Mixing by Barotropic Instability in a Nonlinear Model

KENNETH P. BOWMAN

Climate System Research Program, Department of Meteorology, Texas A&M University, College Station, Texas

PING CHEN

Department of Atmospheric Sciences, University of Washington, Seattle, Washington

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ABSTRACT

A global, nonlinear, equivalent barotropic model is used to study the isentropic mixing of passive tracers by barotropic instability. Basic states are analytical zonal-mean jets representative of the zonal-mean flow in the upper stratosphere, where the observed 4-day wave is thought to be a result of barotropic, and possibly baroclinic, instability. As is known from previous studies, the phase speed and growth rate of the unstable waves is fairly sensitive to the shape of the zonal-mean jet; and the dominant wave mode at saturation is not necessarily the fastest growing mode; but the unstable modes share many features of the observed 4-day wave. Lagrangian trajectories computed from model winds are used to characterize the mixing by the flow. For profiles with both midlatitude and polar modes, mixing is stronger in midlatitudes than inside the vortex; but there is little exchange of air across the vortex boundary. There is a minimum in the Lyapunov exponents of the flow and the particle dispersion at the jet maximum. For profiles with only polar unstable modes, there is weak mixing inside the vortex, no mixing outside the vortex, and no exchange of air across the vortex boundary. These results support the theoretical arguments that, whether wave disturbances are generated by local instability or propagate from other regions, the mixing properties of the total flow are determined by the locations of the wave critical lines and that strong gradients of potential vorticity are very resistant to mixing.

1. Introduction

The 4-day wave is an eastward propagating planetary wave disturbance in the polar winter upper stratosphere. Since it was discovered by Venne and Stanford (1979), a number of authors have used observational (Venne and Stanford 1982; Prata 1984; Lait and Stanford 1988; Manney 1991) and theoretical (Hartmann 1983; Manney et al. 1988; Manney and Randel 1993; Ishioka and Yoden 1993) approaches to investigate the dynamics and origin of the wave. Randel and Lait (1991) summarized the previous observational studies and examined a single month of high-quality data. They noted that the wave has three salient characteristics:

1) It appears in the upper stratosphere in both winter hemispheres, but is stronger in the Southern Hemisphere (SH). The maximum variance is located between 60° and 70° latitude and between approximately 1 and 10 mb.

2) It has an equivalent barotropic vertical structure.

3) It is dominated by wavenumber 1, but actually consists of at least waves 1–4 traveling with the same phase speed to form "warm pools" that circle the pole with roughly a 4-day period (Prata 1984; Lait and Stanford 1988).

On the basis of the observed heat and momentum fluxes, they concluded that, during their period of study, the source of the wave was predominantly barotropic instability, although one event during their period of analysis had a significant baroclinic component.

Several authors have examined instability of idealized and observed climatological zonally averaged jets as a potential source for the 4-day wave. Hartmann (1983) found linearly unstable barotropic modes similar to the 4-day wave for analytical wind profiles. The modes fell into two categories: 1) polar modes and 2) midlatitude modes. The two types of modes are associated with the reversed potential vorticity (PV) gradients on the poleward and equatorward flanks of the jet, respectively. The most unstable polar modes had zonal wavenumbers 1 and 2 with periods on the order of 3–4 days and 1.5–2 days, respectively. The most unstable midlatitude modes were wavenumbers 1 through 3, but the periods were on the order of one week.

Manney et al. (1988) examined the barotropic stability of asymmetric analytical jet profiles and of observed time-averaged profiles at 10, 5, and 2 mb for
SH winter. They found good agreement between the characteristics of the unstable modes and the observed waves. In particular, they noted that during one month when the 4-day wave was not observed in the data, the wind profile produced unstable midlatitude modes at wavenumbers 3 and 4 that grew much faster than the polar modes.

Manney and Randel (1993) used a three-dimensional instability model to study the relative importance of barotropic and baroclinic instability in producing the 4-day wave. They concluded that both horizontal and vertical shear are important to the instability, at least for time-averaged wind profiles, since growth rates are very slow when one shear or the other is removed. They noted that for instantaneous wind profiles, either barotropic or baroclinic instability alone may be sufficient to produce significant growth rates.

Ishioika and Yoden (1993) studied the nonlinear interaction and saturation of unstably growing waves in a barotropic model with Hartmann-like wind profiles. In the nonlinear case they found that the dominant wave usually had smaller wavenumber than the most unstable wave for a given unstable initial condition. Mixing occurred by stretching of the fluid into long narrow filaments that were wrapped up around the vortex or within local eddies until the spatial scale of the filaments was too small to resolve.

The presence of planetary-scale waves with significant amplitudes in the winter upper stratosphere has implications for tracer transport and chemistry. Photochemical lifetimes in sunlight at these levels are typically brief, while lifetimes within the polar night may be much longer. Large amplitude waves can transport trace constituents in and out of the polar night, but may or may not effectively mix tracers on isentropic surfaces.

McIntyre (1989) argued that the stratosphere is well mixed in the “surf zone” by breaking planetary waves, but that the large potential vorticity gradient on the poleward side of the surf zone is resistant to mixing by wavebreaking through the Rossby wave restoring mechanism. Hartmann et al. (1989) and Schoeberl et al. (1989, 1992) analyzed data from the Antarctic Airborne Ozone Expedition and Arctic Airborne Stratosphere Expedition and concluded that little mixing occurs across the boundary of the vortex. Bowman (1993a) used isentropic trajectories computed from nine years of National Meteorological Center stratospheric analyses to show that the large-scale flow induces very little mixing across the boundary of the polar vortices in the lower stratosphere. Chen et al. (1994) found similar results using semi-Lagrangian and contour-advective techniques with U.K. Meteorological Office analyses for 1992. To date, most studies have concentrated on the ozone hole region in the lower stratosphere. In the lower stratosphere the large-scale waves are primarily forced waves that propagate upward from the upper troposphere. In the middle and upper stratosphere, on the other hand, barotropic or baroclinic instability may be an additional in situ source of planetary-scale waves. The purpose of this paper is to examine the evolution of unstable modes in a nonlinear model, and to characterize the isentropic mixing associated with the growing waves at finite amplitude.

2. Methods

a. Model

The model used for the numerical experiments was developed by Salby et al. (1990a). The principal features are reviewed briefly here. By integrating vertically in isentropic coordinates and assuming that the vertical scale of the motion field is large compared to the vertical scale of the mass field, the primitive equations can be reduced to a two-dimensional system similar to the shallow-water equations. The model variables are the column-averaged horizontal wind velocities $u$ and $v$; a thickness variable $h$, which corresponds to the pressure on the lower-bounding material surface through hydrostatic equilibrium; and the potential temperature $\theta$. The model uses the spectral transform method, and the basis functions for $u$, $v$, and $h$ are chosen to be Hough functions, which are the eigenfunctions of the linearized shallow-water equations on the sphere. The $\theta$ equation is solved by using spherical harmonics as basis functions.

Diabatic processes are represented by linear relaxation (Newtonian cooling) to a zonally symmetric basic state in the $\theta$ equation. Experiments are conducted with radiative damping times of 5 and 10 days, and with no radiative damping. Since the advective timescale $\tau = 2\pi a_0 \cos \phi_0 / U_0$ is on the order of 1–2 days, the radiative damping has little effect on the results, although the peak amplitude of the waves is slightly smaller. Therefore, results are presented here from runs with no explicit radiative damping. The model represents small-scale mixing of momentum by an eddy viscosity term in the $u$ and $v$ equations, and also includes a scale-dependent dissipation applied in the spectral domain. For all of the experiments described here, the viscosity is set to zero (no explicit diffusion), the spectral dissipation is proportional to $\nabla^2$, and the dissipation timescale for the largest wavenumbers is 1 hour.

The model can represent motions that are relatively deep, that is, on the order of a scale height or more, which is characteristic of the 4-day wave. All experiments here are carried out at H40 resolution horizontally (roughly equivalent to R40 truncation for spherical harmonic models), which uses a $128 \times 128$ point transform grid to avoid nonlinear aliasing. Salby et al. (1990a,b) have discussed the model behavior and transport properties. Mixing in the Lagrangian sense can occur even for smooth flows (Ottino 1989), and this paper concentrates on the large-scale behavior of the flow and gross properties of mixing. Bowman (1993b) used the model to study mix-
ing due to wave–mean flow interaction in the lower Antarctic stratosphere.

b. Basic and initial states

The basic and initial states used for the instability calculations are taken from the analytical profile of Manney et al. [1988, Eq. (7)] or Hartmann (1983). The model is initialized with a zonally symmetric jet in geostrophic balance. Initially a small random perturbation is added to the velocity field at each grid point. No attempt is made to ensure geostrophic balance or nondivergence in the perturbation.

c. Trajectories

Lagrangian trajectories are computed offline using a standard fourth-order Runge–Kutta scheme with winds saved four times per day (Bowman 1993a). Velocity
components are interpolated to the locations of particles by linear interpolation in space and time. Trajectories near the pole are computed in a local Cartesian coordinate system to avoid problems with the singularity at the pole. Trajectories are calculated for large numbers of particles (typically $128^2 = 16384$) initially arranged on a regular longitude–latitude grid in the Southern Hemisphere.

The precision of the numerical integration scheme is evaluated by varying the time step size and integrating from the same initial condition. These calculations suggest that the error in the position of the particles resulting from numerical truncation error is less than $0.5^\circ$ of great circle arc ($\sim 50$ km) after 10 days.

d. Mixing diagnostics

Mixing can be characterized in several ways. Intuitively, if fluid parcels that are initially close together
become widely scattered throughout the domain after some time, the fluid is likely to have been well mixed. Conversely, if the parcels are not widely scattered, then they have followed similar trajectories and the mixing has likely been weak. The rate of separation of trajectories with nearby initial conditions can be quantified by means of Lyapunov exponents. The separation of fluid parcels in physical space is equivalent to the separation of trajectories in phase space in the study of dynamical systems and chaos. The Lyapunov exponents in a fluid are related to the local stretching deformation of the flow in a Lagrangian sense (i.e., following a fluid parcel). The Lyapunov exponents are defined as
TABLE 1. E-folding time (days), wave period (days), and phase speed (m s$^{-1}$) during the initial exponential growth phase at (a) 79°S and (b) 50°S estimated graphically from amplitude and phase plots.

<table>
<thead>
<tr>
<th>Wave</th>
<th>E-folding time</th>
<th>Period</th>
<th>Phase speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 79°S</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.76</td>
<td>4.0</td>
<td>22.1</td>
</tr>
<tr>
<td>2</td>
<td>1.09</td>
<td>2.5</td>
<td>17.7</td>
</tr>
<tr>
<td>3</td>
<td>2.06</td>
<td>1.7</td>
<td>17.3</td>
</tr>
<tr>
<td>4</td>
<td>1.74</td>
<td>2.5</td>
<td>8.8</td>
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<tr>
<td>5</td>
<td>1.52</td>
<td>1.5</td>
<td>11.8</td>
</tr>
<tr>
<td>6</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>(b) 50°S</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.87</td>
<td>3.5</td>
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</tr>
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</tr>
<tr>
<td>6</td>
<td>2.61</td>
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</tr>
</tbody>
</table>

\[ \lambda_i(X, M_i) = \lim_{t \to \infty} \left[ \frac{1}{t} \ln \left( \frac{|dX|}{|dX|} \right) \right], \quad (1) \]

where $X$ is the initial location, $dX$ is a vector located at $X$ with orientation $M_i$, and $dX$ is the vector at time $t$ (Ottino 1990, p. 116). Positive Lyapunov exponents indicate exponential separation of trajectories, a good indicator of efficient mixing. In practice, the Lyapunov exponents are estimated for finite times by tracking the separation of particles initially located very close together (Pierrehumbert 1991; Pierrehumbert and Yang 1993). A Lyapunov exponent of 0.2 day$^{-1}$, characteristic of the stratospheric surf zone, would indicate an accumulated stretching deformation of a fluid parcel by a factor of $\sim 400$ over the course of a month (Bowman 1993a). Particle separations are periodically renormalized to ensure that the trajectories are representative of local strain rates, rather than of flow in widely separated regions of the fluid; and Lyapunov exponents are computed for two different initial orientations (north-south and east-west).

An additional property of interest, also indicative of mixing, is the meridional dispersion of particles, which can be quantified here by the variance of the meridional displacement from the Lagrangian mean latitude. For a particle with latitude $\eta$, the deviation from the Lagrangian mean is $\eta' = \eta - \langle \eta \rangle$, where angle brackets represent the mean over an ensemble of particles. The dispersion is defined as the variance $\langle \eta'^2 \rangle$ of the latitudes of an ensemble of particles initially located around a latitude circle. This quantity evolves with time as the particles follow their individual trajectories. For linear

![Fig. 4. Zonal-mean wind (a) and zonal-mean potential vorticity (b) as a function of latitude at 10-day intervals during the integration.](image-url)
waves the time derivative of the dispersion can be directly related to diffusion coefficients (Andrews et al. 1987).

e. Diffusive parameterization

Diffusion coefficients \( K_{yy} \) can be estimated from the dispersion statistics. For linear waves the diffusion coefficient is

\[
K_{yy} = \frac{1}{2} \frac{\partial}{\partial t} \left( \eta^2 \right),
\]

(Andrews et al. 1987; Schoeberl et al. 1992; Bowman 1993a). Here, \( K_{yy} \) is computed by using a linear fit to \( \langle \eta^2 \rangle \) as a function of time.

The evolution of the zonal-mean PV in the model is compared with a purely diffusive parameterization,

\[
\frac{\partial \bar{q}}{\partial t} = \frac{1}{a^2 \cos \phi} \frac{\partial}{\partial \phi} \left( K_{yy} \cos \phi \frac{\partial \bar{q}}{\partial \phi} \right),
\]

where \( \bar{q} \) is the zonal-mean potential vorticity, \( a \) is the radius of the earth, \( t \) is time, \( \phi \) is latitude, and \( K_{yy} \) is the diffusion coefficient as a function of latitude. This equation is solved numerically using a Crank–Nicholson finite-difference scheme with 512 grid points pole to pole and 64 time steps per day.

3. Results

a. Unstable modes

Numerical experiments were conducted for a number of jet profiles in Manney et al. (1988) and Hartmann (1983). Results from one experiment using Eq. (7) in Manney \( U_0 = 180 \, \text{m s}^{-1} \), \( B = 10^\circ \), and \( \phi_0 = 60^\circ \), and \( \phi_1 = 45^\circ \) are discussed here. These settings produce a narrow jet with regions of reversed potential vorticity on both flanks of the jet (see Fig. 4a). A profile from Hartmann that produces only a polar mode is also discussed briefly.

The amplitudes of the first six zonal waves in the meridional velocity \( v \) are plotted in Fig. 1 as a function of time and latitude. Two different wave regimes can be seen. Poleward of 70°S the flow is dominated by wave 1, which achieves significant amplitude between days 15 and 20 of the integration. Higher wavenumbers have small amplitudes in this latitude zone. By contrast, wave 1 is nearly absent in midlatitudes, where higher wavenumbers dominate. Waves 2, 3, 4, and 5 are all present with significant amplitudes in this latitude zone. Higher wavenumbers than 5 are also present, but their amplitudes are small. Wave 5 appears first, grows quickly, reaches a peak near day 11, and then decays. Wave 4 appears next, reaches a peak near day 15, and
then decays. Wave 3 appears next, reaches a peak around day 21, and then decays more slowly than the higher wavenumbers. Wave 2 also appears in this latitude zone, but does not reach significant amplitude until after day 20, and does not peak until day 29. The largest amplitude wave in midlatitudes is wave 3.

The waves grow approximately exponentially and then level off abruptly. Wave amplitudes are plotted as a function of time at 79°S in Fig. 2. As indicated in Fig. 1, outside the vortex wave 1 is weak while higher wavenumbers dominate. Growth rates during the initial exponential growth phase (estimated graphically) are given in Tables 1a and 1b for 79° and 50°S. The phase propagation of the waves has been plotted in Hovmöller diagrams (not shown). All modes exhibit eastward phase progression, except wave 2 near the pole, which is quasi-stationary. Wave periods and phase speeds (estimated graphically from the Hovmöller diagrams) are given in Tables 1a and 1b. All the polar modes have phase speeds much less than the jet maximum, so critical lines are located inside the vortex away from the jet maximum. Some midlatitude modes have fast phase speeds, but the fastest modes (wave 1 especially) cease to grow as the equatorward flank of the jet rapidly stabilizes. The small amplitude of wave 1 ($v' < 1$ m s$^{-1}$) prevents it from mixing efficiently near the jet maximum. The period of wave 1 inside the vortex increases to about 5.5 days after the wave amplitude saturates and the jet decelerates.

The evolution of the potential vorticity (PV) field is plotted in Fig. 3 at 2-day intervals from day 12 to day 42. Also plotted is a ring of passive tracer particles initially located near 80°S. Higher wavenumber structure can be seen between days 12 and 24, at which time wavenumber 1 becomes dominant inside the vortex. A single narrow filament of particles has been drawn off of the ring by this time (days 22–26). The relatively high PV air initially in the center of the vortex remains a coherent blob until about day 32. After that time it is permanently pushed off the pole into a narrow ring at about 65°S in a manner similar to that in Ishioka and Yoden (1993). This air remains within the jet maximum and does not mix across the strong potential vorticity gradient between 50° and 60°S. (See the following paragraph.) A number of wavebreaking filaments can be seen outside the vortex in the region that is vigorously mixed by breaking of the smaller-scale waves.

The zonal-mean wind and potential vorticity are shown in Figs. 4a and 4b at 10-day intervals during the integration. As expected, the strength of the jet decreases and the regions of negative potential vorticity gradient are removed by the growing waves. The contributions of each wave regime to the momentum and potential vorticity fluxes are shown in Fig. 5 for wave
Fig. 7. Finite-time estimates of the Lyapunov exponents (a) for E-W oriented and (b) for N-S oriented vectors after 50 days.
that particles are separating subexponentially (e.g., linearly). The Lyapunov exponents are somewhat larger in the midlatitude zone than in the polar zone, which is consistent with the more rapid growth of the midlatitude modes and the presence of more modes simultaneously. There is a definite notch in $\lambda$ at 60°S, which is precisely the peak in the jet. The N–S orientation also has larger $\lambda$ than the E–W. Presumably this is because the shear of the background flow stretches N–S oriented vectors more rapidly than E–W, and flow randomizes the orientations through rotation rather slowly. The difference between the two orientations is approximately equal to the depth of the notch at 60°S (where the shear vanishes), also suggesting that the background shear is responsible. The Lyapunov exponents indicate that there are two regions of mixing in the flow, corresponding to the polar and midlatitude wave modes. It is not clear from the Lyapunov exponents alone whether there is any mixing across the peak in the jet. For this we will turn to the other diagnostic quantities.

The Lagrangian particle dispersion $\langle \eta'^2 \rangle$ is plotted in Fig. 8 as a function of time and latitude for each zonal ring of particles. Each time series has been smoothed with a 5-day running-mean filter. The dispersion remains small until the waves reach significant amplitude, around day 10. At this time the particles rapidly disperse in latitude, more rapidly in midlatitudes than in the polar regions. There is a notch in the particle dispersion at 60°S similar to that seen in the Lyapunov exponents. Within the polar region the dispersion initially increases, but then levels off at less than $50(\degree)^2$, suggesting that the particles are largely confined within about 10° of their initial latitudes. This provides some evidence that particles do mix inside the vortex, but probably do not mix across the jet maximum. The dispersion values outside the vortex are much larger, and it is not clear whether particles are mixing into the vortex, or simply mixed within the midlatitude zone.

This question can be tested by tracking the particles initially inside and outside the vortex to see whether they remain separated. Figures 9a and 9b show the locations on day 50 of particles initially poleward and equatorward of the jet maximum on day 0. The disk of particles initially within the 60° latitude circle contracts during the course of the integration. (Because this is a shallow-water model, the zonal-mean meridional velocity $v$ need not vanish and, in fact, is primarily southward within this latitude region during the integration. The result is convergence toward the pole.) One or two very small filaments of interior air are stretched into the exterior air, and one or two slightly larger filaments are intruded into the vortex. The filamentation does not occur until after day 30 of the integration. The boundary between the two air masses is still quite distinct, however, and the total interchange is a very small fraction of the air inside the vortex.
Fig. 9. Locations on day 50 of particles that were initially (a) south and (b) north of 60°S. Latitude circles are at 30°S and 60°S.
c. Comparison with diffusive parameterization

The diffusion coefficient $K_p$ has the latitudinal structure shown in Fig. 10. This function is simplified in the diffusion model by setting $K_p$ to $0.5 \times 10^5 \text{ m}^2 \text{s}^{-1}$ south of $60^\circ$S, to $1.5 \times 10^5 \text{ m}^2 \text{s}^{-1}$ between $60^\circ$ and $30^\circ$S, and to zero north of $30^\circ$S. Using the zonal-mean PV at day 0 as the initial condition yields the results shown in Fig. 11 at 10-day intervals.

Comparing Fig. 11 with Fig. 4b it can be seen that the diffusive parameterization captures the gross effect of the waves on the basic state. The kink in $\bar{q}$ at $60^\circ$S is due to the stepwise change in $K_p$ at that latitude. The PV extrema are reduced with time as $\bar{q}$ is smoothed in latitude. Diffusion misses a number of important effects, however. The negative potential vorticity gradient on the poleward side of the jet is not removed, even after 50 days, and the transport of PV toward the pole is much weaker than in the model. This is a result of the absence of a PV gradient south of about $75^\circ$S. Diffusion also does not produce the characteristic surf-zone shape, which can be seen in Fig. 4b beginning at about day 30, with flat PV gradients in low and high latitudes and a roughly linear gradient between. Instead it tends to oversmooth and produce an overly wide region of weak PV gradients.

d. Purely polar modes

Experiments were also conducted with basic-state wind profiles that support only unstable polar modes, the midlatitude mode being absent. Under these conditions a slowly growing wavenumber 1 appears, causing weak mixing within the vortex. The wave does not extend into midlatitudes, and no mixing occurs outside the vortex. There is also no exchange of air across the vortex boundary.

4. Conclusions

Numerical experiments with a nonlinear global shallow-water model produce results consistent with linear barotropic instability theory for the idealized zonally averaged initial conditions used. Different unstable wave regimes are associated with the two regions of reversed potential vorticity gradient on the poleward and equatorward flanks of the jet. The midlatitude modes are wavenumbers 2–5, with the larger wavenumbers appearing first and having the largest growth rates. By contrast, the polar mode is predominantly wavenumber 1 and is slower growing than the midlatitude modes. The eddy momentum and potential vorticity fluxes act rapidly to decelerate the jet and eliminate the regions of reversed potential vorticity gradient.

Chaotic mixing occurs on both flanks of the jet around the critical lines for the unstable waves. The
mixing rates, as measured by Lyapunov exponents or Lagrangian dispersion statistics, are larger on the equatorward flank of the jet. There is a distinct minimum in the mixing at the peak of the jet near 60°S.

Trajectory calculations with large ensembles of particles show little exchange of air between the air masses initially poleward and equatorward of the jet maximum.

Using a wind profile that produces unstable modes only within the vortex results in weak mixing, even inside the vortex, and no exchange of air across the vortex boundary. This result is similar to that of Ishioka and Yoden (1993). With a similar basic-state wind profile, they found that the fluid of lower potential vorticity initially centered over the pole was expelled to lower latitudes as a coherent blob. Their description suggests that the high PV air is expelled and mixes with midlatitude air. In our case the high PV air is displaced to the inside edge of the vortex, as shown in Fig. 3. The high PV air does not cross the jet maximum or mix into the region of strong PV gradients on the equatorward flank of the jet.

In a series of unpublished experiments with a linear model of barotropically unstable waves, Chen and J. Holton (personal communication) found that the mixing was confined to very narrow latitude bands around the critical lines for the unstable wave. In this nonlinear model, mixing occurs throughout much wider latitude bands. This is a result of several factors. First, there are multiple unstable waves in the flow with critical lines at different, albeit nearby, latitudes. Second, the zonally averaged flow changes during the course of the integration as a result of the wave activity. This shifts the critical lines and the regions of strong mixing. Third, in this model the waves are allowed to grow to finite amplitude, and the integration continues longer than for the experiments in Chen and Holton. As a result, particles are displaced farther from their initial latitudes, and mixing occurs throughout broader bands.

The most significant result of these experiments is the existence of the barrier to transport between the interior and exterior of the vortex. Despite the large changes in the flow that occur during the 50-day integration, air poleward of the jet maximum remains nearly perfectly separated from midlatitude air. Very little air is exchanged across the jet maximum in either direction. The existence of the barrier can be understood in the following terms. Mixing occurs by filamentation (stretching and folding) at the critical lines for the unstable waves in the flow. This is consistent with earlier experiments and with the theory for nonlinear critical lines. The critical lines are located on either flank of the jet, typically near 50° and 70° latitude, for the unstable waves of this particular profile. This produces two regions of efficient mixing, one on each flank of the jet. Furthermore, the poleward transport of momentum into the vortex by the growing unstable waves quickly accelerates the vortex interior and eliminates the critical lines inside the vortex, reducing the mixing inside the vortex relative to that outside. Between the two critical lines is a region of very large potential vorticity gradient that is highly resistant to deformation through the Rossby wave restoring mechanism (McIntyre 1989). Perturbations in this region produce only small, narrow filaments despite the large deformations that occur. This is an idealized experiment, with simplified dynamics and basic state, but the barrier persists in the presence of multiple unstable modes with a range of spatial scales and full nonlinear dynamics.

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