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On the Existence of Equilibria in Generalized Games[1]

By G. Tian[2]

Abstract: This paper deals with the existence of equilibrium in generalized games (the so-called abstract economies) and Nash equilibrium in games with general assumptions. Preference correspondences, unlike the existing theorems in the literature, need not have open graphs or open lower sections, strategy spaces need not be compact and finite dimensional, the number of agents need not be countable, and preference relations need not be ordered. Thus, our results generalize many of the existence theorems on equilibria in generalized games, including those of Debreu (1952), Shafer and Sonnenschein (1975), Toussaint (1984), Kim and Richter (1986), and Yannelis (1987).

1 Introduction

In the last fifteen years, the classical Debreu (1952) result on the existence of (social) equilibrium in generalized games has been extended in many ways. Shafer and Sonnenschein (1975) and Borglin and Keiding (1976) extended Debreu's result to generalized games with the finite dimensional strategy spaces and the finitely many agents and without ordered preferences. For the infinite dimensional strategy spaces and finite or infinitely many agents case, the existence results were given by Yannelis and Prabhakar (1983), Khan and Vohra (1984), Toussaint (1984), Khan (1986), Khan and Papageorgiou (1987), Yannelis (1987), and Kim, Prikry, and Yannelis (1989) among others. All the existence theorems mentioned above, however, are obtained by assuming that the preference correspondence has an open graph or has open lower sections. Besides, in these models, the strategy sets are assumed to be compact in topological vector spaces. This is a restricted assumption since, in the infinite settings, the set of feasible allocations generally is not compact in any topology of the commodity space. The motivations for economists continually to be interested in setting forth conditions for the existence of equilibria come from the importance of generalized games in the study of markets and other general games and from the restrictions of the existing theorems.

For generalized games with non-compact and infinite dimensional strategy spaces, Tian (1992) proved the equilibrium existence for a countably infinite number of agents by using the quasi-variational inequality approach and a result obtained in

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Tian and Zhou (1991). However, the quasi-variational inequality approach requires that preferences be representable by a utility function. Tian (1990) generalized the existence theorem of Yannelis and Prabhakar (1983) by relaxing the compactness strategy space. This theorem allows for the preferences to be nontotal-nontransitive, but it assumes that the preference correspondences have open lower sections.

In this paper we extend the existence results in the literature for generalized games with preference correspondences which have neither open graphs nor open lower sections. Also, in our framework, strategy spaces may be infinite-dimensional and non-compact, the number of agents may be uncountably infinite, preferences may be nontotal-nontransitive. Thus, our results generalize many of the existence theorems, on equilibria in generalized games by relaxing the compactness of strategy space and the openness of graphs or lower sections of preference correspondences. We also give a result on the existence of Nash equilibria in the conventional games, which extends the results in Nash (1950, 1951), Nikaido and Isoda (1955), Dasgupta and Maskin (1986), and Tian and Zhou (1992) to allow nontotal-nontransitive preferences. It may be remarked that, since the competitive mechanism can be regarded as a generalized game, the results given in this paper can be used to prove the existence of competitive equilibria in economies with a non-compact and infinite dimensional feasible set, an uncountably infinite number of agents, and with inter-dependent, price-dependent, and nontotal-nontransitive preference correspondences which may not have an open graph or open lower sections. Thus many existence theorems on the competitive equilibrium in the literature can be also generalized. For instance, our results can be used to generalize two existence theorems of Kim and Richter (1986) by relaxing the open-lower-sections condition for preference correspondences. Our existence theorems can also be used to give correct proofs for their results (see Remark 1 below).

The paper is organized as follows. Notation and definitions are given in Section 2. The main existence theorems of the paper and their proofs are given in Section 3. The approach adopted in this paper is based on selection-type arguments and was first given by Yannelis and Prabhakar (1983). But the selection theorem we use in this paper is Theorem 3.1''' in Michael (1956). In Section 4, we give some concluding remarks and show how the results obtained in Section 3 are extended to include more general cases.

2 Notation and Definitions

Let X and Y be two topological spaces, and let 2^Y be the collection of all subsets of Y. A correspondence $F: X \to 2^Y$ is said to be *upper hemi-continuous* (in short, u.h.c.) if the set $\{x \in X: F(x) \subset V\}$ is open in X for every open subset V of Y. A correspondence $F: X \to 2^Y$ is said to be *lower hemi-continuous* (in short, l.h.c.) if the set $\{x \in X: F(x) \cap V \neq \emptyset\}$ is open in X for every open subset V of Y. A correspondence $F: X \to 2^Y$ is said to be *continuous* if it is both u.h.c. and l.h.c. A correspondence $F: X \to 2^Y$ is said to have *open lower sections* if the set $F^{-1}(y) = \{x \in X: y \in F(x)\}$ is open in X for every $y \in Y$. A correspondence $F: X \to 2^Y$ is said to have *open upper*

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$X\to 2^Y$ is said to have *open upper*

sections if, for every $x\in X$, $F(x)$ is open in Y. A correspondence $F: X\to 2^Y$ is said to be *closed* if the set $\{(x, y)\in X\times Y: y\in F(x)\}$ is closed in $X\times Y$. A correspondence $F: X\to 2^Y$ is said to have an *open graph* if the set $\{(x, y)\in X\times Y: y\in F(x)\}$ is open in $X\times Y$. Denote by *con* B the convex hull of the set B.

Let I be the set of agents which is any countable or uncountable set. Each agent $i\in I$ has a choice set X_i, a constraint correspondence $A_i: X\to 2^{X_i}$, and a preference correspondence $P_i: X\to 2^{X_i}$, where $X=\prod_{i\in I} X_i$. Denote by X_{-i} and A the product $\prod_{j\in I\setminus\{i\}} X_j$ and the product $\prod_{i\in I} A_i$.

A *generalized game (or an abstract economy)* $\Gamma=(X_i, A_i, P_i)_{i\in I}$ is defined as a family of ordered triples (X_i, A_i, P_i). An *equilibrium* for Γ is an $x^*\in X$ such that $x^*\in A(x^*)$ and $P_i(x^*)\cap A_i(x^*)=\emptyset$ for each $i\in I$.

If $A_i(x)\equiv X_i$, $\forall i\in I$, the generalized game reduces to the conventional game $\Gamma=(X_i, u_i)$ and the equilibrium is called a *Nash equilibrium*.

3 Existence of Equilibrium

In this section we give two existence results on equilibrium for generalized games and one existence result on Nash equilibrium for games. Before proceeding to the main theorems, we state some technical lemmas which were due to Michael (1956, Propositions 2.5–2.6 and Theorem 3.1''').

Lemma 1: Let X and Y be two topological spaces and $\phi: X\to 2^Y$, $\psi: X\to 2^Y$ be correspondences such that

(i) ϕ is l.h.c. and has open upper sections,
(ii) ψ is l.h.c.,
(iii) for all $x\in X$, $\phi(x)\cap\psi(x)\neq\emptyset$.

Then the correspondence $\theta: X\to 2^Y$ defined by $\theta(x)=\phi(x)\cap\psi(x)$ is l.h.c.

Lemma 2: Let X be a topological space and Y be a convex set of a topological vector space, and let $\phi: X\to 2^Y$ be l.h.c. Then the correspondence $\psi: X\to 2^Y$ defined by $\psi(x)=con\,\phi(x)$ is l.h.c.

Lemma 3: Let X be a perfectly normal T_1-topological space and Y be a separable Banach space. Let $\mathscr{D}(Y)$ be the set of all nonempty and convex subsets of Y which are either finite-dimensional or closed or have an interior point. Suppose $F: X\to\mathscr{D}(Y)$ is a l.h.c. correspondence such that $F(x)$ is nonempty and convex for all $x\in X$. Then there exists a continuous function $f: X\to Y$ such that $f(x)\in F(x)$ for all $x\in X$.

The following theorems generalize the results of Debreu (1952), Shafer and Sonnenschein (1975), and Toussaint (1984) by relaxing the compactness of strategy spaces and the openness of graphs or lower sections of preference correspondences. Theorem 2 generalizes Theorem 1 to have a non-compact strategy space. Note that the method of proof in the following is different from those given in Debreu (1952), Shafer and Sonnenschein (1975), and Toussaint (1984) so that we can allow for an infinite number of commodities and a countably or uncountably infinite number of agents.

Theorem 1: Let $\Gamma = (Z_i, A_i, P_i)_{i\in I}$ be a generalized game satisfying for each $i\in I$:

(i) Z_i is a nonempty, compact, and convex subset in $\mathbb{R}^{l_i}$,
(ii) A_i is a continuous correspondence, and $A_i(x)$ is nonempty, compact, and convex for all $x\in Z$,
(iii) P_i is l.h.c. and has open upper sections,
(iv) $x_i \notin con\, P_i(x)$ for all $x\in Z$.

Then Γ has an equilibrium.

Proof: For each $i\in I$, define a correspondence F_i: $Z\to 2^{Z_i}$ by $F_i(x) = A_i(x)\cap con\, P_i(x)$. Let $U_i = \{x\in Z: F_i(x)\neq\emptyset\}$. Since A_i and P_i are l.h.c. in Z, so are they in U_i. Then, by Lemma 2, $con\, P_i$ is l.h.c. in U_i. Also since P_i has open upper sections in Z, so does $con\, P_i$ in Z and thus $con\, P_i$ in U_i. Further, $A_i(x)\cap con\, P_i(x)\neq\emptyset$ for all $x\in U_i$. Hence, by Lemma 1, the correspondence $F_i \mid U_i$: $U_i\to 2^{Z_i}$ is l.h.c. in U_i and for all $x\in U_i$, $F(x)$ is nonempty and convex. Also Z_i is finite dimensional. Hence, by Lemma 3, there exists a continuous function f_i: $U_i\to Z_i$ such that $f_i(x)\in F_i(x)$ for all $x\in U_i$. Note that U_i is open since F_i is l.h.c. Define a correspondence G_i: $Z\to 2^{Z_i}$ by

$$G_i(x) = \begin{cases} \{f_i(x)\} & \text{if } x\in U_i \\ A_i(x) & \text{otherwise} \end{cases}. \tag{1}$$

Then G_i is u.h.c. Thus the correspondence G: $Z\to 2^Z$ defined by $G(x) = \prod_{i\in I} G_i(x)$ is u.h.c. by Lemma 3 in Fan (1952, p. 124) and for all $x\in Z$, $G(x)$ is nonempty, closed, and convex. Hence, by Theorem 1 in Fan (1952, p. 122), there exists a point $x^*\in Z$ such that $x^*\in G(x^*)$. Note that for each $i\in I$, if $x^*\in U_i$, then $x_i^* = f_i(x^*)\in F(x^*) \subset con\, P_i(x^*)$, a contradiction to (iv). Hence, $x^*\notin U_i$ and thus for all $i\in I$, $x_i^*\in A_i(x^*)$ and $A_i(x^*)\cap con\, P_i(x^*) = \emptyset$ which implies $A_i(x^*)\cap P_i(x^*) = \emptyset$. Thus Γ has an equilibrium. ■

Remark 1: Kim and Richter (1986) gave two existence theorems (their Theorems 1 and 2) on competitive equilibrium for exchange economies by using Threorem 6.1 of Yannelis and Prabhakar (1983). But, the proofs of these two theorems are not quite correct since the budget constrains need to satisfy the open-lower-sections condition in order to apply the results of Yannelis and Prabhakar (1983). But the budget constraints clearly do not satisfy this condition under the assumptions imposed in their theorems. However, these theorems can be proved by applying Theorem 1 above and in fact can be generalized by relaxing the open-lower-sections assumption for preference correspondences.

By using similar techniques
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Theorem 2: Let $\Gamma = (X_i, A_i, P_i)_{i\in I}$

(i) X_i is a nonempty and co
(ii) A_i is a continuous corr
convex for all $x\in X$,
(iii) P_i is l.h.c. and has open
(iv) $x_i\notin P_i(x)$ for all $x\in X$,
(v) there exists a non-empty
(v.a) $A_i(x)\cap Z_i\neq\emptyset$ for
$C = \prod_{i\in I} C_i$;
(v.b) for each $x_i\in Z_i\backslash C_i$
$y_i\in P_i(x)$.

Then Γ has an equilibrium.

Proof: Since $C = \prod_{i\in I} C_i$ is compac
values, by Proposition 3.11 in A
$A_i(C)$ is compact and thus Z_i is c
Let $Z = \prod_{i\in I} Z_i$. For each $i\in I$,
$x\in Z$,

$$K_i(x) = A_i(x)\cap Z_i.$$

Then, by Conditions (ii) and (v.a)
Z is compact and A_i is closed
p. 111), then K_i is closed and the
Ekeland (1984, p. 111). Also, no

$$K_i(x) = \begin{cases} A_i(x) & \text{if } x_i\in C \\ A_i(x)\cap Z_i & \text{otherwi} \end{cases}$$

Therefore, the generalized game
Theorem 1 and thus there ex

$P_i(x^*)\cap K_i(x^*) = \emptyset$. Now $x^*\in C$,
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Remark 2: In the literature, the
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et al. (1976, p. 265)) and thus P h

By using similar techniques to those in Tian (1990, 1992), the above theorem can be generalized by relaxing the compactness of the strategy space.

Theorem 2: Let $\Gamma = (X_i, A_i, P_i)_{i \in I}$ be a generalized game satisfying for each $i \in I$:

(i) X_i is a nonempty and convex subset in $\mathbb{R}^{l_i}$,
(ii) A_i is a continuous correspondence and $A_i(x)$ is nonempty, compact, and convex for all $x \in X$,
(iii) P_i is l.h.c. and has open upper sections,
(iv) $x_i \notin P_i(x)$ for all $x \in X$,
(v) there exists a non-empty compact set $C_i \subset X_i$ such that
  (v.a) $A_i(x) \cap Z_i \neq \emptyset$ for all $x \in X_{-i} \times Z_i$, where $Z_i = con\{A_i(C) \cup C_i\}$ with $C = \prod_{i \in I} C_i$;
  (v.b) for each $x_i \in Z_i \setminus C_i$ and $x_{-i} \in X_{-i}$ there exists $y_i \in A_i(x) \cap Z_i$ such that $y_i \in P_i(x)$.

Then Γ has an equilibrium.

Proof: Since $C = \prod_{i \in I} C_i$ is compact and A_i is a u.h.c. correspondence with compact values, by Proposition 3.11 in Aubin and Ekeland (1984, p. 113), we know that $A_i(C)$ is compact and thus Z_i is compact convex.

Let $Z = \prod_{i \in I} Z_i$. For each $i \in I$, define a correspondence $K_i: Z \to 2^{Z_i}$ by, for each $x \in Z$,

$$K_i(x) = A_i(x) \cap Z_i. \tag{2}$$

Then, by Conditions (ii) and (v.a), $K_i(x)$ is nonempty and convex for all $x \in Z$. Since Z is compact and A_i is closed by Proposition 3.7 in Aubin and Ekeland (1984, p. 111), then K_i is closed and therefore is u.h.c. on Z by Theorem 3.8 in Aubin and Ekeland (1984, p. 111). Also, note that

$$K_i(x) = \begin{cases} A_i(x) & \text{if } x_i \in C_i \\ A_i(x) \cap Z_i & \text{otherwise} \end{cases}. \tag{3}$$

Therefore, the generalized game $\Gamma = (Z_i, K_i, P_i \mid Z)_{i \in I}$ satisfies all the assumptions of Theorem 1 and thus there exists $x^* \in Z$ such that $x^* \in K(x^*) = \prod_{i \in I} K_i(x^*)$ and $P_i(x^*) \cap K_i(x^*) = \emptyset$. Now $x^* \in C$, for otherwise Hypothesis (v.b) would be violated, and hence $K(x^*) = A(x^*)$. Therefore, we have $x^* \in A(x^*)$ and $P_i(x^*) \cap A_i(x^*) = \emptyset$ for all $i \in I$. So $x^* \in X$ is an equilibrium of the generalized game Γ. ■

Remark 2: In the literature, the preference correspondence P is assumed either to have an open graph or have open lower sections. It is known that if a correspondence P has an open graph then P has open upper and lower sections (see Bergstrom, et al. (1976, p. 265)) and thus P has open upper sections and is l.h.c. Thus Assumption (iii) in Theorems 1 and 2 on preference correspondences is satisfied if P_i has an open graph or open upper and lower sections.[3]

Remark 3: Observe that in case of a compact convex X_i, Assumption (v) in Theorem 1 is satisfied by $C_i = X_i$. Thus Theorem 2 is indeed a generalization of Theorem 1 by relaxing the compactness condition.

Remark 4: Assumption (v.a) is needed to guarantee the existence of a fixed point of A (cf. Tian (1991)). Assumption (v.b) guarantees the fixed point x^* in C. This assumption is similar to the one imposed by Border (1985, p. 34–35) and is used to prove the existence of maximal elements of preferences on a non-compact set.

When a generalized game reduces to the conventional game, the openness of upper sections of preference correspondences can be removed and we have the following corollary on the existence of Nash equilibrium, which extends many results in the literature.

Corollary 1: Let $\Gamma = (Z_i, P_i)_{i \in I}$ be a game satisfying for each $i \in I$:

(i) Z_i is a nonempty, compact, and convex subset in $\mathbb{R}^{l_i}$,
(ii) P_i is l.h.c.,
(iii) $x_i \notin con P_i(x)$ for all $x \in Z$.

Then Γ has a Nash equilibrium.

Proof: Let $A_i(x) \equiv X_i$ for all $i \in I$. Then A_i is continuous and in particular has open upper sections so that Lemma 1 can be applied to show that the correspondence F_i, defined by $F_i(x) = A_i(x) \cap con P_i(x)$, is l.h.c. The remaining proof of this corollary is the same as that of Theorem 1 and thus is omitted here. ■

4 Concluding Remarks

In Section 3, we have proved the existence of an equilibrium for generalized games with non-compact and finite dimensional strategy spaces, an infinite number of agents, and nontotal-nontransitive preference correspondences which may not have open graphs or open lower sections. These results generalize many results in the existing literature by relaxing the compactness of strategy spaces and the openness of graphs or lower sections of preference correspondences. In the context of the exist-

[3] Remark 4 in Bergstrom, et al. (1976, p. 266) shows that a correspondence with open upper and lower sections may not have an open graph. Also, an example given by Yannelis and Prabhakar (1983, p. 237) shows that a l.h.c. correspondence may not have open lower sections. Thus our assumption on the preference correspondences are indeed weaker than theirs.

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Remark 5: Theorems 1 and 2 c
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The proof of the modified theo
the correspondence $F_i \mid U_i$: $U_i \to$
for all $x \in U_i$ under Assumption
above) can still be applied.

Remark 6: Theorem 1 can be al
cular, the modified theorem can
by weakening Assumption (A.4)
dition:

(A.4)(a)′ For each $t \in T$,
$L_1(\mu, X) \times \mathbb{R}^l$ an

Even the correspondence θ:
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by Yannelis (1987, p. 108)), the
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rrespondences are indeed weaker than

ence theorems appearing in the recent economics literature, perhaps this relaxation is the most notable aspect of our theorems on the existence of equilibria in generalized games. We also give a very general result on the existence of Nash equilibrium for the conventional games. In the following, we want to mention some of the possible extensions of our existence results.

Remark 5: Theorems 1 and 2 can be extended to separable Banach spaces if Assumption (iii) is strengthened to the following condition:

(iii)′ A_i is a continuous correspondence and $A_i(x)$ is compact, convex and has a nonempty interior for all $x \in X$.

The proof of the modified theorems remains the same. One only needs to note that the correspondence $F_i \mid U_i$: $U_i \to 2^{Z_i}$ by $F_i(x) = A_i(x) \cap con P_i(x)$ has an interior point for all $x \in U_i$ under Assumption (iii)′ so that Michael's selection theorem (Lemma 3 above) can still be applied.

Remark 6: Theorem 1 can be also extended to a measure space of agents. In particular, the modified theorem can generalize the existence theorem of Yannelis (1987) by weakening Assumption (A.4)(a) of Yannelis (1987, p. 100) to the following condition:

(A.4)(a)′ For each $t \in T$, $P(t, \cdot)$: $L_1(\mu, X) \to 2^{\mathbb{R}^l}$ has open upper sections in $L_1(\mu, X) \times \mathbb{R}^l$ and is l.h.c.

Even the correspondence θ: $T \times L_1(\mu, X) \to 2^{\mathbb{R}^l}$ defined by $\theta(t, x) = A_i(t, x) \cap con P_i(t, x)$ may not be l.h.c. in X if (A.4)(a) is weakened to (A.4)(a)′ (as indicated by Yannelis (1987, p. 108)), the correspondence θ, by Lemma 1, is l.h.c. in U so that Caratheodory-Type Selection Theorem can still be applied on U.[4] The proof of the modified theorem remains the same. Thus, Condition (A.4)(a) of Yannelis (1987) turned out to be unnecessarily strong. Similarly, the modified theorem can be generalized by relaxing the compactness of strategy spaces.

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Allocation Rules for H
Situations

By A. van den Nouweland, P. B

Abstract: This paper considers hype
agents the economic possibilities are
possibilities are described by a hyperg
the subgroups of agents who can eff
vided for two allocation rules, the M

1 Introduction

For a group $N := \{1, \dots, n\}$ of ag
players who decide to cooperate
player set N and characteristic fu
possibilities are described by a *(cc*
The interpretation is that commu
and all players in H have to be
will implicitly assume that each p
we restrict our attention to hype
convenience we assume that the
$i \in N$. The triple $(N, v, \mathscr{H})$ is call

An example of a hypergrap
example football. Football teams
mittees are also in the board of
which in turn has representatives
sions can be made within each o
between two organizations is p
mon.

Communication situations w
graph were first studied in Mye
restricted games and characterize
in terms of efficiency and fairnes
and their Shapley value, the *My*
situations where the underlying g
value for graph communication

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