Physics 408 Problem Set 4 Due Weds, Sep. 29 at beginning of class

1) Callen 3.3.2

2) Callen 3.4.7

3) Consider a two-dimensional ideal gas. Note, the conducting layer in the MOSFET shown in lecture can in some cases be considered a 2d electron gas, if the confining potential is steep enough to quench excitations in the 3rd direction, and under proper conditions other physical systems such as adsorbed molecules on a surface can also behave as a 2d gas.
a) Assume the 2d gas is a hard-sphere classical gas with only translational kinetic energy, and solve for the multiplicity vs. U, N, and the area. You could refer to the results of last week's homework for this part.
b) Based on your multiplicity, find the entropy.
c) From your result in part (b) derive the equations of state for temperature and for surface tension (which is the 2d analog of pressure). Then, using the Euler equation specifically, find an expression for the chemical potential.

4) a) Consider an ideal gas at a height z above sea level. Find the z-dependent chemical potential, including the effects of the gravitational potential of the earth.
b) Suppose two containers of helium gas are positioned such that one container is at a height z relative to the other one, with both fixed at the same temperature, and with a hollow tube connecting the containers allowing gas to be transported until equilibrium is reached. Find the ratio of particle number densities in the two containers.
c) Generalize the problem to include aerosol particles, a subject of great interest due to the present pandemic. Find the particle diameter, for particles considered to be spherical water droplets, for a set of particles whose density would be reduced by a factor of 10 over a range of 10 m, at 300 K and in very still air. This serves as one estimate of the limiting size of particle that could be easily transmitted, although note that significantly larger particles can also stay aloft for a considerable period due to convection.

5) Find the fundamental equation for a system which obeys the following equations of state:

$$P = -\frac{NU}{NV-2AUV}\,; \quad T = \frac{2Ce^{AU/N}\sqrt{UV}}{N-2AU}\,.$$

6) As proposed by Stephen Hawking and Jacob Bekenstein, it is now generally agreed that gravitational black holes are not inert but rather they can be considered to have an entropy, and a temperature. The entropy is believed to be $S = \frac{8\pi^2 G M^2 k_B}{hc}$ (for the case of no angular momentum or charge). M is the total mass, which you can convert to an internal energy using the relativistic equivalent Mc^2.
a) Find the black hole temperature for such a case.

b) Find the specific heat, and show that it is negative. (Note that this is an unusual situation but a negative specific heat also can occur in other closed gravitational systems since adding heat can paradoxically cause the kinetic energy to decrease through the virial theorem.)

c) What happens to the entropy as T approaches zero?

d) Show that the Euler equation is not obeyed in this situation, and comment on the extensivity of the parameters in this case and its connection to the validity of the Euler equation.