Notes:

Exam: You can make up some points:

1. Choose 2 problems; blank exam is posted.
2. Work out the solutions from scratch on your own.
3. Turn in the problems by Thursday in class or Friday afternoon 3:30-4 PM; I will arrange be at my office or you can email if necessary. Not in my mailbox
4. You will get 60% of the made-up points back.

Reading: This week we continue with chapter 16 (continuum systems densities of states; Debye model for vibrations). Next up, phase transformations (chapters 8-9).

Density of states: for summations involving only ω (or E).

phonons

$$U = \sum_{all\ modes} \frac{\hbar\omega_i}{(e^{\beta\hbar\omega_i} - 1)} \Longrightarrow \mathbf{3}\int_0^\infty \begin{pmatrix} \#k\ states \\ in\ (\omega, \omega + d\omega) \end{pmatrix} \times \frac{\hbar\omega}{(e^{\beta\hbar\omega} - 1)}$$

$$\equiv \int_0^\infty \frac{\hbar\omega D(\omega) d\omega}{(e^{\beta\hbar\omega} - 1)}$$

1) Find # states inside a sphere (octant) in k space: $N(k)$.
   - This includes polarizations.
   - Anisotropic situations: replace sphere by constant-ω surface
2) Convert to ω units: $N(\omega)$.
3) $D(\omega)$ is the derivative, $D(\omega) = dN(\omega)/\mathrm{d}\omega$, equal to total # modes in $(\omega, \omega + d\omega)$.

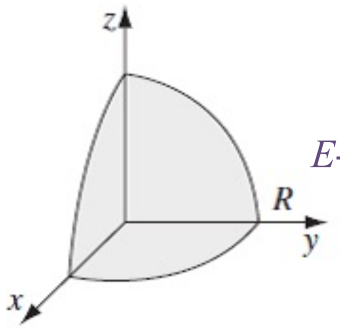
Density of states definition (similar procedure for $D(E)$).

$\Delta k = \frac{\pi}{L}$ -> $V_k = \left(\frac{\pi}{L}\right)^3$ for octant;

$= \left(\frac{2\pi}{L}\right)^3$ for complete sphere traveling-waves

State counting:

cavity modes in a cubic box, dimensions L:



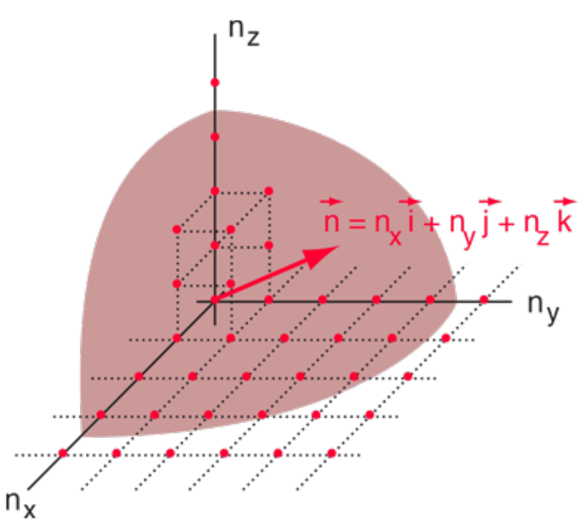
$$E_x \propto cos\left(n_x \frac{\pi}{L} x\right) sin\left(n_y \frac{\pi}{L} y\right) sin\left(n_z \frac{\pi}{L} z\right) \text{ etc.}$$

E-field

$$\Delta k = \frac{\pi}{L} \text{ -> } V_k = \left(\frac{\pi}{L}\right)^3$$

One k-vector per volume element; same as "Phase space volume" $h^3/8$

Octant of sphere; but with 8× state density of traveling-wave momentum space.



Text: abstract space has integer dimensions. Similar counting procedure in chapter 15, hyperspace consideration of multiplicities.
My wavevector-space notation: same n_x, n_y, n_z, multiplied by $\frac{\pi}{L}$ (or $\frac{2\pi}{L}$), otherwise the counting is the same.

Density of states: for summations involving only ω (or E).

phonons

$$U = \sum_{all\ modes} \frac{\hbar\omega_i}{(e^{\beta\hbar\omega_i} - 1)} \Longrightarrow 3\int_0^\infty \begin{pmatrix} \#k\ states \\ in\ (\omega, \omega + d\omega) \end{pmatrix} \times \frac{\hbar\omega}{(e^{\beta\hbar\omega} - 1)}$$

$$\equiv \int_0^\infty \frac{\hbar\omega D(\omega) d\omega}{(e^{\beta\hbar\omega} - 1)}$$

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Result for phonons in isotropic solid:

$$D(\omega) = \frac{3\omega^2 V}{2\pi^2 c^3}$$

$\Delta k = \frac{\pi}{L}$ -> $V_k = \left(\frac{\pi}{L}\right)^3$ for octant;

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Phonons:

$$Z = \prod_{all\ modes} Z_i = \prod_{all\ modes} \sum_{n=0}^{\infty} e^{-\beta n \hbar \omega_i} = \prod_{all\ modes} \frac{1}{1 - e^{-\beta \hbar \omega_i}}$$

(not same as all atoms; factor of 3 here)

(Z_i we saw last time)

$$\langle U \rangle = \sum_{all\ modes} \frac{\hbar \omega_i}{(e^{\beta \hbar \omega_i} - 1)}$$

Phonons:

continuum limit

$$\langle U \rangle = \sum_{all\ modes} \frac{\hbar\omega_i}{\left(e^{\beta\hbar\omega_i}-1\right)} \Longrightarrow \int_0^{\omega_{max}} \frac{\hbar\omega D(\omega)d\omega}{\left(e^{\beta\hbar\omega}-1\right)} = \int_0^{\omega_D} \frac{3V\hbar\omega^3 d\omega}{2\pi^2 c^3\left(e^{\beta\hbar\omega}-1\right)}$$