Implementation in Production Economies with Increasing Returns*

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Abstract

In this paper we investigate incentive aspects of a general price-setting equilibrium principle in production economies with increasing returns or more general types of non-convexities. We do so by introducing the notion of generalized mechanism. We allow preferences and individual endowments to be unknown to the planner. We present a simple generalized mechanism whose social equilibrium allocations coincide with pricing equilibrium allocations. The pricing equilibrium solutions are very general and include marginal pricing equilibrium, loss-free pricing equilibrium, average pricing equilibrium, and voluntary trading equilibrium as special cases. When a pricing equilibrium principle yields Pareto efficient allocations, the mechanism doubly implements the pricing equilibrium correspondence in social and strong social equilibria. Furthermore, the mechanisms work not only for three or more agents, but also for two-agent economies.

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1 Introduction

This paper studies the problem of incentive mechanism design for a general price-setting equilibrium principle in economies with increasing returns to scale or more general types of non-

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convexities. For the general equilibrium approach to allocations of private goods, the most commonly used equilibrium notion is the Walrasian equilibrium principle which is a general equilibrium concept for private-ownership institutions. Since the Walrasian mechanism, in general, is not incentive-compatible even for classical economic environments when the number of agents is finite, many incentive mechanisms have been proposed to implement Walrasian allocations at Nash equilibrium and/or strong Nash equilibrium points such as those in Hurwicz (1979), Schmeidler (1980), Hurwicz, Maskin, and Postlewaite (1995), Postlewaite and Wettstein (1989), Tian (1989, 1992, 1996, 1999), Hong (1995), and Peleg (1996) among others.

For the general variable returns case, however, the Walrasian equilibrium principle may have a problem of failure. It is well known that firms' profit maximizing behavior is inconsistent with the existence of competitive equilibrium in an economy with increasing returns to scale or non-convex technologies. The Walrasian equilibrium principle assumes that production takes place at a price-taking, profit-maximizing point. This precludes the existence of an equilibrium if increasing returns to scale (IRS) are present. Thus, the standard behavioral assumption of profit maximization may be inapplicable in the presence of increasing returns to scale.

To overcome this defect of the Walrasian equilibrium principle, one has to consider alternative solution rules about firms' behavior that can incorporate increasing returns into the setting. One needs to seek alternative mechanisms in the framework that have the generality of the Walrasian model, but in which no convexity assumption is made regarding the individual or the aggregate production sets. Several such solution concepts for economies with increasing returns or more general types of non-convexities have been proposed in literature. In these solution principles, non-convex firms are assumed to follow a pricing rule, while the convex firms may or may not behave competitively. One does not need to distinguish in a priori the convex firms from the non-convex ones or the price-taking firms from the price-setting ones. The economic environment is one in which firms with non-convex production sets, perhaps as a result of regulation, follow pricing rules which do not necessarily guarantee maximum profits. The profit may be distributed among the consumers according to exogenously given rules, as in the Arrow-Debreu model. The pricing equilibrium notion is not radically different from the Walrasian equilibrium solution. In fact, it coincides with the Walrasian equilibrium notion in the case of convex economies under the marginal pricing rule. With the pricing equilibrium notion, all firms are assumed to follow a pre-specified pricing rule when the self-interested behavior of consumers are still described by utility maximization and the market clearing condition is satisfied. If firms adopt price setting behaviors, i.e., if they follow pricing rules which are mappings from the boundaries of their production sets to the price simplex, then under certain assumptions, there exists a price setting equilibrium even in an economy with non-convex technologies, in the sense that (i) consumers maximize their utilities subject to their budget constrains, (ii) all firms set the same prices, and (iii) all markets clear.

Each firm follows a general pricing rule which may be profit maximizing, marginal, or average cost pricing. The two rules of behavior that are considered most often are marginal and average cost pricing rules. The marginal cost pricing doctrine was elaborated on in the nineteen thirties and forties by Allais, Hotelling, Lange, Lerner, Pigou, etc. The most common justification of marginal cost pricing is that a suitable price system and distribution of income allow the decentralization of every Pareto optimal allocation when each producer is instructed to follow the marginal cost pricing rule. The existence of marginal cost pricing equilibria was first studied by Beato (1982) and Mantel (1979), then Beato and Mas-Colell (1985), Bonnisseau and Cornet (1990), Brown and Heal (1982), Brown et al. (1986), and Cornet (1989). The existence of pricing equilibria under other pricing rules, such as the average cost pricing rule has been studied by Brown and Heal (1983, 1985), Dierker et al. (1985), MacKinnon (1979) and Reichert (1986). The existence of a general pricing rule which includes the marginal pricing equilibrium, average rule, and loss-free pricing rule as special cases can be found in Beato (1982), Brown and Heal (1982), Cornet (1988, 1989), Bonnisseau (1988), Bonnisseau and Cornet (1988), Kamiya (1988), Vohra (1988), and Quinzii (1991, 1992).

However, all the studies mainly concerned with welfare properties and the existence and uniqueness of pricing equilibrium, but incentive aspects of the pricing equilibrium principle were neglected. In fact the incentive problem in the presence of non-convexities in production is more severe than one in conventional convex economies (because of the failure of markets). The non-convex firms can be thought of as privately owned public utilities, which are regulated (cf. Brown and Heal (1883, 1985)). Any pricing rule imposed by a regulator then should be incentive compatible. One needs to design mechanisms which implement pricing equilibrium allocations under solution concepts of self-interested behavior of individuals, such as Nash equilibrium strategy. Thus, an important question is whether or not one can design an incentive mechanism that implements a proposed pricing equilibrium rule for non-convex production technologies. This paper attempts to fill this gap.

This paper deals with the problem of the incentive mechanism design for production economies with non-convexities in production technologies. We do so by investigating the incentive aspect of a general price setting solution for general non-convex economies and by introducing the notion of a generalized mechanism in which one agent's feasible message domain depends on the other agents' choices. We allow preferences and individual endowments to be unknown to the planner. We propose a generalized mechanism which implements a general pricing equilibrium rule in social equilibrium. When a pricing rule results in Pareto efficient allocations, this mechanism in fact doubly implements the pricing equilibrium allocations in social and strong social equilibria. That is, in the mechanism not only social equilibrium allocations, but also strong social equilibrium allocations coincide with pricing equilibrium allocations. By double implementation, the solution can cover the situation where agents in coalitions may cooperate and those in other coalitions may not. Thus, the designer does not need to know which coalitions are permissible and, consequently, it allows the possibility for agents to manipulate coalition patterns. Notice that, since the profit maximizing rule is a special case of the general pricing equilibrium principle, this mechanism doubly implements Walrasian allocations in social and strong social equilibria for convex production economies.

The notion of a generalized mechanism is best used for considering implementation of a social choice rule. There are two technical difficulties in using the conventional mechanism approach to consider the implementation problem for non-convex production technologies when one wants to use a well-behaved mechanism that has a finite-dimensional message space. First, in order to have a feasible and continuous mechanism for the case of convex economies, the existing approach constructs a feasible and continuous allocation correspondence with nonempty, closed, and convex values. The allocation outcome at a message is then chosen from the resulting feasible set so that it is closest to the allocation proposed by individual agents. The outcome function is thus single-valued, continuous, and feasible. Such a projecting approach has been used by Postlewaite and Wettstein (1989), Tian (1989, 1992, 1996, 1999), Hong (1995), and Peleg (1996) among others to obtain the continuous and feasible mechanisms. When production sets are non-convex, however, the feasible correspondence is not convex-valued, and a single-valued and continuous mechanism cannot be obtained. Therefore, the approach may not, in general, be applied to the case of non-convex economies.

Secondly, even when one can have a feasible and continuous mechanism for convex economies by using the approach of constructing the feasible correspondence, the resulting mechanism generally is not forthright. Forthrightness requires that, in equilibrium, each agent receives what she has announced as her own consumption bundle, and produces what she has proposed. If a mechanism satisfies forthrightness, then it is easy to compute the equilibrium outcome. This is why the existing feasible and continuous mechanisms mentioned above do not satisfy the forthrightness property.

These two technical problems have led us to adopt the generalized mechanisms approach. By using a generalized mechanism, the designer constructs a feasible message set for each agent, which depends on the other agents' messages. The agents are then asked to report consumption bundles and production plans from the feasible sets of agents, but not from the whole message space. Thus, from the very beginning, one eliminates those unrealistic and/or unfeasible outcomes so that the mechanism becomes simpler, and more intuitive, and also the equilibrium becomes easier to compute. For instance, no projection mapping is needed since agents are required to report messages from feasible sets. Such a generalized mechanism can be particularly useful for non-convex production economies. Furthermore, it may have many desirable properties such as forthrightness, continuity, and feasibility.

The mechanism proposed in the paper has many desired properties. It is feasible with respect to the feasible choice subset of the message space, and forthright, and thus is a natural mechanism using the terminology of Saijo, Tatamitani, and Yamato (1996). Additionally, the mechanism is continuous, is a market type mechanism, uses a finite-dimensional message space, and works not only for three or more agents, but also for two-agent economies. This means it is a unified mechanism irrespective of the number of agents. The modified mechanism does not use the projection approach to determine consumption allocations, the resulting allocation is no longer a rationing scheme. A generalized mechanism, but not a conventional mechanism, should be used when one considers implementation issues by a well-behaved mechanism and/or for economies with non-convex production technologies. This is somewhat similar to the case of studying the existence of competitive equilibrium in the general equilibrium literature, in which the generalized game approach, but not the conventional game, approach has been used to prove the existence of Walrasian equilibrium.

The remainder of this paper is as follows. Section 2 sets up a general model and states the notion of pricing equilibrium, notation, and definitions used for mechanism design. Section 3 gives a mechanism which has the desirable properties mentioned above when preferences and individual endowments are unknown to the designer, but production possibility sets are known. We then prove that this mechanism implements pricing equilibrium allocations in social equilibrium. Concluding remarks are presented in Section 4.

2 The Model

2.1 Economic Environments

We consider production economies in which there are $n \geqq 2$ consumers, J firms, and L commodities.[1] Let $N = \{1, 2, \dots, n\}$ denote the set of consumers. Production technologies of firms are denoted by $\mathcal{Y}_1, \dots, \mathcal{Y}_j, \dots, \mathcal{Y}_J$. We assume that, for $j = 1, \dots, J$, $\mathcal{Y}_j$ is closed, contains 0 (possibility of inaction), and $\mathcal{Y}_j - \mathbb{R}^L_+ \subset \mathcal{Y}_j$ (free-disposal). Thus, $\partial\mathcal{Y}_j$, the boundary of the production set $\mathcal{Y}_j$, is exactly the set of (weakly) efficient production plans of the jth producer, that is,

$$\partial\mathcal{Y}_j = \{y_j \in \mathcal{Y}_j : \nexists y'_j \in \mathcal{Y}_j, y'_j > y_j\}.$$

Each agent's characteristic is denoted by $e_i = (C_i, w_i, R_i)$, where $C_i = \mathbb{R}^L_+$ is the consumption set, $w_i \in \mathbb{R}^L_{++}$ is the true initial endowment vector of commodities, and R_i is the preference ordering defined on $\mathbb{R}^L_+$. Let P_i denote the asymmetric part of R_i (i.e., $a\ P_i\ b$ if and only if $a\ R_i\ b$, but not $b\ R_i\ a$). We assume that R_i is continuous and convex on $\mathbb{R}^L_+$, and strictly monotonically increasing on $\mathbb{R}^L_{++}$.[2]

An economy is the full vector $e = (e_1, \dots, e_n, \mathcal{Y}_1, \dots, \mathcal{Y}_J)$ and the set of all such economies is denoted by E.

An allocation of the economy e is a vector $(x_1, \dots, x_n, y_1, \dots, y_J) \in \mathbb{R}^{L(n+J)}$ such that: (1) $x := (x_1, \dots, x_n) \in \mathbb{R}^{nL}_+$, and (2) $y_j \in \mathcal{Y}_j$ for $j = 1, J$. Denote $y = (y_1, \dots, y_J)$.

An allocation (x, y) is *feasible* if

$$\sum_{i=1}^{n} x_i \leqq \sum_{i=1}^{n} w_i + \sum_{j=1}^{J} y_j. \tag{1}$$

Denote the aggregate endowment, consumption and production by $\hat{w} = \sum_{i=1}^n w_i$, $\hat{x} = \sum_{i=1}^n x_i$, $\hat{y} = \sum_{j=1}^J y_j$, respectively.[3] Then the feasibility condition can be written as

$$\hat{x} \leqq \hat{w} + \hat{y}.$$

An allocation (x, y) is *Pareto-optimal* with respect to the preference profile $R = (R_1, \dots, R_n)$ if it is feasible and there is no other feasible allocation (x', y') such that $x'_i\ P_i x_i$ for all $i \in N$.

[1] As usual, vector inequalities are defined as follows: Let $a, b \in \mathbb{R}^m$. Then $a \geqq b$ means $a_s \geqq b_s$ for all $s = 1, \dots, m$; $a \geq b$ means $a \geqq b$ but $a \neq b$; $a > b$ means $a_s > b_s$ for all $s = 1, \dots, m$.

[2] R_i is convex if, for bundles a and b, $a\ P_i\ b$ implies $\lambda a + (1-\lambda)b\ P_i\ b$ for all $0 < \lambda \leqq 1$. Note that the term "convex" is defined as in Debreu (1959), not as in some recent textbooks.

[3] For notational convenience, "$\hat{a}$"will be used throughout the paper to denote the sum of vectors a_i, i.e., $\hat{a} := \sum a_l$.

2.2 Pricing Equilibrium Allocation

Let $\Delta_{+}^{L-1} = \{t \in \mathbb{R}_{+}^{n} : \sum_{i=1}^{L} t_i = 1\}$ be the $L-1$ dimensional unit simplex.

The behavior of the jth producer is described by its supply correspondence, η_j, which associates with each normalized price vector in $p \in \Delta_{+}^{L-1}$, a subset of $\eta_j(p)$. The production equilibrium condition of the jth producer at the pair $(y_j, p) \in \partial\mathcal{Y}_j \times \Delta_{+}^{L-1}$ is defined by $y \in \eta_j(p)$. Since $\eta_j(p)$ is in general not convex in the presence of increasing returns, there is an alterative approach to describe the behavior of production with the notion of a pricing rule which is the inverse correspondence of η_j. In the absence of convexity assumptions, a pricing rule correspondence may satisfy continuity and convex valuedness properties even when the supply correspondence does not. This is the main reason why most papers on the existence of equilibrium for economies with non-convex production technologies the notion of a pricing rule even though they are equivalent.

A pricing rule for firm j is a correspondence $\phi_j : \partial\mathcal{Y}_j \to\to \Delta_{+}^{L-1}$ defined by $\phi_j(y_j) = \{p \in \Delta_{+}^{L-1} : y_j \in \eta_j(p)\}$. We assume that ϕ_j has a closed graph and that, for all $y_j \in \partial\mathcal{Y}_j$, $\phi_j(y_j)$ is a closed convex cone of vertex 0. The jth firm is said to follow the pricing rule ϕ_j at the pair of a price vector $p \in \Delta_{+}^{L-1}$ and an efficient production plan $y_j \in \partial\mathcal{Y}_j$ if $p \in \phi_j(y_j)$. Such (p, y_j) is then called a production equilibrium. Denoted by $PE = \{(p, y) : \Delta_{+}^{L-1} \times \prod_{j=1}^{J} \partial\mathcal{Y}_j : p \in \cap_{j=1}^{J} \phi(y_j)\}$ the set of all such production equilibria of economy e.

It may be remarked that the above general notion of the supply correspondence or the pricing rule contains the following rules as special cases.

(1) The profit maximizing rule: $\eta_j(p) = \{y_j \in \mathcal{Y}_j : p \cdot y_j \geqq p \cdot y_j' \text{ for all } y_j' \in \mathcal{Y}_j\}$.

(2) The marginal pricing rule: $MC_j(y_j) = N_{Y_j}(y_j)$ where $N_{Y_j}(y_j)$ is Clarke's normal cone to $\mathcal{Y}_j$ which is a generalization of the notion of the marginal rate of transformation for the producer in the absence of smoothness and convexity assumptions (cf. Clark (1975)).

(3) The average pricing rule: $AC_j(y_j) = \{p \in \mathbb{R}_{+}^{L} : p \cdot y_j = 0\}$.

(4) The voluntary trading pricing rule: $VT_j(y_j) = \{p \in \mathbb{R}_{+}^{L} : p \cdot y_j \geqq p \cdot y_j'$ for all $y_j' \in \mathcal{Y}_j$ such that $y_j' \leqq y_{j+}\}$, where $y_{j+} = \max\{0, y_j\}$. Note that the voluntary trading rule implies, in particular, cost minimization.

(5) The loss-free pricing rule: $LF_j(y_j) = \{p \in \mathbb{R}_{+}^{L} : p \in \phi_j(y_j) \text{ imlies } p \cdot y_j \geqq 0\}$.

The i-th consumer's wealth function is denoted by $r_i : \mathbb{R}_{+}^{L} \times \mathbb{R}_{+}^{L} \times \prod_{j=1}^{J} \partial\mathcal{Y}_j \to \mathbb{R}_{++}$. This abstract wealth structure clearly encompasses the case of a private ownership economy for which $r_i(w_i, p, y_1, \ldots, y_J) = p \cdot w_i + \sum_{i=1}^{n} \theta_{ij} p \cdot y_j$, where $\theta_{ij} \in \mathbb{R}_{+}$ are the profit shares of private firms j, $j = 1, \ldots, J$, satisfying $\sum_{i=1}^{n} \theta_{ij} = 1$. It is assumed that $r_i : \mathbb{R}_{+}^{L} \times \mathbb{R}_{+}^{L} \times \prod_{j=1}^{J} \partial \mathcal{Y}_j \to \mathbb{R}_{++}$ is increasing in w_i, continuous, $\sum_{i=1}^{n} r_i(w_i, p, y) = \sum_{i=1}^{n} r_i(w_i, p, y_1, \ldots, y_J) = p \cdot (\sum_{j=1}^{f} y_j + \sum_{i=1}^{n} w_i)$, $r_i(w_i, tp, y_1, \ldots, y_J) = t r_i(w_i, p, y_1, \ldots, y_J)$ for all $(p, y) \in \mathbb{R}_{+}^{L} \times \prod_{j=1}^{J} \partial \mathcal{Y}_j$, $p \cdot (\sum_{j=1}^{f} y_j + \sum_{i=1}^{n} w_i) > 0$ for $(p, y) \in PE$, and $p \cdot (\sum_{j=1}^{f} y_j + \sum_{i=1}^{n} w_i) > 0$ implies that $r_i(w_i, p, y_1, \ldots, y_J) > 0$.

A pricing equilibrium of the whole economy is a list of consumption plans (x_i^*), a list of production plans (y_j^*), and a price vector p^* such that (a) every consumer maximizes his preferences subject to his budget constraint, (b) every firm follows his pricing rule, i.e., $p^* \in \phi_j(y_j^*)$ for all $j = 1, \ldots, J$, and (c) the excess demand over supply is zero. The nature of the equilibrium (b) is the main difference to the Walrasian model. Formally, we have the following definition.

An allocation $z^* = (x^*, y^*) = (x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_J^*) \in \mathbb{R}_{+}^{nL} \times \mathcal{Y}$ is a *pricing equilibrium allocation* for an economy e if it is feasible and there is a price vector $p^* \in \mathbb{R}_{+}^{L}$ such that

(1) $p^* \cdot x_i^* \leqq p^* \cdot r_i(w_i, p^*, y^*)$ for all $i \in N$;

(2) for all $i \in N$, $x_i \; P_i \; x_i^*$ implies $p^* \cdot x_i > r_i(w_i, p^*, y^*)$; and

(3) $y_j^* \in \partial \mathcal{Y}_j$ and $p^* \in \phi_j(y_j^*)$ for $j = 1, \ldots, J$.

Let $PR(e)$ denote the set of all such allocations.

The detailed discussions on the settings of the model in economies with increasing returns and the existence of pricing equilibrium for the general setting can be found in Beato (1982), Brown and Heal (1982), Cornet (1988, 1989), Bonnisseau (1988), Bonnisseau and Cornet (1988), Kamiya (1988), Vohra (1988), and Quinzii (1991, 1992).

An advantage of the notion of pricing equilibrium is that it is quite flexible in allowing prices or quantities to be exogenous variables or endogenous variables. It allows price-taking behavior, such as in the Walrasian model. It also allows price-setting behavior such as the models of Dierker, and Guesnerie and Neuefeind (1985).

In this paper, we consider implementation of pricing equilibrium allocations by designing a feasible and continuous mechanism which implements the above pricing equilibrium allocations. In doing so, we need make the following indispensable assumption.

Assumption 1 (Interiority of Preferences): For all $i \in N, x_i \; P_i \; x_i'$ for all $x_i \in \mathbb{R}_{++}^{L}$, and for all $x_i' \in \partial \mathbb{R}_{+}^{L}$, where $\partial \mathbb{R}_{+}^{L}$ is the boundary of $\mathbb{R}_{+}^{L}$.

Remark 1 Assumption 1 cannot be dispensed. Like the Walrasian correspondence, one can show that the pricing equilibrium correspondence violates Maskin's (1995) monotonicity condition on the boundary of the consumption space, and thus it cannot be Nash implemented by an individually feasible and balanced mechanism. However, by changing Condition 2 in the pricing equilibrium allocation to the following condition:

> Conditioin (2′): for all $i \in N$, $x_i \ P_i \ x_i^*$ implies either $p^* \cdot x_i > r_i(w_i, p^*, y^*)$ or $x_i > \sum_{i=1}^n w_i + \sum_{j=1}^n y_j^*$,

we can modify the pricing equilibrium principle to the constrained pricing equilibrium principle. Now the constrained pricing equilibrium correspondence satisfies Maskin's monotonicity condition, and in fact, it can be shown that the mechanism presented below implements constrained pricing allocations in social equilibrium without Assumption 1. It is clear that every pricing equilibrium allocation is a constrained pricing equilibrium allocation, and the reverse may not be true. However, it can be proven that the constrained pricing equilibrium correspondence coincides with the pricing equilibrium correspondence for interior allocations.

Remark 2 The family of Cobb-Douglas utility functions satisfies Assumption 1.

2.3 Mechanism

Let M_i denote the i-th agent's message domain. Its elements are written as m_i and are called messages. Let $M = \prod_{i=1}^n M_i$ denote the message space. Denoted by $h : M \to \mathbb{R}_+^{L(n+J)}$ the outcome function, or more explicitly, $h(m) = (X_1(m), \ldots, X_n(m), Y_1(m), \ldots, Y_J(m))$. Then the mechanism consists of $\langle M, h \rangle$ which is defined on E.

A message $m^* = (m_1^*, \ldots, m_n^*) \in M$ is said to be a *Nash equilibrium* of the mechanism $\langle M, h \rangle$ for an economy e if, for all $i \in N$ and $m_i \in M_i$,

$$X_i(m^*) \ R_i \ X_i(m_i, m_{-i}^*), \tag{2}$$

where $(m_i, m_{-i}^*) = (m_1^*, \ldots, m_{i-1}^*, m_i, m_{i+1}^*, \ldots, m_n^*)$. $h(m^*)$ is then called a *Nash (equilibrium) allocation* of the mechanism for the economy e. Denoted by $V_{M,h}(e)$ the set of all such Nash equilibria and by $N_{M,h}(e)$ the set of all such Nash equilibrium allocations.

A mechanism $\langle M, h \rangle$ is said to *Nash-implement* pricing equilibrium allocations on E, if, for all $e \in E$, $N_{M,h}(e) = PR(e)$.

A *coalition* C is a non-empty subset of N.

A message $m^* = (m_1^*, \ldots, m_n^*) \in M$ is said to be a *strong Nash equilibrium* of the mechanism $\langle M, h \rangle$ for an economy e if there does not exist any coalition C and $m_C \in \prod_{i \in C} M_i$ such that for all $i \in C$,

$$X_i(m_C, m_{-C}^*) \ P_i \ X_i(m^*). \tag{3}$$

$h(m^*)$ is then called a *strong Nash (equilibrium) allocation* of the mechanism for the economy e. Denoted by $SV_{M,h}(e)$ the set of all such strong Nash equilibria and by $SN_{M,h}(e)$ the set of all such strong Nash equilibrium allocations.

A mechanism $\langle M, h\rangle$ is said to doubly *implement* pricing equilibrium allocations on E, if, for all $e \in E$, $SN_{M,h}(e) = N_{M,h}(e) = PR(e)$.

A mechanism $\langle M, h\rangle$ is said to be *continuous*, if the outcome function h is continuous on M.

In some cases, an agent's strategy domain may depend on the other agents' choices. The resulting mechanism (game form) is thus similar to the so-called abstract economies or generalized games studied by Debreu (1952) and Shafer and Sonnenschein (1975). This kind of mechanism design problem is also similar to the implementation problem when partial verification of private information is possible, which was studied by Green and Laffont (1986). Partial verification occurs when the message space is a correspondence that varies with economic environments. We will call this type of mechanism a generalized mechanism, and like Debreu (1952) call resulting equilibrium a social equilibrium. In what follows, we will first give the general definition of generalized mechanism and related equilibrium concepts. We then present a specified generalized mechanism which doubly implements equilibrium allocations.

Given a message space M, the choice of messages of agent i is restricted to a subset $G_i(m) \subset M_i$, called the *feasible strategy set*; each agent i chooses $m_i \in G_i(m)$ so as to maximize preferences. Denoted $G : M \to 2^M$ by $G(m) = \prod_{i \in N} G_i(m)$. The *generalized mechanism* is defined by $\langle M, G, h\rangle$. Note that, if $G_i(m) = M_i$ for all $m_i \in M_i$ and $i \in N$, the generalized mechanism reduces to the conventional mechanism defined in Section 2.

A message $m^* = (m_1^*, \ldots, m_n^*) \in M$ is said to be a *social equilibrium* of the generalized mechanism $\langle M, G, h\rangle$ for an economy e if, (1) $m^* \in G(m^*)$ and (2) for all $i \in N$ and all $m_i \in G_i(m^*)$,

$$X_i(m^*) \; R_i \; X_i(m_i, m_{-i}^*). \tag{4}$$

Denoted by $SE_{M,h}(e)$ the set of all such social equilibria and by $SEA_{M,h}(e)$ the set of all such social equilibrium allocations.

A message $m^* = (m_1^*, \ldots, m_n^*) \in M$ is said to be a *strong social equilibrium* of the generalized mechanism $\langle M, G, h\rangle$ for an economy e if, (1) $m^* \in G(m^*)$ and (2) there does not exist any coalition C and $m_C \in \prod_{i \in C} G_i(m^*)$ such that for all $i \in C$,

$$X_i(m_C, m_{-C}^*) \; P_i \; X_i(m^*). \tag{5}$$

Denoted by $SSE_{M,h}(e)$ the set of all such strong social equilibria and by $SSEA_{M,h}(e)$ the set of all such strong social equilibrium allocations.

A generalized mechanism $\langle M, G, h\rangle$ is said to doubly *implement* pricing equilibrium allocations in social and strong social equilibria on E, if, for all $e \in E$, $SSEA_{M,h}(e) = SEA_{M,h}(e) = PR(e)$.

A mechanism $\langle M, h\rangle$ is said to be *feasible with respect to the feasible set* $G(M)$, if, for all $m \in G(M)$, (1) $X(m) \in \mathbb{R}^{nL}_{+}$, (2) $Y_j(m) \in \mathcal{Y}_j$ for $j = 1, \ldots, J$, and (3) $\sum_{i=1}^{n} X_i(m) \leqq \sum_{i=1}^{n} w_i + \sum_{j=1}^{J} Y_j(m)$, where, $G(M) = \prod_{i=1}^{n} G_i(M)$ with $G_i(M) = \{m_i \in M_i : \ \exists m_{-i} \in M_{-i}\ s.t.\ m_i \in G_i(m)\}$.

3 Implementation of Pricing Equilibrium Allocations

3.1 The Description of the Generalized Mechanism

In the following we will present a feasible and continuous mechanism which implements pricing equilibrium allocations in social equilibrium, and further doubly implements pricing equilibrium allocations in social and strong social equilibria when they are Pareto efficient. It is well known by now that, to have feasible implementation, the designer has to know some information about individual endowments. That is, we have to require that an agent cannot overstate his own endowment although he can understate his endowment. This requirement is necessary to guarantee the feasibility even at disequilibrium points. The intuition here is straightforward: if a mechanism allows agents to overstate their endowments, then it allows for infeasible outcomes–it will sometimes attempt to allocate more than possible, given the true aggregate endowment. Note that, when goods are physical goods, this requirement can be guaranteed by asking agents to *exhibit* their reported endowments to the designer.

The message space of the mechanism is defined as follows. For each $i \in N$, let the message domain of agent i be of the form

$$M_i = (0, w_i] \times PE \times \mathbb{R}^{L}_{+} \times \mathbb{R}_{++} \times (0, 1]. \tag{6}$$

A generic element of M_i is $m_i = (v_i, p_i, y_{i1}, ..., y_{iJ}, z_i, \gamma_i, \eta_i)$ whose components have the following interpretations. The component v_i denotes a profession of agent i's endowment, the inequality $0 < v_i \leqq w_i$ means that the agent cannot overstate his own endowment; on the other hand, the endowment can be understated, but the claimed endowment v_i must be positive. The component $(p_i, y_i) \in PE$ is the profile of agent i's proposed production equilibria, which is used to determine the wealth of agent $i-1$, where $i-1$ is read to be n when $i = 1$. The component z_i is a consumption proposed by agent i. The component γ_i is a shrinking index of agent i used to shrink the consumption of other agents. The component η_i is the penalty index of agent i for avoiding some undesirable social equilibria.

We point out that we need to impose some restrictions on choices of p_i, y_i, and z_i. We require that, for each i, $(p_i, y_i) \in PE(m)$ with y_i being a feasible production plan, and z_i is a feasible consumption bundle that satisfies the proposed budget constraint and the proposed total supply constraint. Thus, for each agent i, the feasible choice set $G_i(m)$ is defined by

$$G_i(m) = (0, w_i] \times PE(m) \times B_i(m) \times \mathbb{R}_{++} \times (0, 1], \tag{7}$$

where the feasible production equilibrium set $PE(m)$ is defined by

$$PE(m) = \{(p, y) \in PE : \sum_{i=1}^{n} v_i + y_1 > 0\} \tag{8}$$

and the feasible consumption set $B_i(m)$ is defined by

$$B_i(m) = \{x_i \in \mathbb{R}^L_+ : p_{i+1} \cdot x_i \leqq r_i(v_i, p_{i+1}, y_{i+1}) \; \& \; x_i \leqq \sum_{i=1}^{n} v_i + y_1 \; \} \tag{9}$$

which is clearly nonempty for all $m \in G(M)$.

Before we formally define the outcome function of the mechanism, we give a brief description and explain why the generalized mechanism works. For each announced message $m \in M$, the designer first determine the production outcome $Y(m)$ that is proposed by a designated consumer, say consumer 1. The production level used to determine the wealth of consumer i and the price vector facing each consumer i are determined by the production plan and the price level proposed by consumer $i+1$ so that each individual takes his wealth and prices as given and cannot change them by changing his own messages. A preliminary consumption outcome $x_i(m)$ is determined by the product of what he/she announces and a penalty multiplier that gives an incentive for all agents to announce the same production plan and price vector when $m \in G(M)$, and zero otherwise (see Eq. (13) below). To obtain the feasible consumption outcome $X(m)$, we need to shrink the preliminary outcome consumption $x_i(m)$ in some way that will be specified below. We will show that the generalized mechanism constructed in such a way will have the properties we desire. In addition, at equilibrium, all consumers unanimously agree on a production plan and maximize their preferences, all firms are at equilibrium, and all individuals take the prices of goods as given. The generalized mechanism will be shown to implement the pricing equilibrium allocations.

Now we formally present the outcome function of the mechanism. Define the feasible production outcome to be the production plan proposed by any agent, say agent1's proposed efficient production plan. That is, define the feasible production outcome function $Y : M \to \mathbb{R}^{LJ}_{++}$ by

$$Y(m) = y_1. \tag{10}$$

To determine each agent's wealth at m that is independent of his own proposed production profile, define agent i's production profile outcome function $Y_i : M \to \mathbb{R}^{JL}$ by

$$Y_i(m) = y_{i+1} \tag{11}$$

and define agent i's proposed price vector $p_i : G(M) \to \mathbb{R}^{JL}_{++}$ by

$$p_i(m) = p_{i+1}, \tag{12}$$

where $n+1$ is to be read as 1.

Agent i's wealth at m is given by $r_i(v_i, p_i(m), Y_i(m^*))$. Note that although $Y(\cdot)$, $Y_i(\cdot)$, and $p_i(\cdot)$ are functions of the proposed production plan and price vector announced by an agent only, for simplicity, we write $Y(\cdot)$, $Y_i(\cdot)$, and $p_i(\cdot)$ as functions of m without loss of generality. Also notice that every individual takes prices of goods as given.

Define agent i's preliminary consumption outcome function x_i: $M \to \mathbb{R}^L_+$ by

$$x_i(m) = \begin{cases} \frac{1}{1+\eta_i(\|y_i - y_{i+1}\| + \|p_i - p_{i+1}\|)} z_i & \text{if } m \in G(M) \\ 0 & \text{otherwise} \end{cases},$$

Define the γ-correspondence A: $M \to 2^{\mathbb{R}_+}$ by

$$A(m) = \{\gamma \in \mathbb{R}_+ : \gamma\gamma_i \leq 1 \ \forall i \in N \ \& \ \gamma \sum_{i=1}^{n} \gamma_i x_i(m) \leqq \hat{v} + \hat{Y}(m)\}. \tag{13}$$

Let $\bar{\gamma}(m)$ be the largest element of $A(m)$, i.e., $\bar{\gamma}(m) \in A(m)$ and $\bar{\gamma}(m) \geq \gamma$ for all $\gamma \in A(m)$.

Finally, define agent i's outcome function for consumption goods X_i: $M \to \mathbb{R}^L_+$ by

$$X_i(m) = \bar{\gamma}(m)\gamma_i x_i(m), \tag{14}$$

which is agent i's consumption resulting from the strategic configuration m.

Thus the outcome function $(X(m), Y(m))$ is continuous on m and feasible on $G(M)$.

Remark 3 The mechanism works not only for three or more agents, but also for a two-agent world.

3.2 Implementation Result

The remainder of this section is devoted to proving the following theorem.

Theorem 1 *For the class of production economic environments E, specified in Section 2, if the following assumptions are satisfied:*

(1) $w_i > 0$ *for all* $i \in N$*;*

(2) *For each* $i \in N$*, preference orderings,* R_i*, are continuous and convex on* $\mathbb{R}^L_+$*, strictly increasing on* $\mathbb{R}^L_{++}$*, and satisfy the Interiority Condition of Preferences;*

(3) *For each* $j = 1, \ldots, J$*, the production set* $\mathcal{Y}_j$ *is nonempty, closed, and* $0 \in \mathcal{Y}_j$*.*

(4) *The wealth function* $r_i : \mathbb{R}^L_+ \times \mathbb{R}^L_+ \times \prod_{j=1}^J \partial\mathcal{Y}_j \to \mathbb{R}_{++}$ *is increasing in* w_i*, continuous,* $\sum_{i=1}^n r_i(w_i, p, y) = p \cdot (\sum_{j=1}^f y_j + \sum_{i=1}^n w_i)$*,* $r_i(w_i, tp, y_1, \ldots, y_J) = tr_i(w_i, p, y_1, \ldots, y_J)$ *for all* $(p, y) \in \mathbb{R}^L_+ \times \prod_{j=1}^J \partial\mathcal{Y}_j$*, and* $r_i(w_i, p, y_1, \ldots, y_J) > 0$ *for all* $(p, y) \in PE$*,*

then the above defined mechanism, which is continuous, feasible, and forthright, and uses a finite-dimensional message space, implements pricing equilibrium allocations on E in social equilibrium. Furthermore, it doubly implements pricing equilibrium allocations in social and strong social equilibria on E when the pricing equilibrium allocations are also Pareto efficient.

Proof. The proof of Theorem 1 consists of the following three propositions which show the equivalence among social equilibrium allocations, strong social equilibrium allocations, and pricing equilibrium allocations. Proposition 1 below proves that every social equilibrium allocation is pricing equilibrium allocation. Proposition 2 below proves that every pricing equilibrium allocation is a social equilibrium allocation. Proposition 3 below proves that every social equilibrium allocation is a strong social equilibrium allocation when the pricing equilibrium allocations are also Pareto efficient.

To show these propositions, we first prove the following lemmas.

Lemma 1 *Suppose* $x_i(m)\ P_i\ x_i$*. Then agent* i *can choose a very large* γ_i *such that* $X_i(m)\ P_i\ x_i$*.*

Proof: If agent i declares a large enough γ_i, then $\bar{\gamma}(m)$ becomes very small (since $\bar{\gamma}(m)\gamma_i \leqq 1$) and thus almost nullifies the effect of other agents in $\bar{\gamma}(m)\sum_{i=1}^n \gamma_i x_i(m) \leqq \sum_{i=1}^n v_i + \sum_{j=1}^J Y_j(m)$. Thus, $X_i(m) = \bar{\gamma}(m)\gamma_i x_i(m)$ can arbitrarily approach to $x_i(m)$ as agent i wishes. From $x_i(m)\ P_i\ x_i$ and continuity of preferences, we have $X_i(m)\ P_i\ x_i$ if agent i chooses a very large γ_i. Q.E.D.

Lemma 2 *If $m^* \in SE_{M,h}(e)$, then $X(m^*) \in \mathbb{R}^{nL}_{++}$.*

Proof: We argue by contradiction. Suppose $X(m^*) \in \partial\mathbb{R}^{nL}_{+}$. Then there is some $i \in N$ such that $X_i(m^*) \in \partial\mathbb{R}^{L}_{+}$. Since $r_i(v_i^*, p_i(m^*), Y_i(m^*)) > 0$ by assumption and $\sum_{i=1}^{n} v_i^* + \sum_{j=1}^{J} Y_j(m^*) > 0$ by construction, then there is some $x_i \in \mathbb{R}^{L}_{++}$ such that $p_i(m^*) \cdot x_i \leqq r_i(v_i^*, p_i(m^*), Y_i(m^*))$, $x_i < \sum_{i=1}^{n} v_i^* + \sum_{j=1}^{J} Y_j(m^*)$, and $x_i \ P_i \ X_i(m^*)$ by interiority of preferences. Then $x_i \in B_i(m_i, m_{-i}^*)$. Now suppose that agent i chooses $z_i = x_i$, $\gamma_i > \gamma_i^*$, and keeps other components of the message unchanged. We have $m_i \in G_i(m_i, m_{-i}^*)$ and $x_i(m_i, m_{-i}^*) = \frac{1}{1+\eta_i^*(\|p_i - p_{i+1}^*\| + \|y_i - y_{i+1}^*\|)} x_i > 0$. Then, we have $x_i(m_i, m_{-i}^*) \ P_i \ X_i(m^*)$ by interiority of preferences. Therefore, by Lemma 1, $X_i(m_i, m_{-i}^*) \ P_i \ X_i(m^*)$ if agent i chooses a very large γ_i. This contradicts $m^* \in SV_{M,h}(e)$ and thus we must have $X_i(m^*) \in \mathbb{R}^{L}_{++}$ for all $i \in N$. Q.E.D.

Lemma 3 *If m^* is a social equilibrium, then $p_1^* = p_2^* = \ldots = p_n^*$, and $y_1^* = y_2^* = \ldots = y_n^*$. Consequently, $p_1(m^*) = p_2(m^*) = \ldots = p_n(m^*) := p(m^*)$, and $Y_1(m^*) = Y_2(m^*) = \ldots = Y_n(m^*) = Y(m^*)$.*

Proof: Suppose, by way of contradiction, that $y_i^* \neq y_{i+1}^*$ or $p_i^* \neq p_{i+1}^*$ for some $i \in N$. Then agent i can choose a smaller $\eta_i < \eta_i^*$ in $(0, 1]$ so that his consumption becomes larger and he would be better off by monotonicity of preferences. Hence, no choice of η_i could constitute part of a social equilibrium strategy when $y_i^* \neq y_{i+1}^*$ or $p_i^* \neq p_{i+1}^*$. Thus, we must have $p_1^* = p_2^* = \ldots = p_n^*$ and $y_1^* = y_2^* = y_n^*$ for all $i \in N$ at social equilibrium. Therefore, $p_1^* = p_2^* = \ldots = p_n^*$, and $y_1^* = y_2^* = \ldots = y_n^*$. Consequently, we have $p_1(m^*) = p_2(m^*) = \ldots = p_n(m^*) := p(m^*)$, and $Y_1(m^*) = Y_2(m^*) = \ldots = Y_n(m^*) := Y(m^*)$. Q.E.D.

Lemma 4 *Suppose $X(m^*) \in \mathbb{R}^{nL}_{++}$ for some $m^* \in G(m^*)$ and there is $x_i \in \mathbb{R}^{L}_{++}$ for some $i \in N$ such that $p(m^*) \cdot x_i \leqq r_i(v_i^*, p(m^*), Y(m^*))$ and $x_i \ P_i \ X_i(m^*)$. Then there is some $m_i \in G_i(m_i, m_{-i}^*)$ such that $X_i(m_i, m_{-i}^*) \ P_i \ X_i(m^*)$.*

Proof: Since $X(m^*) > 0$, $X_i(m^*) < \sum_{j \in N} v_j^* + \sum_{j=1}^{J} Y_j(m^*)$. Let $x_{\lambda i} = \lambda x_i + (1-\lambda) X_i(m^*)$. Then, by convexity of preferences, we have $x_{\lambda i} \ P_i \ X_i(m^*)$ for any $0 < \lambda < 1$. Also $x_{\lambda i} \in \mathbb{R}^{L}_{++}$, $p(m^*) \cdot x_{\lambda i} \leqq r_i(v_i^*, p(m^*), Y(m^*))$, and $x_{\lambda i} < \sum_{k=1}^{n} v_k^* + \sum_{j=1}^{J} Y_j(m^*)$ when λ is sufficiently close to 0. Then $x_{\lambda i} \in B_i(m_i, m_{-i}^*)$. Now suppose agent i chooses $z_i = x_{\lambda i}$, $\gamma_i > \gamma_i^*$, and keeps other components of the message unchanged, then $m_i \in G_i(m_i, m_{-i}^*)$ and $x_i(m_i, m_{-i}^*) = z_i = x_{\lambda i}$. Thus, we have $x_i(m_i, m_{-i}^*) \ P_i \ X_i(m^*)$. Therefore, by Lemma 1, agent i can choose a very large γ_i such that $X_i(m_i, m_{-i}^*) \ P_i \ X_i(m^*)$. Q.E.D.

Lemma 5 *If m^* is a social equilibrium, then $v_i^* = w_i$ for all $i \in N$.*

Proof: Suppose, by way of contradiction, that $v_i^* \neq w_i$ for some $i \in N$. Then $p(m^*) \cdot X_i(m^*) \leqq r_i(v_i, p(m^*), Y(m^*)) < r_i(w_i, p(m^*), Y(m^*))$, and thus there is some $x_i > X_i(m^*)$ such that $p(m^*) \cdot x_i \leqq r_i(w_i, p(m^*), Y(m^*))$ and $x_i \; P_i \; X_i(m^*)$. Since $X(m^*) \in \mathbb{R}^{nL}_{++}$ by Lemma 2, there is some $m_i \in G_i(m_i, m^*_{-i})$ such that $X_i(m_i, m^*_{-i}) \; P_i \; X_i(m^*)$ by Lemma 4. This contradicts $(X(m^*), Y(m^*)) \in SEA_{M,h}(e)$. Q.E.D.

Lemma 6 *If $(X(m^*), Y(m^*)) \in SEA_{M,h}(e)$, then $p(m^*) \cdot X_i(m^*) = r_i(w_i, p(m^*), Y(m^*))$. Consequently, the feasibility condition must hold with equality, i.e., $\hat{X}(m^*) = \hat{w} + \hat{Y}(m^*)$.*

Proof: Suppose, by way of contradiction, that $p(m^*) \cdot X_i(m^*) < r_i(w_i, p(m^*), Y(m^*))$. Then, there is $x_i > X_i(m^*)$ such that $p(m^*) \cdot x_i \leqq r_i(w_i, p(m^*), Y(m^*))$ and $x_i \; P_i \; X_i(m^*)$ by monotonicity of preferences. Since $X(m^*) \in \mathbb{R}^{nL}_{++}$ by Lemma 2, there is some $m_i \in G_i(m_i, m^*_{-i})$ such that $X_i(m_i, m^*_{-i}) \; P_i \; X_i(m^*)$ by Lemma 4. This contradicts $(X(m^*), Y(m^*)) \in N_{M,h}(e)$. Consequently, the feasibility condition must hold with equality. Otherwise, there exists $i \in N$ such that $p(m^*) \cdot X_i(m^*) < r_i(w_i, p(m^*), Y(m^*))$. Q.E.D.

Lemma 7 *If $(X(m^*), Y(m^*)) \in SEA_{M,h}(e)$, then $\bar{\gamma}(m^*)\gamma_i^* = 1$ for all $i \in N$ and thus $X(m^*) = x(m^*)$.*

Proof: This is a direct corollary of Lemma 6. Suppose $\bar{\gamma}(m^*)\gamma_i^* < 1$ for some $i \in N$. Then $X_i(m^*) = \bar{\gamma}(m^*)\gamma_i^* x_i(m^*) < x_i(m^*)$, and therefore $p(m^*) \cdot X_i(m^*) < r_i(w_i, p(m^*), Y(m^*))$. But this is impossible by Lemma 6. Q.E.D.

Lemma 8 *The generalized mechanism satisfies the forthrightness property, i.e., for every social equilibrium m^*, $Y(m^*) = y_i^*$, $p(m^*) = p_i^*$, $v^* = w_i$, and $X_i(m^*) = z_i^*$ for all $i \in N$.*

Proof: This is a consequence of Lemmas 3, 5, and 7. By Lemma 3, $Y(m^*) = y_i^*$, $p(m^*) = p_i^*$, and thus $x_i(m^*) = z_i^*$ for all $i \in N$. By Lemmas 5, $v_i^* = w_i$. By Lemma 7, $X(m^*) = x(m^*)$. Thus, we have $X_i(m^*) = z_i^*$ for all $i \in N$. Q.E.D.

Proposition 1 *If the mechanism defined above has a social equilibrium m^*, then the social equilibrium allocation $(X(m^*), Y(m^*))$ is a pricing allocation with price vector $p(m^*)$, i.e., $SEA_{M,h}(e) \subseteq PR(e)$.*

Proof: Let m^* be a social equilibrium. We need to prove that $(X(m^*), Y(m^*))$ is a pricing equilibrium allocation with $p(m^*)$ as a price vector. Note that, by construction, the mechanism is feasible, and the budget constraint holds with equality by Lemma 6. So we only need to show that each individual is maximizing his/her preferences subject to his/her budget constraint.

Suppose, by way of contradiction, that there is some $x_i \in \mathbb{R}^L_+$ such that $x_i \; P_i \; X_i(m^*)$ and $p(m^*) \cdot x_i \leqq r_i(w_i, p(m^*), Y(m^*))$. Since $X(m^*) \in \mathbb{R}^{nL}_{++}$ by Lemma 2, there is some $m_i \in G_i(m_i, m^*_{-i})$ such that $X_i(m_i, m^*_{-i}) \; P_i \; X_i(m^*)$ by Lemma 4. This contradicts $(X(m^*), Y(m^*)) \in SEA_{M,h}(e)$. Thus, $(X(m^*), Y(m^*))$ is a pricing equilibrium allocation.

Proposition 2 *If (x^*, y^*) is a pricing allocation with $p^* \in \Delta^{L-1}_+$ as the price vector, then there is a social equilibrium m^* such that $Y(m^*) = y^*$, $p(m^*) = p^*$, and $X_i(m^*) = x^*_i$ for $i \in N$, i.e., $PR(e) \subseteq SEA_{M,h}(e)$.*

Proof: We first note that $x^* \in \mathbb{R}^L_{++}$ by interiority of preferences. Also, by the strict monotonicity of preference orderings, the normalized price vector p^* must be in Δ^{L-1}_{++}. We need to show that there is a message m^* such that (x^*, y^*) is a social equilibrium allocation. For each $i \in N$, define $m^*_i = (v^*, p^*_i, y^*_{i1}, \ldots, y^*_{iJ}, z^*_i, \gamma^*_{i1}, \eta^*_i)$ by $v^*_i = w_i$, $p^*_i = p^*$, $(y^*_{i1}, \ldots, y^*_{iJ}) = (y^*_1, y^*_2, \ldots, y^*_j)$, $z^*_i = x^*_i$, $\gamma^*_i = 1$, and $\eta^*_i = 1$. Then, it can be easily verified that $Y(m^*) = y^*$, $p_i(m^*) = p^*$, and $X_i(m^*) = x^*_i$ for all $i \in N$. Since $p(m_i, m^*_{-i}) = p_i(m^*)$ and $Y_i(m_i, m^*_{-i}) = Y_i(m^*) = Y(m^*)$ for all $m_i \in G_i(m_i, m^*_{-i})$, then we have

$$p(m^*) \cdot X_i(m_i, m^*_{-i}) \leqq r_i(w_i, p(m_i, m^*_{-i}), Y_i(m_i, m^*_{-i})) = r_i(w_i, p(m^*), Y(m^*)) \qquad (15)$$

for all $m_i \in G_i(m_i, m^*_{-i})$. Thus, $X_i(m_i, m^*_{-i})$ satisfies the budget constraint for all $m_i \in G_i(m_i, m^*_{-i})$. Thus, we must have $X_i(m^*) \; R_i \; X_i(m_i, m^*_{-i})$, or it contradicts the fact that $(X(m^*), Y(m^*))$ is a pricing allocation. So $(X(m^*), Y(m^*))$ must be a social equilibrium allocation. Q.E.D.

Proposition 3 *Suppose the set of pricing equilibrium allocation is a subset of Pareto efficient allocations. Then, every social equilibrium m^* of the mechanism defined above is a strong social equilibrium, that is, $SEA_{M,h}(e) \subseteq SSEA_{M,h}(e)$.*

Proof: Let m^* be a social equilibrium. By Proposition 1, we know that $(X(m^*), Y(m^*))$ is a pricing allocation with $p(m^*)$ as the price vector. Then $(X(m^*), Y(m^*))$ is Pareto optimal and thus the coalition N cannot be improved upon by any $m \in M$. Now for any coalition C with $\emptyset \neq S \neq N$, choose $i \in C$ such that $i + 1 \notin C$. Then no strategy played by C can change $p(m)$ and $Y(m)$ since they are determined by m_{i+1}. Furthermore, because $(X(m^*), Y(m^*)) \in P(e)$ and

$$p(m^*) \cdot X_i(m_C, m^*_{-C}) \leqq r_i(w_i, p(m^*), Y(m_C, m^*_{-C})) = r_i(w_i, p(m^*), Y(m^*)), \qquad (16)$$

$X_i(m^*)$ is the maximal consumption in the budget set of i, and thus S cannot improve upon $(X(m^*), Y(m^*))$. Q.E.D.

By Proposition 1 and Proposition 2, we know that $SEA_{M,h}(e) = PR(e)$, which means the mechanism implements the pricing equilibrium allocations in social equilibrium. Furthermore, since every strong social equilibrium is clearly a social equilibrium, by combining Propositions 1-3, we know that $SSEA_{M,h}(e) = SEA_{M,h}(e) = PR(e)$ for all $e \in E$ and thus the proof of Theorem 1 is completed. Q.E.D.

4 Concluding Remarks

In this paper we have considered the incentive aspect of the pricing equilibrium solution for general production economies which allow for non-convexity of production sets. We presented a simple generalized mechanism which implements pricing equilibrium allocations when preferences and endowments are unknown to the designer. When a pricing rule results in Pareto efficient allocations, this generalized mechanism, in fact, doubly implements the pricing equilibrium allocations in social and strong social equilibria. The double implementation covers the case where agents in some coalitions may cooperate and in other coalitions may not, when such information is unknown to the designer. This combining solution concept, which characterizes agents' strategic behavior, may bring about a state which takes advantage of both the social equilibrium and the strong social equilibrium, so that it may be easy to reach and hard to leave. The mechanism constructed in the paper is well-behaved in the sense that it is feasible, continuous, and forthright. In addition, unlike most mechanisms proposed in literature, it gives a unified mechanism which is irrespective of the number of agents.

We end the paper by mentioning a limitation of the paper. We only consider the incentive mechanism design by assuming that production technologies are known to the designer so that we are able to construct the mechanism with a finite-dimensional message space. How to design an incentive compatible mechanism that implements a pricing rule when production sets are non-convex and unknown to the designer is a hard and unsolved problem.

References

1. Berge, C., 1963. Topological Spaces, Translated by E.M. Patterson, New York: Macmillan.

2. Beato, P., 1982. The Existence of marginal cost pricing equilibrium with increasing returns. The Quarterly Journal of Economics 389, 669-688.

3. Beato, P., Mas-Colell, A., 1985. On marginal cost pricing with given tax subsidy rule. Journal of Economic Theory 37, 356-365.

4. Bonnisseau, J. M., 1988. On two existence results in economies with increasing returns. Journal of Mathematical Economics 17, 193-207.

5. Bonnisseau, J. M. B. Cornet (1988). "Existence of Equilibria when Firms Follow Bounded Losses Pricing Rules," *Journal of Mathematical Economics* 17, 119-147.

6. Bonnisseau, J. M., Cornet, B., 1990. Existence of marginal cost pricing equilibrium in an economy with serval noncnvex firms. Econometrica 58, 661-682.

7. Brown, D. J., Heal, G. M., 1982. Existence, local uniqueness and optimality of a marginal cost pricing with increasing returns. Social Science Working Papers 415. California Institute of Technology, CA.

8. Brown D. J., Heal, G. M., 1983. Marginal vs. average cost pricing in the presence of a public monopoly. American Economic Review 73, 189-193.

9. Brown D. J., Heal, G. M., 1985. The optimality of regulated pricing: A general equilibrium analysis. Lecture Notes in Economics and Mathematical Systems, Vol. 244, Advances in Equilibrium Theory. Springer-Verlag, June 1985.

10. Brown D. J., Heal, G. M., Khan, M. A., Vohra R., 1986. On a general existence theorem for marginal cost pricing equilibria. Journal of Economic Theory 38, 371-379.

11. Clark, F., 1975. Generalized gradients and applications. Transactions of the American Mathematical Society 205, 230-234.

12. Cornet, B., 1988. General equilibrium theory and increasing returns. Journal of Mathematical Economics 17, 103-118.

13. Cornet, B., 1989. Existence of equilibria in economies with increasing returns. In: B. Cornet and H. Tulkens, (Eds.), Contributions to Operations Research: The Twentieth Anniversary of CORE. The MIT Press.

14. Debreu, G., 1952. A social equilibrium existence theorem. Proceedings of the National Academy of Sciences of the U. S. A. 38, 386 393.

15. Dicker, E., Guesnerie, R., Neuefeind W., 1985. General equilibrium where some firms follow special pricing rules. Econometrica 53, 1369-1393.

16. Green, J., Laffont, J-J., 1986. Partially verifiable information and mechanism deign. Review of Economic Studies 53, 447-456.

17. Hong, L., 1995. Nash Implementation in production economy. Economic Theory 5, 401-417.

18. Hurwicz, L., 1979. Outcome function yielding Walrasian and Lindahl allocations at Nash equilibrium point. Review of Economic Studies 46, 217-225.

19. Hurwicz L, Maskin E, Postlewaite A., 1995. Feasible Nash implementation of social choice rules when the designer does not know endowments or production sets. In: J. O. Ledyard (Eds.), The Economics of Informational Decentralization: Complexity, Efficiency, and Stability. Essays in Honor of Stanley Reiter Kluwer Academic Publishers.

20. Kamiya, K., 1988. Existence and uniqueness of equilibria with increasing returns. Journal of Mathematical Economics 17, 149-178.

21. MacKinnon, J., 1979. Computing equilibria with increasing returns. European Economic Review 12, 1-16.

22. Mantel, R., 1979. Equilibrio con rendimiento crecientes a escala. Anales de la Asociation Argentina de Economia Politica 1, 271-283.

23. Mas-Colell, A., 1985. Theory of General Economic Equilibrium - A Differentiable approach. Cambridge University Press, Cambridge.

24. Maskin, M., 1999. Nash equilibrium and welfare optimality. Review of Economic Studies 66, 23-38.

25. Peleg, B., 1996. A continuous double implementation of the constrained Walrasian equilibrium. Economic Design 2, 89-97.

26. Postlewaite, A., Wettstein, D., 1989. Continuous and feasible implementation. Review of Economic Studies 56, 603-611.

27. Quinzii, M., 1991. Efficiency of marginal cost pricing equilibria. In: M. Majundar (ed),Equilibrium and Dynamics: Essays in Honor of David Gale. St. Martin's Press, New York, 260-286.

28. Quinzii, M., 1992. Increasing Returns and Efficiency. Oxford University Press, New York.

29. Reichert, J., 1986. Strategic market behavior. Ph.D. Dissertation. Yale University, New Haven, CT.

30. Schmeidler, D., 1980. Walrasian analysis via strategic outcome functions. Econometrica 48, 1585-1593.

31. Shafer, W., Sonnenschein H., 1975. Equilibrium in abstract economies without ordered preferences. Journal of Mathematical Economics 2, 345 348.

32. Saijo, T., Tatamitani, Y., Yamato T., 1996. Toward natural implementation. International Economic Review 37, 949-980.

33. Tian, G., 1989. Implementation of the Lindahl correspondence by a single-valued, feasible, and continuous mechanism. Review of Economic Studies 56, 613-621.

34. Tian, G., 1992. Implementation of the Walrasian correspondence without continuous, convex, and ordered preferences. Social Choice and Welfare 9, 117-130.

35. Tian, G., 1996. Continuous and feasible implementation of rational expectation Lindahl allocations. Games and Economic Behavior 16, 135-151.

36. Tian, G., 1999. Double implementation in economies with production technologies unknown to the designer. Economic Theory 13, 689-707.

37. Vohra, R. 1988. On the existence of equilibria in economies with increasing returns. Journal of Mathematical Economics 17, 179-192.

38. Yoshihara, N., 1999. Natural and double implementation of public ownership solutions in differentiable production economies. Review of Economic Design, 4, 127-151.