Notes:

Homework : Due next Tuesday. (I am not sure yet about presentations, depends upon timing.)

Last class day: Weds. Dec 8.

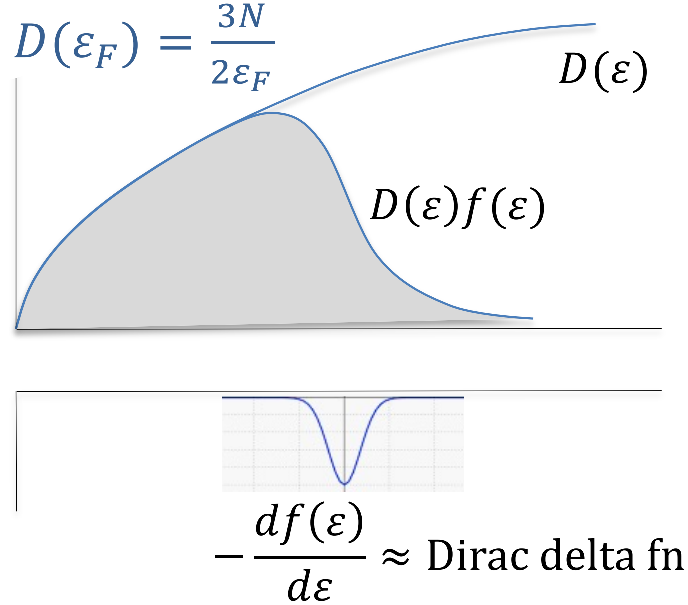
Final Exam: Friday Dec. 10, 12:30 PM, in room 203. Exam will be comprehensive, with no particular focus on new material. A formula sheet will be allowed, similar to the previous exam. More details about this tomorrow.

Fermi gas T dependence:

$$D(\varepsilon) = \frac{V}{(2\pi^2)}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon} \qquad \varepsilon_F = \frac{\hbar^2}{(2m)}\left(3\pi^2\frac{N}{V}\right)^{2/3}$$

& note, $D(\varepsilon_F) = \frac{3N}{2\varepsilon_F}$

$$f(\varepsilon) = \frac{1}{\left(1 + e^{(\varepsilon-\mu)/kT}\right)}$$



$-\frac{df}{d\mu}$

$$N = \int D(\varepsilon)f(\varepsilon)d\varepsilon = -\int\left[\int_{-\infty}^{\varepsilon} D(\varepsilon')d\varepsilon'\right]\frac{df}{d\varepsilon}d\varepsilon$$

by parts

$$N = \int\left[\int_{-\infty}^{\mu} D(\varepsilon')d\varepsilon'\right]\frac{df}{d\mu}d\varepsilon + \int [D(\mu)](\varepsilon-\mu)\frac{df}{d\mu}d\varepsilon + \int\left[\frac{dD(\mu)}{d\mu}\right]\frac{(\varepsilon-\mu)^2}{2}\frac{df}{d\mu}d\varepsilon + \cdots$$

Taylor expand square bracket

0, odd function

$$N \approx \int_{-\infty}^{\mu} D(\varepsilon')d\varepsilon' + (kT)^2\frac{\pi^2}{6}\left[\frac{dD(\mu)}{d\mu}\right]$$

Yesterday's result

$$\left.\frac{\partial\mu}{\partial T}\right]_N = -\left.\frac{\partial N}{\partial T}\right]_\mu \Big/ \left.\frac{\partial N}{\partial \mu}\right]_T = -\frac{Vk^2T2\frac{\pi^2}{6}\left[\frac{dD(\mu)}{d\mu}\right]}{VD(\mu) + (small)}$$

Fermi gas T dependence:

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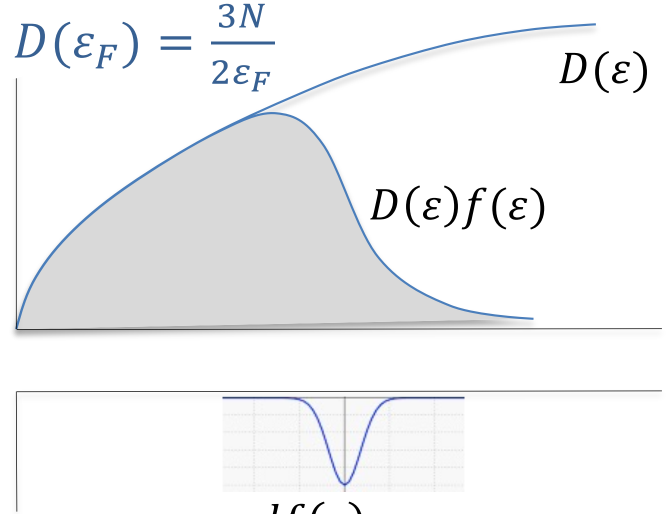

$$-\frac{df(\varepsilon)}{d\varepsilon} \approx \text{Dirac delta fn.}$$

$$f(\varepsilon) = \frac{1}{(1 + e^{(\varepsilon-\mu)/kT})}$$

$$\left.\frac{\partial\mu}{\partial T}\right]_N = -\frac{Vk^2T2\frac{\pi^2}{6}\left[\frac{dD(\mu)}{d\mu}\right]}{VD(\mu) + (small)}$$

with $\frac{dD(\mu)}{d\mu} \to \frac{3N}{4\varepsilon_F^2}$ and $D(\mu) \to \frac{3N}{2\varepsilon_F}$

So,

$$\mu \cong \mu(T=0) + T\left.\frac{\partial\mu}{\partial T}\right]_N$$

Fermi gas T dependence:

$$D(\varepsilon) = \frac{V}{(2\pi^2)}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon} \qquad \varepsilon_F = \frac{\hbar^2}{(2m)}\left(3\pi^2\frac{N}{V}\right)^{2/3}$$

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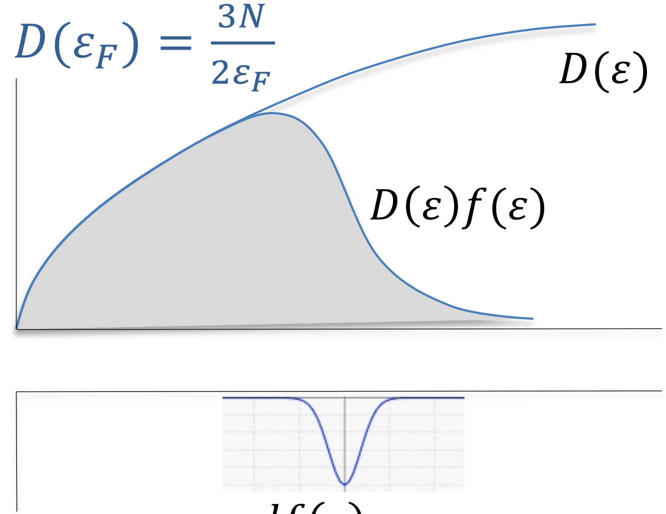


$$-\frac{df(\varepsilon)}{d\varepsilon} \approx \text{Dirac delta fn.}$$

$$f(\varepsilon) = \frac{1}{(1 + e^{(\varepsilon-\mu)/kT})}$$

$$\left.\frac{\partial\mu}{\partial T}\right]_N = -\frac{Vk^2T2\frac{\pi^2}{6}\left[\frac{dD(\mu)}{d\mu}\right]}{VD(\mu) + (small)}$$

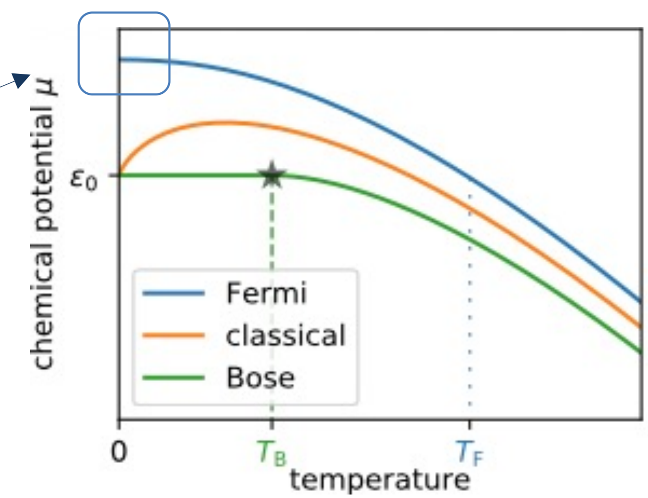
$\left[\frac{dD(\mu)}{d\mu}\right] = \frac{3N}{4\varepsilon_F^2}$, $\quad D(\mu) = \frac{3N}{2\varepsilon_F}$

So,

$$\mu \cong \mu(T=0) + T\left.\frac{\partial\mu}{\partial T}\right]_N = \varepsilon_F - k_B^2\frac{\pi^2}{12}\frac{T^2}{\varepsilon_F^2}$$

$$\mu \cong \varepsilon_F\left(1 - \frac{\pi^2}{12}\left[\frac{T}{T_F}\right]^2\right)$$

(reminder, this is for fixed-N case)



Fermi gases:

$$\mu \cong \varepsilon_F \left(1 - \frac{\pi^2}{12}\left[\frac{T}{T_F}\right]^2\right)$$

similar expansion for energy:

$$-\frac{df}{d\mu}$$

$$E = \int \varepsilon D(\varepsilon) f(\varepsilon) d\varepsilon = -\int \left[\int_{-\infty}^{\varepsilon} \varepsilon' D(\varepsilon') d\varepsilon'\right] \frac{df}{d\varepsilon} d\varepsilon$$

by parts

$$E = \int \left[\int_{-\infty}^{\mu} \varepsilon' D(\varepsilon') d\varepsilon'\right] \frac{df}{d\mu} d\varepsilon + \int [\mu D(\mu)](\varepsilon - \mu) \frac{df}{d\mu} d\varepsilon$$

$$+ \int \left[\frac{d[\mu D(\mu)]}{d\mu}\right] \frac{(\varepsilon - \mu)^2}{2} \frac{df}{d\mu} d\varepsilon + \cdots$$

Taylor expand square bracket

$$E \approx \int_{-\infty}^{\varepsilon_F} \varepsilon D(\varepsilon) d\varepsilon + \cancel{[\varepsilon_F D(\varepsilon_F)]}(\mu - \varepsilon_F) + (kT)^2 \frac{\pi^2}{6} D(\varepsilon_F) + \cancel{(kT)^2 \frac{\pi^2}{6}} D(\varepsilon_F) \left.\frac{dD}{d\mu}\right]_{\varepsilon_F}$$

2 terms cancel

$$E \approx E_o + (kT)^2 \frac{\pi^2 N}{4\varepsilon_F}$$

Fermi gases:

$$E \approx E_o + (kT)^2 \frac{\pi^2 N}{4\varepsilon_F}$$

$$C_V = N k_B \frac{\pi^2}{2} \frac{T}{T_F} \equiv \gamma T$$

- Characteristic result for Fermion systems: exclusion principle strongly reduces thermal excitations.

- In metals, *C* much smaller than *phonon* term, except for very low *T*, where $C_V = \gamma T + \beta T^3$ (recognized as *Fermi* + *Debye* terms).

- *Entropy* also remains very small, for $T \ll T_F$ degenerate situation (e.g. typically *all temperatures* for metals).

Saw this before:

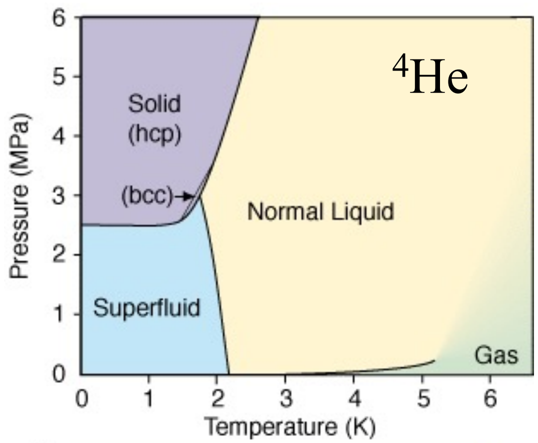
& recall, $\frac{L}{T} = \Delta S$

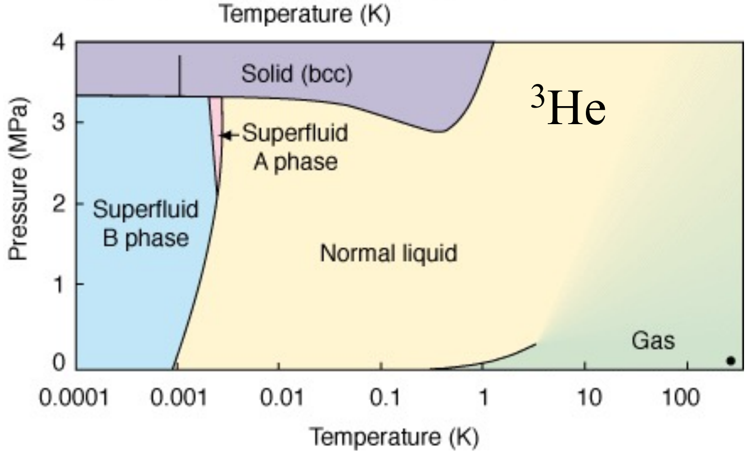
- Clausius-Clapeyron relation

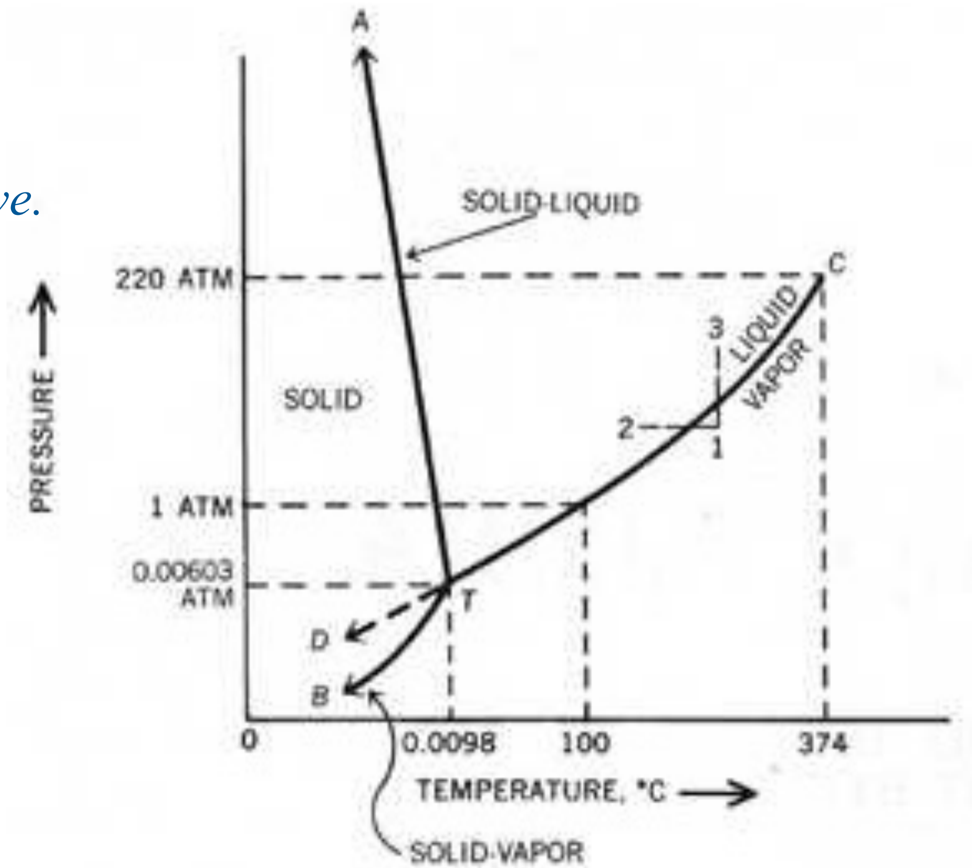
$$dG = -SdT + VdP$$

$$\frac{dP}{dT} = \frac{L}{T\Delta V}$$

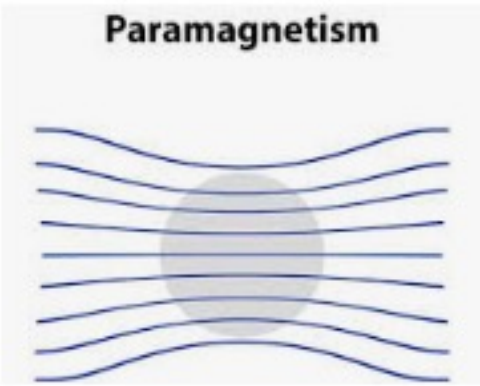
Describes slope *following coexistence curve.*







Pauli Paramagnetism:



Can show,

$$\chi_P = \frac{1}{V}\mu_B^2 D(\varepsilon_F)$$

$$= \frac{3N\mu_B^2}{2Vk_BT_F}$$

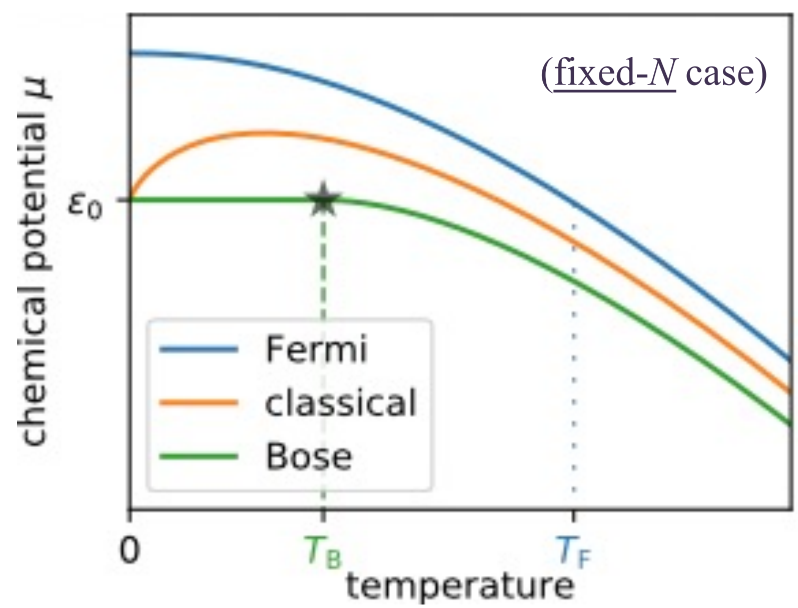
- Recall Curie law, $\chi_C = \frac{N\mu_B^2}{V4k_BT}$

  (*nondegenerate* independent spins)
- χ_P much smaller: Pauli exclusion strongly limits probability for promotion of particles to unfilled states, inhibits magnetism.
- HW problem: interactions can overcome the large KE barrier.

Bose gases; Bose-condensation

$$N = \int_0^\infty D(\varepsilon)\frac{1}{e^{\beta(\varepsilon-\mu)}-1}d\varepsilon$$

Bose distribution showed last week



Note, $\varepsilon > \mu$ required, all states, required from partition function derivation

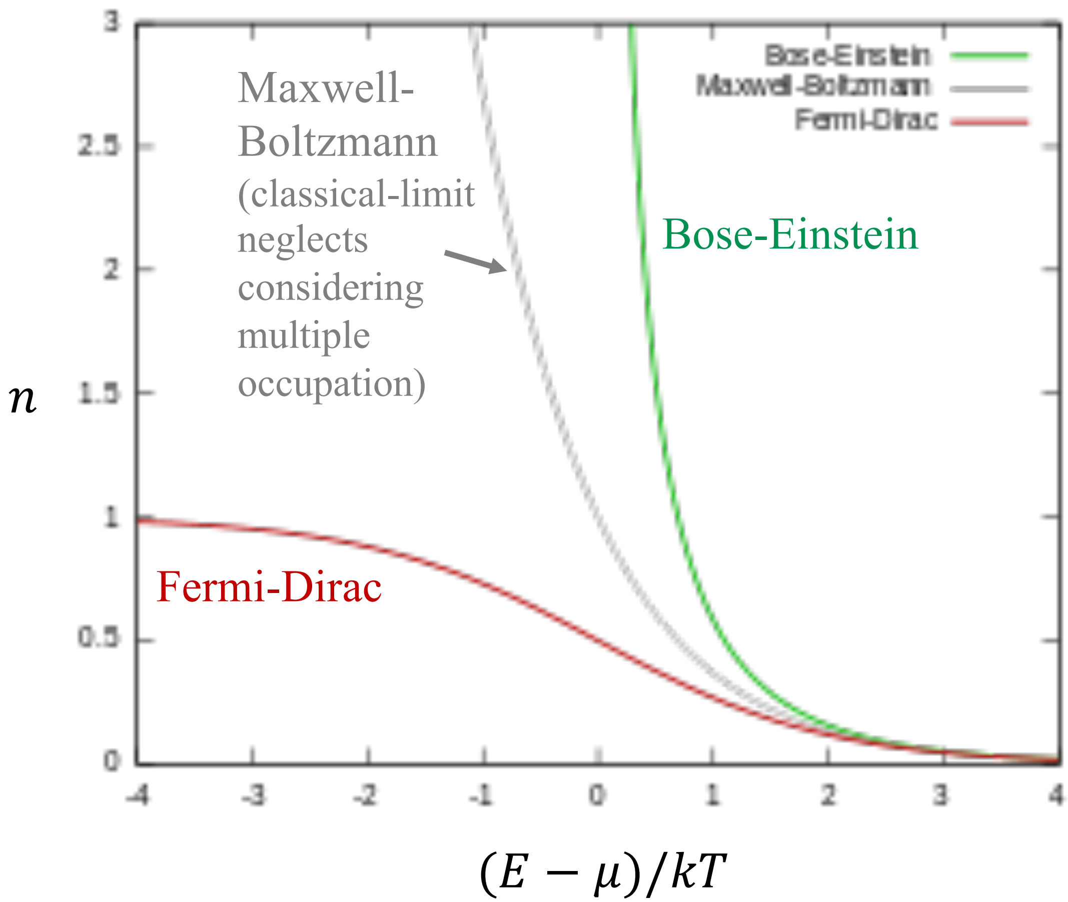
$D(\varepsilon)$ is same function we use for Fermions (except with spin factor removed if $J = 0$ Bosons).

[& assuming here, lowest energy state is $\varepsilon_o \equiv 0$.]

(Plot I showed last week)

$$n_{BE} = \frac{1}{(e^{(E-\mu)/kT} - 1)}$$

Bose-Einstein distribution = $\langle N \rangle$ for a single eigenstate.



Bose gases; Bose-condensation

$$N = \int_0^\infty D(\varepsilon)\frac{1}{e^{\beta(\varepsilon-\mu)}-1}d\varepsilon$$

- High T, requires increasingly more negative μ to conserve N.
- We will see, Bose condensation below well-defined T_c (2nd order phase transition): large number occupies ground state at low T.
- Weakly interacting gas cases: particles all same Q.M. phase;

$$\Psi = \psi_o(r_1)\psi_o(r_2)\,\psi_o(r_3)\ldots$$