Physics 408 Problem Set 9   Due Thurs, Nov 11 at beginning of class

due date changed as noted in class. Also see problem 6, now with wording updated.

1) Callen problem 16.2.6, with the additions:
(i) In your answers include the ground state energies per atom for atoms A and B given explicitly as E_{A0} and E_{B0}.
(ii) Add part (c): Identify the low-temperature limit for your answer in part a, and show that it has the expected form, and identify the high-temperature limiting form for the heat capacity, and discuss why this result is also expected from a physical standpoint.

2) Callen 16.7.2, with the added second part:
(b) Starting with the formal result for the Debye-model entropy established in this problem, consider the low-T limit. As explained in class, in this limit for the energy the upper limit of the integral can be replaced by infinity since the decay of the exponential function make the cutoff frequency irrelevant; the same holds for the integrals involved here. Find the entropy in this limit, and as a result show that the ratio ST/U is the same as for thermal photons in a blackbody cavity; I showed this in my slides earlier in the term. This result comes about because of the equivalence of the phonon and photon excitations.

3) a) Consider a gas which obeys the van der Waals equation of state. Could a factorized partition function such as in equation 16.17 of the text (or possibly with a $1/N!$ factor for indistinguishable particles) be used to represent the properties of this gas? Explain. (Note, recall our discussion of the physical meaning of the a parameter in the vdW equation of state.)
b) There are some materials in which a portion of the atoms are loosely bonded to their neighbors. This can happen in some crystals which have relatively open "cages" in which ions can be positioned, or in ice crystals where hydrogens are constrained rather weakly along one direction, a situation which also leads to its nonzero entropy at $T = 0$, mentioned earlier in the term. In these situations the vibrational behavior can be modeled by assuming the Einstein model applies to some of the vibrational modes and the Debye model applies to the rest. Suppose that a solid has N_E Einstein modes, and N_D Debye modes. What do you expect for the high-temperature specific heat in this situation?

4) Consider a distribution of relativistic particles where the thermal energies is such that the mass can be neglected: $\varepsilon = \sqrt{p^2c^2 + m^2c^4} \cong pc$. The large gravitational energy in some collapsed stars leads to such a situation, and for example the crossover from classical to relativistic behavior sets the limit for white dwarf stability.
a) Consider a set of N identical relativistic particles of one particle type, in the dilute limit so that our ideal-gas approximations are valid. Write down the general form of the partition function.
b) Solve for the energy vs. T in the high-temperature limit.
c) Repeat the process for the case of a 1-D relativistic ideal gas. (Electrons in carbon nanotubes in some cases can be treated in an equivalent way.) Find the partition function, and solve for the energy vs. T.
d) As a result, identify a modified equipartition theorem which applies for a Hamiltonian which is linear in the generalized coordinates, rather than quadratic.

5) In a ferromagnet with all spins aligned in the same direction at low temperatures, the quantized normal modes are magnons: local disturbances in spin orientation lead to oscillating

spin waves. The quantized waves have discrete energies $\hbar\omega$ similar to photons or phonons, and the quantized spin waves are called magnons. Magnons can carry information, but without the dissipation that comes with electric currents; this is a possible basis for spintronics as a more efficient platform for next-generation integrated circuits.

a) Unlike phonons, magnons have a dispersion relation $\omega = \alpha k^2$, with α a constant. Also consider that there are two polarizations similar to photons. Calculate the density of modes $D(\omega)$ for this case.

b) Find the temperature exponent for the low-temperature specific heat, e.g. $C \propto T^n$.

6) Ionization of hydrogen: I treated this situation earlier by determining the chemical potentials, but the problem can be treated naturally in the Canonical formalism. Consider for simplicity a 1 cm^3 volume containing one H atom. Assume that the possible states are, (i) the non-ionized atom at a relative energy –13.6 eV, (ii) the ionized electron, an ideal gas. For simplicity assume that the atom, and the ionized proton, are fixed in place so their motion contributes no degrees of freedom. (The masses are the same so their density of states integrals nearly cancel, making this a good approximation.)

(a) At a temperature of 5000 K, calculate the ratio of probability of finding the atom ionized, using the simplified picture that only the Boltzmann factor matters with the ionized atom assumed to have relative energy E = 0, vs. the atom at energy –13.6 eV.

(b) Repeat, but using the more realistic approximation that the ionized electron probability is the sum of the ideal-gas probabilities vs energy, normalized by the partition function. For this you need the density of states in energy for the electron (ideal gas form). Integrate over available energy from zero to infinity to find the total probability that the electron occupies one of the ideal-gas states. Rather than calculating the partition function, find instead the ratio of the ionized vs. non-ionized atom probability, for which Z drops out.

(c) Compare to the result I derived in class; show that they are the same, within a factor of 2 since in our previous treatment the electron spin was not included. [Added note: a further refinement of this calculation should properly include the proton spin, which gives a factor of 2 further increase in the likelihood of ionization, since the density of available ionized states is enhanced; we are neglecting this term for simplicity.]