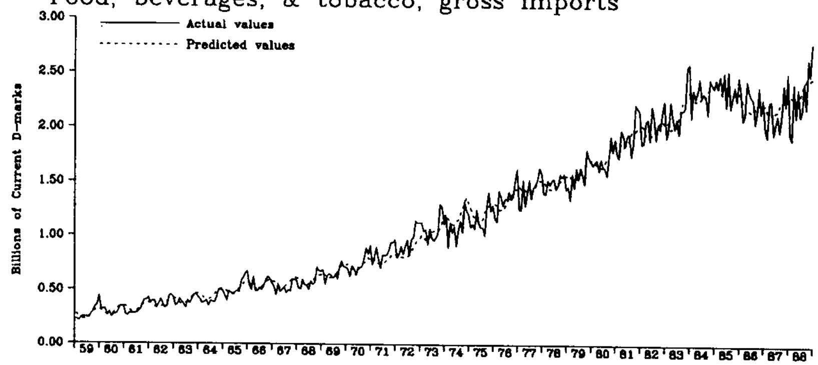
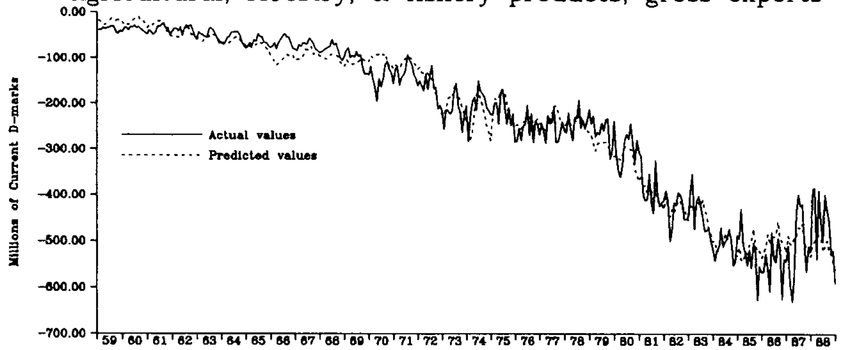
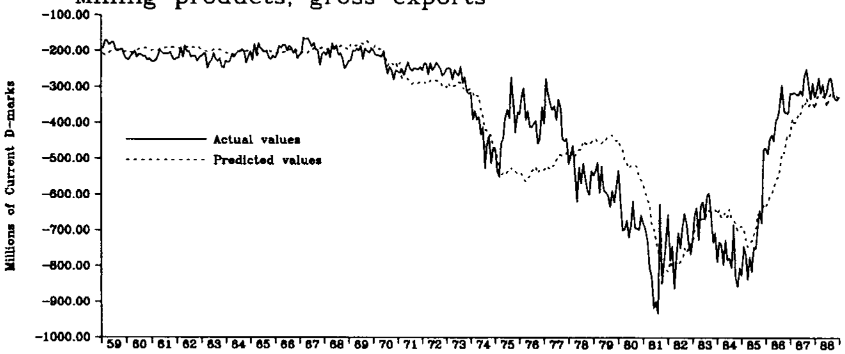


Figure 5c: Predicted and Actual Values from NLLS Estimation

Agricultural, forestry, & fishery products, gross exports



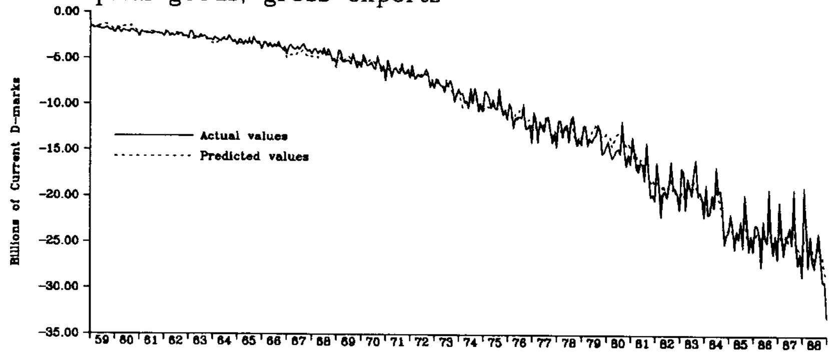
Mining products, gross exports



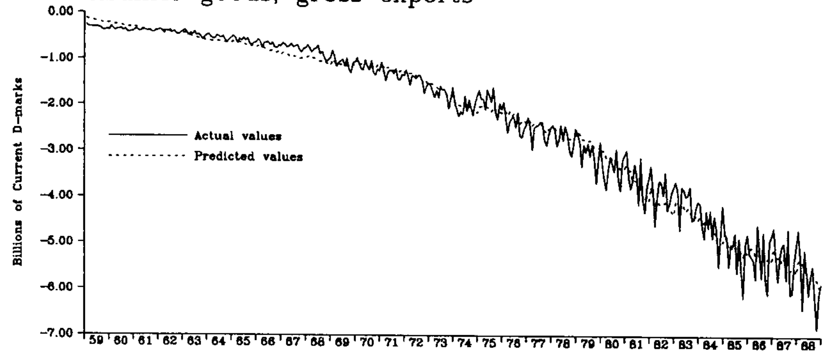
Basic materials, gross exports

Figure 5d: Predicted and Actual Values from NLLS Estimation

Capital goods, gross exports



Consumer goods, gross exports



Food, beverages, & tobacco, gross exports

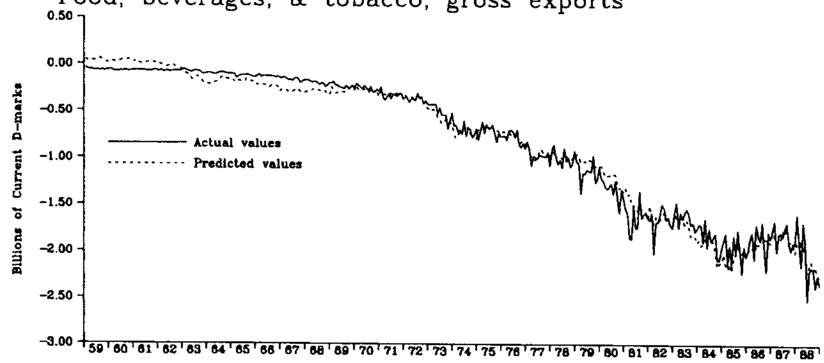
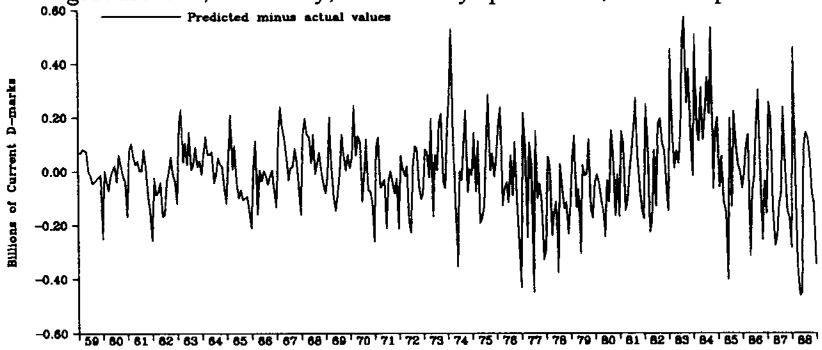


Figure 6a: Residuals from GML Estimation

Agricultural, forestry, & fishery products, net imports

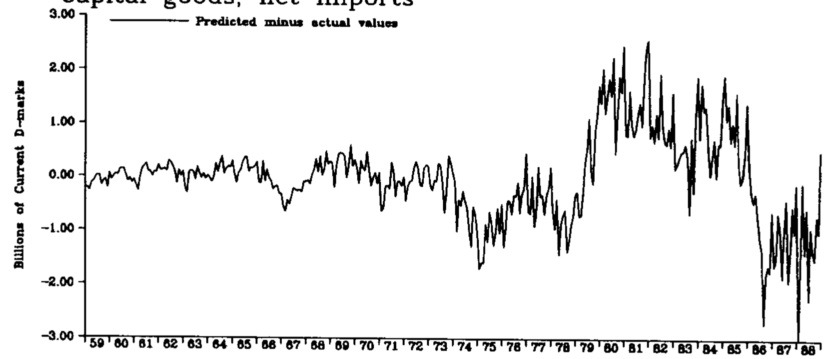


Mining products, net imports

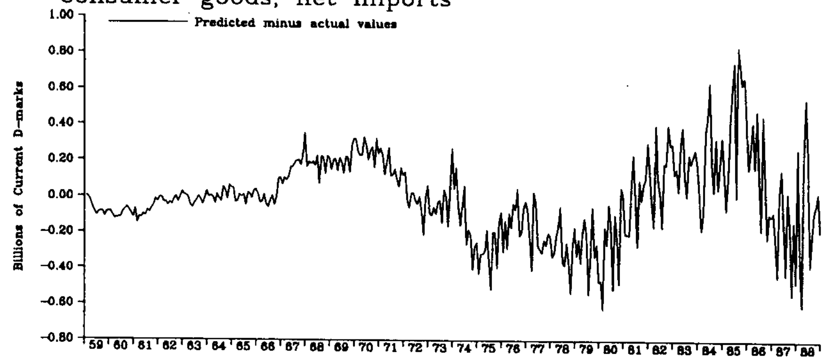
Basic materials, net imports

Figure 6b: Residuals from GML Estimation

Capital goods, net imports



Consumer goods, net imports



Food, beverages, & tobacco, net imports

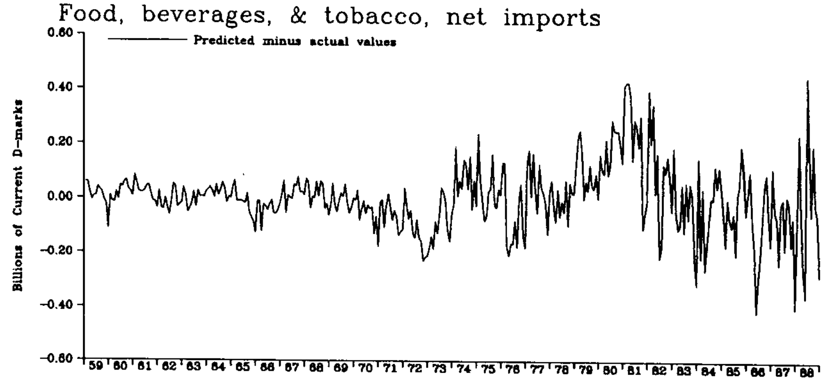
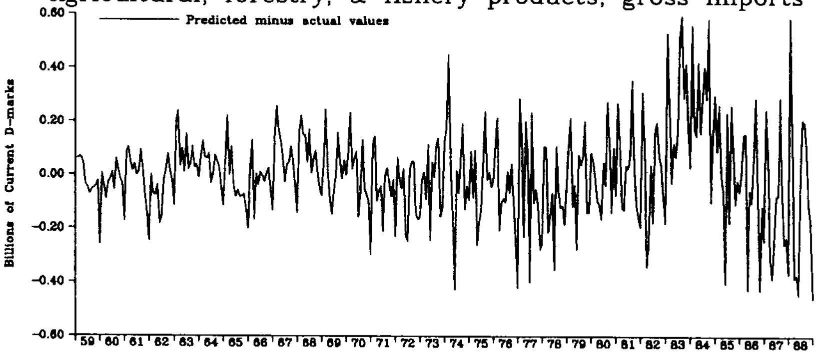
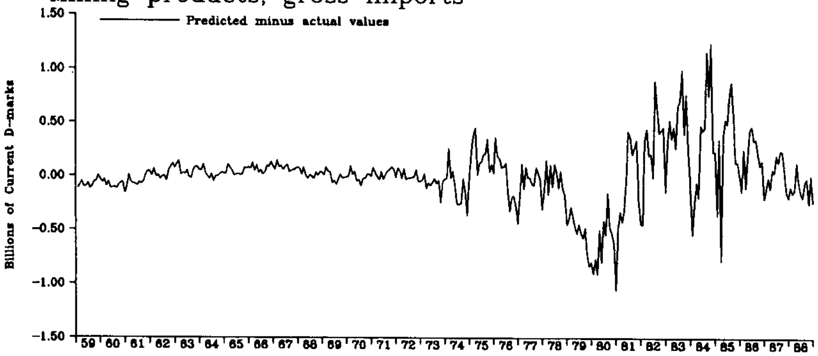


Figure 7a: Residuals from GML Estimation

Agricultural, forestry, & fishery products, gross imports



Mining products, gross imports



Basic materials, gross imports

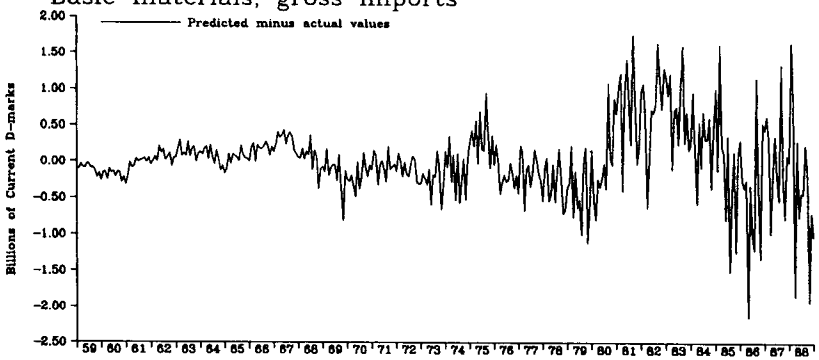
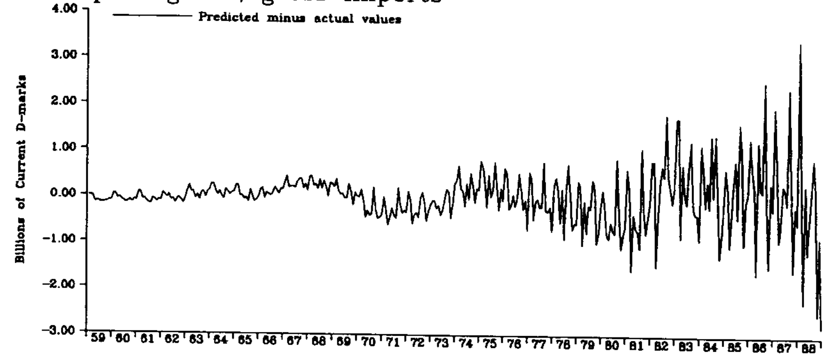
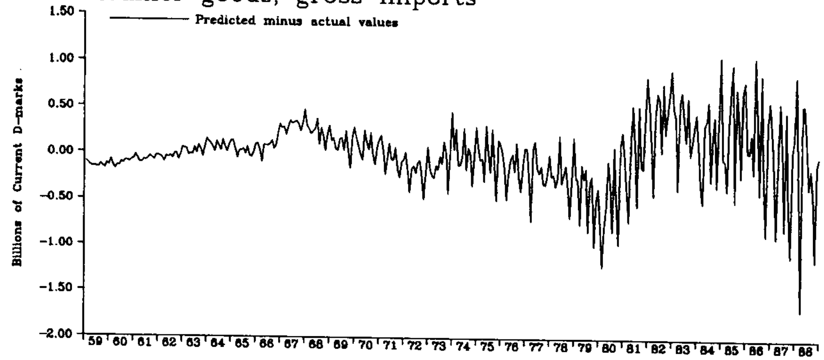


Figure 7b: Residuals from GML Estimation

Capital goods, gross imports



Consumer goods, gross imports



Food, beverages, & tobacco, gross imports

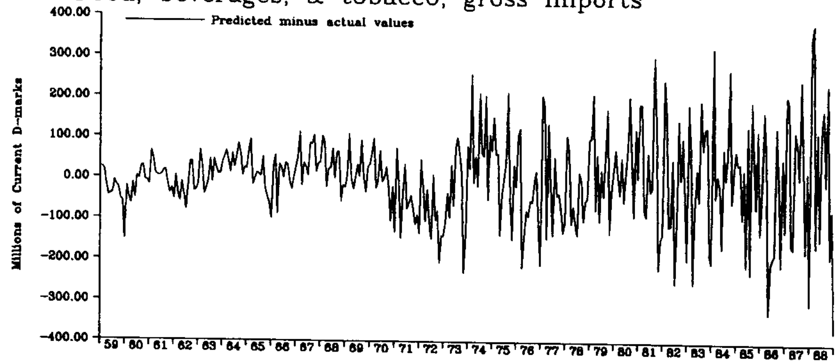
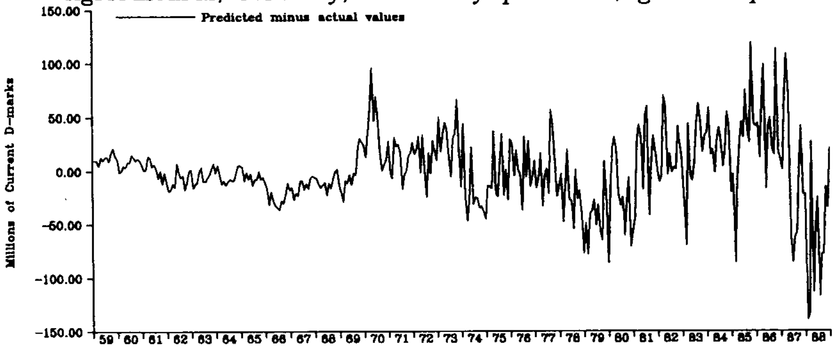
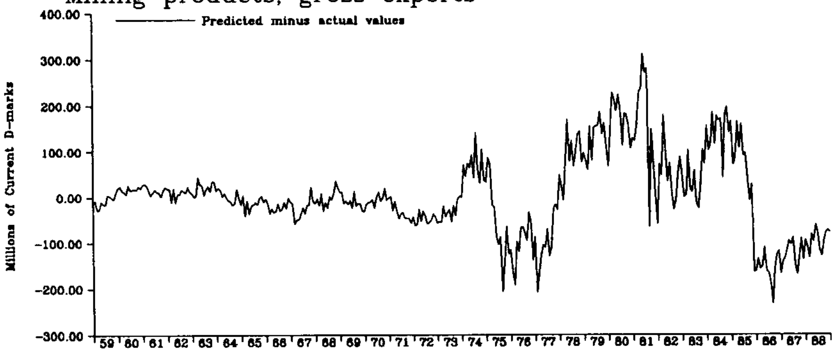


Figure 7c: Residuals from GML Estimation

Agricultural, forestry, & fishery products, gross exports



Mining products, gross exports



Basic materials, gross exports

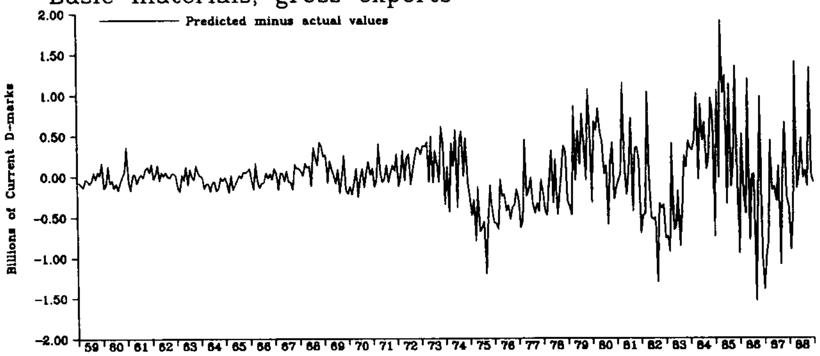
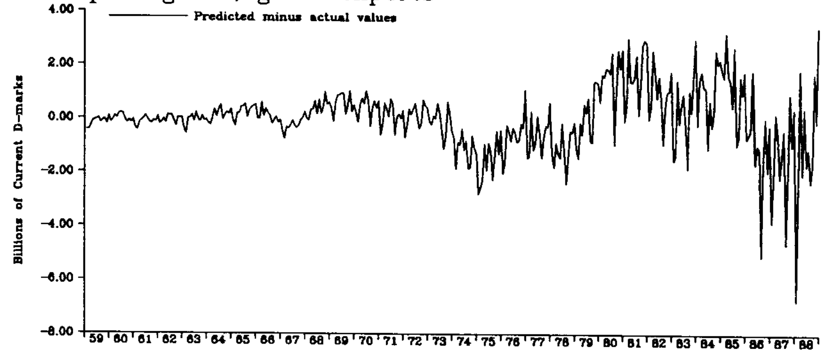
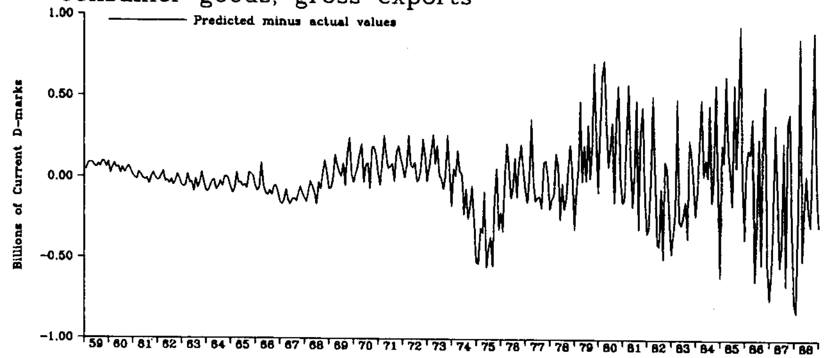


Figure 7d: Residuals from GML Estimation

Capital goods, gross exports



Consumer goods, gross exports



Food, beverages, & tobacco, gross exports

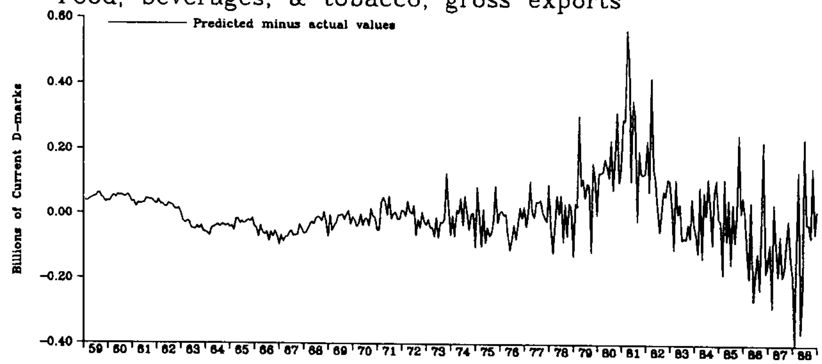
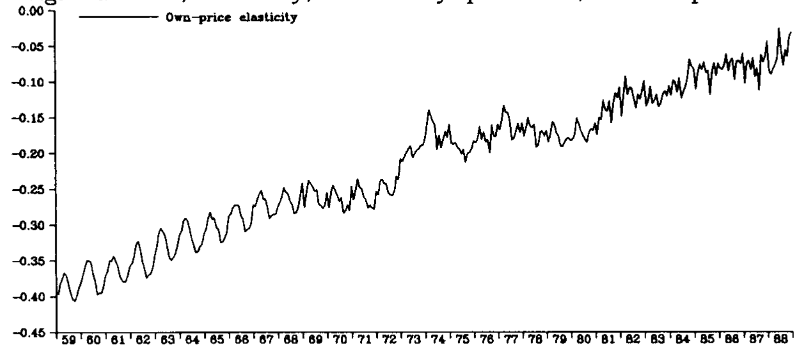


Figure 8a: Own-Price Elasticities from GML Estimation

Agricultural, forestry, & fishery products, net imports



Basic materials, net imports

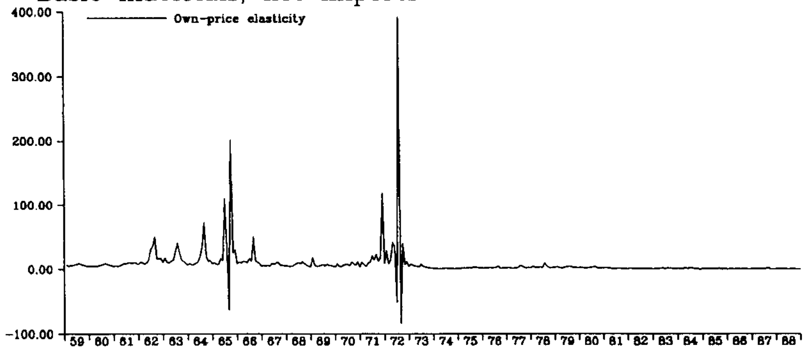
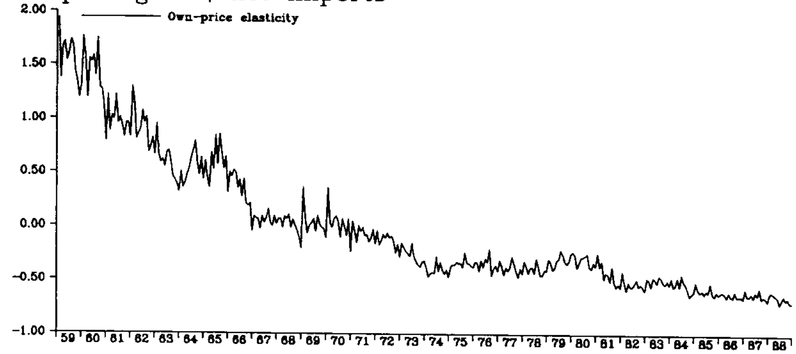
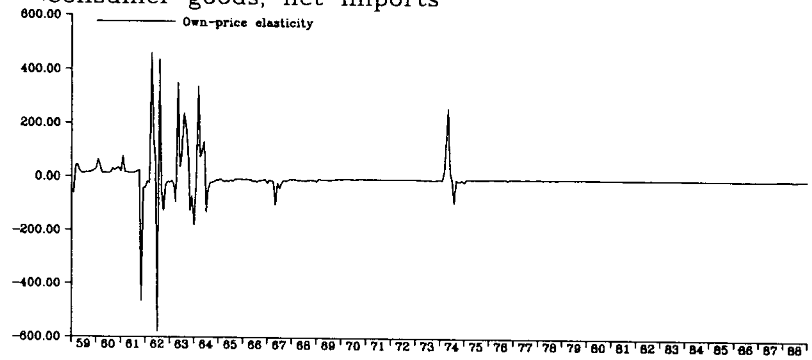


Figure 8b: Own-Price Elasticities from GML Estimation

Capital goods, net imports



Consumer goods, net imports



Food, beverages, & tobacco, net imports

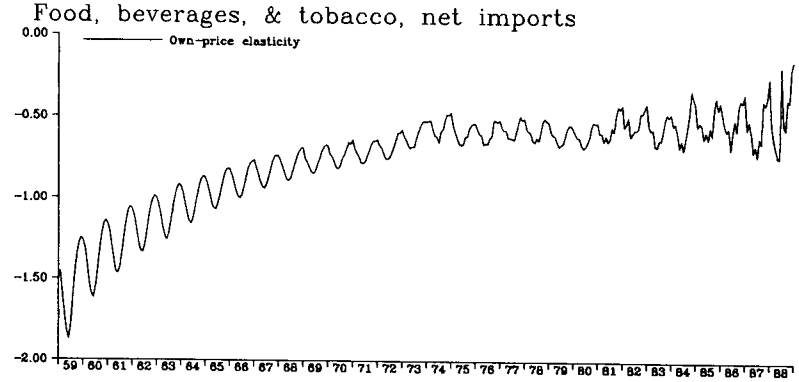
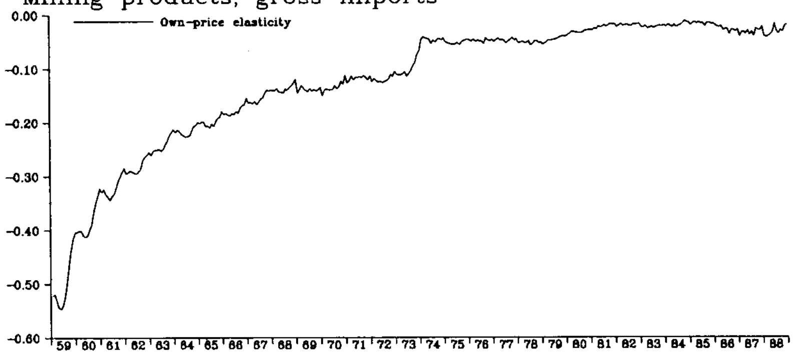


Figure 9a: Own-Price Elasticities from GML Estimation

Agricultural, forestry, & fishery products, gross imports

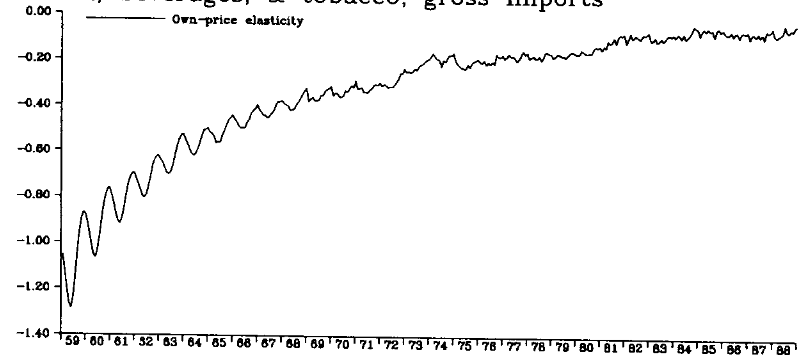
Mining products, gross imports



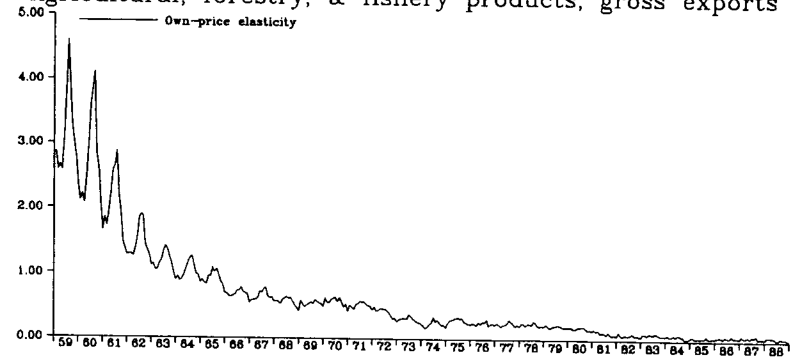
Basic materials, gross imports

Figure 9b: Own-Price Elasticities from GML Estimation

Food, beverages, & tobacco, gross imports



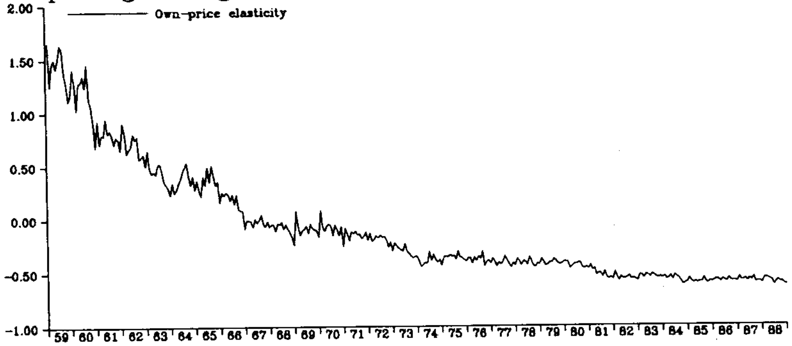
Agricultural, forestry, & fishery products, gross exports



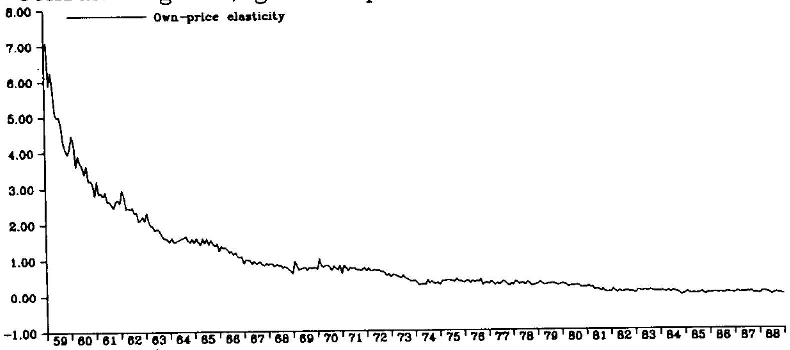
Basic materials, gross exports

Figure 9c: Own-Price Elasticities from GML Estimation

Capital goods, gross exports



Consumer goods, gross exports



Food, beverages, & tobacco, gross exports

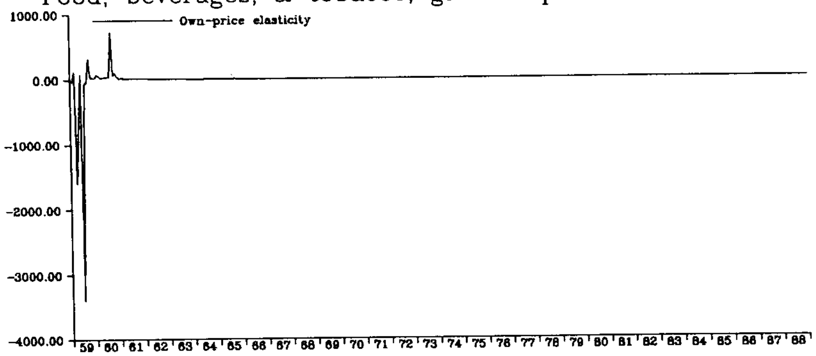
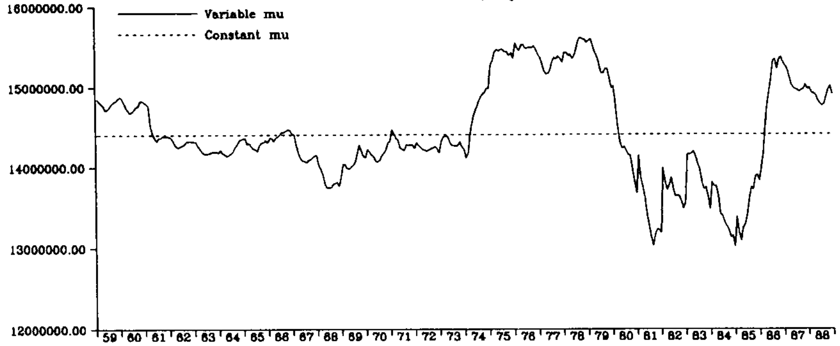


Figure 10: Comparison of $\sum_{k=1}^{m} p_{tk}\left[\hat{\gamma}_{tk}+\exp\left(\hat{a}_k+\frac{\hat{v}_{kk}}{2}\right)\right]$ **and** $-D_{\min}+\exp(\hat{\nu})$

Variable and constant μ_t for net trades



Variable and constant μ_t for gross trades