Implementation of Walrasian Allocations in Economies with Infinite Dimensional Commodity Spaces

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Abstract

This paper considers the problem of implementing constrained Walrasian allocations for exchange economies with infinitely many commodities and finitely many agents. The mechanism we provide is a feasible and continuous mechanism whose Nash allocations and strong Nash allocations coincide with constrained Walrasian allocations. This mechanism allows not only preferences and initial endowments but also coalition patterns to be privately observed, and it works not only for three or more agents, but also for two-agent economies, and thus it is a unified mechanism which is irrespective of the number of agents.

1 Introduction

This paper considers the problem of double implementation of constrained Walrasian allocations in Nash and strong Nash equilibria using a feasible and continuous mechanism for pure exchange economies with infinitely many commodities and finitely many agents.

*Financial support from the Texas Advanced Research Program as well as from the Private Enterprise Research Center and the Lewis Faculty Fellowship at Texas A&M University is gratefully acknowledged.

It is by now widely acknowledged that the finite-dimensional setting is too restrictively modeled in many economic situations such as intertemporal decisions, uncertainty, and differentiated commodities. Infinite dimensional models have become prominent in economics and finance because they capture nature aspects of the world that cannot be examined in finite dimensional models. It has become clear that economic models capable of addressing real policy questions must be both stochastic and dynamic. A dynamic model requires infinite dimensional spaces. If time is modeled as continuous, then time series of economic data reside in infinite dimensional functional spaces. Even if time is modeled as being discrete, we are forced to use infinite dimensional models when we are to make realistic models of money or growth. Other features of the world that arguably requires infinite dimensional modeling are uncertainty and commodity differentiation.

Incentive mechanism theory in general and implementation theory in particular, however, have ignored the economic environment with infinite-dimensional settings. There are numerous papers on implementation of (constrained) Walrasian allocations in various solution concepts in implementation literature, including Hong (1995), Hurwicz (1972, 1979), Hurwicz, Maskin, and Postlewaite (1995), Nakamura (1990), Peleg (1996), Postlewaite and Wettstein (1989), Schmeidler (1980), Suh (1994), and Tian (1990, 1999, 2000). All the mechanisms mentioned above only work with the finite-dimensional economic environments, and there are no mechanisms given in the literature which implements Pareto and individually rational allocations for economies with infinite-dimensional commodity spaces.

This paper gives a mechanism that Nash implements constrained Walrasian allocations for exchange economies with an infinite-dimensional commodity space. The commodity space we adopt are all the sequence spaces $l_p(1 \leqq p \leqq \infty)$ and the Lebesgue spaces $L_p(1 \leqq p \leqq \infty)$. Thus, we allow for commodity spaces which are general enough to include most of the spaces used economic analysis. It will be noted that our implementation result holds on a very large domain of economic environments. Only the strict monotonicity condition is assumed. No continuity and convexity assumptions on preferences are needed, and further, preferences may be nontotal or nontransitive. The mechanism is simple and natural. It is a type of \market game" and thus it is similar to the Walras rule: the strategies of the mechanism are \prices" and \quantities", and agents' consumptions are chosen from their budget sets. The \natural" mechanism design provides at least a partial response to a common concern about much of the implementation literature, namely that the implementing mechanisms are highly unrealistic and impossible for a real player to use. In addition, the mechanism works not only for three or more agents, but also for a two-agent world.

We should emphasize that the continuous and feasible mechanism design in the in¯nite-dimensional setting di®ers in important ways from the continuous and feasible mechanism design in the ¯nite-dimensional setting. There are mainly four di±culties that arise in in¯nite-dimensional settings, but do not arise in ¯nite-dimensional setting: (1) the feasible sets may not be compact; (2) there may not exist a compact and convex subset of the positive cone of the dual space; and (3) the wealth map may not be jointly continuous as a function of quantities and prices so that the feasible correspondence may not continuous, and (4) the projection of a point to a convex compact set may not be unique so that the feasible outcome function may not be single-valued. The ¯rst three di±culties are needed to be solved in order to show the feasible correspondence is continuous so that the outcome function obtained from the projection mapping to the feasible set is continuous. The fourth di±culty should be solved for the outcome function to be a single-valued function.

To understand why these di±culties arise in the in¯nite-dimensional setting, we discuss brie°y below these issues which distinguish a feasible and continuous mechanism design in the in¯nite-dimensional setting from the feasible and continuous mechanism design in the ¯nite-dimensional setting, and the ways in which we deal with them.

The ¯rst of these di±culties is that the feasible set may not generally be compact in a given topology of the commodity space, in many cases, the best we can hope for is that they will be compact in some weaker topology. In order to be sure that optimization solutions exist, we will need to assume that such a weak topology does indeed exist and the feasible correspondence are suitably continuous in this weak topology. These assumptions will be satisfied for the class of infinite-dimensional spaces we consider.

The second difficulty concerns the existence of a compact and convex subset of the positive cone of the dual space. In the finite-dimensional setting, the unit simplex is convex and compact. In the infinite-dimensional setting, however, such a compact and convex set may not exist. To be sure the existence of such a set so that we can prove that the feasible correspondence is continuous, we adopt an assumption introduced by Mas-Colell (1986) that, together with other assumptions we make in the papers, can guarantee the existence of a convex and compact subset of a positive cone.

The third difficulty is that the wealth map $p \cdot x$ may not be jointly continuous. We need this joint continuity of the wealth map to prove that the feasible correspondence is continuous. In the finite-dimensional setting, this map is jointly continuous. In the infinite-dimensional setting, in order that the feasible correspondence is compact-valued and there exist a convex and compact subset of positive cone L^*_+, we are led to consider a weak topology on the commodity space. Unfortunately, as shown by Mas-Colell and Zame (1992), such a pair of choices may lead to failure of joint continuity. Thus, we need to make sure that the wealth map is jointly continuous for the commodity space under consideration.[1]

The fourth of these difficulties is about the single-valuedness of the feasible outcome function. In the finite-dimensional setting, the single-valued and feasible outcome function can be obtained by a projection mapping from each proposed allocation that may not be feasible to the feasible correspondence. In other words, the outcome determined by the mechanism is the point in the feasible set that is closest to the proposed allocation, i.e., which minimizes the distance between the proposed outcome and any point in the feasible set for a given message. It is known that such a projection in the finite-dimensional setting is unique if and only if the

[1] The second and third difficulties do not arise, if the feasible sets have nonempty interior. In such a case, one can easily prove the feasible correspondence is continuous.

feasible set is closed and convex. Furthermore, the projection is continuous (cf Mas-Colell (1985)). In the infinite-dimensional setting, however, the projection is generally not unique. In order to be sure that single-valued projection exists, we will need to assume the functional spaces are either the sequence spaces or Lebesgue spaces. Fortunately, we can prove the projection is unique for either the sequence spaces or Lebesgue spaces.[2] In fact, as we will show, these four difficulties can be solved with suitable choices of a compatible topologies for these two classes of infinite-dimensional spaces.

The remainder of the paper is organized as follows. Section 2 presents notions, definitions, and solution concepts which will be used in the paper. Section 3 presents a specific mechanism that is feasible and continuous. Section 4 proves the equivalence among Nash allocations, strong Nash allocations, and constrained Walrasian allocations and the mechanism doubly implements the constrained Walrasian correspondence. Finally, some concluding remarks are given in Section 5.

2 Notation and Definitions

2.1 Economic Environments

To make this paper relatively self-contained, we begin with a brief review of the basic properties of ordered normed spaces.

A normed space is a real vector space L equipped with a norm, i.e., a function $\|\cdot\| : L \to [0, \infty)$ such that $\|x\| = 0$ if and only if $x = 0$; $\|\lambda x\| = |\lambda| \|x\|$ for all $x \in L$ and $\lambda \in \Re$; $\|x + y\| \leqq \|x\| + \|y\|$ for all $x, y \in L$:

By the dual space of L we mean the space L^* of continuous linear functionals $p : L \to \Re$. As usual, if $p \in L^*$, then value $\langle x, p \rangle$ will also be denoted by $p \cdot x$, i.e., $\langle x, p \rangle = p \cdot x$. The dual space of L is also a normed space, when equipped with the norm $\|p\| = \sup\{\|p \cdot x\| : x \in L, \|x\| \leqq 1\}$.

In addition to the normed topologies, we shall be interested in several other topologies on normed spaces. The weak topology on L, denoted by $\sigma(L, L^*)$, is the

[2] It seems this has only been proved for Hilbert space (cf. Gariepy and Ziemer (1995)).

weakest topology with respect to which all the elements of L^* are continuous. The weak-star topology on L^*, denoted by $\sigma(L^*; L)$, is the weakest topology on L^* with respect to which all the elements of L are continuous.

A normed lattice (or called normed Riesz space) is a normed space L, together with a partial order $\leq$ on L (i.e., a reflexive, antisymmetric, transitive relation on L) that satisfies: $x \leq y$ implies $x + z \leq y + z$ for all $x; y; z \in L$; $x \leq y$ implies $\lambda x \leq \lambda y$ for all $x; y \in L$ and $\lambda \in \Re_{++}$; every pair of elements, $x; y \in L$ has a supremum (least upper bound) $x \vee y$ and an infimum (greatest lower bound) $x \wedge y$; $|x| \leqq |y|$ implies $\|x\| \leqq \|y\|$, where $|x|$ is the absolute value of x that is defined by $|x| = x_+ + x_-$ with $x_+ = x \vee 0$ (positive part of x) and $x_- = x \wedge 0$ (negative part of x). The notation $x \geq y$ is, of course, equivalent to $y \leq x$. Also, $x > y$ means $x \geq y$ and $x \neq y$. x is positive if $x \geq 0$ and $x \neq 0$. Denote by $L_+ = \{a \in L : 0 \leq a\}$ the set of all positive elements of L; L_+ is referred to the positive cone. L_+ may sometimes be called the nonnegative cone.

In a Riesz pair $\langle L; L' \rangle$, a positive vector $x \in L_+$ is strictly positive, written $x >> 0$, if $px > 0$ for each $0 < p \in L'$. Denote by $L_{++} = \{a \in L : 0 < a\}$ the set of all strictly positive elements L; L_+ is referred to be the strictly positive cone. A strictly positive vector is also called a quasi-interior point. An equivalent characterization is that an element $x \in L$ is strictly positive if and only if the sequence $\{kx \wedge y\}$ converges in norm to y as k tends to infinity for each $y \in L_+$. Note that if the positive cone L_+ of L has non-empty (norm) interior, then the set of strictly positive elements coincides with the interior of L_+. However, many Banach lattices contains strictly positive elements even though the positive cone L_+ has an empty interior. For more information about normed spaces and normed lattices, we refer to Aliprantis and Border (1994).

We formalize the notion of an economy in the usual way. Exchange economies under consideration have a commodity space L that is the one either in the family of $L_p(S; \Sigma; \mu)$-spaces or in the family of l_p-spaces, where $1 \leqq p \leqq \infty$, S is the set of states, Σ the set of a σ-algebra of subsets of S, and μ a finite and positive measure defined on Σ and $L_p(S; \Sigma; \mu)$ is the class of all μ-measurable functions f for which

$\int_\Sigma |f|^p d\mu < \infty$ for $1 \leqq p < \infty$ and L_∞ is the space all μ-measurable functions with finite essential supremum in which the L_∞-norm is defined $\|f\|_\infty = \operatorname{ess\,sup} f = \sup\{t : \mu(\{x : |f(x)| \geqq t\}) > 0\}$. When time is modeled as a sequence of discrete dates, one may use a space l_p. For instance, space l_∞ plays a major role in the neoclassical theory of growth and discreet-time dynamic macroeconomic models. If there is an exhaustible resource in a setting, the l_1 may be an appropriate setting for the time series. When function spaces arise in models of uncertainty or time is continuous, one may use a space $L_p(S, \Sigma, \mu)$.

For $x, y \in L_p$, define $x = y$ if $x(s) = y(s)$ for μ-almost all $s \in S$. We can endow L_p with the following partial order relation $\leq$. We say $x \leq y$ if $x(s) \leqq y(s)$ for μ-almost all $s \in S$. We say $x < y$ if $x \leq y$ and there exists a μ nonnull subset $E \in S$ such that $x(s) < y(s)$ for μ-almost all $s \in E$. We say $x << y$ if $x(s) < y(s)$ for μ-almost all $s \in S$. Thus, the Lebesgue vector spaces L_p $(1 \leqq p \leqq \infty)$ with the above defined partial orders are clearly Banach lattices.

We assume that there are n agents in economies. Denote by $N = \{1, 2, \ldots, n\}$ the set of agents. Each agent's characteristic is denoted by $e_i = (P_i, \hat\omega_i)$, where P_i is the strict (irreflexive) preference defined on L_+ which may be nontotal or nontransitive,[3] and $\hat\omega_i \in L_{++}$ is the initial endowment of the agent. We assume preference relation P_i is strictly monotonic in the sense that $(x_i + v_0) P_i x_i$ for any $v_0 \in L_+ \setminus \{0\}$. To require well-behaved preferences that they admit supporting prices, we assume that the canonical conjugate R_i of P_i is uniformly proper on the order interval $[0, \hat\omega]$, where $\hat\omega = \sum_{i=1}^n \hat\omega_i$.[4]

[3] If we define the binary relation R_i by $a R_i b$ if and only if $\neg\, b P_i a$ where $\neg$ stands for "it is not the case that", then R_i is the weak (reflexive) preference and is called the `canonical conjugate' of P_i (see Kim and Richter (1986). If concepts used in this paper such as Nash equilibrium and the constrained Walrasian allocations are interpreted in terms of the R_i, then the results obtained in this paper for P_i are, in particular, valid for the R_i.

[4] The concept of properness was introduced by Mas-Colell (1986). The preference relation R_i is said to be proper at x with respect to the total endowment $\hat\omega$, if there is an open cone Γ_x at 0, containing $\hat\omega$, such that $x - \Gamma_x$ does not intersects the preferred set $\{x' \in L_+ : x' R_i x\}$, i.e., if $x' R_i x$, then $x - x' \notin \Gamma_x$. The interpretation is that the total endowment is desirable, in the sense that loss of an amount $\alpha\hat\omega$ ($\alpha > 0$) cannot be compensated for by an additional amount αz of any

An economy is the full vector $e = (e_1, \ldots, e_n)$ and the set of all such economies is denoted by E.

It will not generally be true in the infinite dimensional setting that the feasible consumption set is compact in the norm topology of the commodity space. To avoid this difficulty, we should explicitly assume the existence of a Hausdorff vector space topology τ such that the feasible consumption set is compact. Thus we have the following notion.

A Hausdorff topology τ, on the Banach lattice L, will be called compatible if

(a) τ is weaker than the norm topology of L,

(b) τ is a vector space (i.e., the vector space operations on L are continuous in the topology τ),

(c) all order intervals $[0, z]$ in L are τ-compact.

The topology will vary according to the underlying Banach lattice L; it may be the norm topology itself, or the weak topology, or the weak-star. For instance, if the commodity space is the Lebesgue space L_p, $1 \leq p < \infty$, the compatible topology will be the weak topology. This follows from the fact that the Lebesgue space L_p, $1 \leq p < \infty$, are normed vector lattices with continuous norm, order intervals are weakly compact (see Aliprantis and Burkinshaw (1985, Theorem 12.9). If the commodity space is L_∞ (l_∞), the compatible topology will be the weak* topology. Recall that Alaoglu's theorem implies that order intervals are weak* compact (see Aliprantis and Burkinshaw (1985, Theorem 9.20)). Finally, if the commodity space is the space of sequences l_p, $1 \leq p < \infty$, the compatible topology will be the norm topology.[5] This follows from the standard result that order intervals in l_p, $1 \leq p < \infty$, are norm compact.

commodity bundle z, if z is sufficiently small. The preference relation P_i is said to be uniformly with respect to $\hat{w}$ on the order interval $[0, \hat{w}]$ if it is proper at every $w \in [0, \hat{w}]$, and we can choose the properness cone independently of w.

[5] If the commodity space is l_1, the weak and norm topologies have the same compact set, so there is certainly nothing to be gained by taking for τ the weak topology. This is one of the few settings in which the feasible consumption will be norm compact.

2.2 The Constrained Walrasian Allocations

By an allocation, we mean an n-tuple $x := (x_1, \ldots, x_n) \in L^n$.

An allocation $x \in L^n$ is feasible if $x_i \in L_+$ for all $x_i \in L_+$, and

$$\sum_{i=1} x_i \leqq \sum_{i=1} \hat{\omega}_i ;$$

An allocation $x^* = (x_1^*, x_2^*, \ldots, x_n^*) \in L^n$ is a Walrasian allocation for an economy e if there is a price vector $p^* \in L_+^* \setminus \{0\}$ such that

(1) $p^* \cdot x_i^* \leqq p^* \cdot \hat{\omega}_i$ for all $i \in N$,

(2) for all $i \in N$, there does not exist $x_i \in L_+$ such that

(2.a) $x_i \, P_i \, x_i^*$;

(2.b) $p^* \cdot x_i \leqq p^* \cdot \hat{\omega}_i$;

(3) $\sum_{j=1}^{n} x_j \leqq \sum_{j=1}^{n} \hat{\omega}_j$.

The $n+1$-tuple $(x_1^*, \ldots, x_n^*, p^*)$ is then called a Walrasian equilibrium. Denote by $W(e)$ the set of all Walrasian allocations.

An allocation $x^* = (x_1^*, x_2^*, \ldots, x_n^*) \in L^n$ is a constrained Walrasian allocation for an economy e if there is a price vector $p^* \in L_+ \setminus \{0\}$ such that

(1) $p^* \cdot x_i^* \leqq p^* \cdot \hat{\omega}_i$ for all $i \in N$,

(2) for all $i \in N$, there does not exist $x_i \in L_+$ such that

(2.a) $x_i \, P_i \, x_i^*$;

(2.b) $p^* \cdot x_i \leqq p^* \cdot \hat{\omega}_i$;

(2.c) $x_i \leqq \sum_{j=1}^{n} \hat{\omega}_j$,

(3) $\sum_{j=1}^{n} x_j \leqq \sum_{j=1}^{n} \hat{\omega}_j$.

The $n+1$-tuple $(x_1^*, \ldots, x_n^*, p^*)$ is then called a constrained Walrasian equilibrium. Denote by $W_c(e)$ the set of all such allocations.

Remark 1 From the above definition, we can see that every ordinary Walrasian allocation (competitive equilibrium allocation) is a constrained Walrasian allocation and that a constrained Walrasian allocation differs from a Walrasian allocation only in the way that each agent maximizes his preferences not only subject to his budget constraint but also subject to total endowments available to the economy.

An allocation x is Pareto-efficient with respect to strict preference profile $P = (P_1, \ldots, P_n)$ if it is feasible and there does not exist another feasible allocation x' such that $x'_i \, P_i \, x_i$ for all $i \in N$.

An allocation x is individually rational with respect to P if $\neg \, \hat{w}_i \, P_i \, x_i$ for all $i \in N$.[6]

It can be easily shown that every constrained Walrasian allocation is Pareto-efficient and individually rational.[7]

An coalition C is a non-empty subset of N.

A group of agents (a coalition) $C \subset N$ is said to block an allocation x if there exists some allocation (x', y') such that

(i) $\sum_{i \in C} x'_i \leqq \sum_{i \in C} \hat{w}_i$,

(ii) $x'_i P_i x_i$ for all $i \in C$.

A feasible allocation x is said to be in the core of e if there does not exist any coalition C that can improve upon x.

Note that an allocation cannot be improved upon by N if and only if it is Pareto efficient, and an allocation cannot be improved upon by any single person if and only if it is individually rational. Also every constrained Walrasian allocation is in the core of e.

[6]This definition coincides with the conventional definition when P_i is the asymmetric part of a reflexive, transitive, and total preference R_i.

[7]For weak preferences, Thomson (1985) showed that a constrained Walrasian allocation may not be (regular) Pareto-efficient (i.e., there is no way of making everyone at least well off and one person better off) even if preferences satisfy local non-satiation. However, when preferences satisfy strict monotonicity, it is (regular) Pareto-efficient by Theorem 2.iv of Tian (1988).

2.3 Mechanism

Let F be a social choice rule, i.e., a correspondence from E to the commodity space L. In the rest of the paper, we will use the constrained Walrasian correspondence as a social choice rule.

Let M_i denote the i-th message (strategy) domain. Its elements are written as m_i and called messages. Let $M = \prod_{i=1}^{n} M_i$ denote the message (strategy) space. Let $X : M \to L$ denote the outcome function, or more explicitly, $X_i(m)$ is the i-th agent's outcome at m. A mechanism consists of $\langle M, X\rangle$ defined on E. A message $m^* = (m_1^*, \dots, m_n^*) \in M$ is a Nash equilibrium (NE) of the mechanism $\langle M, h\rangle$ for an economy e if for any $i \in N$ and for all $m_i \in M_i$,

$$\neg X_i(m^*/m_i, i) \, P_i \, X_i(m^*), \tag{1}$$

where $(m^*/m_i, i) = (m_1^*, \dots, m_{i-1}^*, m_i, m_{i+1}^*, \dots, m_n^*)$. The outcome $X(m^*)$ is then called a Nash (equilibrium) allocation. Denote by $V_{M,h}(e)$ the set of all such Nash equilibria and by $N_{M,h}(e)$ the set of all such Nash (equilibrium) allocations.

A mechanism $\langle M, h\rangle$ fully Nash-implements the constrained Walrasian correspondence W_c on E if, for all $e \in E$, $N_{M,h}(e) = W_c(e)$.

Remark 2 Note that the above definition which was due to Hurwicz [5, p. 219] allows the social choice correspondence W_c and the set of Nash equilibria to be empty for the main purpose of this paper is to study the equivalence of the constrained Walrasian correspondence and the set of Nash equilibrium allocations under the minimal possible assumptions.[8] A stronger definition of full Nash-implementation used in the literature is that not only $N_{M,h}(e) = F(e)$ but also $N_{M,h}(e) \neq \emptyset$ for all $e \in E$. Thus, if we restrict the domain of W_c to the one on which W_c is nonempty-valued, our results, to be presented below, will be equivalent for both definitions.

A message $m^* = (m_1^*, \dots, m_n^*) \in M$ is said to be a strong Nash equilibrium of the mechanism $\langle M, h\rangle$ for an economy $e \in E$ if there does not exist any coalition C

[8] Of course, if we impose more assumptions on preferences, by using the results such as in Zame (1987), one can prove the existence of constrained Walrasian equilibria.

and $m_C \in \prod_{i \in C} M_i$ such that for all $i \in C$,

$$X_i(m_C, m^*_{-C}) \, P_i \, X_i(m^*). \tag{2}$$

$X(m^*)$ is then called a strong Nash (equilibrium) allocation of the mechanism for the economy e. Denote by $SV_{M,X}(e)$ the set of all such strong Nash equilibria and by $SN_{M,X}(e)$ the set of all such strong Nash (equilibrium) allocations.

The mechanism $\langle M, h \rangle$ is said to doubly implement the constrained Walrasian correspondence W_c on E, if, for all $e \in E$, $SN_{M,N}(e) = N_{M,X}(e) = W_c(e)$.

A mechanism $\langle M, h \rangle$ is individually feasible if $X(m) \in L_+$ for all $m \in M$.

A mechanism $\langle M, h \rangle$ is weakly balanced if for all $m \in M$

$$\sum_{j=1} X_j(m) \leqq \sum_{j=1} \mathring{w}_j. \tag{3}$$

A mechanism $\langle M, h \rangle$ is feasible if it is individually feasible and weakly balanced.

Sometimes we say that an outcome function is individually feasible, balanced, or continuous if the mechanism is individually feasible, balanced, or continuous.

3 A Feasible and Continuous Mechanism

In this section, we present a simple feasible and continuous mechanism which doubly implements the constrained Walrasian correspondence on E. The mechanisms we use in the paper is reminiscent from the those given in Tian (1992, 2000) in the finite-dimensional context.

Let $\Delta \subset L^*_+$ be a weak*-compact and convex set such that $p \cdot \mathring{w} = 1$ for every $p \in \Delta$. By Theorem 9.1 in Mas-Colell and Zame (1991), such a set exists.

For each $i \in N$, let the message domain of agent i be of the form

$$M_i = (0, \mathring{w}_i] \times \Delta \times L^n, \tag{4}$$

where $(0, \mathring{w}_i] = \{w_i \in L_+ : 0 << w_i \leq \mathring{w}_i\}$. A generic element of M_i is $m_i = (w_i, p_i, x_{i1}, \ldots, x_{in})$ whose components have the following interpretations. The component w_i denotes a profession of agent i's endowment, the inequality $0 << w_i \leqq \mathring{w}_i$

means that the agent cannot overstate his own endowment bundle; on the other hand, the endowment can be understated, but the claimed endowment w_i must be strictly positive. The component p_i is the price vector proposed by agent i and is used as a price vector of agent $i-1$, where $i-1$ is read to be n when $i=1$. The component x_{ij} is interpreted as the trade that agent i is willing to make to agent j (a negative x_{ij} means agent i wants to get $-x_{ij}$ amount of goods from agent j).

Define agent i's price vector $p_i : M \to \Delta$ by

$$p_i(m) = p_{i+1}; \tag{5}$$

where $n+1$ is to be read as 1. Note that although $p_i(\cdot)$ is a function of proposed price vector announced by agent $i+1$, for simplicity, we can write $p(\cdot)$ as a function of m without loss of generality.

Define a feasible correspondence $B : M \to\to L^n_+$ by

$$\begin{aligned} B(m) \;=\; \{x \in L^n_+ : \sum_{i=1} x_i \leqq \sum_{i=1} w_i \;\&\\ p_i(m) \cdot x_i \leqq \frac{1}{1 + \|p_i - p_i(m)\|} p_i(m) \cdot w_i \;\forall i \in N\}; \end{aligned} \tag{6}$$

which is nonempty, convex, and τ-compact for all $m \in M$ by the set is norm bounded by the total endowments and τ-closed. We will show the following lemma in the Appendix.

Lemma 1 $B(\cdot)$ is τ-continuous on M.

Let $\hat{x}_j = \sum_{i=1}^n x_{ij}$ which is the sum of contributions that agents are willing to make to agent j and $\hat{x} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$.

The outcome function $X : M \to L^n_+$ is given by

$$X(m) = \{x \in L^n_+ : \min_{x \in B(m)} \|x - \hat{x}\|\}; \tag{7}$$

which is the closest to $\hat{x}$.

We then have the following lemma.

Lemma 2 $X(\cdot)$ is a single-valued continuous function.

Proof. Since the distance function $d(\hat{x}, y) = \|\hat{x} - y\|$ is continuous in y, we know that d will reach its maximum on $B(m)$. Thus, X is a nonempty correspondence. We want to show X is in fact a single-valued function on B. If $\hat{x} \in B(m)$, then $d(\hat{x}; B(m)) = 0$, and thus $X(m) = \hat{x}$. So we only consider the case where $\hat{x} \notin B(m)$.

Suppose by way of contradiction that there are two points x_1 and x_2 in $B(m)$ such that $\|x_1 - \hat{x}\| = \|x_2 - \hat{x}\| = d(\hat{x}; B(m))$ for some $m \in M$. Since $B(m)$ is convex, the convex combination $x_\lambda = \lambda x_1 + (1 - \lambda)x_2 \in B(m)$ with $0 < \lambda < 1$, and thus, by Minkowski's inequality, we have

$$\|x_\lambda - \hat{x}\| = \|\lambda(x_1 - \hat{x}) + (1 - \lambda)(x_1 - \hat{x})\| \leq \|\lambda(x_1 - \hat{x})\| + \|(1 - \lambda)(x_1 - \hat{x})\| = d(\hat{x}; B(m)):$$

Thus, we must have

$$\|\lambda(x_1 - \hat{x}) + (1 - \lambda)(x_2 - \hat{x})\| = \|\lambda(x_1 - \hat{x})\| + \|(1 - \lambda)(x_2 - \hat{x})\|$$

Notice that the Minkowski's inequality become equality if and only if there is some $t \geq 0$ such that

$$\lambda(x_1 - \hat{x}) = (1 - \lambda)t(x_2 - \hat{x}):$$

Taking the norm on both sides and noting that $\|(x_1 - \hat{x})\| = \|(x_2 - \hat{x})\|$, we must have

$$\lambda = (1 - \lambda)t:$$

Consequently, we have

$$x_1 - \hat{x} = x_2 - \hat{x}$$

and therefore $x_1 = x_2$, a contradiction. Thus, X is single-valued.

Finally, since $B(m)$ is a continuous correspondence, then, by Berge's Maximum Theorem (Berge (1963)), we know X is a upper hemi-continuous correspondence. However, since X is also single-valued, and thus it is a continuous function on B.

Also, since $X(m) \in L^n_+$ and

$$\sum_{i=1} X_i(m) \leq \sum_{i=1} \hat{w}_i \tag{8}$$

for all $m \in M$, the mechanism is feasible and continuous.

Remark 3 Note that the above mechanism does not depend on the number of agents. Thus it is a unified mechanism which works for two-agent economies as well as for economies with three or more agents. For two-agent economies, only the feasible and continuous mechanism which Nash implements the constrained Walrasian correspondence was given by Nakamura (1990). Here we give an even simpler feasible and continuous mechanism which doubly implements the constrained Walrasian correspondence not only for economies with a finite dimensional consumption space but also for economies with an infinite dimensional consumption space.

4 Results

The remainder of this paper is devoted to the proof of equivalence among Nash allocations, strong Nash allocations, and constrained Walrasian allocations. Proposition 1 below proves that every Nash allocation is a constrained Walrasian allocation. Proposition 2 below proves that every constrained Walrasian allocation is a Nash allocation. Proposition 3 below proves that every Nash equilibrium is a strong Nash equilibrium. To show these results, we first prove the following lemmas.

Lemma 3 *If $m^* \in V_{M,X}(e)$, then $p_1^* = p_2^* = \cdots = p_n^*$, and thus $p_1(m^*) = p_2(m^*) = \cdots = p_n(m^*) = p^*$ for some $p^* \in \Delta$.*

Proof: Suppose, by way of contradiction, that $p_i^* \neq p_{i+1}^*$ (i.e., $p_i^* \neq p_i(m^*)$) for some $i \in N$. Then $p_i(m^*) \cdot X_i(m^*) \leqq \frac{1}{1+\|p_i^* - p_i(m^*)\|} p_i(m^*) \cdot w_i^* < p_i(m^*) \cdot w_i^*$, and thus there is $x_i \in L_+$ such that $p_i(m^*) \cdot x_i \leqq p_i(m^*) \cdot w_i^*$ and $x_i \, P_i \, X_i(m^*)$ by strict monotonicity of preferences. Now if agent i chooses $p_i = p_i(m^*)$, $x_{ii} = x_i - \sum_{t \neq i} x_{ti}^*$, $x_{ij} = -\sum_{t \neq i} x_{tj}^*$ for $j \neq i$, and keeps w_i^* unchanged, then $(0, \ldots, 0, x_i, 0, \ldots, 0) \in B(m_i, m_{-i}^*)$, and thus $X_i(m_i, m_{-i}^*) = x_i$. Therefore, $X_i(m_i, m_{-i}^*) \, P_i \, X_i(m^*)$. This contradicts $X(m^*) \in N_{M,X}(e)$. Thus we must have $p_1^* = p_2^* = \cdots = p_n^*$, and therefore $p_1(m^*) = p_2(m^*) = \cdots = p_n(m^*) = p^*$ for some $p^* \in \Delta$. Q.E.D.

Lemma 4 *If $m^* \in N_{M,X}(e)$, then $w_i^* = \hat{w}_i$ for all $i \in N$.*

Proof: Suppose, by way of contradiction, that $w_i^* \neq \hat{w}_i$ for some $i \in N$. Then $p_i(m^*) \cdot X_i(m^*) \leqq p_i(m^*) \cdot w_i^* < p_i(m^*) \cdot \hat{w}_i$, and thus there is $x_i \in L_+$ such that $p_i(m^*) \cdot x_i \leqq p_i(m^*) \cdot \hat{w}_i$ and $x_i \; P_i \; X_i(m^*)$ by strict monotonicity of preferences. Now if agent i chooses $w_i = \hat{w}_i$, $x_{ii} = x_i - \sum_{t \neq i} x_{ti}^*$, $x_{ij} = -\sum_{t \neq i} x_{tj}^*$ for $j \neq i$, and keeps p_i^* unchanged, then $(0, \ldots, 0, x_i, 0, \ldots, 0) \in B(m_i, m_{-i}^*)$, and thus $X_i(m_i, m_{-i}^*) = x_i$. Hence, $X_i(m_i, m_{-i}^*) \; P_i \; X_i(m^*)$. This contradicts $X(m^*) \in N_{M,X}(e)$ and thus $w_i^* = \hat{w}_i$ for all $i \in N$. Q.E.D.

Lemma 5 If $X(m^*) \in N_{M,X}(e)$, then $p_i(m^*) \cdot X_i(m^*) = p_i(m^*) \cdot \hat{w}_i$.

Proof: Suppose, by way of contradiction, that $p_i(m^*) \cdot X_i(m^*) < p_i(m^*) \cdot \hat{w}_i$ for some $i \in N$. Then there is $x_i \in L_+$ such that $p_i(m^*) \cdot x_i \leqq p_i(m^*) \cdot \hat{w}_i$ and $x_i \; P_i \; X_i(m^*)$ by strict monotonicity of preferences. Now if agent i chooses $x_{ii} = x_i - \sum_{t \neq i} x_{ti}^*$, $x_{ij} = -\sum_{t \neq i} x_{tj}^*$ for $j \neq i$, and keeps p_i^* and w_i^* unchanged, then $(0, \ldots, 0, x_i, 0, \ldots, 0) \in B(m_i, m_{-i}^*)$, and thus $X_i(m_i, m_{-i}^*) = x_i$. Hence, $X_i(m_i, m_{-i}^*) \; P_i \; X_i(m^*)$. This contradicts $X(m^*) \in N_{M,X}(e)$. Q.E.D.

Proposition 1 If the mechanism $\langle M, X \rangle$ defined above has a Nash equilibrium m^* for $e \in E$, then $X(m^*)$ is a constrained Walrasian allocation with p^* as a competitive equilibrium price vector, i.e., $N_{M,X}(e) \subset W_c(e)$ for all $e \in E$.

Proof. Let m^* be a Nash equilibrium. Then $X(m^*)$ is a Nash equilibrium allocation. We wish to show that $X(m^*)$ is a constrained Walrasian allocation. By Lemmas 2 – 4, $p_1(m^*) = \cdots = p_n(m^*) = p^*$ for some $p^* \in \Delta$, $w_i^* = \hat{w}_i$, and $p(m^*) \cdot X_i(m^*) = p(m^*) \cdot \hat{w}_i$ for all $i \in N$. Also, by the construction of the mechanism, we know that $X(m^*) \in L_+^n$ and $\sum_{j=1}^n X_j(m^*) \leqq \sum_{j=1}^n \hat{w}_j$. So we only need to show that each individual is maximizing his/her preferences. Suppose, by way of contradiction, that for some agent i, there exists some $\hat{x}_i \in L_+$ such that $\hat{x}_i \leqq \sum_{j=1}^n \hat{w}_j$, $p(m^*) \cdot \hat{x}_i \leqq p(m^*) \cdot \hat{w}_i$, and $\hat{x}_i \; P_i \; X_i(m^*)$. Let $x_{ii} = \hat{x}_i - \sum_{t \neq i} x_{ti}^*$, $x_{ij} = -\sum_{t \neq i} x_{tj}^*$ for $j \neq i$, and keep p_i^* and w_i^* unchanged, then $(0, \ldots, 0, \hat{x}_i, 0, \ldots, 0) \in B(m_i, m_{-i}^*)$, and thus $X_i(m_i, m_{-i}^*) = \hat{x}_i$. Therefore, we have $X_i(m_i, m_{-i}^*) \; P_i \; X_i(m^*)$. This contradicts $X(m^*) \in N_{M,X}(e)$. So $X(m^*)$ is a constrained Walrasian allocation. Q.E.D.

Proposition 2 If $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ is a constrained Walrasian allocation with a competitive equilibrium price vector $p^* \in \Delta$ for $e \in E$, then there exists a Nash equilibrium m^* of the mechanism $\langle M, X \rangle$ defined above such that $X_i(m^*) = x_i^*$, $p_i(m^*) = p^*$, for all $i \in N$, i.e., $W_c(e) \subset N_{M,X}(e)$ for all $e \in E$.

Proof. Since preferences satisfy the strict monotonicity condition and x^* is a constrained Walrasian allocation, we must have $p^* \in \Delta$, $\sum_{j=1}^n x_j^* \leqq \sum_{j=1}^n \hat{w}_j$ and $p^* \cdot x_i^* = p^* \cdot \hat{w}_i$ for $i \in N$. Now for each $i \in N$, let $m_i^* = (p^*, x_{i1}^*, \ldots, x_{in}^*)$, where $x_{ii}^* = x_i^*$ and $x_{ij}^* = 0$ for $j \neq i$.

Then x^* is an outcome with p^* as a price vector, i.e., $X_i(m^*) = x_i^*$ for all $i \in N$, and $p_i(m^*) = p^*$. We show that m^* yields this allocation as a Nash allocation. In fact, agent i cannot change $p_i(m^*)$ by changing his proposed price (i.e., $p_i(m_i, m_{-i}^*) = p_i(m^*)$ for all $m_i \in M_i$). Announcing a different message m_i by agent i may yield an allocation $X(m_i, m_{-i}^*)$ such that $X_i(m_i, m_{-i}^*) \in L_+$ and

$$p(m^*) \cdot X_i(m_i, m_{-i}^*) \leqq p(m^*) \cdot \hat{w}_i. \tag{9}$$

Now suppose, by way of contradiction, that m^* is not a Nash equilibrium. Then there are $i \in N$ and m_i such that $X_i(m_i, m_{-i}^*)\ P_i\ X_i(m^*)$. Since $X_i(m_i, m_{-i}^*) \leqq \sum_{i=1}^n \hat{w}_i$, we must have, by the definition of the constrained Walrasian allocation, $p(m^*) \cdot X_i(m_i, m_{-i}^*) > p(m^*) \cdot \hat{w}_i$. But this contradicts the budget constraint (9). Thus we have shown that agent i cannot improve his/her utility by changing his/her own message while the others' messages remain fixed for all $i \in N$. Hence x^* is a Nash allocation. Q.E.D.

Proposition 3 Every Nash equilibrium m^* of the mechanism defined above is a strong Nash equilibrium, that is, $N_{M,X}(e) \subseteq SN_{M,X}(e)$.

Proof: Let m^* be a Nash equilibrium. By Proposition 1, we know that $X(m^*)$ is a constrained Walrasian allocation with $p(m^*)$ as a price vector. Then $X(m^*)$ is Pareto optimal and thus the coalition N cannot be improved upon by any $m \in M$. Now for any coalition C with $\emptyset \neq C \neq N$, choose $i \in C$ such that $i + 1 \notin C$. Then no strategy played by C can change the budget set of i since $p_i(m)$ is determined by p_{i+1}. Furthermore, because $X(m^{¤})$ 2 $W_c(e)$, it is the preference maximizing consumption with respect to the budget set of i, and thus C cannot improve upon $X(m^{¤})$. Q.E.D.

Since every strong Nash equilibrium is clearly a Nash equilibrium, then by combining Propositions 1-3, we have the following theorem.

Theorem 1 For the class of exchange economies E, there exists a feasible and continuous mechanism which doubly implements the constrained Walrasian correspondence. That is, $N_{M;X}(e) = SN_{M;X}(e) = W_c(e)$ for all e 2 E.

5 Concluding Remarks

This paper gives a simple mechanism which doubly implements the constrained Walrasian correspondence in Nash and strong Nash equilibrium for economies with in¯nitely many commodities. In¯nite-dimensional commodity spaces arise naturally when we consider economic activity over an in¯nite time horizon, or with uncertainty about the possible in¯nite number of states of the world, or in a setting where an in¯nite variety of commodity characteristics are possible. The mechanism we give is feasible, continuous, and allows coalition patterns, preferences and endowments to be unknown to the designer. Furthermore, preferences under consideration may not be total, transitive, continuous, and convex preferences. In addition, unlike most mechanisms proposed in the literature, it gives a uni¯ed mechanism which is irrespective of the number of agents.

Though this paper only considers double implementation of the constrained Walrasian correspondence for economies with in¯nite-dimensional spaces, one can similarly consider implementation of other social choice rules such as Lindahl allocations for economies with in¯nitely many commodities.

Appendix

Proof of Lemma 1: First note that Δ is weak* compact. Also, if $x_k \to x$ in the norm topology of L, $p_k \to p$ in the weak* topology of L^* and $\{p_k\}$ is order bounded, $p_k \cdot x_k \to p \cdot x$ [Yannelis and Zame (1986, Lemma A, p. 107)]. Then $B(\cdot)$ has closed graph by the continuity of $p_i(\cdot)$ and $p_i(\cdot) \cdot x$. Since the range space of the correspondence $B(\cdot)$ is weakly bounded by the total endowments $\sum_{i=1}^n w_i$, it is weakly compact. Thus, $B(\cdot)$ is upper hemi-continuous on M. So we only need to show that $B(m)$ is also lower hemi-continuous at every $m \in M$. Let $m \in M$, $x = (x_1, \ldots, x_n) \in B(m)$, and let $\{m_k\}$ be a sequence such that $m_k \to m$, where $m_k = (m_1^k, \ldots, m_n^k)$ and $m_i^k = (w_i^k, p_i^k, z_{i1}^k, \ldots, z_{in}^k)$. We want to prove that there is a sequence $\{x_k\}$ such that $x_k \to x$, and, for all k, $x_k \in B(m_k)$, i.e., $x_k = (x_{1k}, \ldots, x_{nk}) \in L_+^n$, $p_i(m_k) \cdot x_{ik} \leq \frac{1}{1+\|p_i^k - p_i(m_k)\|} p_i(m_k) \cdot w_i^k$ for all $i \in N$, and $\sum_{i \in N} x_{ik} \leq \sum_{i \in N} w_i^k$. We first prove that there is a sequence $\{\hat{x}_k\}$ such that $\hat{x}_k \to x$, and, for all k, $\hat{x}_k \in L_+^n$ and $p_i(m_k) \cdot \hat{x}_{ik} \leq \frac{p_i(m_k) \cdot w_i^k}{1+\|p_i^k - p_i(m_k)\|}$ for all $i \in N$. For each $i \in N$, two cases will be considered.

Case 1. $p_i(m) \cdot x_i < \frac{p_i(m) \cdot w_i}{1+\|p_i - p_i(m)\|}$. Hence, for all k larger than a certain integer k', we have $p_i(m_k) \cdot x_i < \frac{p_i(m_k) \cdot w_i^k}{1+\|p_i^k - p_i(m_k)\|}$ by noting that $p_i(\cdot)$ is continuous. Let $\hat{x}_{ik} = x_i$ for all $k > k'$ and $\hat{x}_{ik} = 0$ for $k \leq k'$. Then, we have $p_i(m_k) \cdot \hat{x}_{ik} < \frac{p_i(m_k) \cdot w_i^k}{1+\|p_i^k - p_i(m_k)\|}$.

Case 2. $p_i(m) \cdot x_i = \frac{p_i(m) \cdot w_i}{1+\|p_i - p_i(m)\|}$. Note that, since $p_i(m) > 0$ and $w_i > 0$ for all i, we must have $x_i > 0$. Let $\omega_i = \frac{p_i(m) \cdot w_i}{1+\|p_i - p_i(m)\|}$ and $\omega_{ik} = \frac{p_i(m_k) \cdot w_i^k}{1+\|p_i^k - p_i(m_k)\|}$. Define $\hat{x}_{ik}$ as follows:

$$\hat{x}_{ik} = \begin{cases} \frac{\omega_{ik}}{p_i(m_k) \cdot x_i} x_i & \text{if } \frac{\omega_{ik}}{p_i(m_k) \cdot x_i} \leq 1 \\ x_i & \text{otherwise} \end{cases}.$$

Then $\hat{x}_{ik} \leq x_i$, and $p_i(m_k) \cdot \hat{x}_{ik} \leq \frac{p_i(m_k) \cdot w_i^k}{1+\|p_i^k - p_i(m_k)\|}$. Also, since $\frac{\omega_{ik}}{p_i(m_k) \cdot x_i} \to \frac{\omega_i}{p_i(m) \cdot x_i} = 1$, we have $\hat{x}_{ik} \to x_i$. Thus, in both cases, there is a sequence $\{\hat{x}_k\}$ such that $\hat{x}_k \to x$, and, for all k, $\hat{x}_k \in L_+^n$ and $p_i(m_k) \cdot \hat{x}_{ik} \leq \frac{p_i(m_k) \cdot w_i^k}{1+\|p_i^k - p_i(m_k)\|}$ for all $i \in N$.

We now show that there is a sequence $\{\check{x}_k\}$ such that $\check{x}_k \to x$, and, for all k, $\check{x}_k \in L_+$ and $\sum_{i \in N} \check{x}_{ik} \leq \sum_{i \in N} w_i^k$.

We first show this for the sequence spaces l_p. There are two cases will be considered for each component of vector $x = (x^1; x^2; \dots; x^t; \dots)$ with $1 \leqq t < \infty$.

Case 1. $\sum_{i \in N} x_i^t < \sum_{i \in N} w_i^t$. Hence, for all k larger than a certain integer k', we have $\sum_{i \in N} x_i^t < \sum_{i \in N} w_i^{tk}$. For each $i \in N$, let $\tilde{x}_{ik}^t = x_i^t$ for all $k > k'$ and $\hat{x}_{ik}^t = 0$ for $k \leqq k'$. Then, we have $\sum_{i \in N} x_{ik}^t < \sum_{i \in N} w_i^{tk}$.

Case 2. $\sum_{i \in N} x_i^t = \sum_{i \in N} w_i^t$. Note that, since $w_i > 0$ for all i, we must have $\sum_{i \in N} x_i^t > 0$. For each $i \in N$, define $\tilde{x}_{ik}^t$ as follows:

$$\tilde{x}_{ik}^t = \begin{cases} \frac{\sum_{i \in N} w_i^{tk}}{\sum_{i \in N} x_i^t} x_i & \text{if } \frac{\sum_{i \in N} w_i^{tk}}{\sum_{i \in N} x_i^t} \leqq 1 \\ x_i^t & \text{otherwise} \end{cases}.$$

Then $\hat{x}_{ik}^t \leqq x_i^t$, and $\sum_{i \in N} \tilde{x}_{ik}^t \leqq \sum_{i \in N} w_i^{tk}$. Also, since $\frac{\sum_{i \in N} w_i^{tk}}{\sum_{i \in N} x_i^t} \to \frac{\sum_{i \in N} w_i^{tk}}{\sum_{i \in N} x_i^t} = 1$, we have $\hat{x}_{ik}^t \to x_i^t$. Thus, in both cases, there is a sequence $\{\hat{x}_k\}$ such that $\hat{x}_k \to x$, and, for all k, $\hat{x}_k \in L_+$ and $\sum_{i \in N} x_i^k \leqq \sum_{i \in N} w_i^k$. Here $\hat{x}_k = (\hat{x}_k^1; \hat{x}_k^2; \dots)$.

Similarly, we can show this for the Lebesgue spaces L_p by considering two cases: (1) $\sum_{i \in N} x_i(s) < \sum_{i \in N} w_i(s)$ and (2) $\sum_{i \in N} x_i(s) = \sum_{i \in N} w_i(s)$ for each $s \in S$.

Finally, let $x_k^0 = \min(\tilde{x}_k; \hat{x}_k)$ with $x_{ik}^0 = \min(\tilde{x}_{ik}; \hat{x}_{ik})$ for $i = 1; \dots; n$. Then $x_k^0 \to x$ since $\tilde{x}_k \to x$ and $\hat{x}_k \to x$. Also, for every k larger than a certain integer $\tilde{k}$, we have $x_k^0 \geqq 0$, $\sum_{i \in N} x_{ik}^0 \leqq \sum_{i \in N} w_i^k$ because $x_k^0 \leqq \tilde{x}_k$ and $\sum_{i \in N} \tilde{x}_{ik} \leqq \sum_{i \in N} w_i^k$, and $p_i(m_k) \cdot x_{ik}^0 \leqq \frac{p_i(m_k) \cdot w_i^k}{1 + \|p_i^k - p_i(m_k)\|}$ for all $i \in N$ by noting that $x_{ik}^0 \leqq \hat{x}_{ik}$. Let $x_k = x_k^0$ for all $k > \tilde{k}$ and $x_k = 0$ for $k \leqq \tilde{k}$. Then, $x_k \to x$, and $x_k \in B(m_k)$ for all k. Therefore, the sequence $\{x_k\}$ has all the desired properties. So $B_x(m)$ is lower hemi-continuous at every $m \in M$. Q.E.D.

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