Physics 408 Problem Set 1 Due Thurs, Sep. 9 at beginning of class

1) Consider a system with a fundamental relation defining the entropy as $S = \alpha V^{1/4} U^{3/4}$, where α is a constant. Note that the relation is independent of the particle number for this particular system (the number is indefinite, I didn't leave it out).
a) Find the temperature and pressure vs. *U* and *V*.
b) Reverse these to express the *S, U,* and *P* vs the temperature.
c) Consider an isothermal compression of this system (e.g. *T* = constant), in which the volume is reduced by the amount ΔV. Find the work done on this system, and the heat that must flow into the system to maintain the constant temperature.

2) Callen 1.8-3

3) Consider the process treated in class, in which a monatomic ideal is allowed to expand freely after a piston is suddenly displaced, followed by an adiabatic, controlled compression back to the original volume. Consider a reverse process, in which an insulated cylinder contains *N* atoms of the same gas, in the initial state with pressure P_0 and volume V_0, and then (1) a piston compresses the gas adiabatically to 1/3 its initial volume and (2) the piston is suddenly retracted, allowing the gas to freely expand back to volume V_0.
a) Find the work done on the gas in process (1).
b) Find the entropy change of the gas in process (1) and in process (2).
c) Determine the highest temperature found for the gas during this sequence.

4) Consider throwing a 6-sided die, a total of N times.
(a) Find a formula for the probability that half the throws yield 6, and half yield some other number.
(b) Find the average number of throws that would yield 6, in N tries.
(c) Find the relative size of the standard deviation (the RMS width discussed in class divided by the mean number of throws) for differences about this mean value, for the case that N = 600.

5) The 6-sided die can be mapped onto a magnet in which each atom has a J=5/2 magnetic moment, having 6 spin levels, with no external field so that the 6 levels are degenerate, and with each magnetic moment acting independently. Since degenerate and noninteracting, the magnetic quantum numbers will be randomly distributed.
a) Find the spin entropy associated with a 1-mole magnet containing such unmagnetized spins.
b) Suppose that there are actually small interactions, so that at 300 K the magnet becomes ferromagnetic (all moments lined up in the same direction, thus all having identical magnetic quantum numbers). Also suppose (not very realistically) that this transformation happens suddenly at the 300 K transition temperature, e.g. all ordered below 300 K, all random above 300 K. How much heat energy is liberated when this transformation occurs? (This is a latent heat, and such a substance could be very effective for a next-generator refrigerator or air conditioner; on this point compare your result to the 22 kJ/mole latent heat of evaporating freon.)

6) Considering the sun's radiation on earth as a heat flow process as discussed in class, note that the temperature of the sun to be considered in such a process is that of the outer surface where the escaping radiation is emitted, with T ~ 6000 K.

a) Estimate the net entropy increase per year (in J/K) associated with the solar radiative heat hitting a 1 m^2 area of the earth's surface balanced by the corresponding amount of the sun's radiation leaving its surface.

b) It is useful to compare this entropy change with the entropy that might be associated with the use of this solar energy for various processes. For example, consider the case that the work of photosynthesis causes grass to grow on this particular m^2 of surface. An overriding contribution to the entropy is that the carbon in CO_2 from the atmosphere (an ideal gas) is converted to cellulose (essentially a solid). The gas has much larger entropy than the solid, due mostly to the much larger space that gas molecules can randomly explore, so consider that the cellulose entropy to be essentially zero. I displayed a solution for the ideal gas entropy in class. Estimate the number of moles per year that might be involved in such a carbon-capture process for the 1 m^2 area, and determine the amount of entropy that correspondingly disappears. Compare to your answer for (a). Note: the process of developing increasingly organized structures by civilization, the evolution of more highly intelligent beings, etc. perhaps also correspond to a reduction in entropy (though likely a rather small value), but when considering the overall entropy change the process described in part (a) must also be considered.