REPUTATION AND OPTIMAL CONTRACT
FOR CENTRAL BANKERS *

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Abstract

We implement optimal economic outcomes at the lowest social cost by combining reputation and contracting mechanisms to overcome the time-inconsistency problem of monetary policy associated with an inflation bias. We characterize the condition under which the reputation force alone induces a central bank to behave in a socially optimal way. When this condition fails, an incentive contract is invoked, whose cost is significantly reduced by the presence of the reputation force. The contract poses a penalty threat that is a concave function of wage growth, which in equilibrium is tied to expected rather than realized inflation, with a global maximum that provides the least upper bound on all threatened penalties. This bound can be used as a uniform penalty threat to achieve optimal economic outcomes and still, for moderate to large shocks, its magnitude can be much smaller than the size of the transfers required by the standard contracts that are linear functions of realized inflation rates. Further, under both the concave and the uniform penalty threats, the central bank will behave in the socially optimal way and no transfer is materialized in equilibrium. Thus our hybrid mechanism solves the time-inconsistency problem while leaves the central bank with complete discretion to respond to new circumstances, without any reputation cost or penalty threatened by the contract actually invoked along the equilibrium path.

Keywords: Monetary policy, time consistency, reputation, optimal contract

JEL classification: E52, E58

*We wish to thank the editor, an associate editor, and two anonymous referees for very useful suggestions that have improved the paper’s exposition substantially. We also thank Tao Wu for helpful comments. The second author gratefully acknowledges financial support from the National Natural Science Foundation of China (NSFC-70773073), the Program to Enhance Scholarly Creative Activities at Texas A&M University, and from Cheung Kong Scholars Program at the Ministry of Education of China.
1. INTRODUCTION

An agent’s behavior often disobeys his optimal plan, as the plan made in the past may no longer look optimal today. The inconsistency of optimal plans has long been studied in the literature, at least since Strotz (1956), who analyzed an individual decision-maker. When the agent is a government policymaker, there can be a big stake on the table concerning public welfare. On the one hand, the public’s perceptions of how policy will be conducted can influence its decisions. On the other hand, after the public forms expectations based on the promised policy course and makes decisions accordingly, the policymaker may find itself in a situation facing an incentive to act in a way that is inconsistent with the originally planned. Thus, absent a commitment device, the policymaker may find it now optimal to exercise discretion and deviate from the planned course or planned response to new circumstances. The public may foresee that the policymaker’s incentives and choices will change, and thus would not be misled in the first place, or for long. This time inconsistency of optimal discretionary policy can lead to suboptimal outcomes. This is the case even when the policymaker is benevolent and aims to maximize public welfare.

Many areas in policymaking are subject to the time-inconsistency problem. Since the seminal work of Kydland and Prescott (1977) and Barro and Gordon (1983a), the time-inconsistency issue has been at and continues to take center stage in the monetary policy landscape. Particular attention has been paid to the problem that inflation can exceed the socially desirable rate under optimal discretionary policy without a systematic gain in the level of output or employment. Due to various rigidities in the economy, a central bank may perceive a short-run tradeoff between unexpected inflation and unemployment. If households and firms expect low inflation and set wages and prices accordingly, a favorable tradeoff arises, as the marginal benefit of inflation now exceeds its marginal cost, and the benevolent central bank may have an incentive to implement expansionary monetary policy to increase employment and output. However, such benefit is, at best, only temporary. Once the higher inflation is recognized, the public will revise its inflation expectations and push wages and prices up. Feared for causing a recession, the central bank may not do better than to fulfill the higher inflation expectations. Hence, the central bank’s attempt to increase public welfare will end up producing an average inflation bias without an average gain in the level of employment or output in the long run.

The root of the problem lies in the misalignment of a central bank’s incentives over time. Several approaches have been proposed to mitigate the time-inconsistency problem and inflation bias by increasing the marginal cost of inflation as perceived by the central bank when setting its policy instrument. Barro and Gordon (1983b), and many others that followed (e.g., Backus and
Driffill, 1985; Tabellini, 1988; Al-Nowaihi and Levine, 1994; Li, Liu and Tian, 2008), showed that, under some conditions, a central bank’s concern about losing its reputation in the future due to deviations from the socially desirably inflation today could help eliminate the inflation bias. These conditions may not always hold. Another avenue, as proposed by Rogoff (1985), and also examined by many others (e.g., Canzoneri, 1985; Garfinkle and Oh, 1993; Lohmann, 1992), is for the government to delegate monetary policymaking to an independent and conservative central banker who is more averse to inflation than the general public. This delegation can help reduce the inflation bias, but only at a cost of distorting the stabilization role of monetary policy in response to shocks. An alternative approach, first analyzed by Walsh (1995a), and later extended by many others (e.g., Persson and Tabellini, 1993; Jansen, 2000; Huang and Padila, 2002; Walsh, 2003a), models monetary policy delegation as an incentive contract between the government and the central bank, which removes the inflation bias without generating a stabilization distortion. The cost of such contract can be forbiddingly large or small, as it necessarily involves a monetary transfer in each period between taxpayers and the central banker as a linear function of realized inflation, which can be unboundedly large or small depending on the size of the shocks, regardless of the fact that the central bank’s responses to the shocks are socially optimal. This makes the contract either practically or politically difficult to implement, as many researchers noted (e.g., Persson and Tabellini, 1993; Svensson, 1997; Chortareas and Miller, 2003).

We cast the time-consistency problem in discretionary monetary policymaking as a dynamic mechanism design problem viewed through the lens of the incentive mechanism design literature (e.g., Hurwicz, 1972; Maskin, 1999; Tian, 1989, 1990). To that end, we unify and extend two existing approaches by embedding a reputation mechanism into the design of optimal contracts for central bankers. Our hybrid mechanism combining reputation forces and penalty threats solves the time-inconsistency problem faced by a central banker and implements optimal economic outcomes at the lowest social cost.

Our results can be easily summarized. We characterize the necessary and sufficient condition under which the reputation force from the public single-handily induces a central bank to behave in a socially optimal way. An incentive contract from the government is then invoked when this condition fails, and the embodied penalty threat is just large enough to offset the central bank’s temporary gain from deviating from the socially desirable behavior over its future reputation cost. Hence, although in this case the reputation force alone is not strong enough to, by itself, overcome the time inconsistency and inflation bias of optimal discretionary policy, it significantly reduces the cost of the incentive contract that is called for joining force to implement the socially
optimal outcome. In contrast to the standard contracting literature, the contract presented here poses a penalty threat that is a concave function of wage growth, which in equilibrium is tied to expected rather than realized inflation, with a global maximum that provides the least upper bound on all threatened penalties, regardless of the size of shocks or aggregate economic conditions. We show that this bound, which depends on deep parameters but not economic variables, can be used as a uniform penalty threat to achieve optimal economic outcomes and still, for moderate or large shocks, its magnitude can be much smaller than the size of the transfers required by the standard contracts that are linear functions of realized inflation rates. Moreover, under both the concave and the uniform penalty threats, the central bank will behave in the socially optimal way and no transfer is in fact materialized in equilibrium. In sum, our hybrid mechanism eliminates the time inconsistency and inflation bias, while leaves the central bank with complete discretion to respond to shocks or the arrival of new information, without any reputation cost or penalty threatened by the contract actually invoked along the equilibrium path. In this sense, our unified approach suggests an incentive-compatible, cost-efficient mechanism for delegating monetary policymaking to achieve socially optimal outcomes.

From a methodological viewpoint, our normative analysis illustrates the advantage of using a unified framework for thinking about central bank incentives in the conduct of monetary policy. To make our point transparent, we have followed the classical time-inconsistency literature that originated as a positive attempt to understand why, for long periods of time, average inflation rates in many countries exceeded what seemed to be the socially desirable rates of inflation. As is known from this literature, an inflation bias can arise with a central bank attempting to rectify an inefficiently low employment or output level brought by various distortions. Some researchers argued that central banks knowing its inflationary consequence might not exercise such ambitious discretion, but behave with prudent discretion (e.g., McCallum, 1995, 1997; Blinder, 1998, 2000). As Walsh (2003b) pointed out, “such a view ignores the basic problem; even central banks that want to do the right thing may face the choice of either inflating or causing a recession.” Indeed, as shown by a number of recent studies, an inflation bias can arise even with a prudent central bank if it faces a policy control error, an uncertain economic outlook and asymmetric concern about recession versus expansion, or a nonlinear tradeoff between inflation and unemployment (e.g., Jordan, 2001a,b; Cukierman, 2002; Cukierman and Gerlach, 2003; Cukierman and Muscatelli, 2008; Tambakis, 2009), or if it needs to learn about the underlying conditions.

These include monopolistic distortions in the product market, monopoly union distortions in the labor market, distortionary taxes, and political pressures from incumbent politicians, while incentives for inflation can also depend on the desire for seigniorage and sovereignty. See Walsh (2003b), among others, for more discussions.
structure of the economy (e.g., Sargent, 1999; Sargent, Williams, and Zha, 2006).

On the positive ground, time-series and cross-country empirical evidence suggests that the
time-inconsistency problem associated with an inflation bias of discretionary policymaking may
help explain the behavior of actual inflation observed in the United States and other developed
countries during some historical episodes, such as the Great Inflation of the 1960s to the mid
1980s. The Great Moderation experienced in later years led to a long period of relatively low
inflation and, as a result, concerns about time inconsistency for policymakers receded. Yet the
observed inflation is an equilibrium phenomenon, and it may remain low for extended periods
of time when mechanisms for tackling the time-inconsistency problem discussed above, coupled
with a more favorable macroeconomic environment absent large or frequent shocks, play a role in
determining the equilibrium. Indeed, a large number of studies find that the reforms of central
banks undertaken in many countries as of the late 1980s, which have led to greater central bank
independence and adoption of inflation targeting rules for the conduct of monetary policy, are
arguably an important factor underlying the recent period of low inflation. Incidently, some of
the reforms, like the type of penalty rules incorporated into New Zealand’s Reserve Bank Act of
1989, can be thought of as mimicking partially an optimal contract prescribed by the standard
contracting literature, and Svensson (1997) demonstrated a connection between such optimal
linear contract and inflation targeting, which is known to face a tradeoff between bias reduction
and stabilization response, similar to the one we have seen earlier in discussing Rogoff’s model.

From this perspective, one contribution of our paper is to provide a convenient starting point for
thinking about how the design of monetary policymaking institutions may be further improved.

Further, notwithstanding the relatively low inflation in the recent years, the period has not
been without inflation scares, say, the run up in inflation prior to the 1990 recession and the
preemptive policy intervention in 1994. The recent contributions by Sargent (1999) and Sargent,
Williams, and Zha (2006) show how a well-meaning central bank’s perceptions about the short-

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3See, among others, Walsh (2003b) and Cukierman (2008) for some comprehensive surveys of these studies.
See, also, Bernanke and Woodford (2005) for a collection of recent contributions on this subject.
4See, for example, Persson and Tabellini (1993) and Walsh (1995b) for more discussions on this. See, also,
Beestma and Jensen (1996), and Herrendorf and Lockwood (1997).
5Given the large amount of discussion over the past several years about whether inflation targeting by central
banks hurts employment, it is no accident that the issue of time inconsistency and inflationary bias in monetary
policymaking, and the time-consistency problem in the broader context of policymaking, have been discussed
intensively in a number of recent Federal Reserve publications and speeches delivered by Fed officials. See, for
example, Christiano and Fitzgerald (2003), King (2006), Plosser (2007), and Dotsey (2008).
run tradeoff between inflation and unemployment can change over time, as new shocks arrive, and as its policy actions affect the data-generating process (through their impacts on private sector’s expectations and behavior) and thus also the prospect of exploiting the inflation-output tradeoff. In effect, the central bank may be tempted to try the experiment repeatedly, giving rise to a recurrent inflation bias, as the interactions of shocks and its beliefs with the data-generating process evolve over time. Hence, the apparent conquest of inflation may be temporary in nature, and the equilibrium can alternate between periods of low inflation and periods of high inflation. Their theory provides a coherent account for the rise and fall of American inflation, such as those observed during the Great Inflation and Moderation. Incidently, and also from a long-term perspective, Christiano and Fitzgerald (2003) find that the basic Kydland-Prescott and Barro-Gordon mechanism is broadly consistent with the key features of the inflation-unemployment relationship during the entire twentieth century. They thus urge that the fundamental ideas of time-inconsistency and inflationary bias as embodied in the Kydland-Prescott and Barro-Gordon framework should be incorporated into modern models for monetary policy analysis. From this perspective, our present paper may not just represent a theoretical curiosity, but provide a set of important insights for thinking about the robust design of monetary policy.

In addition to the large body of literature mentioned above, our work is also related to some recent studies on the time inconsistency of discretionary monetary policy associated with a general inflationary bias. Acknowledging the shortcomings of the optimal linear contracts prescribed by the standard contracting literature, ones that we have already discussed earlier, Athey, Atkeson, and Kehoe (2005) apply the Kydland-Prescott and Barro-Gordon framework to study an optimal social contract for a central bank and government that takes the form of an inflation cap, which specifies an upper limit on the permitted inflation rate. The contract features an optimal degree of tradeoff between bias reduction and stabilization response, by determining an optimal degree of discretion left with the central bank, which is decreasing in the severity of the time-inconsistency problem. If the time-inconsistency problem is sufficiently severe, then the concern about surprise inflation by the society will dictate the contract to leave the central bank with no discretion to respond to the arrival of new information. Chari and Kehoe (2008) extend the Kydland-Prescott and Barro-Gordon model to a monetary union. They show that, without a commitment technology or reputation mechanism to remove the time-inconsistency problem faced by the central monetary authority, its discretionary conduct of monetary policy will result in higher inflation for all the member countries in the union. They also show that constraints imposed on nonmonetary policies can alleviate but not eliminate the
problem of time inconsistency and inflation bias. Like Athey, Atkeson, and Kehoe (2005), Chari and Kehoe (2008) do not study how reputation mechanisms may interact with other mechanisms in solving the problem when the reputation forces by themselves may not be strong enough to accomplish the task, as we do in this paper.

The rest of the paper is organized as follows. We set up the model and describe the time-inconsistency problem faced by a central bank in Section 2. In Section 3, we first characterize the condition under which a reputation mechanism alone implements optimal economic outcomes, and we then develop a hybrid mechanism that invokes, when this condition fails, an optimal incentive contract to join the reputation force to implement the optimal policy outcomes at the lowest social cost. We provide some concluding remarks in Section 4.

2. THE MODEL

2.1. The Economy

We consider a standard new classical framework with a Lucas-type aggregate supply relation of the following form,

\[ y_t - \bar{y} = a(\pi_t - \pi^e_t) + x_t, \]

where \( y_t \) is output, \( \bar{y} \) is the natural rate of output, \( \pi_t \) is inflation, \( \pi^e_t \) is the public’s expectations of inflation, \( x_t \) is a supply shock that is identically and independently distributed over time with a zero mean and a finite standard deviation \( \sigma_x \), and \( a \) is a positive parameter that measures the sensitivity of the change in output from the natural rate due to unanticipated inflation.

Equation (1) can be thought of as arising from a constant-returns-to-scale aggregate production function where firms hire labor to produce output and the growth rate of nominal wage \( w_t \) is contracted based on the expect inflation rate \( \pi^e_t \): if actual inflation is in fact higher (lower) than expected, then real wage will be lower (higher) than expected, and firms will expand (reduce) employment and output will be higher (lower) than the natural rate, assent any supply shock.

The parameter \( a \) should then be linked to deep parameters in the underlying economy governing technology and market structure.

To focus on the nature of incentives with which the monetary authority should be faced, we assume that the central bank and government share the same \textit{ex ante} preferences over variations of inflation and output from their target levels, as specified by a quadratic loss function,

\[ L_t = \frac{1}{2}[\pi_t^2 + \theta(y_t - \bar{y} - k)^2]. \]
Without loss of generality, and as is standard in the literature, we have assumed a zero inflation target, and used a positive $k$ to symbolize the idea that the various distortions in the economy as discussed in the introduction can result in an inefficiently low employment or output level $\bar{y}$ that the central bank attempts to rectify. The positive parameter $\theta$ measures the weight placed on output stabilization relative to inflation stabilization. Such policy objective can be derived from the first principle by approximating the utility function of a representative household in the underlying economy, with the parameter $\theta$ being linked to deep parameters governing the household’s preferences.

For simplicity, and without loss of insight, we abstract from issues concerning the choice of monetary policy instrument and the associated control errors, and assume that the central bank can control the actual rate of inflation directly. This is a standard assumption in the literature.

As for the sequence of events in a period $t$, we also follow the lead of the literature: first, workers and firms negotiate a nominal wage rate based on what they expect inflation to be, as described in the first paragraph of this section in justifying equation (1); then, the supply shock $x_t$ is realized; finally, after observing the realized shock, the central bank chooses inflation $\pi_t$ to minimize (2), taking as given the private sector’s behavior, as prescribed by (1), and the public’s inflation expectations, as determined by $\pi^e_t = E(\pi_t)$, where $E$ denotes the public’s expectations operator taken over the distribution of $x_t$, which is assumed to be a common knowledge, as well as the resultant nominal wage contracts set at the first stage between firms and workers that are assumed to stay binding.\footnote{For some potential empirical evidence on such information advantage of the central bank over the public, see the study by Romer and Romer (2000) based on the experience of the U.S. Federal Reserve. This assumption is also meant to capture the idea that monetary policy decisions can be made more frequently than can most private decisions on wages and prices and thus monetary policy actions can be taken to respond to economic disturbances before the private agents will have the chance to renegotiate all the prevailing nominal contracts. The assumption of rational expectations implies an expected loss function for the public as $L^p = E[\pi_t - \pi^e_t]^2$, and, given its understanding of the central bank’s policymaking process, the public’s choice of $\pi^e_t$ is optimal.}

In light of our discussions above, we can use (1) to rewrite (2) as a loss function of the actual inflation chosen by the central bank and the inflation expectations held by the public,

$$L(\pi_t, \pi^e_t) = \frac{1}{2}\{\pi^2_t + \theta[a(\pi_t - \pi^e_t) + x_t - k]\}^2,$$

(3)

to reflect the game-theoretical feature of this model economy.
2.2. Commitment versus Discretionary Monetary Policy

It is useful to contrast here three cases of monetary policy practice. In either case, given the linear-quadratic structure of (1)-(2), or the quadratic structure of (3), a corresponding optimal monetary policy takes the form of a linear-affine function of the supply shock, that is, it takes the form \( \pi_t = \bar{\pi} + a_1 x_t \). As we will show below, in all of the three cases, policy’s response to the shock \( x_t \) is the same, characterized by an identical value for \( a_1 \). The three cases differ in their implied values for \( \bar{\pi} \). Under the commitment policy, \( \bar{\pi} = 0 \) and average inflation is zero. Under the cheating or discretionary policy, \( \bar{\pi} > 0 \) and there is an average inflation bias. This difference has important implications for social welfare.

2.2.1. Commitment Policy

We proceed by considering first a benchmark case in which the central bank can pre-commit to a policy rule of the linear-affine type (e.g., by announcing and sticking to some particular values of the two parameters \( \bar{\pi} \) and \( a_1 \)) before firms and workers form their inflation expectations and sign into nominal wage contracts (and thus before the realization of the shock \( x_t \)). The central bank’s problem is to choose \( \bar{\pi} \) and \( a_1 \) to minimize \( E[\mathcal{L}(\pi_t, \pi_e^t)] \), where \( E \) is the expectations operator taken over the distribution of \( x_t \), subject to \( \pi_t = \bar{\pi} + a_1 x_t \), treating the public’s inflation expectations \( \pi_e^t \) as the best response to its (announced) policy rule which are also ratified by rational expectations, that is, taking into account \( \pi_e^t = E(\pi_t) = E(\bar{\pi} + a_1 x_t) = \bar{\pi} \). The solution is \( \bar{\pi}^R = 0 \) and \( a_1 = -a\theta/(1 + a^2\theta) \). This gives rise to a socially desirable policy rule,

\[
\pi_t^R = -\frac{a\theta}{1 + a^2\theta} x_t. \tag{4}
\]

The unconditional expectation of the social loss under this commitment policy is given by

\[
E[\mathcal{L}(\pi_t^R, E\pi_t^R)] = \frac{1}{2} \theta k^2 + \frac{1}{2} \frac{\theta}{1 + a^2\theta} \sigma_x^2. \tag{5}
\]

With a commitment technology, the central bank’s choice of inflation after observing a realization of the supply shock conforms to the policy rule prescribed by (4).

2.2.2. Cheating Policy

Absent a commitment technology, however, the central bank may no longer have an incentive to implement the policy rule (4) once firms and workers form their inflation expectations based on it and lock into a nominal wage contract accordingly, as a favorable tradeoff now arises and the central bank can implement instead an inflationary policy to increase employment and output.
The central bank’s problem after observing the realization of $x_t$ is to choose $\pi_t$ to minimize (3), taking as given the public’s inflation expectations $\pi^e_t = \pi^R_t = 0$ as based upon the promised policy course (4). The solution is

$$\pi^C_t = \frac{a\theta}{1 + a^2\theta}(k - x_t),$$

which implies a positive inflation bias $\bar{\pi}^C = \frac{a\theta k}{1 + a^2\theta}$. The unconditional expectation of the social loss under this cheating policy is given by

$$E[\mathcal{L}(\pi^C_t, \pi^R_t)] = \frac{1}{2} \frac{\theta}{1 + a^2\theta} k^2 + \frac{1}{2} \frac{\theta}{1 + a^2\theta} \sigma_x^2,$$

which is indeed lower than that in (5) under the commitment policy. Thus, all agents would be better off if all could be fooled.

2.2.3. Discretionary Policy

The assumption of rational expectations implies that the public foresees that the central bank’s incentives and choices will change, and thus would not be misled in the first place. This is to say that, absent a commitment technology, the policy rule (4) would not be credible. The central bank’s problem after observing the realization of $x_t$ is to choose $\pi_t$ to minimize (3), treating the public’s inflation expectations $\pi^e_t$ as a state variable that is then ratified by rational expectations, that is, with $\pi^e_t = E(\pi_t)$ in equilibrium. The solution is

$$\pi^D_t = a\theta k - \frac{a\theta}{1 + a^2\theta} x_t,$$

which implies a positive inflation bias $\bar{\pi}^D = a\theta k$. The unconditional expectation of the social loss under this discretionary policy is given by

$$E[\mathcal{L}(\pi^D_t, \pi^D_t)] = \frac{1}{2} \theta(1 + a^2\theta) k^2 + \frac{1}{2} \frac{\theta}{1 + a^2\theta} \sigma_x^2,$$

which is in fact higher than that in (5) under the commitment policy. Thus, the time-inconsistency of optimal discretionary policy can lead to suboptimal outcomes under rational expectations.

3. DYNAMIC MECHANISM DESIGN

The analysis in Section 2.2 illustrates how public expectations may shape equilibrium outcomes. We cast the time-consistency problem as a dynamic mechanism design problem by embedding this role of expectations into a repeated-game framework in which current policy action affects the public’s expectations about future policy practice. If at a given date the public believes the
socially desirable policy rule will be followed and forms its expectations accordingly, then the central bank can use a surprise inflation to reduce the unemployment rate and the social loss for the current period. But, once the public realizes the central bank has deviated from the rule, it will revise its expectations about how policy will be conducted in the future, and the economy will then end up with the bad discretionary outcomes, with higher inflation but no systematic gain in employment and thus extra social losses for the future periods. This loss of credibility in the future due to a current deviation from the rule represents a reputation cost of inflation on the part of the central bank. Whether this future cost is large enough to offset the current benefit from cheating depends on how much the future is discounted and how long the central bank will lose its reputation after cheating, along with other characteristics of the economy.

In what follows, we first characterize the necessary and sufficient condition under which the reputation cost imposed by the public is sufficiently large so that the central bank will not cheat but follow the socially desirable policy rule. We then describe a hybrid mechanism that invokes, when this condition fails, an optimal incentive contract from the government to join the reputation force to implement the optimal policy outcome at the lowest social cost.

3.1. Reputation Mechanism

We postulate the following strategy for the public in forming its expectations about how policy will be conducted: if the central bank followed the socially desirable rule in the previous period, then the public believes it will continue to follow the rule in the current period; but, if it turns out the central bank actually deviates from the rule in the current period, then the public will expect the inflation rate that would arise under the discretionary policy for the next $P$ periods. In short, the central bank can cheat successfully for one period, which will then be followed by $P$ periods of the bad discretionary outcomes.$^7$

Assuming a subjective time discount factor $\beta \in (0, 1)$. For a central bank that maintains its reputation at $t$, the tension between sticking to and deviating from the rule can be illustrated by rewriting the difference in the present values of the social losses in periods $t$ through $t+P$ under the two policy strategies as the difference between the current benefit (in terms of a reduced social loss in period $t$) from using a cheating policy $\pi_t$ and the present value of the future costs

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$^7$The punishment length $P$ may be interpreted in two alternative ways. First, once it cheats, it will take the central bank $P$ periods to rebuild its reputation. Second, the average length of wage contracts is $P$ periods and, thus, following a surprise inflation workers will lock into a higher nominal wage for the next $P$ periods.
(in terms of increased social losses in periods \( t + 1 \) through \( t + P \)) due to credibility loss,

\[
D^t_{t+P}(\pi_t) \equiv \sum_{i=0}^{P} \beta^i \mathcal{L}(\pi_{t+i}, E\pi_{t+i}^R) - \left[ \mathcal{L}(\pi_t, E\pi_t^R) + \sum_{i=1}^{P} \beta^i \mathcal{L}(\pi_{t+i}, E\pi_{t+i}^D) \right]
\]

\[
= \left[ \mathcal{L}(\pi_t^R, E\pi_t^R) - \mathcal{L}(\pi_t, E\pi_t^R) \right] - \sum_{i=1}^{P} \beta^i \left[ \mathcal{L}(\pi_{t+i}^D, E\pi_{t+i}^D) - \mathcal{L}(\pi_{t+i}^R, E\pi_{t+i}^R) \right].
\]

(10)

Using a similar analysis as in Section 2.2.2 we can verify that, conditioning on cheating, the central bank will cheat at \( \pi_t^C \). Thus, the central bank will cheat if and only if

\[
E_t \left[ D^t_{t+P}(\pi_t^C) \right] > 0,
\]

(11)

where \( E_t \) denotes the central bank’s expectations operator conditional on the realization of \( x_t \).

That is to say that, the central bank will cheat if and only if the expected benefit outweighs the expected cost of cheating. Using a similar analysis as in Section 2.2, we can compute the conditional expectation of the benefit-cost difference in (11) as

\[
E_t \left[ D^t_{t+P}(\pi_t^C) \right] = \frac{a^2 \theta^2 k^2}{2} \left( \frac{1}{1 + a^2 \theta} - \sum_{i=1}^{P} \beta^i \right),
\]

(12)

where we note that the terms related to the supply shock \( x_t \) and its second moment \( \sigma_x^2 \) are all canceled out. Thus, the central bank will cheat if and only if \( (1 + a^2 \theta)^{-1} > \sum_{i=1}^{P} \beta^i \).

In other words, we have

**Proposition 1** The reputation mechanism in the repeated game between the public and the central bank overcomes the time-inconsistency problem of monetary policy associated with an inflation bias and implements the socially optimal outcome if and only if \( \sum_{i=1}^{P} \beta^i \geq (1 + a^2 \theta)^{-1} \).

The condition in Proposition 1 is less likely to hold, the more is the future discounted (i.e., the smaller is \( \beta \)), or the shorter is the punishment length \( P \). Indeed, if \( \beta \) is less than \((2 + a^2 \theta)^{-1}\), then this condition never holds no matter how long the punishment length is; while if \( \beta \) is close to 1, then this condition always holds no matter how short the punishment length is. We have

**Corollary 1** The reputation mechanism in the repeated monetary policy game between the public and the central bank implements the socially optimal outcome no matter how short the punishment length \( P \) is, as long as \( \beta \) is close to 1; while it alone can’t implement the socially optimal outcome no matter how long the punishment length \( P \) is, provided \( \beta < (2 + a^2 \theta)^{-1} \).

A central message from Proposition 1 and Corollary 1 is that, when the discount factor \( \beta \) is small or the punishment length \( P \) is short, the reputation force by itself may not be strong enough
to induce the central bank to behave in the socially optimal way. In fact, if the quadratic loss
function in (2) is replaced by a quadratic-linear loss function, $\mathcal{L}_t = 0.5\pi_t^2 + \theta(y_t - \bar{y}_t)$, as originally
used by Barro and Gordon (1983b) in their reputation model, the reputation mechanism alone
will fail to implement the socially optimal outcome for any discount factor $\beta \in [0, 1]$.
We show below how combining the reputation force in such a case with a central bank incentive contract
can help induce the socially optimal outcome in a cost-efficient way.

3.2. Hybrid Mechanism of Reputation and Optimal Contract

Our analysis in Section 3.1 shows that, when $\sum_{i=1}^{P} \beta_i < (1 + a^2\theta)^{-1}$, the reputation force from
the public will not be strong enough to single-handily solve the time-inconsistency problem of
monetary policy and, in consequence, the central bank’s attempt to increase public welfare will
end up producing an average inflation bias with no systematic gain in employment or output,
resulting in bad policy outcomes and low social welfare. Our task in this section is to show that,
an incentive contract from the government can be invoked in this case to join the reputation
force from the public to implement the optimal economic outcome at the lowest social cost.
To this end, we will take a dynamic mechanism design approach to determine how the optimal
contract should be designed and how it can be combined with the reputation force in creating
the right incentives for the central bank to carry out the optimal monetary policy rule. We shall
use a principal-agent framework for this analysis: the government is the principal, whose goal
is to implement the optimal monetary policy rule, and the central bank is the agent, to which
the government delegates the task of implementing this goal.

Before presenting the details of our results, it is useful to highlight the main features of our
hybrid mechanism combining a reputation force and an optimal central bank contract. First,
the contract part of the mechanism is muted when the reputation force alone can induce the
central bank to behave in the socially optimal way, that is, when $\sum_{i=1}^{P} \beta_i \geq (1 + a^2\theta)^{-1}$.

Second, the contract part of the hybrid mechanism is invoked only when the reputation
force alone can’t induce the socially optimal outcome, that is, only when $\sum_{i=1}^{P} \beta_i < (1 + a^2\theta)^{-1}$.
In contrast to the standard contracting literature that typically abstracts from any reputation
force, however, the presence of the reputation force in this case still plays an important role in
solving the problem even though it alone can’t do the whole work, as it significantly reduces the
cost of the contract that is called for joining force to implement the socially optimal outcome.

Third, the contract part of the hybrid mechanism poses a penalty threat from the government

\footnote{For more discussions about potentially other problems with the pure reputation approach, see, for example, Walch (2003b).}
that shares a similar behavioral spirit as that of the reputation force from the public. If the central bank has followed the optimal monetary policy rule in the previous period, then it is expected to also follow the rule in the current period. If it does indeed, then it maintains its reputation with the public and no penalty is imposed from the government either. If it turns out that the central bank deviates from the rule in the current period, then it loses its reputation with the public while also incurs a penalty from the government in accordance with the term specified by the contract. In contrast to the cost associated with the standard contract, which can be forbiddingly large as it involves a transfer in each period as a linear function of realized inflation that can be unboundedly large depending on the size of the supply shock even though the central bank’s policy practice is socially optimal, the threatened penalty posed by the contract in our hybrid mechanism is a concave function of nominal wage, which is contracted based on expected rather than realized inflation in equilibrium and is thus tied to surprise inflation but independent of the size of the supply shock, and it will be materialized only if $\bar{\pi} > 0$ in the light of the behavior rule embedded in the contract (and in the reputation mechanism as well).

Fourth, the penalty threat posed by the contract in the hybrid mechanism is cost efficient. It is just large enough to offset at any given date the central bank’s expected temporary gain from deviating from the optimal monetary policy rule over its future expected reputation cost. In other words, it is just large enough to offset at any given date $t$ the expected benefit-cost difference from using any cheating policy $\pi_t$, $E_t[D_t^{t+P}(\pi_t)]$. Thus, our hybrid mechanism that combines the reputation force from the public and the penalty threat from the government implements the optimal economic outcome at the lowest social cost.

Fifth, the penalty threat posed by the contract in the hybrid mechanism has a unique global maximum, which provides the least upper bound on all threatened penalties, regardless of the size of the supply shock or aggregate economic condition. This bound depends on deep parameters but not economic variables. It can thus be used as a uniform penalty threat to implement the optimal outcome and still, for a moderate or large shock, its size can be much smaller than that of the transfer required by the standard contract.

Finally, our hybrid mechanism is incentive compatible. The reputation force and the penalty threat reinforce with each other to increase the cost of inflation perceived by the central bank to discourage it from using the cheating policy and thus to eliminate the time inconsistency and inflation bias. Under both the concave and the uniform penalty threats, the central bank will behave in the socially optimal way and no transfer is materialized in equilibrium. Thus our hybrid mechanism solves the time-inconsistency problem while leaves the central bank with
complete discretion to respond to the supply shock, without any reputation cost or penalty threatened by the contract actually invoked along the equilibrium path.

We proceed now to formally describe our hybrid mechanism. Consider a central bank that maintains its reputation at $t$. If the central bank deviates from the optimal monetary policy rule in period $t$, then it will lose its reputation with the public for the next $P$ periods while it will also incur a penalty from the government as specified by the incentive contract. The penalty scheme is designed as to be contingent on the growth rate of nominal wage, which in equilibrium is tied to the expected rate of price inflation. Recall that optimal monetary policy takes the form of a linear-affine function of the supply shock, \( \pi_t = \bar{\pi} + a_1 x_t \), in all cases of policy practice, given the linear-quadratic structure of (1)-(2), or the quadratic structure of (3). It is also useful to recall here, and as we reconfirm below as well, that, in all cases, optimal policy’s response to $x_t$ is the same, characterized by the identical $a_1 = -a\theta/(1 + a^2\theta)$, and the cases differ only in their implied values for $\bar{\pi}$: under the commitment policy $\bar{\pi} = 0$, while under the cheating or discretionary policy $\bar{\pi} > 0$. These observations, together with our discussions in the preceding paragraphs, suggest that the threatened penalty can be thought of as effectively a function of a surprise inflation $\bar{\pi}$, and thus can be denoted $W(\bar{\pi})$. The present value of losses in periods $t$ through $t + P$ for the central bank that uses a cheating policy $\pi_t$ in period $t$ is then

\[
\mathcal{L}(\pi_t, E\pi_{t+1}^R) + \sum_{i=1}^{P} \beta^i \mathcal{L}(\pi_{t+i}^D, E\pi_{t+i}^D) + W(\bar{\pi}).
\]

(13)

On the other hand, if the central bank sticks to the optimal monetary policy rule all the time, then it will continue to hold its reputation with the public and no threatened penalty from the government will be materialized, and thus the present value of its losses in the corresponding $1 + P$ periods is simply

\[
\sum_{i=0}^{P} \beta^i \mathcal{L}(\pi_{t+i}^R, E\pi_{t+i}^R).
\]

(14)

To design the optimal penalty scheme for use in the central bank contract, it is useful to first define for any date $t$ the set of cheating policies under which the expected benefit from cheating outweighs the expected cost of cheating in the absence of any penalty threat from a government contract for the central bank. This set is given by

\[
\Pi_t = \left\{ \pi_t : E_t \left[ D_t^{1+P}(\pi_t) \right] > 0 \right\},
\]

(15)

which can be called the cheating set at $t$ absent a central bank contract. Here, we may recall that $D_t^{1+P}(\pi_t)$ measures the difference between the current benefit from using a cheating policy $\pi_t$ in period $t$ and the present value of the future costs due to the subsequent loss of the central
bank’s reputation with the public in periods $t + 1$ through $t + P$. Since the central bank will never use any cheating policy outside $\Pi_t$ even without a central bank contract, the design of an optimal central bank contract should have an eye only on the cheating policies in $\Pi_t$.

Several considerations should be taken into account when designing the central bank contract. First, given a penalty scheme $W(\bar{\pi})$, were the central bank to cheat in period $t$, it would cheat optimally by choosing a cheating policy in $\Pi_t$ to minimize the conditional expectation of (13), or, equivalently, to minimize the conditional expectation of the difference between (13) and (14), that is, to

$$\text{Minimize } E_t \left[ W(\bar{\pi}) - D^{t+P}_t(\pi_t) \right] \text{ over } \pi_t \in \Pi_t.$$  \hspace{1cm} (16)

Second, the threatened penalty $W(\bar{\pi})$ must be large enough to make the conditional expectation of the difference between (13) and (14) nonnegative for all $\pi_t \in \Pi_t$ and for all $t$, that is, to make the minimization result in (16) nonnegative for all $t$, so that the central bank would never cheat.

Third, the threatened penalty $W(\bar{\pi})$ should be as small as possible so that on average, taking into account all possible realizations of the supply shock, the minimal difference between (13) and (14) over all $\pi_t \in \Pi_t$ is just zero at all $t$, that is, it should be small enough to make the minimal unconditional expectation of the difference between (13) and (14) over all $\pi_t \in \Pi_t$ equal to zero at all $t$. These considerations lead to the following definition of an optimal central bank contract:

**Definition 1** A central bank contract $W(\bar{\pi})$ is said to be an optimal contract that implements the socially optimal monetary policy rule if it satisfies the following three conditions:

1. (Optimal Cheating): $\min_{\pi_t \in \Pi_t} E_t \left[ W(\bar{\pi}) - D^{t+P}_t(\pi_t) \right]$ for all $t$.
2. (Incentive Compatibility): $\min_{\pi_t \in \Pi_t} E_t \left[ W(\bar{\pi}) - D^{t+P}_t(\pi_t) \right] \geq 0$ for all $t$.
3. (Efficient Contract): $\min_{\pi_t \in \Pi_t} E_t \left[ W(\bar{\pi}) - D^{t+P}_t(\pi_t) \right] = 0$ for all $t$.

In the above definition, Condition 1 is known as the central bank’s rational (optimal) choice condition that says that conditional on cheating the central bank will choose an optimal cheating policy. Condition 2 is known as the incentive compatibility requirement that discourages the central bank from deviating from the socially optimal policy rule so that the central bank’s interest is compatible with the government’s (and the public’s) interest for any realization of the shock. Condition 3 is known as the efficient contract condition under which the penalty specified by the contract is the lowest penalty that just discourages the central bank from cheating.\(^9\)

\(^9\)Note that Condition 1 is not the same as Condition 3. For instance, adding a positive constant to the penalty
We are now ready to characterize an optimal central bank contract. Recall here again that, in all cases of policy practice, optimal monetary policy takes the form of a linear-affine function of the supply shock, \( \pi_t = \bar{\pi} + a_1 x_t \), as implied by the linear-quadratic structure of (1)-(2), or the quadratic structure of (3). For Condition 1, the central bank’s optimal choice of \( a_1 \) and \( \bar{\pi} \) for a cheating policy yields the following two first order conditions,

\[
(1 + \theta a^2) \pi_t x_t + \theta a (x_t - k) x_t = 0, \\
\frac{\partial W(\bar{\pi})}{\partial \bar{\pi}} = 0.
\]

(17) (18)

Taking the unconditional expectation of (17), we obtain \( [(1 + a^2 \theta) a_1 + a \theta] \sigma^2_x = 0 \), which, given \( \sigma^2_x > 0 \), yields the solution for \( a_1 \) as \( a_1 = -a \theta / (1 + a^2 \theta) \). Thus, the policy’s response to the supply shock \( x_t \) is the same as that of the socially optimal policy rule. In other words, an optimal cheating policy takes the form \( \pi_t = \bar{\pi} + \pi^R_t \), and the cheating policy differs from the socially optimal policy rule only by an average surprise inflation bias \( \bar{\pi} \). Substituting this form for an optimal cheating policy \( \pi_t = \bar{\pi} - a \theta / (1 + a^2 \theta) x_t \) into (18), we get

\[
\frac{\partial W(\bar{\pi})}{\partial \bar{\pi}} = \theta a k - (1 + \theta a^2) \bar{\pi},
\]

(19)

where we note that the terms related to the supply shock \( x_t \) are canceled out. Integrating (19) then yields

\[
W(\bar{\pi}) = W^0 + \theta a k \bar{\pi} - \frac{1}{2} (1 + \theta a^2) \bar{\pi}^2,
\]

(20)

where \( W^0 \) is an integration constant.

Using the form for an optimal cheating policy \( \pi_t = \bar{\pi} - [a \theta / (1 + a^2 \theta)] x_t \) obtained above, we can also compute the following conditional expectation

\[
E_t \left[ W(\bar{\pi}) - D_t^{l+P}(\pi_t) \right] = \frac{1}{2} (1 + \theta a^2) \bar{\pi}^2 - \theta a k \bar{\pi} + \frac{1}{2} \theta^2 a^2 k^2 \sum_{i=1}^{P} \beta^i + W(\bar{\pi}),
\]

(21)

where we note that the terms related to the supply shock \( x_t \) and its second moment \( \sigma^2_x \) are all canceled out. Substituting (20) into (21), we get

\[
E_t \left[ W(\bar{\pi}) - D_t^{l+P}(\pi_t) \right] = \frac{1}{2} \theta^2 a^2 k^2 \sum_{i=1}^{P} \beta^i + W^0.
\]

(22)

Thus, any contract in (20) with \( W^0 \geq -0.5 \theta^2 a^2 k^2 \sum_{i=1}^{P} \beta^i \) would satisfy Condition 2 and thus be incentive compatible, under which the central bank can’t do any better from cheating than following the socially optimal policy rule.
To obtain an efficient contract, we can take the unconditional expectation of (22), which in fact coincides with the conditional expectation as is revealed by the expression, and set it to zero to make sure that Condition 3 is satisfied. This gives rise to a unique value of $W^0$ under which the penalty specified by the contract is the lowest penalty that just discourages the central bank from cheating. This value is $W^0 = -0.5\theta^2a^2k^2\sum_{i=1}^{P} \beta^i$.

To summarize, the optimal central bank contract is given by

$$W(\bar{\pi}) = \begin{cases} \theta ak\bar{\pi} - \frac{1}{2}(1 + \theta a^2)\bar{\pi}^2 - \frac{1}{2}\theta^2a^2k^2\sum_{i=1}^{P} \beta^i & \text{if } \bar{\pi} \in \Pi_t \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

The central bank will be penalized by the amount of $W(\bar{\pi})$ if it uses a cheating policy $\pi_t \in \Pi_t$ at $t$, but will not be penalized if it follows the socially optimal policy rule, or if it uses a cheating policy $\pi_t$ that is not in $\Pi_t$ (which it will never do).

Recall that, $\Pi_t$, the cheating set at $t$ absent a central bank contract, is nonempty if and only if $\sum_{i=1}^{P} \beta^i < (1 + a^2\theta)^{-1}$. When $\sum_{i=1}^{P} \beta^i \geq (1 + a^2\theta)^{-1}$, the reputation force alone implements the socially optimal monetary policy rule, so $\Pi_t$ is an empty set and no central bank contract is called for: this can be seen by noticing that $W(\bar{\pi}) \leq 0$ for any surprise inflation $\bar{\pi}$ in this case.

On the other hand, we notice that $W(\bar{\pi}) > 0$ if and only if $\sum_{i=1}^{P} \beta^i < (1 + a^2\theta)^{-1}$ and

$$g_1 = -a\theta k\left[1 - \sqrt{1 - (1 + \theta a^2)\sum_{i=1}^{P} \beta^i}\right] < \bar{\pi} < \frac{a\theta k\left[1 + \sqrt{1 - (1 + \theta a^2)\sum_{i=1}^{P} \beta^i}\right]}{1 + \theta a^2} \equiv g_2.$$

Therefore, the optimal central bank contract in this case can alternatively be characterized by the cheating set (absent a central bank contract) in terms of surprise inflation rates, $\Pi = [q_1, q_2]$, along with the penalty function,

$$W(\bar{\pi}) = \begin{cases} \theta ak\bar{\pi} - \frac{1}{2}(1 + \theta a^2)\bar{\pi}^2 - \frac{1}{2}\theta^2a^2k^2\sum_{i=1}^{P} \beta^i & \text{if } \bar{\pi} \in \Pi \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

We summarize the above results in the following proposition.

**Proposition 2** Suppose $\sum_{i=1}^{P} \beta^i < (1 + a^2\theta)^{-1}$ so the reputation force alone can’t implement the socially optimal outcome. The hybrid mechanism combining the reputation force and the optimal central bank contract specified by (23) or (24) in the repeated monetary policy game among the public, the central bank, and the government implements the socially optimal outcome.

To appreciate Proposition 2 it is worth mentioning that, although the reputation force can’t single-handily implement the socially optimal monetary policy rule when $\sum_{i=1}^{P} \beta^i < (1 + a^2\theta)^{-1}$, its presence in the hybrid mechanism reduces the minimal penalty prescribed by the optimal
central bank contract that is required to be put in force in order to deter the central bank from deviating from the rule. This reduction in the contract cost is given by the central bank’s loss from its lost reputation in the future if it cheats at the present date \( t \),

\[
E_t \sum_{i=1}^{P} \beta^i \left[ \mathcal{L}(\pi^D_{t+i}, E\pi^D_{t+i}) - \mathcal{L}(\pi^R_{t+i}, E\pi^R_{t+i}) \right] = \frac{1}{2} \theta^2 \sigma^2 k^2 \sum_{i=1}^{P} \beta^i. \tag{25}
\]

This is the last term deducted in the penalty function (23) or (24). The more the central bank cares about the future (the larger the discount factor \( \beta \) is), or the longer the punishment length \( P \) is, the greater is this reputation effect. In fact, when the central bank is short-sighted and doesn’t care about the future at all, that is, when \( \beta = 0 \), or, when the punishment length \( P \) is zero, this reputation effect vanishes. The cheating set absent a central bank contract in this case takes a simple form \( \Pi = (0, 2\theta a k / (1 + \theta a^2)) \), and the hybrid mechanism becomes a pure contract mechanism,

\[
W(\bar{\pi}) = \begin{cases} 
\theta a k \bar{\pi} - \frac{1}{2} (1 + \theta a^2) \bar{\pi}^2 & \text{if } \bar{\pi} \in \left(0, \frac{2\theta a k}{1 + \theta a^2}\right), \\
0 & \text{otherwise},
\end{cases} \tag{26}
\]

which is a new pure contract mechanism that differs from the existing contract schemes in the literature. It is clear that the hybrid mechanism (23) or (24) is the combination of the reputation force (25) and the pure contract mechanism (26).

By its design, our hybrid approach combining the reputation force from the public and the penalty threat from the government provides a most efficient, incentive compatible mechanism to implement the socially optimal monetary policy rule. The threatened penalty embedded in the contract part of the mechanism is a concave function of surprise inflation, the size of which is reduced by the reputation effect, and which just offsets the expected benefit-cost difference from using an optimal cheating policy \( \pi_t = \bar{\pi} - \left[a\theta/(1 + a^2\theta)\right]x_t \),

\[
W(\bar{\pi}) = E_t[D^t_{t+P}(\pi_t)] = -\frac{1}{2} (1 + \theta a^2) \bar{\pi}^2 + \theta a k \bar{\pi} - \frac{1}{2} \theta^2 a^2 k^2 \sum_{i=1}^{P} \beta^i, \tag{27}
\]

leaving the central bank with no incentive to cheat at any given point in time. Thus, our hybrid mechanism implements the optimal economic outcome at the lowest social cost. In particular, the contract cost in our hybrid mechanism is uniformly lower than that of the standard central bank contract proposed in the literature, such as the optimal linear contract of Walsh (1995a). This is shown by Figure 1. The convex curve in the figure displays the expected cost-benefit difference \(-E_t[D^t_{t+P}(\pi_t)]\) from using an optimal cheating policy \( \pi_t \) at \( t \). The threatened penalty is prescribed by the concave curve, which is the reflection mapping of \(-E_t[D^t_{t+P}(\pi_t)]\) for \( \bar{\pi} \in (g_1, g_2) \) and zero elsewhere. These two curves are mirror images since \( W(\bar{\pi}) - E_t[D^t_{t+P}(\pi_t)] = 0 \) for all
Figure 1: $\theta = 1, a = 1, k = 5, P = 1, \beta = .2, g_1 = 0.5635, \text{ and } g_2 = 4.4365$

$\bar{\pi} \in (g_1, g_2)$, as the threatened penalty just offsets the expected benefit-cost difference from using a cheating policy. The threatened penalty is zero elsewhere since the central bank will never use a cheating policy outside $\Pi = (g_1, g_2)$. In consequence, and as can be seen from the figure, the threatened penalty scheme in our hybrid mechanism always lies below the straight line, which represents the cost of the optimal linear contract in Walsh (1995a).

Figure 1 also reveals another important feature of the optimal central bank contract in our hybrid mechanism: the threatened penalty posed by the contract has a unique global maximum. Indeed, as we can verify, provided $\sum_{i=1}^{P} \beta_i < (1 + a^2 \theta)^{-1}$, we have $\pi^C_t \in \Pi_t \neq \emptyset$, or, equivalently, $\pi^C = a\theta k / (1 + a^2 \theta) \in \Pi \neq \emptyset$, where we recall that $\pi^C_t$ is the optimal cheating policy absent a central bank contract, as specified by (6), and the penalty function in (23) or (24) reaches its unique global maximum at $\pi^C$ with

$$W(\pi^C) = \frac{1}{2} \frac{\theta^2 a^2 k^2}{1 + \theta a^2} - \frac{1}{2} \theta^2 a^2 k^2 \sum_{i=1}^{P} \beta_i. \tag{28}$$

Intuitively, since absent any penalty threat the expected benefit-cost difference from cheating is maximized at $\pi^C_t$ with the maximum being $E_t[D^i_t(\pi^C_t)]$, as shown by (12), a maximal penalty threat must be posed against $\pi^C_t$, as prescribed by (28), which is equal to (12), in order to
prevent this cheating policy from ever being used. In other words, the peak of the concave curve is the mirror image of the trough of the convex curve in Figure 1.

The right hand side of (28) provides the least upper bound on all threatened penalties. This bound depends on the model’s fundamental parameters but not economic variables. One may use it as a uniform penalty threat to punish the central bank whenever it cheats. That is, one may penalize the central bank by the amount of (28) when it chooses any \( \bar{\pi} \in \Pi = (g_1, g_2) \). Clearly, this uniform penalty threat implements the socially optimal monetary policy rule. Still, for moderate or large supply shocks, the magnitude of this flat penalty can be smaller than the size of the transfer required by the standard contract in the literature, such as the penalty transfer required by the optimal linear contract of Walsh (1995a), which is given by \( \theta a_k \pi_t \). This is true even when we ignore the reputation effect. For instance, setting \( \beta = 0 \) or \( P = 0 \) so that the reputation effect is absent, we can verify that the linear transfer required by Walsh’s (1995a) contract is greater than the flat penalty threat prescribed by (28) for all \( \pi_t > \bar{\pi} C / 2 \).

Not only is the transfer required by the standard contract greater, it is also necessarily materialized in every period. This is so since such a transfer is specified as a linear function of realized inflation, which in turn is a linear function of the supply shock. Unless the shock is muted, a transfer between the taxpayers and the central banker is necessarily involved. The transfer can be forbiddingly large if a large shock is realized. This is so even though the central bank’s response to the shock is socially optimal. This can make the standard contract either practically or politically difficult to implement, as pointed out by several researchers. In contrast, not only is the threatened penalty prescribed by the contract in our hybrid mechanism smaller, it is also not materialized in equilibrium. This is so since the penalty is effectively a function of surprise inflation, and our mechanism eliminates the time inconsistency and inflation bias so that the central bank always follows the socially optimal monetary policy rule and never has an incentive to use a surprise inflation. In fact, even under the uniform penalty threat, the central bank will behave in the socially optimal way, and no reputation cost or penalty threatened by the contract will actually be invoked along the equilibrium path. Thus, our hybrid mechanism implements the optimal economic outcome at the lowest social cost.

The cost-efficient feature of our hybrid mechanism has yet another implication. That is, it may render our mechanism potentially stable and robust with regard to various information imperfections and measurement errors. For instance, if the central bank’s observation of the supply shock is subject to some noises or measurement errors, or, if its choice of inflation is subject to some control errors, then its perceived socially optimal response to a supply shock
may deviate from the true socially optimal response. In this case, the cost-efficient feature of our mechanism ensures that the materialized penalty transfer will be kept to its minimal.

All that said, our hybrid mechanism does share an important common feature with the pure reputation mechanism and the pure contract mechanism. That is, while it completely overcomes the time-inconsistency problem of monetary policy associated with an inflation bias, it also leaves the central bank with complete discretion to respond to new circumstances. In this sense, our unified mechanism suggests an incentive-compatible and cost-efficient approach for delegating monetary policymaking to achieve socially optimal outcomes.

4. CONCLUSION

Since the seminal contributions of Kydland and Prescott (1977) and Barro and Gordon (1983a), various approaches have been proposed to solve the problem of time inconsistency and inflation bias in discretionary monetary policymaking. The existing literature has typically assessed the alternative solutions separately. We have shown in this paper how combining different approaches can have an advantage in tackling the problem. Our hybrid mechanism that unites a reputation force from the public with a central bank incentive contract from the government overcomes the problem and implements socially desirable outcomes in a cost-efficient way.

To make the point punchy, we have followed the lead of much of the time-inconsistency literature and adopted a new classical model with a Lucas-type supply curve. An important recent literature of monetary policy uses a New Keynesian model. In such a model, discretionary monetary policymaking produces not only the classical inflation bias concerning the average inflation rate, but also a stabilization bias concerning the variability of the inflation rate. The presence of the dual biases makes it difficult to tackle the time-inconsistency problem. This is perhaps why a commitment technology for a central bank is assumed in much of the New Keynesian literature, so that the time-inconsistency issue is avoided. Studies that do confront the time-inconsistency problem typically focus on the stabilization bias, while the inflation bias is assumed away. This is usually done by postulating output and employment subsidies to offset the monopolistic distortions in the product and labor markets. As Benigno and Woodford (2005a, b) pointed out, such assumption of output and employment subsidies, rather than positive tax rates on sales, payrolls, and wage income, is for technical convenience but unrealistic.

A few studies in the New Keynesian literature do focus on the inflation bias concerning the average inflation rate. King and Wolman (2004) show that modest or extreme inflation rates can arise from non-fundamental (sunspot) sources under the discretionary conduct of
monetary policy in a New Keynesian model. Adam and Billi (2005, 2008) show how introducing a Rogoff (1985) type conservative central bank into a New Keynesian model may help overcome the inflation bias, and perhaps even a public spending bias associated with discretionary fiscal policymaking, at a low cost of distorting the stabilization role of monetary policy in response to shocks. How reputation and contacting mechanisms or their combined forces may help overcome the time-inconsistency problem in a New Keynesian model remains an important open question. We leave our investigation in this question to future research.
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