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Completely Feasible and Continuous Implementation of the Lindahl Correspondence with a Message Space of Minimal Dimension*

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This paper deals mainly with the problem of designing mechanisms whose Nash allocations coincide with the Lindahl allocations. It goes beyond the previously existing literature in that it uses outcome functions that are individually feasible, balanced, and continuous, and further, has a message space of minimal dimension and thus is informationally efficient. *Journal of Economic Literature* Classification Numbers: 022, 025, 026. © 1990 Academic Press, Inc.

1. Introduction

In the literature, the mechanisms yielding Pareto efficient allocations for the public goods economies have the disadvantage that they do not guarantee complete feasibility (i.e., individual feasibility and balancedness) and/or continuity at disequilibrium points. Groves and Ledyard [1] were the first to propose a mechanism which yields Pareto efficient Nash allocations. Their mechanism, however, is neither individually rational nor individually feasible. Hurwicz [4] exhibited a quasi-game which yields Lindahl allocations and thus results in Pareto-efficient and individually rational allocations at Nash equilibria. But the mechanism is not balanced. Hurwicz [3] gave another mechanism with a balanced and smooth outcome function whose Nash allocations are precisely the Lindahl alloca-

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tions. The mechanism, however, is not individually feasible. Walker [14] is a modification of Hurwicz [3] which not only has all properties of Hurwicz's mechanism but also has the advantage of using a minimal-sized message space. Unfortunately, like Hurwicz's mechanism, it is not individually feasible, either. Hurwicz, Maskin, and Postlewaite [6] presented a mechanism Nash-implementing the constrained Lindahl correspondence. Their mechanism is individually feasible and balanced but discontinuous, and uses a huge message space. For people's motivations to require that a mechanism be individually rational, completely feasible, and continuous, and have a message space of lower dimensions, see Tian [11] or Groves and Ledyard [2].

A similar situation prevailed with regard to the Nash-implementation of the Walrasian correspondence until Postlewaite and Wettstein [9] designed a mechanism with an individually feasible, weakly balanced, and continuous outcome function which fully Nash-implements the constrained Walrasian correspondence, though their mechanism is not balanced[1] and uses a message space of dimension higher than the minimal dimension required for implementation of the (constrained) Walrasian correspondence (see [10]).

An appropriate question then is whether or not mechanisms exist which are individually feasible, balanced, and continuous and which fully Nash-implement a Pareto-efficient and individually rational correspondence over a class of economies. If the answer is positive, is it possible to have a message space of minimal dimension?[2]

This paper answers both questions in the affirmative for a class of public goods economies. If the designer knows the individual endowments, but it is nevertheless required that the outcome function be informationally decentralized, then we will prove that there exists such a mechanism.[3] The conclusion is shown to be true for any economy in which there are at least three agents, with individual preferences satisfying the usual regularity assumptions (in particular, strict monotonicity and convexity) and, in addition, the condition that any interior allocation is strictly preferred to a boundary allocation. Though this note only considers Nash-implementation of the Lindahl correspondence, we think some of the techniques developed in the paper can be applied to implementation of other social choice correspondences.[4]

The plan of the note is as follows. A public goods model and the mechanism with the desirable properties are presented in Section 2. The main results and their proofs are given in Section 3. In Section 4 we will give some remarks on possible extensions of the results obtained in Section 3.

[1] The difference between balancedness and weak balancedness becomes important when implementation requires that nonequilibrium messages be used to compute the allocations. For example, if a mechanism which is not balanced is actually used, say an iterative process which was terminated prior to the attainment of an equilibrium, then it is possible that the allocation at a nonequilibrium point may be less than the total endowments and thus some resources are not completely used (free disposal occurs) though preferences of agents are strictly monotone increasing.

[2] From Theorem 2.4 of Reichelstein and Reiter [10] we know that Nash implementation is always at least as costly, in message space size, as (decentralized) realization. Thus, if we can design a mechanism which implements some social choice correspondence and has a message space whose dimension is the same as the minimal dimension required for decentralized realization of the correspondence, we know that the mechanism has a message space of minimal dimension for implementation.

[3] In the case where the initial endowments are private information, Tian [13] gave a feasible (i.e., individually feasible and weakly balanced) and continuous mechanism which implements the Lindahl correspondence. This case, of course, certainly increases the size of the message space but reduces the information requirements on the designer.

2. Public Goods Model and Mechanism

2.1. *Economic Environments*

In order to give the clearest possible exposition, we shall only deal with economies with one private and one public good, x being private and y public.[5] There are n agents. Denote by $N = \{1, 2, ..., n\}$ the set of agents. Each agent has an initial endowment of the private good w_i and a preference ordering R_i defined on $\mathbb{R}^2_+$ (P_i the strictly preference ordering). We assume that there is no initial endowment of the public good, but the public good can be produced from the private good in a constant returns to scale production process. With suitable normalization, the input–output ratio can be set equal to one, so that one unit of the private good can produce one unit of the public good. Thus the feasibility constraint becomes

$$\sum_{i=1}^{n} x_i + y \leqslant \sum_{i=1}^{n} w_i. \tag{1}$$

As above, $e_i = (w_i, R_i)$ is called the characteristic of consumer i and the full vector $e = (e_1, ..., e_n)$ is called an economy. The set of all such economies is denoted by E. The following assumptions are made on E:

[4] In [11, Chap. 6] we have used similar techniques to give a completely feasible (not merely feasible) and continuous mechanism which Nash-implements the constrained Walrasian correspondence.

[5] In Section 4 we will indicate the results obtained in this note can be extended to an arbitrary number of public and private goods.

Assumption 1. $n \geqslant 3$.[6]

Assumption 2. $w_i > 0$ for all $i \in N$.

Assumption 3. The preference R_i is reflexive, transitive, total, and convex[7] on $\mathbb{R}^2_+$ as well as strictly monotone increasing on $\mathbb{R}^2_{++}$.

Assumption 4. For all $i \in N$, $(x_i, y)\ P_i\ (x'_i, y')$ for all $(x_i, y) \in \mathbb{R}^2_{++}$ and $(x'_i, y') \in \partial\mathbb{R}^2_+$, where $\partial\mathbb{R}^2_+$ is the boundary of $\mathbb{R}^2_+$.

2.2. *Lindahl Allocations*

An allocation $(x, y) = (x_1, ..., x_n, y)$ is *completely feasible* for an economy e if it is individually feasible (i.e., $(x, y) \in \mathbb{R}^{n+1}_+$) and balanced (i.e., (1) holds with equality).

An allocation (x^*, y^*) is a *Lindahl allocation* for an economy e, if it is completely feasible and there are personalized prices $q_i^* \in \mathbb{R}_+$, one for each i, such that (a) $x_i^* + q_i^* y^* \leqslant w_i$ for all $i \in N$; (b) $(x_i, y)\ P_i\ (x_i^*, y^*)$ implies $x_i + q_i^* y > w_i$ for all $i \in N$; (c) $\sum_{i=1}^n q_i^* = 1$. Denote by $L(e)$ the set of all such allocations.

Remark 1. If only the above assumptions are imposed—in particular, if continuity of preferences is not assumed—the correspondence $L(e)$ may be empty. In the following sections, we only consider those economies that guarantee existence of Lindahl allocations.

Let F be a social choice rule, i.e., a correspondence from E to the set Z of resource allocations. In the rest of the paper, we will assume that the social choice rule is the Lindahl correspondence.

2.3. *The Completely Feasible and Continuous Mechanism*

Let M_i denote the ith message domain, whose elements are written as m_i. Let $M = \prod_{i=1}^n M_i$ denote the message space and $h: M \to Z$ denote the outcome function, or more explicitly, $h_i(m) = (X_i(m), Y(m))$, where $X_i(m)$ is the ith agent's outcome function for the private good and $Y(m)$ is the outcome function for the public good. A mechanism is defined by (M, h).

A message $m^* = (m_1^*, ..., m_n^*) \in M$ is a *Nash equilibrium* (NE) of the mechanism $(M; h)$ for an economy e if for any $i \in N$,

$$h_i(m^*)\ R_i(e)\ h_i(m^*/m_i, i) \qquad \text{for all} \quad m_i \in M_i, \tag{2}$$

where $(m^*/m_i, i) = (m_1^*, ..., m_{i-1}^*, m_i, m_{i+1}^*, ..., m_n^*)$. The $h(m^*)$ is then called a *Nash (equilibrium) allocation*. Denote by $N_{M,h}(e)$ the set of all such Nash (equilibrium) allocations.

A mechanism (M, h) (fully) *Nash-implements* the social choice rule F on E, if, for all $e \in E$, $N_{M,h}(e) \neq \varnothing$ and $N_{M,h}(e) \subseteq F(e)$ $(N_{M,h}(e) = F(e))$.

In the following, we present a specific mechanism with a completely feasible and continuous outcome function[8] which fully Nash-implements the Lindahl correspondence for a class of economies with one private good and one public good.

Let $M_i = \mathbb{R}$ whose elements m_i are interpreted as the taxes that agent i are willing to pay. Let $A = (a_{ij})$ be an $n \times n$ coefficient matrix with $a_{ii} = 0$ for all $i \in N$, $\sum_{i=1}^n A_{i.} = 0$, where $A_{i.}$ is the ith row of A. We assume that $\sum_{j=1}^n |a_{ij}| > 0$ for all $i \in N$ and the row vectors $A_{i.}$'s are linearly independent of the vector with ones.

The personalized price column vector of public good $q(m): M \to \mathbb{R}^n$ is defined as, for all $m \in \mathbb{R}^n$,

$$q(m) = b + Am, \tag{3}$$

where b is an $n \times 1$ column vector with $\sum_{i=1}^n b_i = 1$. Then we have

$$\sum_{i=1}^n q_i(m) = 1 \tag{4}$$

for all $m \in M$.

Remark 2. Note that $q_i(m)$ in (3) is independent of m_i by definition. Also the $q(m)$ is a very general form of personalized price vector and contains the personalized price vectors specified by Hurwicz [3] and Walker [14] as special cases.

Let

$$N'(m) = \{i: i \in N \text{ and } q_i(m) > 0\} \tag{5}$$

and

$$a(m) = \min_{i \in N'(m)} \frac{w_i}{q_i(m)}. \tag{6}$$

Since $\sum_{i=1}^n q_i(m) = 1 > 0$, $N'(m)$ is a nonempty subset of N for all $m \in M$. The $a(m)$ is an upper bound which guarantees the individual feasibility for each choice of m and is simply called the feasible upper bound.

[6] Kwan and Nakamura [7] recently proved that there are no balanced and continuous mechanisms which implement the (constrained) Walrasian correspondence and the (constrained) Lindahl correspondence for two-agent economies.

[7] Preference R_i is convex if, for a, b, c in C_i and $0 < \lambda \leqslant 1$, $c = \lambda a + (1-\lambda)b$, the relation $a\ P_i\ b$ implies $c\ P_i\ b$.

[8] That is, the outcome function is continuous and yields individually feasible and balanced allocations for all $m \in M$.

The Y-outcome function for the public good $Y(m): M \to \mathbb{R}_+$ is defined as

$$Y(m) = \begin{cases} \tilde{m} & \text{if } 0 \leqslant \tilde{m} \leqslant a(m) \\ a(m) & \text{if } \tilde{m} > a(m), \\ 0 & \text{if } \tilde{m} < 0 \end{cases} \tag{7}$$

where $\tilde{m} = \sum_{i=1}^{n} m_i$. Thus $Y(m)$ is single-valued and continuous at every $m \in M$.

An interpretation of this formulation is that if the total taxes that the agents are willing to pay were between zero and the feasible upper bound, the level of public good to be produced would be exactly the total taxes; if the total taxes were less than zero, no public good would be produced; if the total taxes exceeded the feasible upper bound, the level of the public good would be equal to the feasible upper bound.

The ith agent's tax-outcome function $T_i(m): M \to \mathbb{R}$ is given by

$$T_i(m) = q_i(m)\, Y(m). \tag{8}$$

The ith agent's X-outcome function for the private good $X_i(m): M \to \mathbb{R}_+$ is given by

$$X_i(m) = w_i - T_i(m) = w_i - q_i(m)\, Y(m), \tag{9}$$

which means the budget constraint holds with equality for all m.

The outcome function is clearly continuous on M. Also since $(X_i(m), Y(m)) \in \mathbb{R}_+^2$ and

$$\sum_{i=1}^{n} X_i(m) + Y(m) = \sum_{i=1}^{n} w_i \tag{10}$$

for all $m \in M$, it is individually feasible and balanced. Thus the mechanism is completely feasible and continuous on M.

3. The Main Results

The remainder of this note is devoted to the proof of equivalence between Nash allocations and Lindahl allocations. We first prove the following lemmas.

Lemma 1. *If $\tilde{m} \geqslant 0$ and $w_i - q_i(m)\, \tilde{m} \geqslant 0$ for all $i \in N$, then $Y(m) = \tilde{m}$, where $\tilde{m} = \sum_{i=1}^{n} m_i$.*

The proof of the lemma is left to readers.

Lemma 2. *If $(X(m^*), Y(m^*)) \in N_{M,h}(e)$, then $(X_i(m^*), Y(m^*)) \in \mathbb{R}_{++}^2$ for all $i \in N$.*

Proof. We argue by contradiction. Suppose $(X_i(m^*), Y(m^*)) \in \partial\mathbb{R}_+^2$. Then $X_i(m^*) = 0$ for some $i \in N$ or $Y(m^*) = 0$. Consider the quadratic equation

$$y = \frac{w^*}{2a(y+c)}, \tag{11}$$

where $w^* = \min_{i \in N} w_i$; $a = \max_{i,j \in N} |a_{ij}|$; $c = b^* a^{-1} + 2 \sum_{i=1}^{n} |m_i^*|$, where $b^* = \max_{i \in N} |b_i|$. The larger root of (11) is positive and denoted by $\tilde{y}$. Suppose that player i chooses his/her message $m_i = \tilde{y} - \sum_{j \neq i}^{n} m_j^*$. Then $\tilde{y} = m_i + \sum_{j \neq i}^{n} m_j^* > 0$ and

$$\begin{aligned} w_j - q_j(m^*/m_i, i)\, \tilde{y} &= w_j - \left[b_j + \sum_{s \neq i}^{n} a_{js} m_s^* + a_{ji} \left(\tilde{y} - \sum_{s \neq i}^{n} m_s^* \right) \right] \tilde{y} \\ &\geqslant w_j - \left[b^* + a \left(2 \sum_{s=1}^{n} |m_s^*| + \tilde{y} \right) \right] \tilde{y} \\ &= w_j - a \left(\tilde{y} + b^* a^{-1} + 2 \sum_{s=1}^{n} |m_s^*| \right) \tilde{y} \\ &= w_j - w^*/2 \geqslant w_j/2 > 0 \end{aligned} \tag{12}$$

for all $j \in N$. So $Y(m^*/m_i, i) = \tilde{y} > 0$ by Lemma 1 and $X_j(m^*/m_i, i) = w_j - q_j(m^*/m_i, i)\, Y(m^*/m_i, i) = w_j - q_j(m^*/m_i, i)\, \tilde{y} > 0$ for all $j \in N$. Thus we have $(X_i(m^*/m_i, i), Y(m^*/m_i, i))\ P_i\ (X_i(m^*), Y(m^*))$ by Assumption 4, which contradicts the hypothesis $(X(m^*), Y(m^*)) \in N_{M,h}(e)$. Q.E.D.

Lemma 3. *If $(X(m^*), Y(m^*)) \in N_{M,h}(e)$, then $Y(m^*)$ is an interior point of $[0, a(m)]$ and thus $Y(m^*) = \sum_{i=1}^{n} m_i^*$.*

Proof. By Lemma 2, $Y(m^*) > 0$. So we only need to show $Y(m^*) < a(m^*)$. Suppose, by way of contradiction, that $Y(m^*) = a(m^*)$. Then $X_j(m^*) = w_j - q_j(m^*)\, Y(m^*) = w_j - q_j(m^*)\, a(m^*) = w_j - w_j = 0$ for at least some $j \in N$. But $X(m^*) > 0$ by Lemma 2, a contradiction. Q.E.D.

We now prove the main results of this section in the following propositions.

Proposition 1. *Under Assumptions 1–4, if the mechanism has a Nash equilibrium m^*, then $(X(m^*), Y(m^*))$ is a Lindahl allocation with $(q_1(m^*), \ldots, q_n(m^*))$ as the Lindahl price vector; i.e., $N_{M,h}(e) \subseteq L(e)$.*

Proof. Let m^* be a Nash equilibrium. Now we prove that $(X(m^*), Y(m^*))$ is a Lindahl allocation with $(q_1(m^*), ..., q_n(m^*))$ as the Lindahl price vector. Since the mechanism is completely feasible and $\sum_{i=1}^n q_i(m^*)=1$ as well as $X_i(m^*)+q_i(m^*)\, Y(m^*)=w_i$ for all $i \in N$, we only need to show that each individual is maximizing his/her preference. Suppose, by way of contradiction, that there is some $(x_i, y) \in \mathbb{R}^2_+$ such that $(x_i, y)\, P_i\, (X_i(m^*), Y(m^*))$ and $x_i+q_i(m^*)\, y \leqslant w_i$. Because of monotonicity of preferences, it will be enough to confine ourselves to the case of $x_i+q_i(m^*)\, y=w_i$. Let $(x_{i\lambda}, y_\lambda)=(\lambda x_i+(1-\lambda)\ X_i(m^*), \lambda y+(1-\lambda)\ Y(m^*))$. Then by convexity of preferences we have $(x_{i\lambda}, y_\lambda)\, P_i\, (X_i(m^*), Y(m^*))$ for any $0<\lambda<1$. Also $(x_{i\lambda}, y_\lambda) \in \mathbb{R}^2_+$ and $x_{i\lambda}+q_i(m^*)\, y_\lambda=w_i$.

Suppose that player i chooses his/her message $m_i=y_\lambda-\sum_{j\neq i}^n m_j^*$. Since $Y(m^*)=\sum_{j=1}^n m_j^*$ by Lemma 3, $m_i=y_\lambda-Y(m^*)+m_i^*$. Thus as $\lambda \to 0$, $y_\lambda \to Y(m^*)$, and therefore $m_i \to m_i^*$. Since $X_j(m^*)=w_j-q_j(m^*)\, Y(m^*)>0$ for all $j \in N$ by Lemma 2, we have $w_j-q_j(m^*/m_i, i)\, y_\lambda>0$ for all $j \in N$ as λ is a sufficiently small positive number. Therefore, $Y(m^*/m_i, i)=y_\lambda$ by Lemma 1 and $X_i(m^*/m_i, i)=w_i-q_i(m^*)\, Y(m^*/m_i, i)=w_i-q_i(m^*)\, y_\lambda=x_{i\lambda}$. From $(x_{i\lambda}, y_\lambda)\, P_i\, (X_i(m^*), Y(m^*))$, we have $(X_i(m^*/m_i, i), Y(m^*/m_i, i))\, P_i\, (X_i(m^*), Y(m^*))$. This contradicts $(X(m^*), Y(m^*)) \in N_{M,h}(e)$. Q.E.D.

PROPOSITION 2. *Under Assumptions 1–3, if (x^*, y^*) is a Lindahl allocation with the Lindahl price vector $q^*=(q_1^*, ..., q_n^*)$, then there is a Nash equilibrium m^* of the mechanism such that $X_i(m^*)=x_i^*$, $q_i(m^*)=q_i^*$, for all $i \in N$, $Y(m^*)=y^*$; i.e., $L(e) \subseteq N_{M,h}(e)$.*

Proof. We need to show that there is a message m^* such that (x^*, y^*) is a Nash allocation. Let m^* be the solution of the following linear equations system:[9]

$$\sum_{i=1}^n m_i=y^*, \qquad Am=q^*-b. \tag{13}$$

Then $X_i(m^*)=x_i^*$, $Y(m^*)=y^*$, and $q_i(m^*)=q_i^*$ for all $i \in N$. Thus from $(X(m^*/m_i, i), Y(m^*/m_i, i)) \in \mathbb{R}^2_+$ and $X_i(m^*/m_i, i)+q_i(m^*)\, Y(m^*/m_i, i)=w_i$ for all $i \in N$ and $m_i \in M_i$, we have $(X_i(m^*), Y(m^*))\, R_i\, (X_i(m^*/m_i, i), Y(m^*/m_i, i))$. Q.E.D.

Remark 3. Since the minimal dimension required for decentralized realization of the Lindahl correspondence is $(L+K-1)n$, where L is the number of private goods and K is the number of public goods (see, e.g., [5]), then from Footnote 2 the mechanism presented in this note has a message space of minimal dimension and thus is informationally efficient.

[9] The solution exists by the assumptions on the matrix A.

Summarizing the above discussions, we have

THEOREM 1. *Let E be a class of economies with two goods (one private good and one public good) and a constant-returns technology whose elements e satisfy Assumptions 1–4. Then there exists a completely feasible and continuous mechanism with a message space of minimal dimension which fully Nash-implements the Lindahl correspondence.*

4. SOME REMARKS

In this section we will give some remarks on possible extensions. First, the mechanism presented in the above sections can be generalized to include economies with arbitrary numbers of private and public goods and to more general production relations. Tian [11] made a generalization which is similar to those proposed by Groves and Ledyard [1] or Walker [14]. In those types of mechanisms, private goods are allocated through competitive markets and public goods and taxing rules are determined by the mechanism (government) according to consumers' messages. For the case that private goods are allocated by a mechanism rather than through competitive markets, see Tian [13].

Second, Proposition 1 is based on Assumption 4. Tian [12] showed that the Lindahl correspondence violates Maskin's [8] monotonicity condition in the absence of Assumption 4 and thus cannot be Nash-implemented by a completely feasible mechanism. Nevertheless, we can still have some extended results.

COROLLARY 1. *Under Assumptions 1–3, (a) if the mechanism has a unique Nash equilibrium allocation for economy e, then $N_{M,h}(e)=L(e)$; (b) if $(X(m^*), Y(m^*)) \in \operatorname{Int} N_{M,h}(e)$, then $(X(m^*), Y(m^*)) \in L(e)$.*

The proof of (a) is trivial. The proof of (b) is almost the same as that of Proposition 1. We only need to note that if $(X(m^*), Y(m^*)) \in \operatorname{Int} N_{M,h}(e)$, then $Y(m^*)=\sum_{i=1}^n m_i^*$ and $(X(m^*/m_i, i), Y(m^*/m_i, i)) \in \mathbb{R}^2_{++}$ for all i when m_i is sufficiently close to m^*.

From Corollary 1 and Proposition 2, we have

COROLLARY 2. *Let $\tilde{E}$ be a class of economies with two goods (one private good and one public good) and a constant-returns technology whose elements e satisfy Assumptions 1–3 such that $N_{M,h}(e) \subseteq \mathbb{R}^{n+1}_{++}$ for all $e \in \tilde{E}$. Then the mechanism presented in the previous section is Nash-implementable on $\tilde{E}$.*

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Application for Membership. Applications should be addressed to:

Corresponding Secretary
Society for the Promotion of Economic Theory
Cornell University
402 Uris Hall
Ithaca NY 14853-7601 USA

The applicant should indicate his institutional affiliation and provide a *complete* address to be used for all Society communications. Applications must be accompanied by a check (U.S. dollars *drawn on a U.S. bank*) for 1990 dues.

Inquiries. Questions about the Society should be addressed to the Corresponding Secretary. The telephone number is (607) 255-4878. The Society receives electronic mail through BITNET number JET@CORNELLC. Our FAX number is (607) 255–8838. All communications should be clearly addressed to the attention of SPET.