Physics 408 Problem Set 5   Due Weds, Oct 6 at beginning of class

(corrected version Oct. 5, 5 PM)

1) Callen 3.5.2

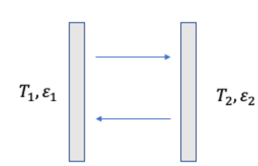
2) Callen 3.5.6

3) a) In class I showed how to calculate the average photon number, which I labeled $\langle N \rangle$, as well as the average internal energy, for the thermal radiation in a blackbody cavity. Set up the calculation of the average photon number as an integral over *frequency*, similar to the way this was done for the energy. Your result contains the *photon number density*, e.g. the number of photons in an infinitesimal frequency slice $d\omega$. The photon number density exhibits a peak, qualitatively similar to the peak in the energy spectrum. Find the peak frequency, ω_{peak}, where the density of states is a maximum, in terms of $k_B T/\hbar$.
b) Considering room temperature to be 300 K, find the position of the density of states maximum (e.g. ω_{peak}) in units of eV.
c) At a given temperature, the distribution of thermal photons will include photons with much larger energy than the typical photon energy, with small probability. As an example at room temperature it is in principle possible to detect a thermal photon strong enough to ionize hydrogen in its ground state (13.6 eV or higher). Find this probability by computing the relative numbers of photons in this energy range at room temperature.

4) For this problem you may want to utilize the multi-variable calculus expressions I showed in class, and also read the last section of chapter 3. Consider a system subject to a state equation interconnecting the three variables P, T, and V. The constant-*P* thermal expansion coefficient is defined as $\alpha = \left(\frac{1}{V}\right)\frac{\partial V}{\partial T}\Big)_P$, and the constant-*T* compressibility is defined as $\kappa_T = -\left(\frac{1}{V}\right)\frac{\partial V}{\partial P}\Big)_T$.
(a) Show that $\frac{\partial \alpha}{\partial P}\Big)_T = -\frac{\partial \kappa_T}{\partial T}\Big)_P$.
(b) Show that $\alpha/\kappa_T = \frac{\partial P}{\partial T}\Big)_V$.
(c) Find α and κ_T explicitly for an ideal gas and show that the relationship (b) is upheld.

5) a) Compute the parameters α and κ_T defined in the last problem for the case of a van der Waals gas.
b) For the van der Waals gas there is a critical point (discussed in ch. 9), which is the highest pressure for which a divide between the gas phase and liquid phase exists. In terms of the van der Waals parameters the pressure, volume, and temperature at this point are shown to be, $V_c = 3Nb$, $T_c = \frac{8a}{27k_B b}$ and $P_c = a/(27b^2)$. Show that the constant-*T* compressibility and the constant-*P* thermal expansion coefficient both go to infinity at this point.

6) Consider two sheets of material facing each other as shown. By suitable arrangements of shields and perfect reflectors, assume that all of the radiation leaving one surface arrives at the other surface (or equivalently we could make one surface completely surround the other

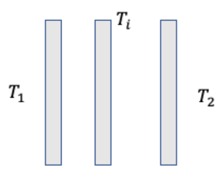


one, in all directions in space). The temperatures are T_1, T_2 and the emissivities $\varepsilon_1, \varepsilon_2$. As noted in class thermal radiation is such that $I = \varepsilon\sigma T^4$, In addition, the reflectivity of a surface relates to the emissivity as, $r = 1 - \varepsilon$.

a) Find a general expression for the intensity of radiation flowing from left to right, in terms of the 4 given quantities. Show that if the temperatures are equal, that there is no net flow of radiation in either direction, showing that the thermal equilibrium condition (which we established from general principles) is upheld for the case of thermal radiation, even for the case that the emissivities are not symmetric.

b) In the case that the temperatures are not equal, find the conditions on $\varepsilon_1, \varepsilon_2$ that yield the maximum energy transfer, and find the intensity of the net transmitted radiation in that case.

c) Consider the situation of the second figure, where an intermediate shield at temperature T_i is inserted. For this case assume that all surfaces have the same emissivity ε, and assume all radiation leaving the left-most sheet arrives on the left surface of the intermediate shield, and vice-versa, with a similar situation for the surfaces on the right. The intermediate shield is held in place by very thin supports which are completely insulating, so its only heat exchange is by radiation. Find the temperature T_i once equilibrium is reached, and find the ratio of the net heat flow between the two sides in this situation vs. the situation where the shield is removed.



d) Finally, consider the situation where there is a series of *N* shields, again thermally insulated and arranged such that all radiation leaving one shield strikes its immediate neighbors. Find the ratio of the net heat flow in this case, vs the situation with no shields. This is the principle of "superinsulation", an arrangement of thin aluminized sheets that is sometimes used for cryogenics.