

Physics 408 Problem Set 3 Due Weds, Sep. 22 at beginning of class

1) Callen 2.8.1

2) Callen 2.7.2

3) As described in class, for this problem we treat the ideal gas in the microcanonical ensemble, in terms of a phase space integration. For a classical gas of N particles the multiplicity will be simply, $\frac{1}{N!h^{3N}} \iiint d^{3N}r d^{3N}p$, where the integral covers all possible regions of *phase space* consistent with the total energy U , and the factorial in the denominator accounts for multiple counting of identical particles. This comes about since the phase space “volume” of one state is h^{3N} , consistent with the uncertainty principle. Also assume U consists of only the translational energy of the hard-sphere gas. The real-space integrals can be directly evaluated in terms of the total volume. This leaves the momentum integrals, which together correspond that of a $3N$ dimensional hypersphere.

(a) I discussed in class evaluating the number of states in a thin shell between U and $U - \Delta U$. Instead, for this problem evaluate the number of states in the *entire* hypersphere from energy = 0 to U , which turns out to give the same result in the limit of large N , due to the sharpness of the distribution. In terms of the maximum energy U what is the radius of the hypersphere in momentum space? You will have to look up the volume of a hypersphere – do this and evaluate the multiplicity of states in the sphere in terms of U .

(b) As a measure of the sharpness of the distribution, determine an energy U' which gives a multiplicity which is 10 times larger than the result obtained in part (a), for the same volume of gas. For the case that $N = 1$ mole of atoms, find to leading order the quantity $[U'/U - 1]$, which should be a very small number! This shows how sharply the states “pile up” in a very thin shell close to the edge, when treating many-body energies rather than single-particle energies.

4) Consider the Einstein-oscillator approximation for the vibrating crystal we have discussed.

a) Find a relation for the entropy as a function of the temperature. Show that S approaches zero as T approaches zero, in agreement with the 3rd law. Also determine the leading temperature-dependence of S at high temperatures, and explain why this behavior could be expected, based on the behavior of the specific heat.

b) Solve for the chemical potential vs. temperature, and make a plot.

5) Consider a system with the fundamental equation, $U = \alpha \frac{S^5}{NV^3}$, where α is a constant.

a) Find the three equations of state for this system.

b) For a constant- N isothermal process, determine the form of the P - V curves (e.g. for an ideal gas we would have $PV = \text{constant}$, here there will be a different relation between P and V).

c) For a constant- N adiabatic process, similarly determine the form of the P - V curves.