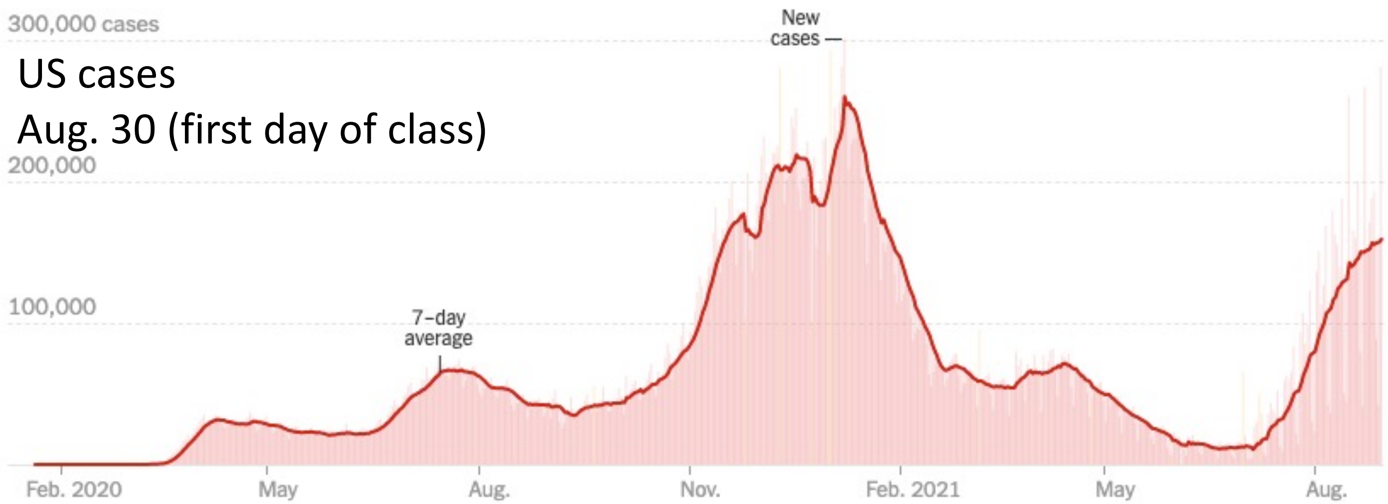
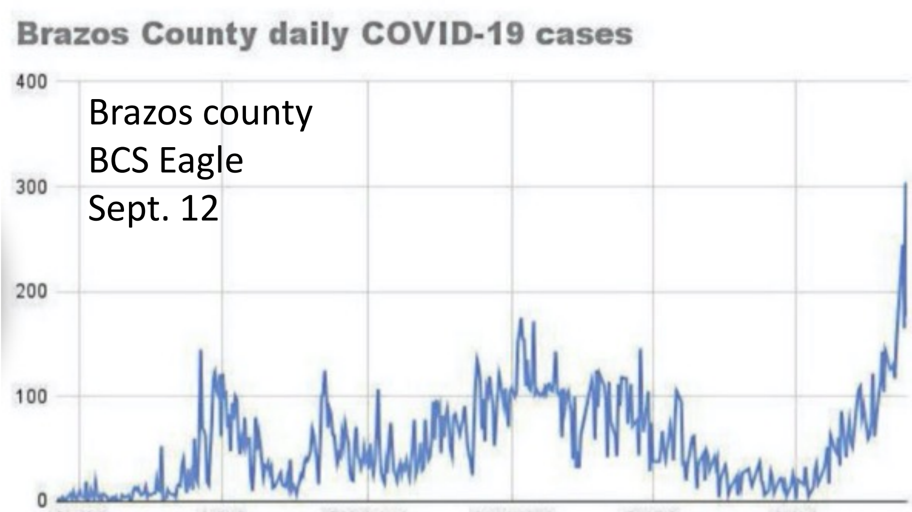
Notes for today

- Reading: This week finishing chapter 15, then we continue with (the remainder of) chapter 2.
- Homework 2: Volunteer still needed for problem 5.
- Lecture recordings etc.: Reminder again that you should let me know if you have a Covid quarantine (or other University excuse). I can share lecture recording or a zoom link to view the lecture in real time.
- Enable the microphone

Covid safety:



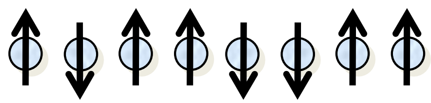


Local cases: Currently worst ever
They will get better!
For now we all must do what we can to minimize the danger.

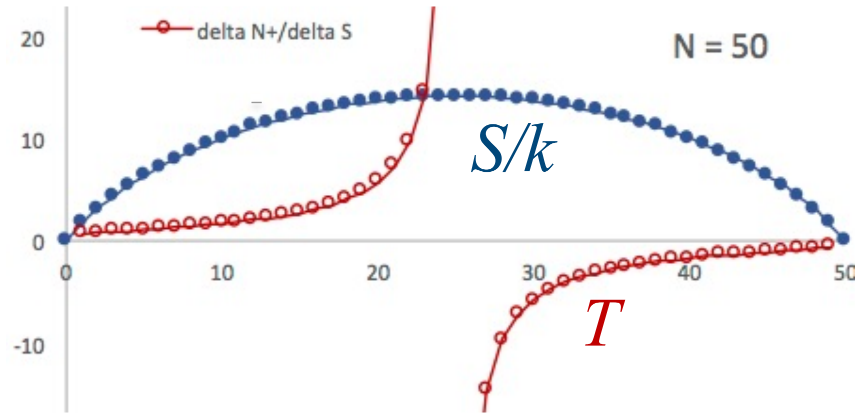
spin-1/2 non-interacting paramagnet

(microcanonical ensemble solution):

$U = \pm\mu B$ per atom -> $U = \mu B(N^- - N^+) = \mu B(2N^- - N)$



$$S = k_B ln\left(\frac{(N)!}{(N^+)!\,(N^-)!}\right)$$



• *microcanonical* means constant-*U*, *V*, and *N* conditions (*canonical* means constant-*T, V, N*). More on this later; formal treatment applicable to different ensembles starts ch. 5; canonical formalism discussed ch. 16.

• Plan: – Determine entropy *S* (*U,N*) (there is no *V* in this case).
– Then from *S* (*U,N*) determine *T* and other physical properties.

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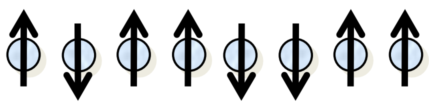

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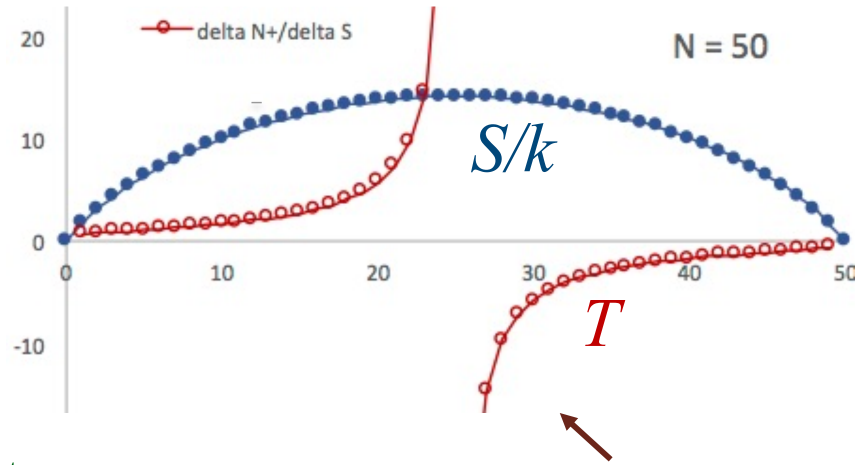

$$\frac{N^-}{N^+} = e^{-{2\mu B}/{kT}}$$

True in large-N limit;
Also this is equivalent to
Boltzmann distribution or
Canonical probability
distribution (later chapter)

Note negative T solutions:
Stable & behave normally
vs heating and cooling,
can show.

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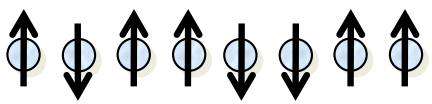

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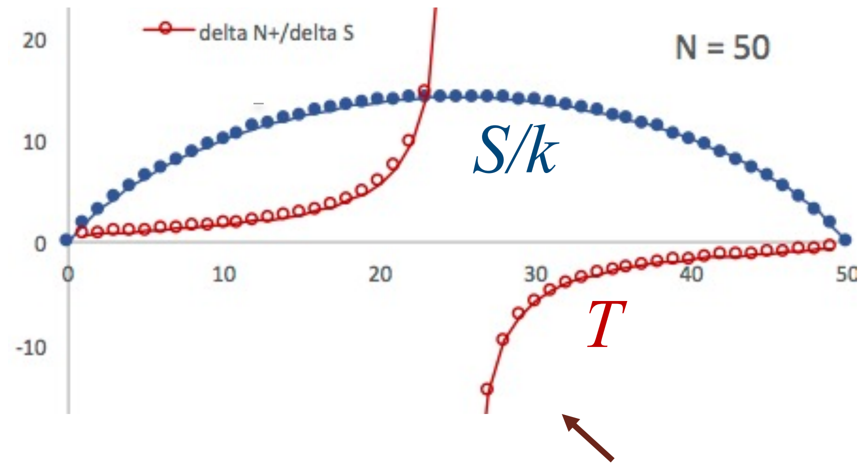

$$\frac{N^-}{N^+} = e^{-2\mu B/kT}$$

solving:

$$U = -\mu BNtanh(\mu B/kT)$$

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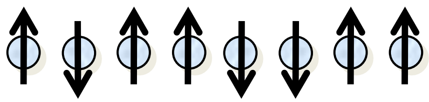
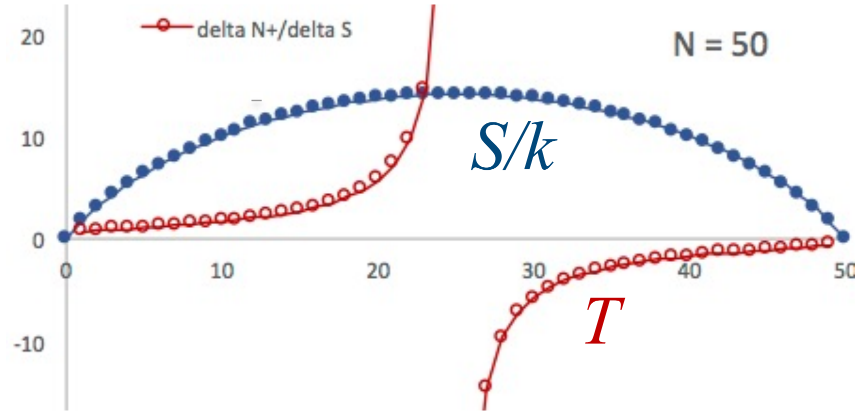


$$S = k_B ln\left(\frac{(N)!}{(N^{+})!\,(N^{-})!}\right)$$

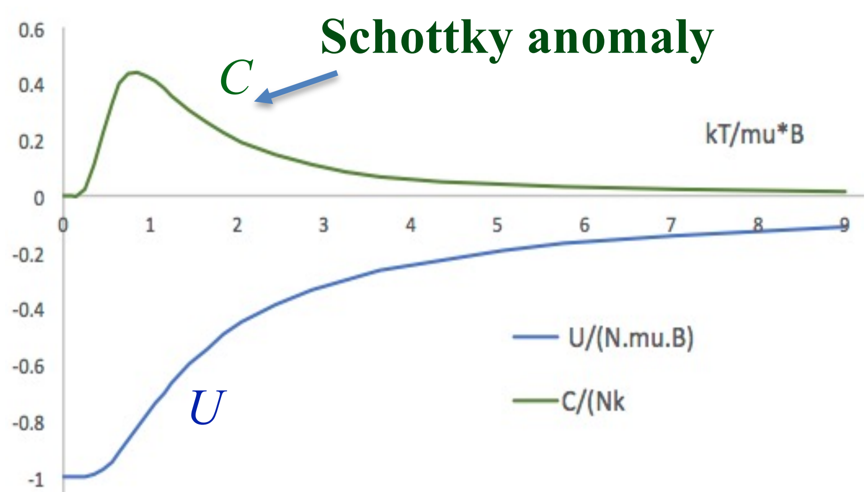


$$\frac{N^{-}}{N^{+}} = e^{-{2\mu B}/{kT}}$$

solving:

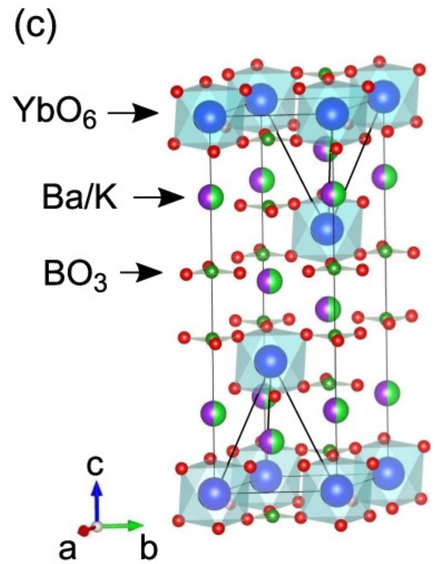
$$U = -\mu BNtanh(\mu B/kT)$$

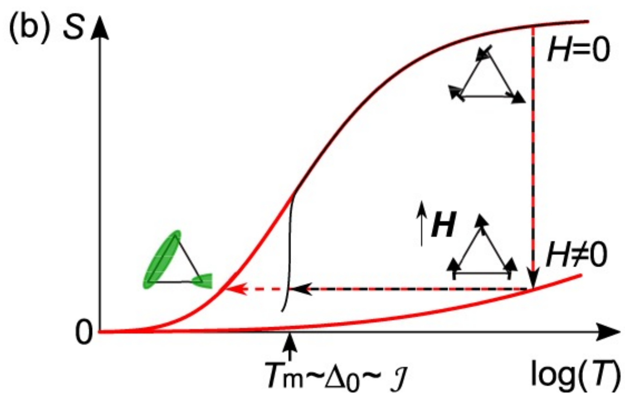
$$C = \frac{\partial U}{\partial T} = Nk_B\frac{(2\mu B/kT)^2}{(e^{\mu B/kT} + e^{-\mu B/kT})^2}$$

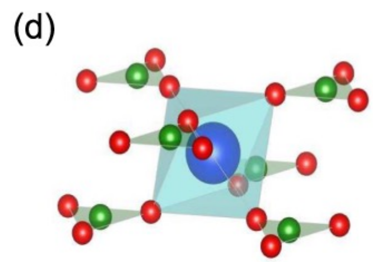


Paramagnetic cooling (Adiabatic Demagnetization)

- One example, recent work proposing improved cooling material
- “Paramagnet” in this context means material has “free” magnetic moments that don’t order

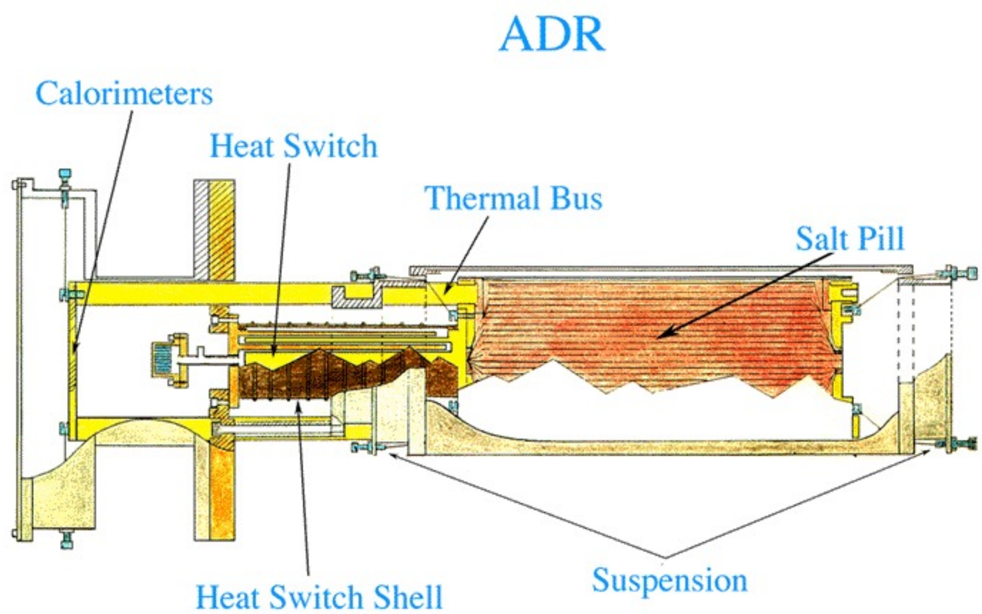






Tokiwa, Y., Bachus, S., Kavita, K. *et al.* Frustrated magnet for adiabatic demagnetization cooling to milli-Kelvin temperatures. *Commun Mater* **2,** 42 (2021).

NASA adiabatic demagnetization refrigerator for cooling x-ray detectors &c.
“Salt” is the paramagnet [ferric ammonium sulfate, has iron atomic moments].



Hypersphere counting argument (text)

• Problem: choose arbitrary U, will generally include <u>no</u> microstates due to discreteness of state energies.
• Entropy $S(U,V,N)$: smooth function in principle, but really not differentiable due to discrete counting; may be very rough on fine scale.
• Solution: choose a range of energies, between $(U\text{–}\Delta U)$ and U rather than fixed U.

For large N: find that for arbitrarily small ΔU, result includes same number of states as for $\Delta U=U$!

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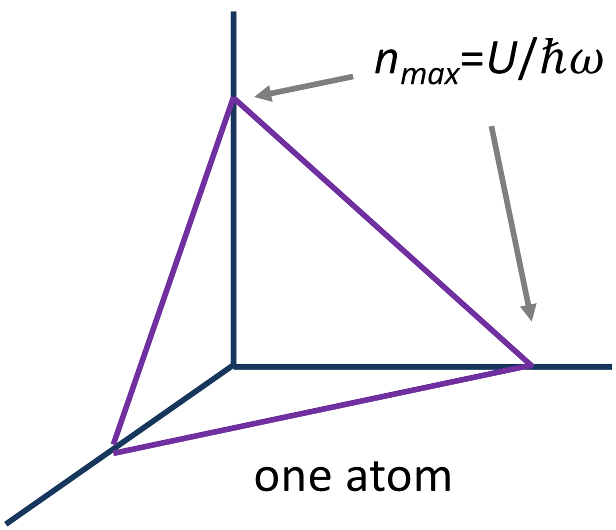
Einstein solid



$$\Omega = \frac{1}{6}(U/\hbar\omega)^3$$

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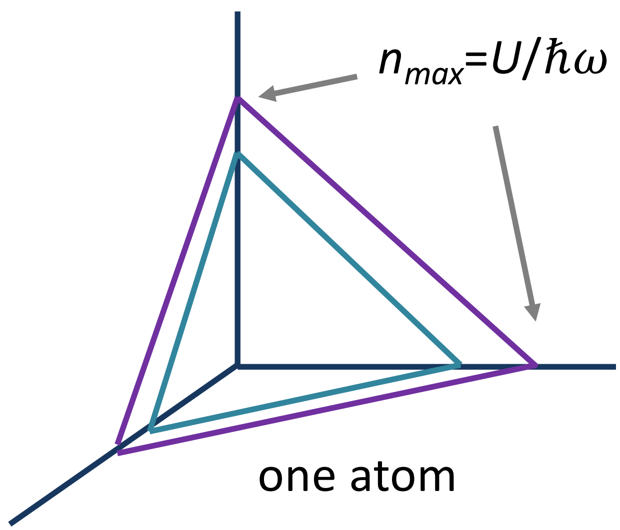

$$\Omega = \frac{1}{6}(U/\hbar\omega)^3 \quad \Rightarrow \quad \frac{1}{(3N)!}(U/\hbar\omega)^{3N}$$