

Chapter 14

Periodic Motion

PowerPoint® Lectures for
University Physics, 14th Edition
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Lectures by Jason Harlow

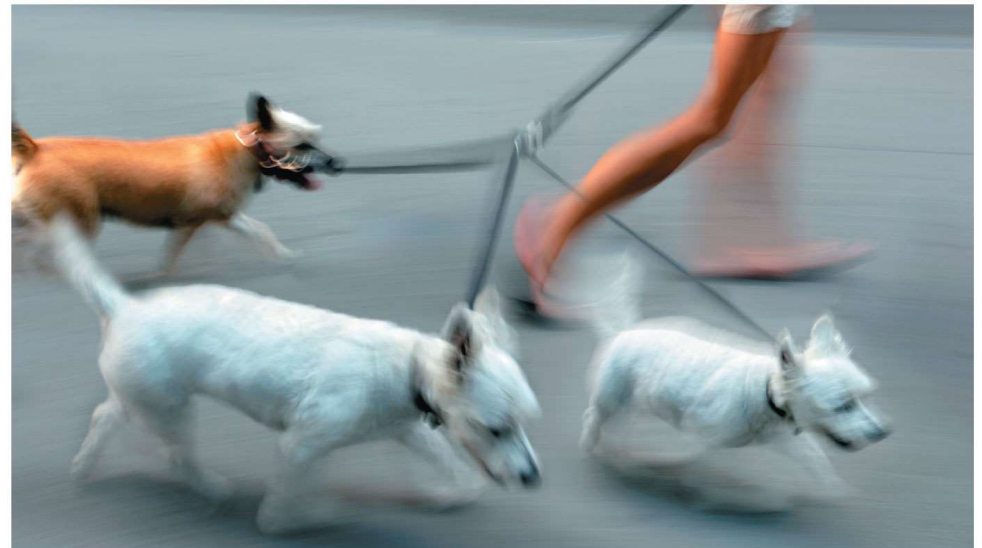
Learning Goals for Chapter 14

Looking forward at ...

- how to describe oscillations in terms of amplitude, period, frequency, and angular frequency.
- how to apply the ideas of **simple harmonic motion** to different physical situations.
- how to analyze the motions of a pendulum.
- what determines how rapidly an oscillation dies out.
- how a driving force applied to an oscillator at a particular frequency can cause a very large response, or **resonance**.

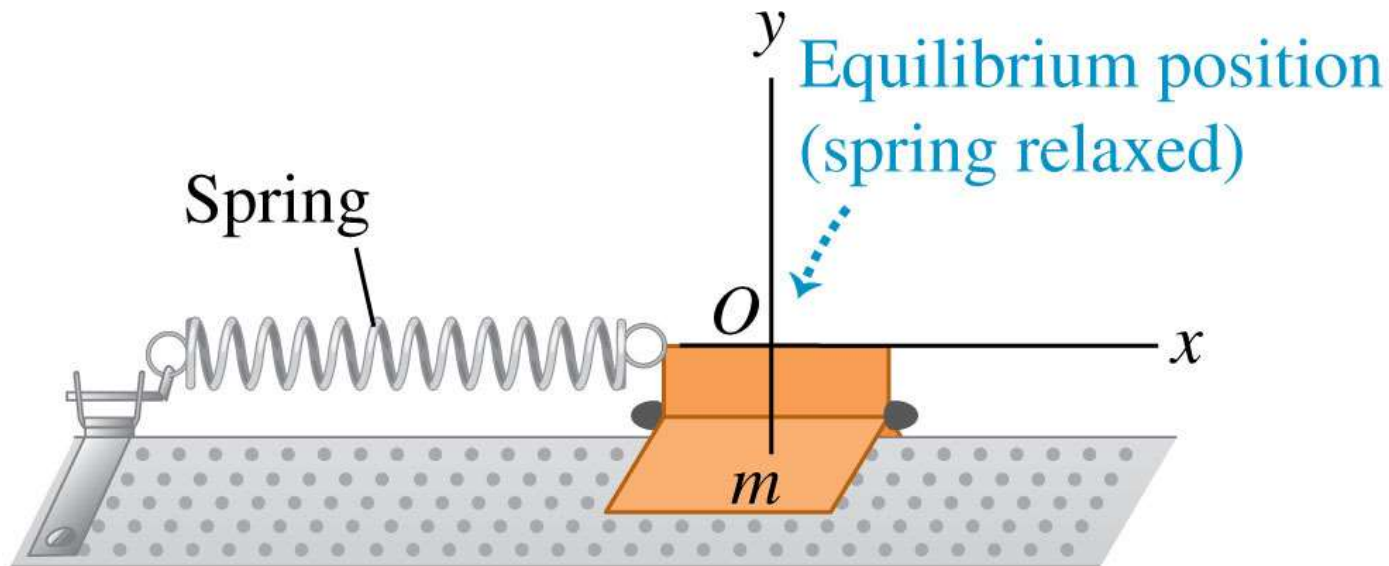
Introduction

- What would happen if you doubled a pendulum's mass?
- Why do dogs walk faster than humans? Does it have anything to do with the characteristics of their legs?
- Many kinds of motion (such as a pendulum, musical vibrations, and pistons in car engines) repeat themselves. We call such behavior **periodic motion** or **oscillation**.



What causes periodic motion?

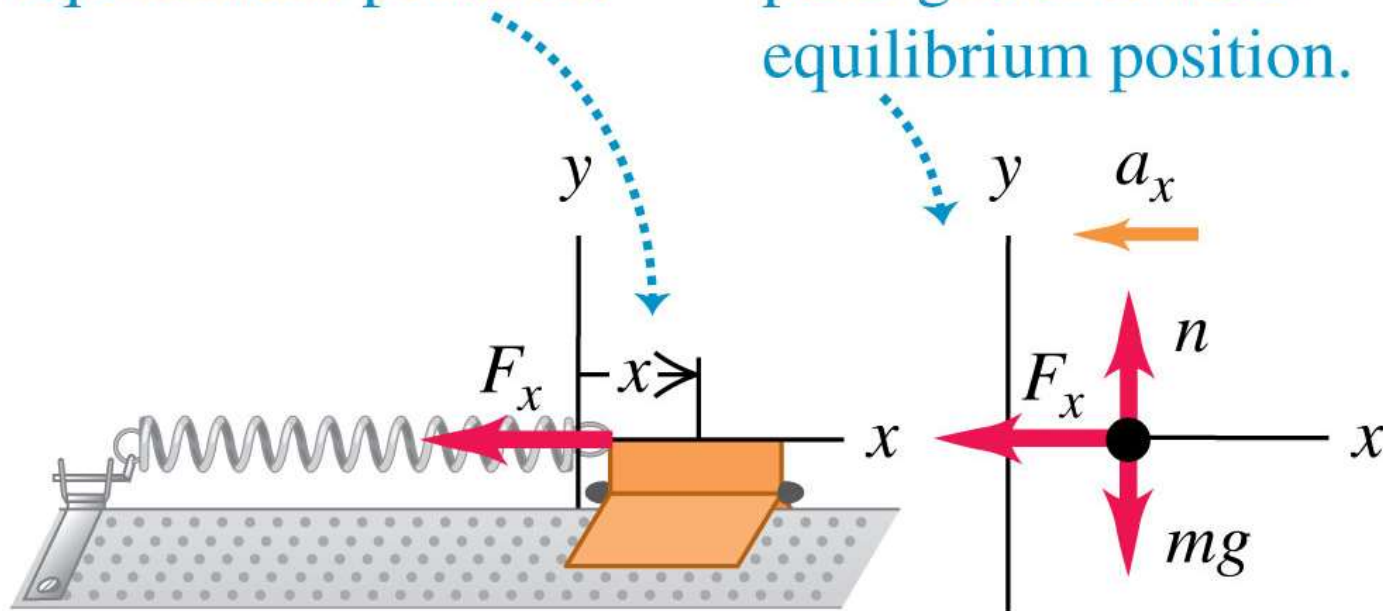
- If a body attached to a spring is displaced from its equilibrium position, the spring exerts a **restoring force** on it, which tends to restore the object to the equilibrium position.
- This force causes **oscillation** of the system, or **periodic motion**.



What causes periodic motion?

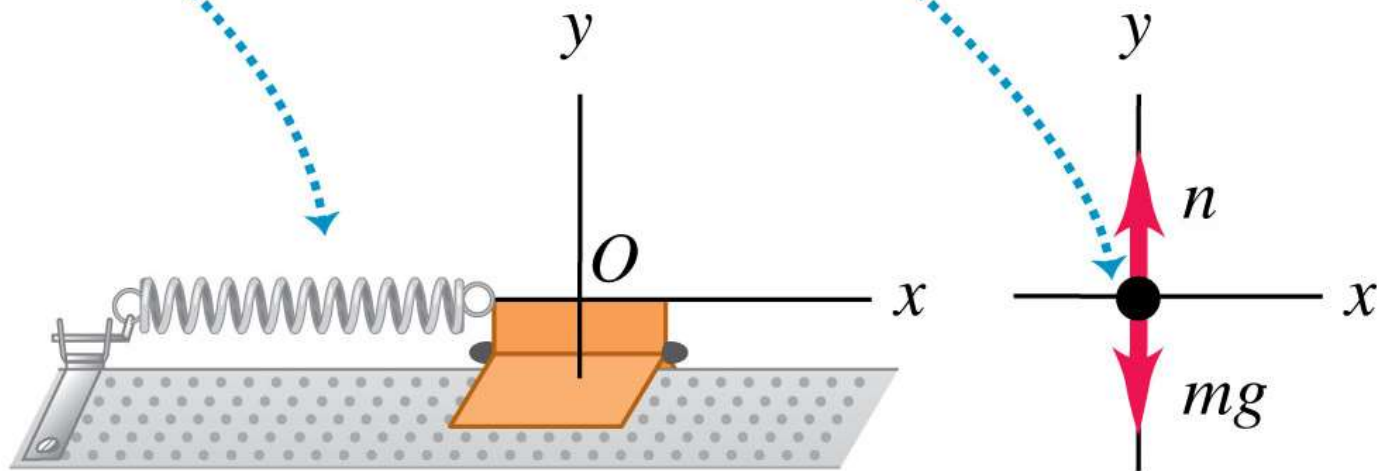
$x > 0$: glider displaced to the right from the equilibrium position.

$F_x < 0$, so $a_x < 0$: stretched spring pulls glider toward equilibrium position.



What causes periodic motion?

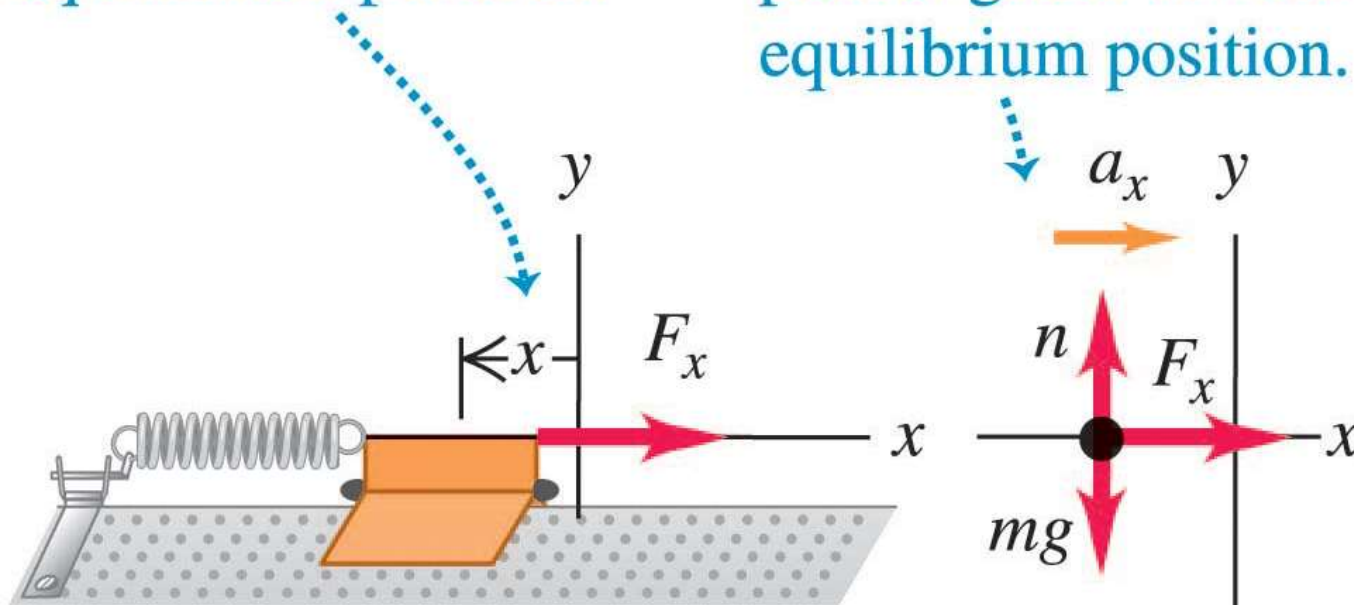
$x = 0$: The relaxed spring exerts no force on the glider, so the glider has zero acceleration.



What causes periodic motion?

$x < 0$: glider displaced to the left from the equilibrium position.

$F_x > 0$, so $a_x > 0$: compressed spring pushes glider toward equilibrium position.

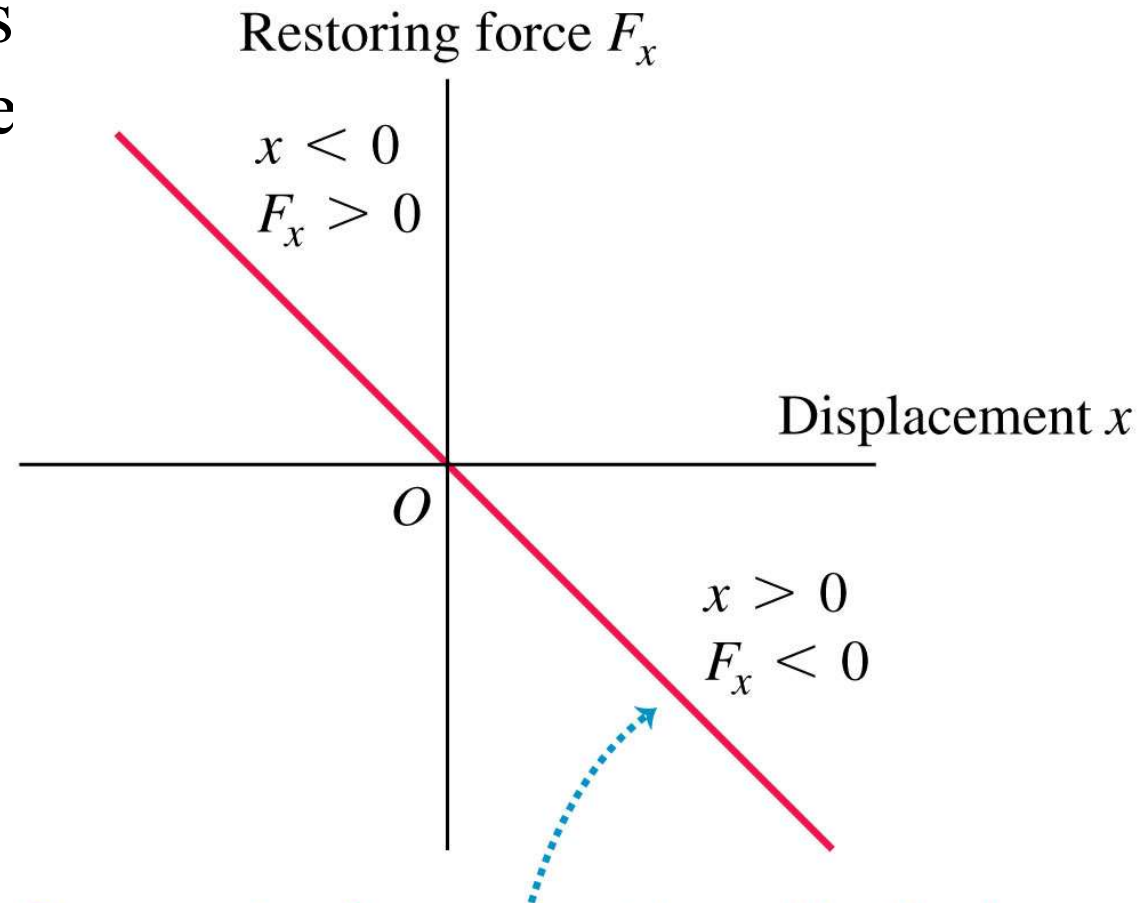


Characteristics of periodic motion

- The **amplitude**, A , is the maximum magnitude of displacement from equilibrium.
- The **period**, T , is the time for one cycle.
- The **frequency**, f , is the number of cycles per unit time.
- The **angular frequency**, ω , is 2π times the frequency:
 $\omega = 2\pi f$.
- The frequency and period are reciprocals of each other:
 $f = 1/T$ and $T = 1/f$.

Simple harmonic motion (SHM)

- When the restoring force is *directly proportional* to the displacement from equilibrium, the resulting motion is called **simple harmonic motion (SHM)**.

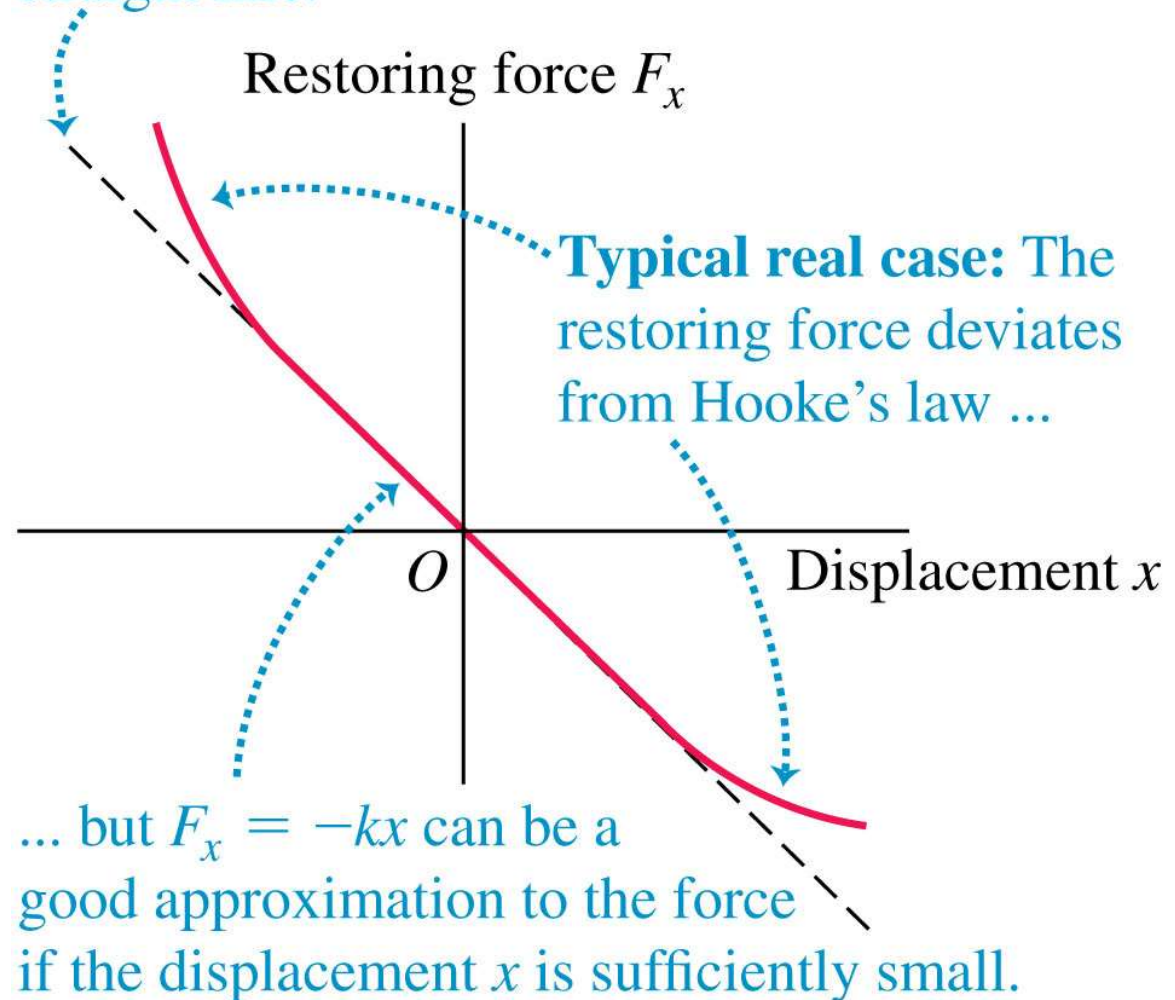


The restoring force exerted by an idealized spring is directly proportional to the displacement (Hooke's law, $F_x = -kx$): the graph of F_x versus x is a straight line.

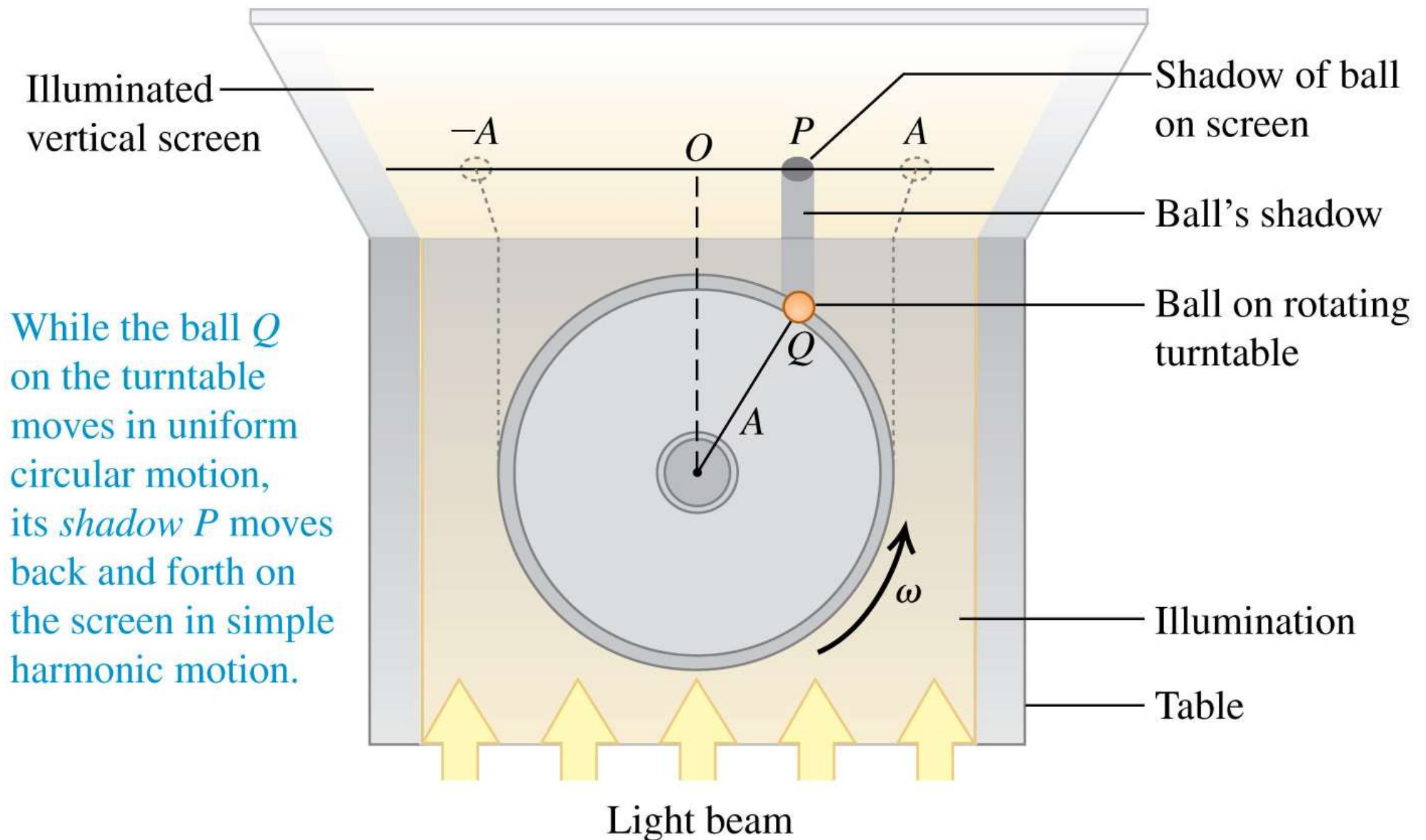
Simple harmonic motion (SHM)

- In many systems the restoring force is approximately proportional to displacement if the displacement is sufficiently small.
- That is, if the amplitude is small enough, the oscillations are approximately simple harmonic.

Ideal case: The restoring force obeys Hooke's law ($F_x = -kx$), so the graph of F_x versus x is a straight line.

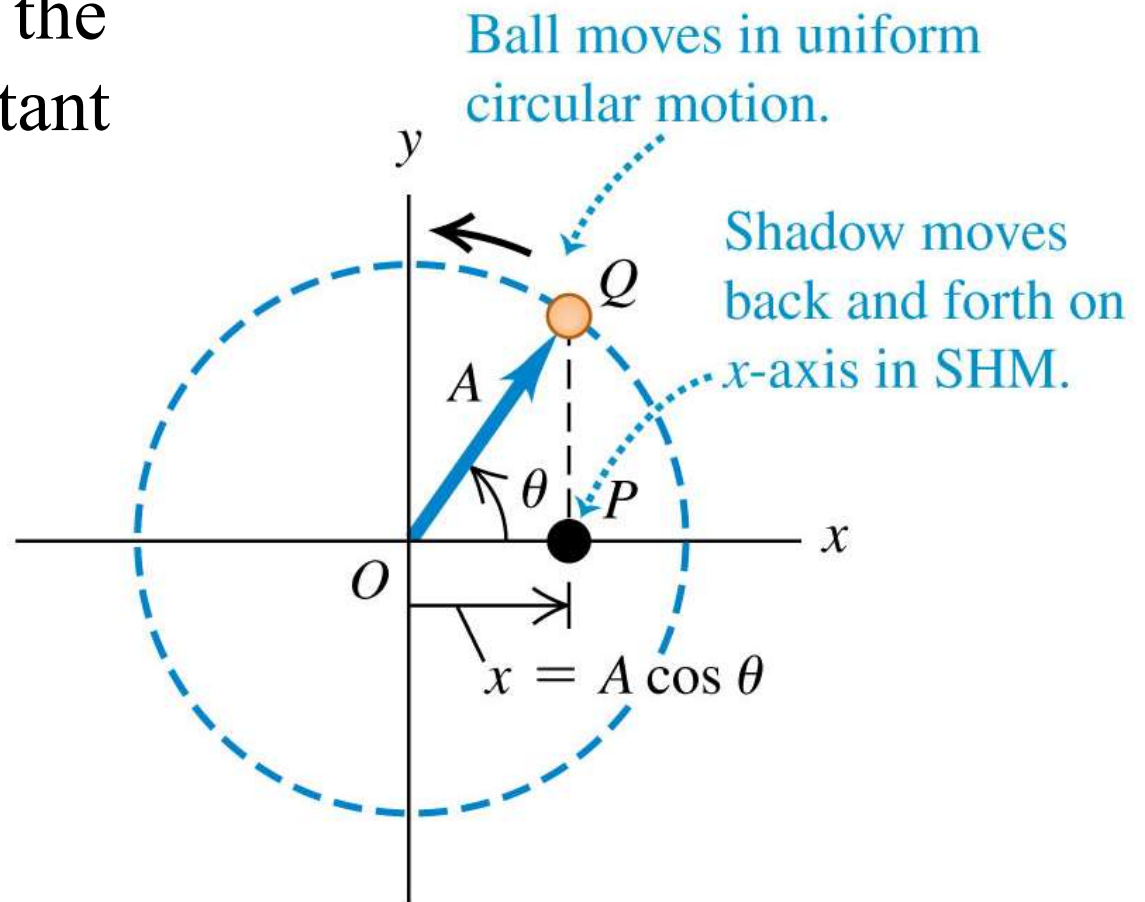


Simple harmonic motion viewed as a projection



Simple harmonic motion viewed as a projection

- The circle in which the ball moves so that its projection matches the motion of the oscillating body is called the **reference circle**.
- As point Q moves around the reference circle with constant angular speed, vector OQ rotates with the same angular speed.
- Such a rotating vector is called a **phasor**.



Characteristics of SHM

- For a body of mass m vibrating by an ideal spring with a force constant k :

Angular frequency for simple harmonic motion $\omega = \sqrt{\frac{k}{m}}$

Force constant of restoring force k
Mass of object m

Frequency for simple harmonic motion $f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

Angular frequency ω
Force constant of restoring force k
Mass of object m

Period for simple harmonic motion $T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$

Frequency f
Angular frequency ω
Mass of object m
Force constant of restoring force k

Characteristics of SHM

Tines with large mass m :
low frequency $f = 128 \text{ Hz}$



Tines with small mass m :
high frequency $f = 4096 \text{ Hz}$

- The greater the mass m in a tuning fork's tines, the lower the frequency of oscillation, and the lower the pitch of the sound that the tuning fork produces.

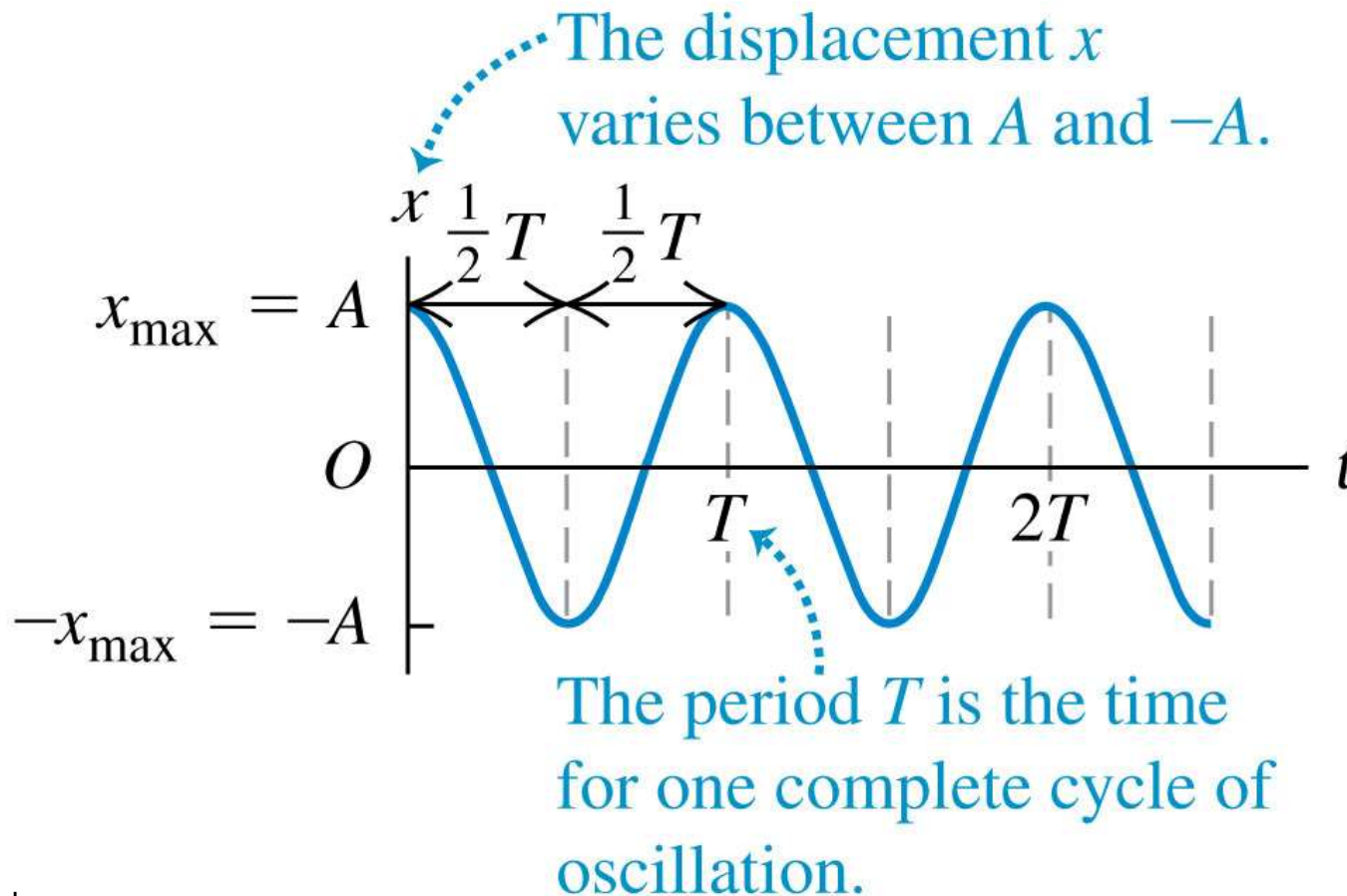
Displacement as a function of time in SHM

- The displacement as a function of time for SHM is:

Displacement in simple harmonic motion as a function of time

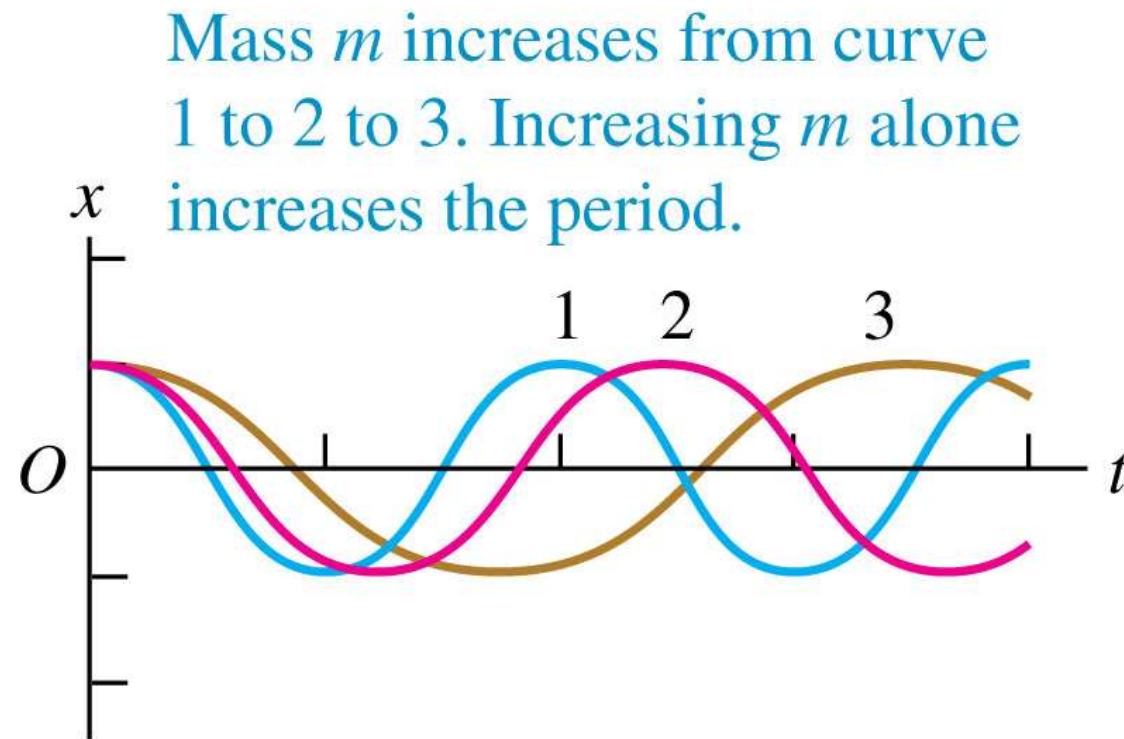
$$x = A \cos(\omega t + \phi)$$

Amplitude A
Time t
Phase angle ϕ
Angular frequency $\omega = \sqrt{k/m}$



Displacement as a function of time in SHM

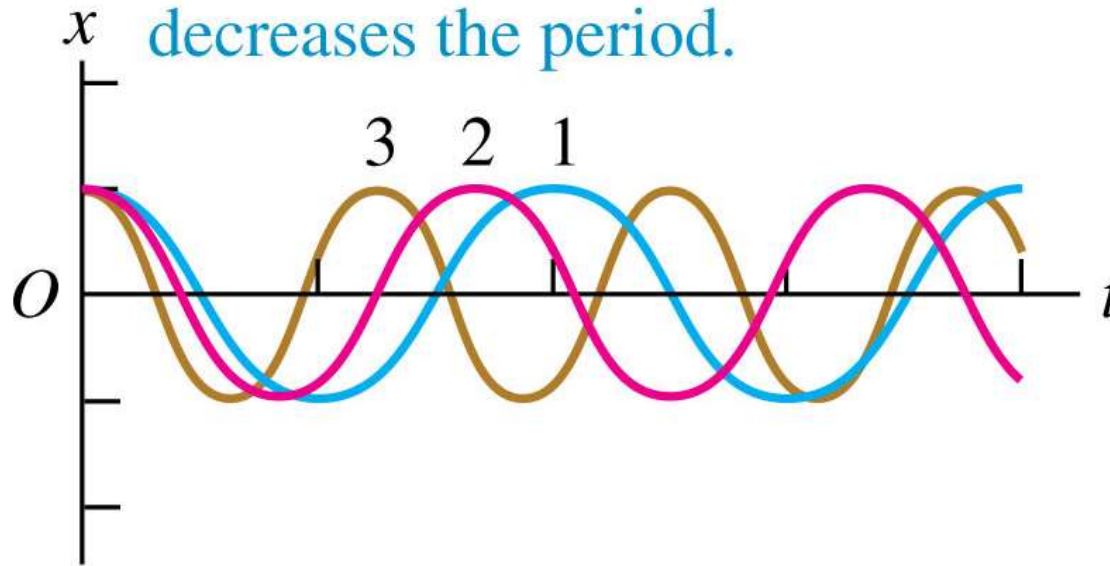
- Increasing m with the same A and k increases the period of the displacement vs time graph.



Displacement as a function of time in SHM

- Increasing k with the same A and m decreases the period of the displacement vs time graph.

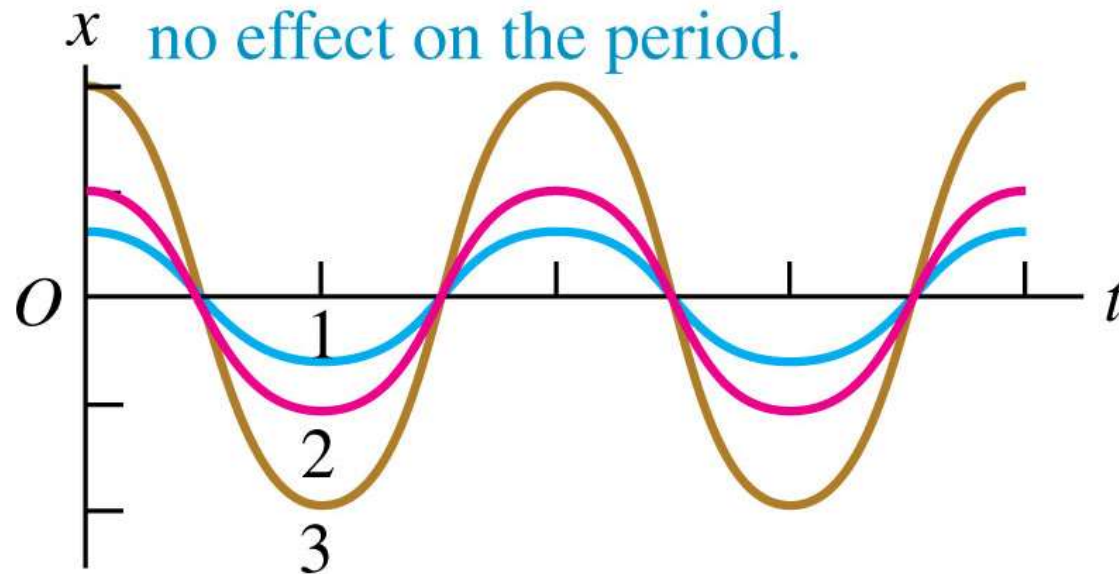
Force constant k increases from curve 1 to 2 to 3. Increasing k alone decreases the period.



Displacement as a function of time in SHM

- Increasing A with the same m and k does not change the period of the displacement vs time graph.

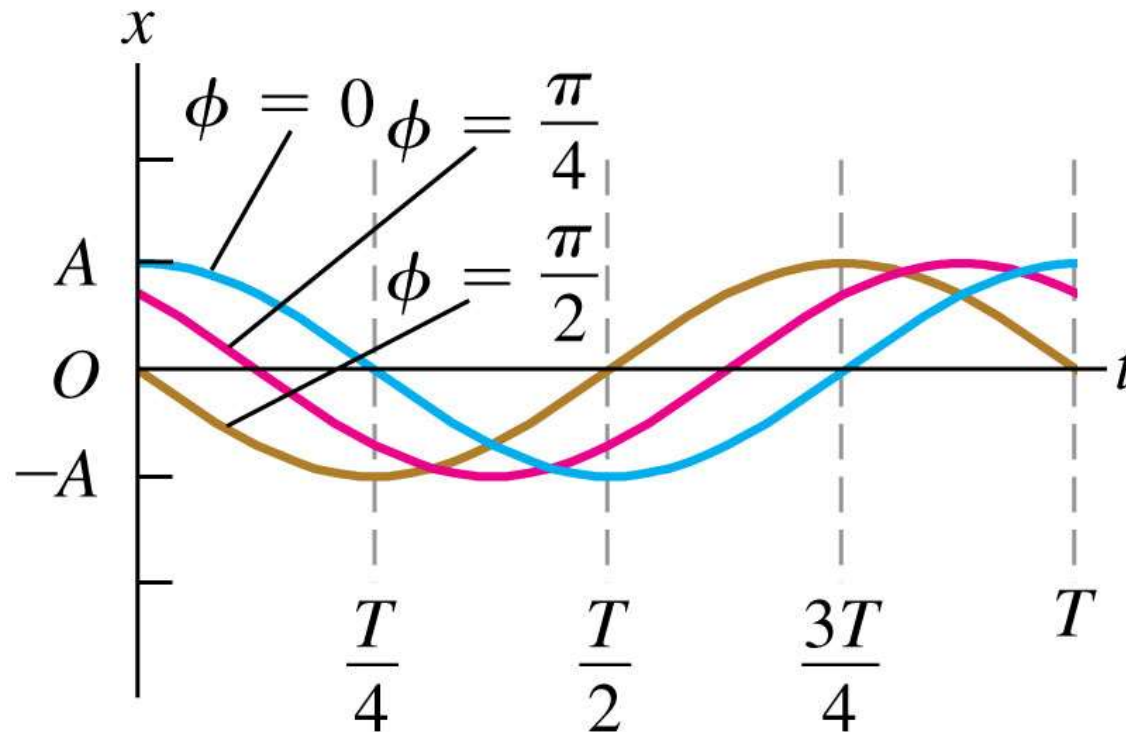
Amplitude A increases from curve 1 to 2 to 3. Changing A alone has no effect on the period.



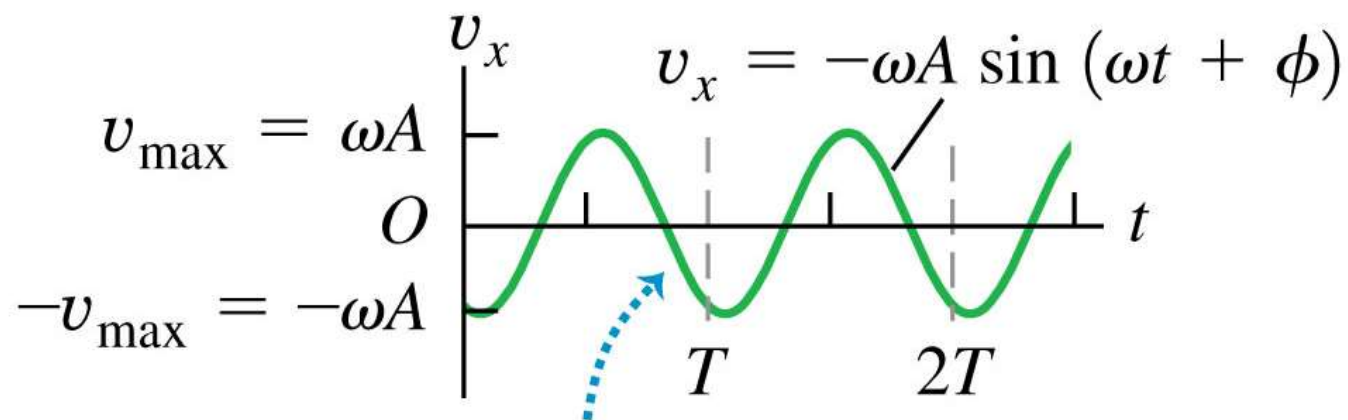
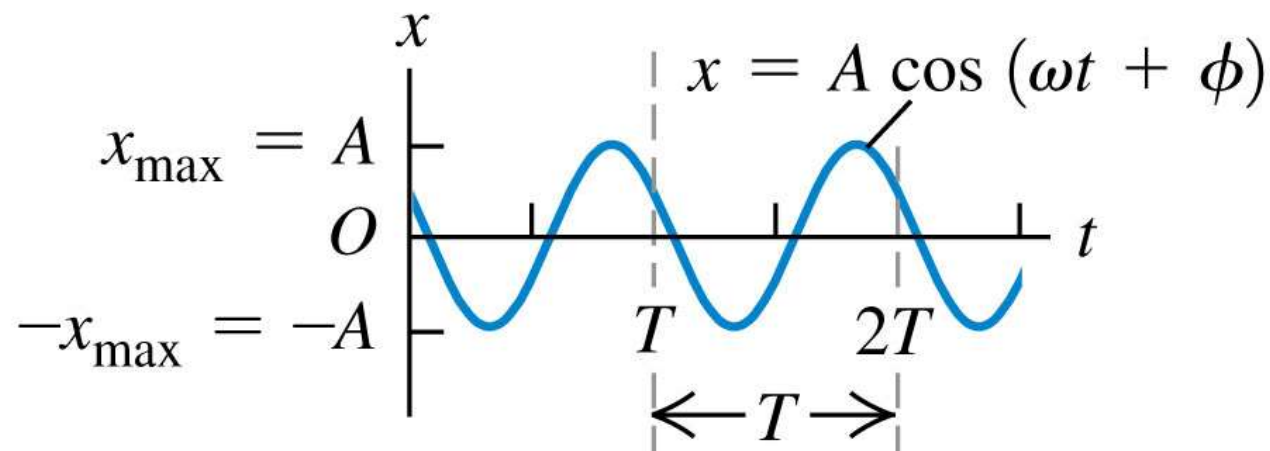
Displacement as a function of time in SHM

- Increasing ϕ with the same A , m , and k only shifts the displacement vs time graph to the left.

These three curves show SHM with the same period T and amplitude A but with different phase angles ϕ .

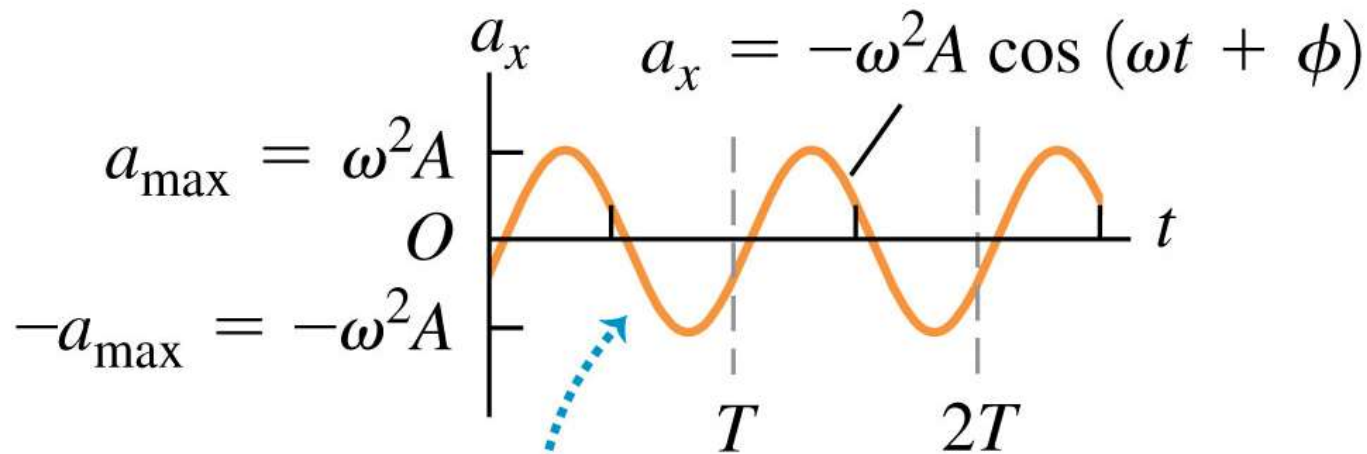
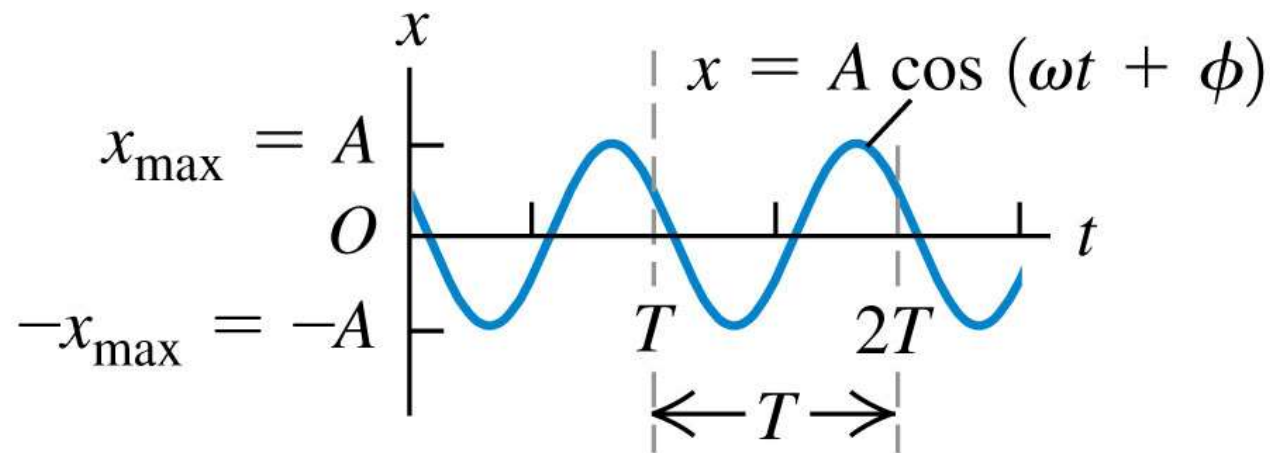


Graphs of displacement and velocity for SHM



The v_x - t graph is shifted by $\frac{1}{4}$ cycle from the x - t graph.

Graphs of displacement and acceleration for SHM

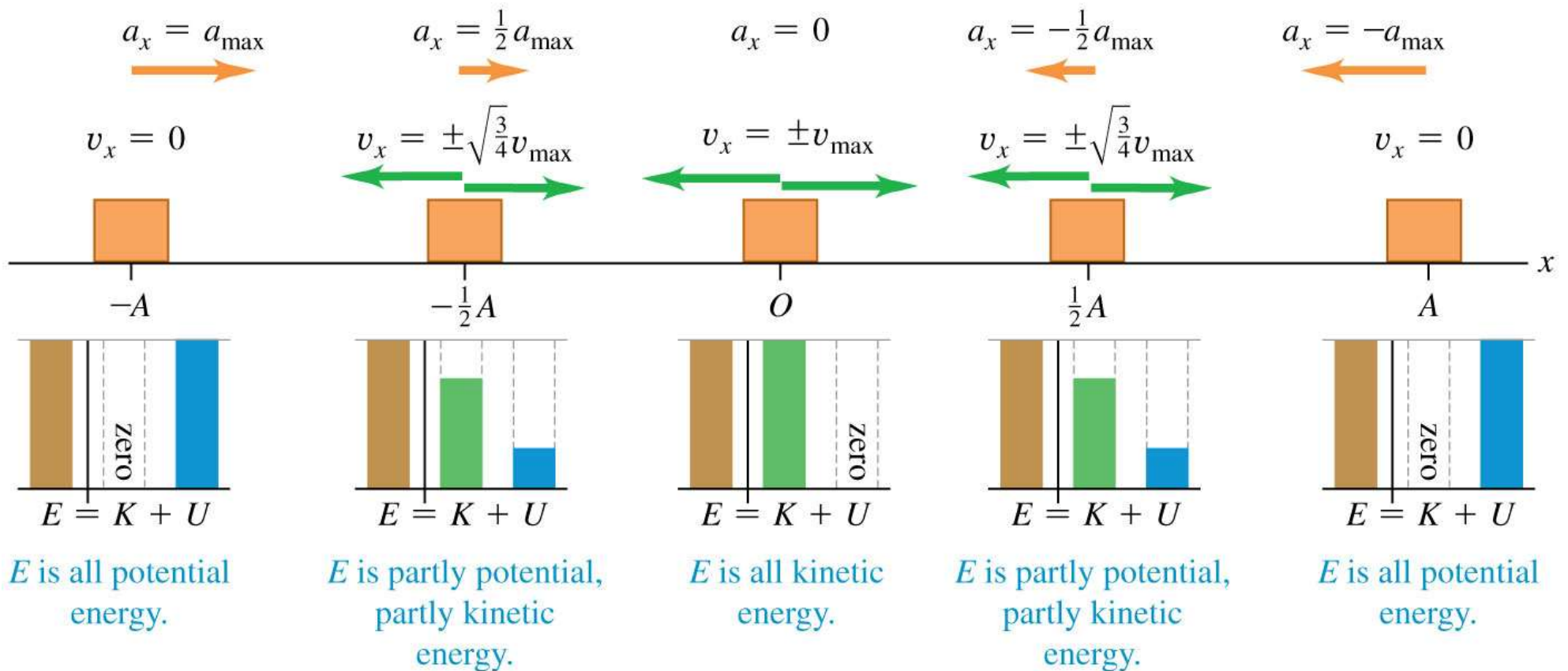


The a_x - t graph is shifted by $\frac{1}{4}$ cycle from the v_x - t graph and by $\frac{1}{2}$ cycle from the x - t graph.

Energy in SHM

- The total mechanical energy $E = K + U$ is conserved in SHM:

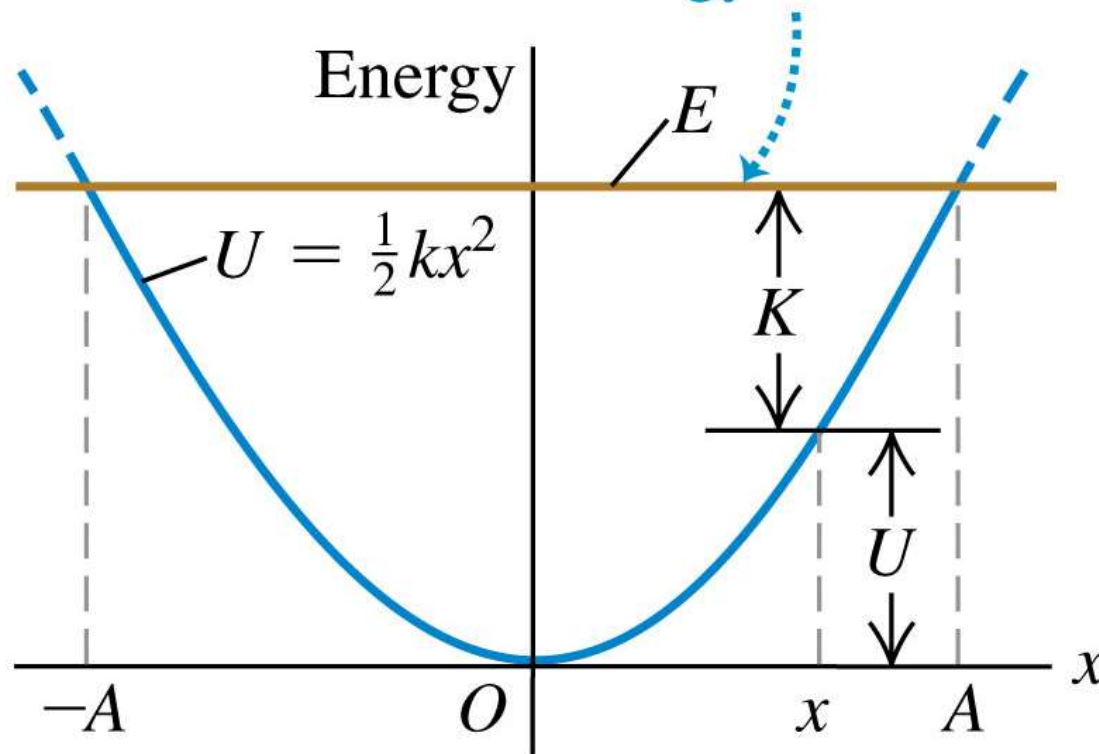
$$E = \frac{1}{2} m v_x^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2 = \text{constant}$$



Energy diagrams for SHM

- The potential energy U and total mechanical energy E for a body in SHM as a function of displacement x .

The total mechanical energy E is constant.

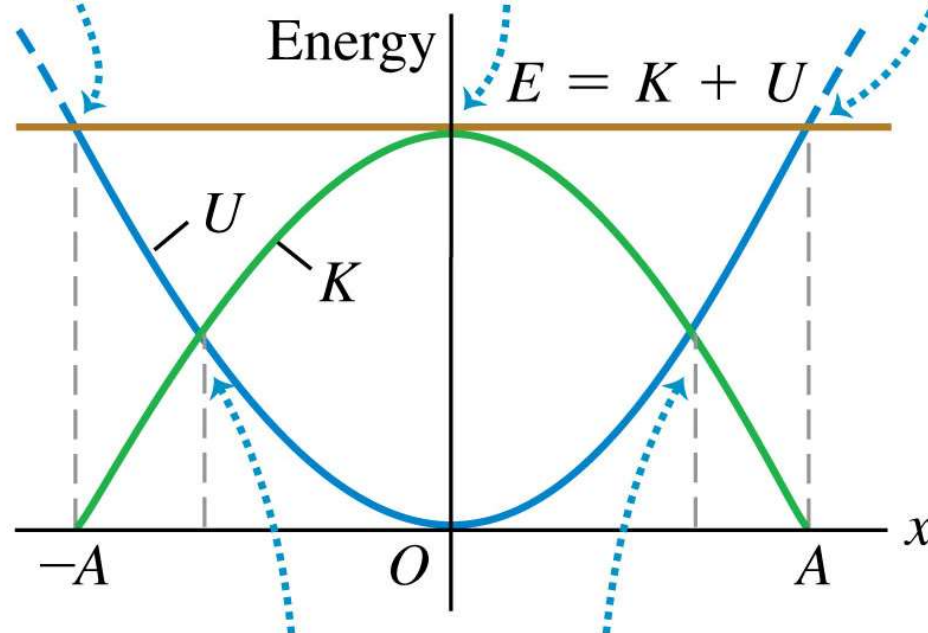


Energy diagrams for SHM

- The potential energy U , kinetic energy K , and total mechanical energy E for a body in SHM as a function of displacement x .

At $x = \pm A$ the energy is all potential; $K = 0$.

At $x = 0$ the energy is all kinetic; $U = 0$.



At these points the energy is half kinetic and half potential.

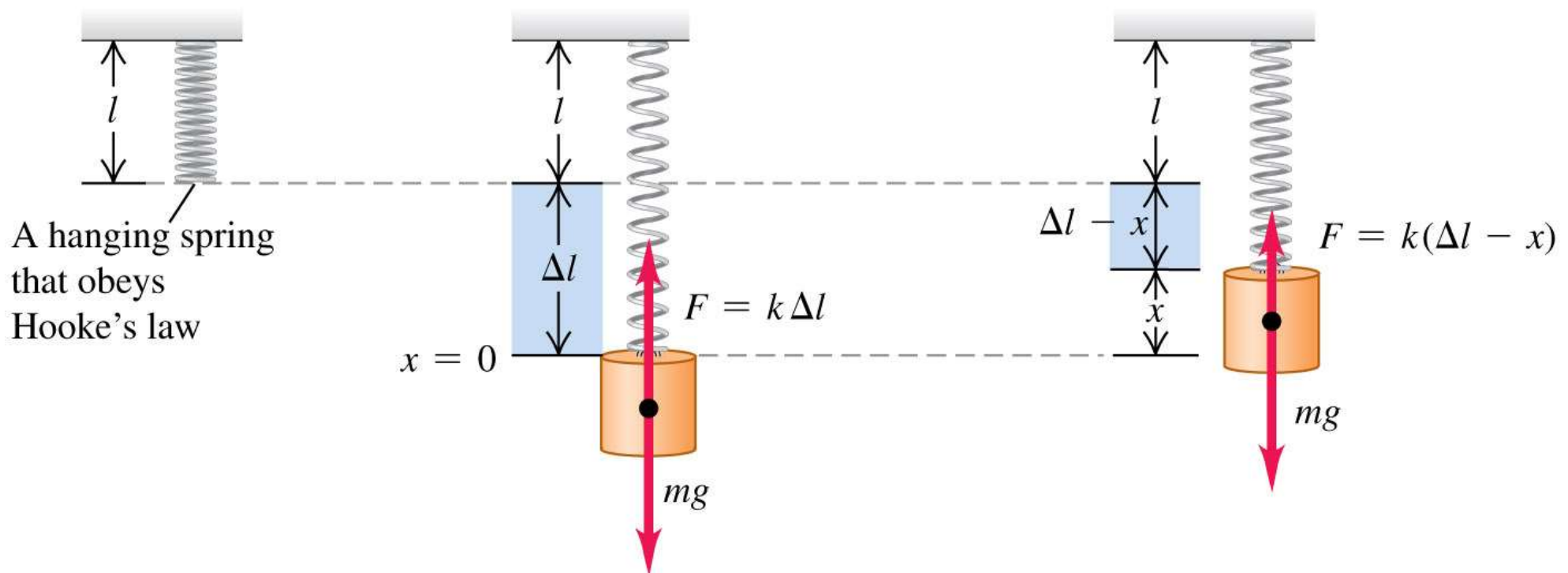
Vertical SHM

- If a body oscillates vertically from a spring, the restoring force has magnitude kx . Therefore the vertical motion is SHM.

(a)

(b) A body is suspended from the spring. It is in equilibrium when the upward force exerted by the stretched spring equals the body's weight.

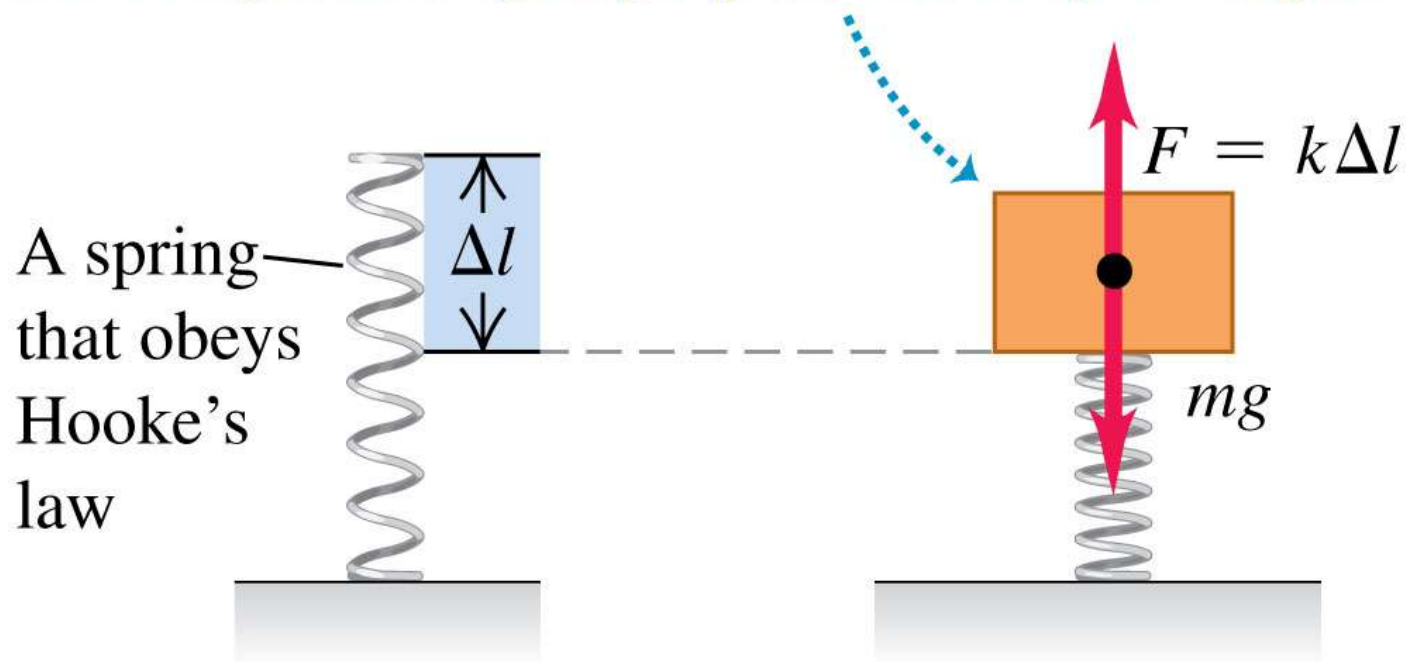
(c) If the body is displaced from equilibrium, the net force on the body is proportional to its displacement. The oscillations are SHM.



Vertical SHM

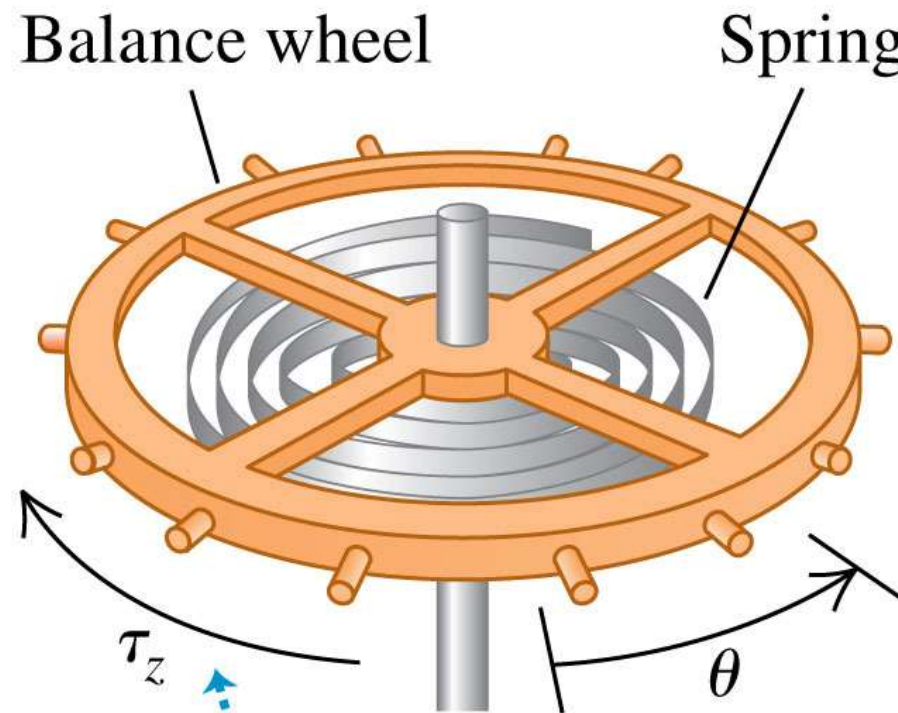
- If the weight mg compresses the spring a distance Δl , the force constant is $k = mg/\Delta l$.

A body is placed atop the spring. It is in equilibrium when the upward force exerted by the compressed spring equals the body's weight.



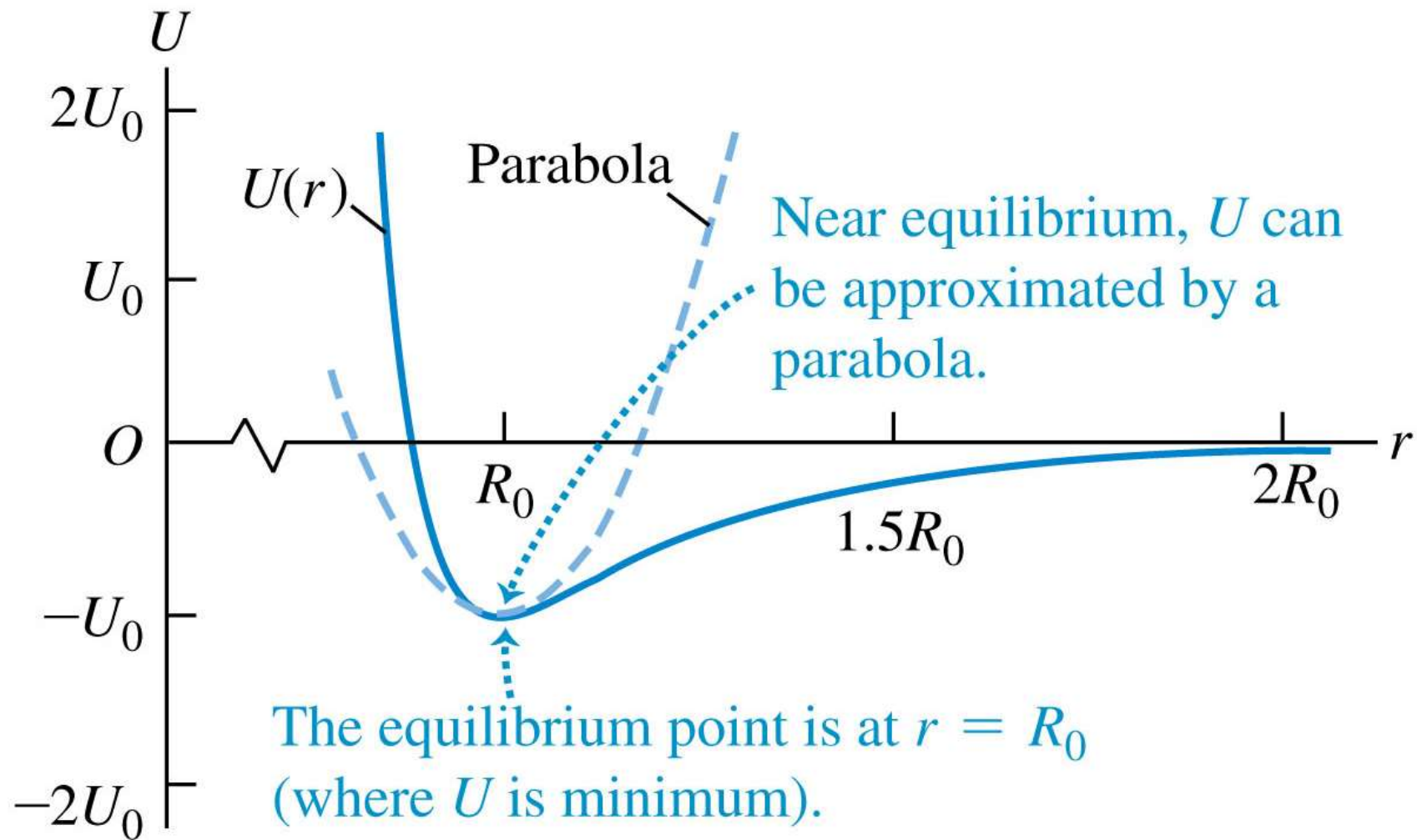
Angular SHM

- A coil spring exerts a restoring torque $\tau_z = -\kappa\theta$, where κ is called the **torsion constant** of the spring.
- The result is *angular* simple harmonic motion.



The spring torque τ_z opposes the angular displacement θ .

Potential energy of a two atom system

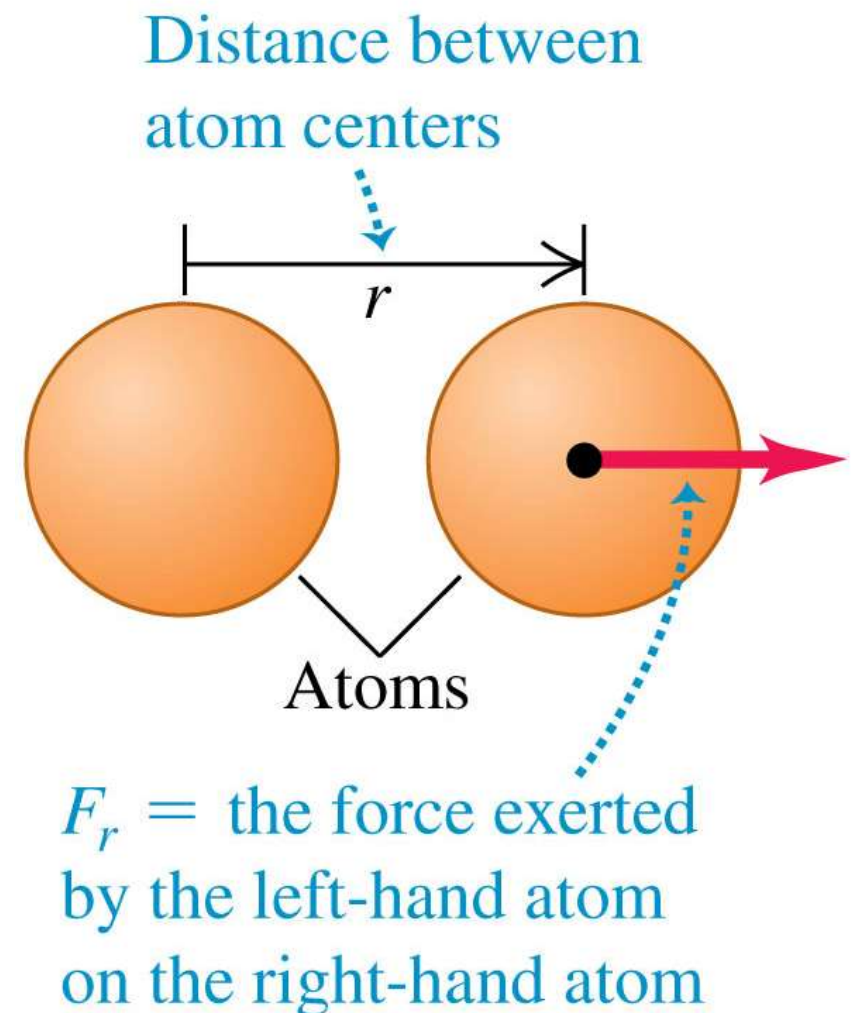


Vibrations of molecules

- Shown are two atoms having centers a distance r apart, with the equilibrium point at $r = R_0$.
- If they are displaced a small distance x from equilibrium, the restoring force is approximately

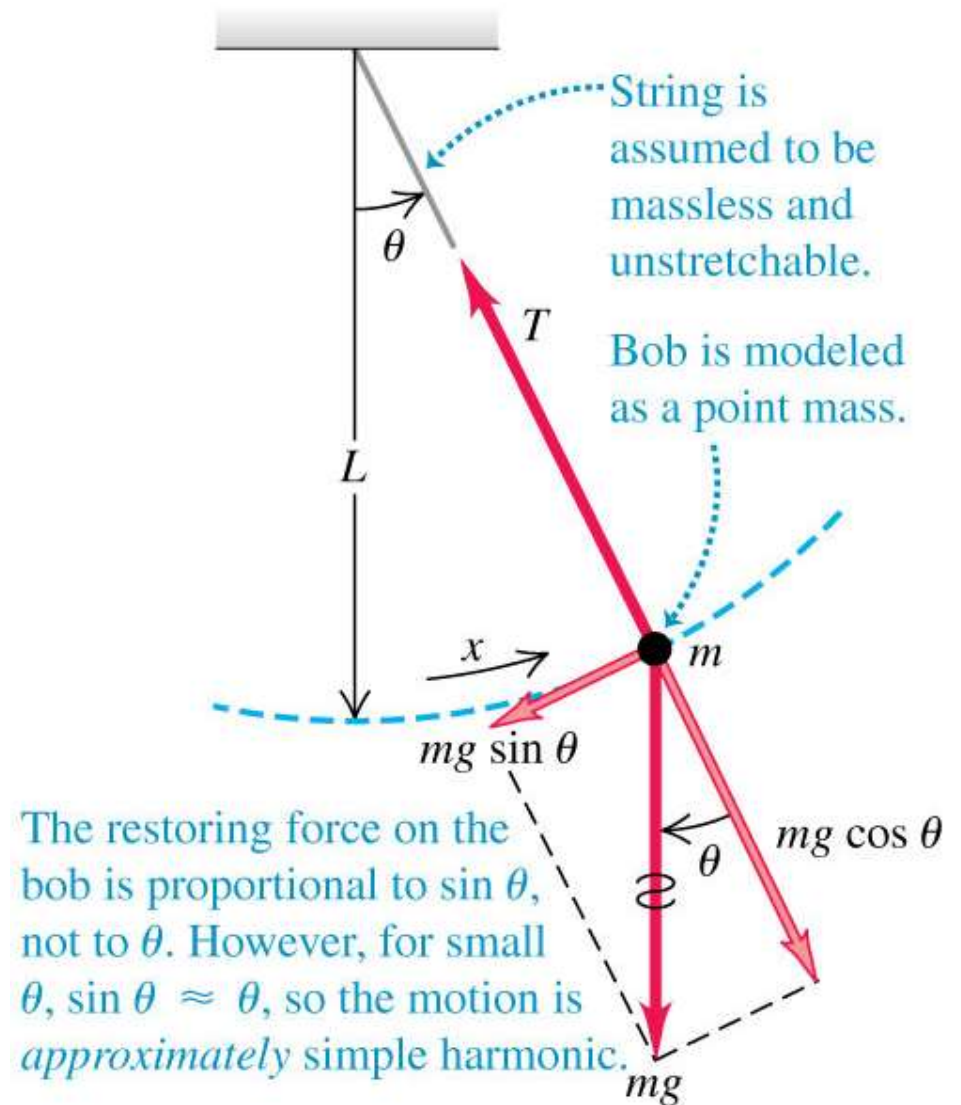
$$F_r = -(72U_0/R_0^2)x$$

- So $k = 72U_0/R_0^2$, and the motion is SHM.



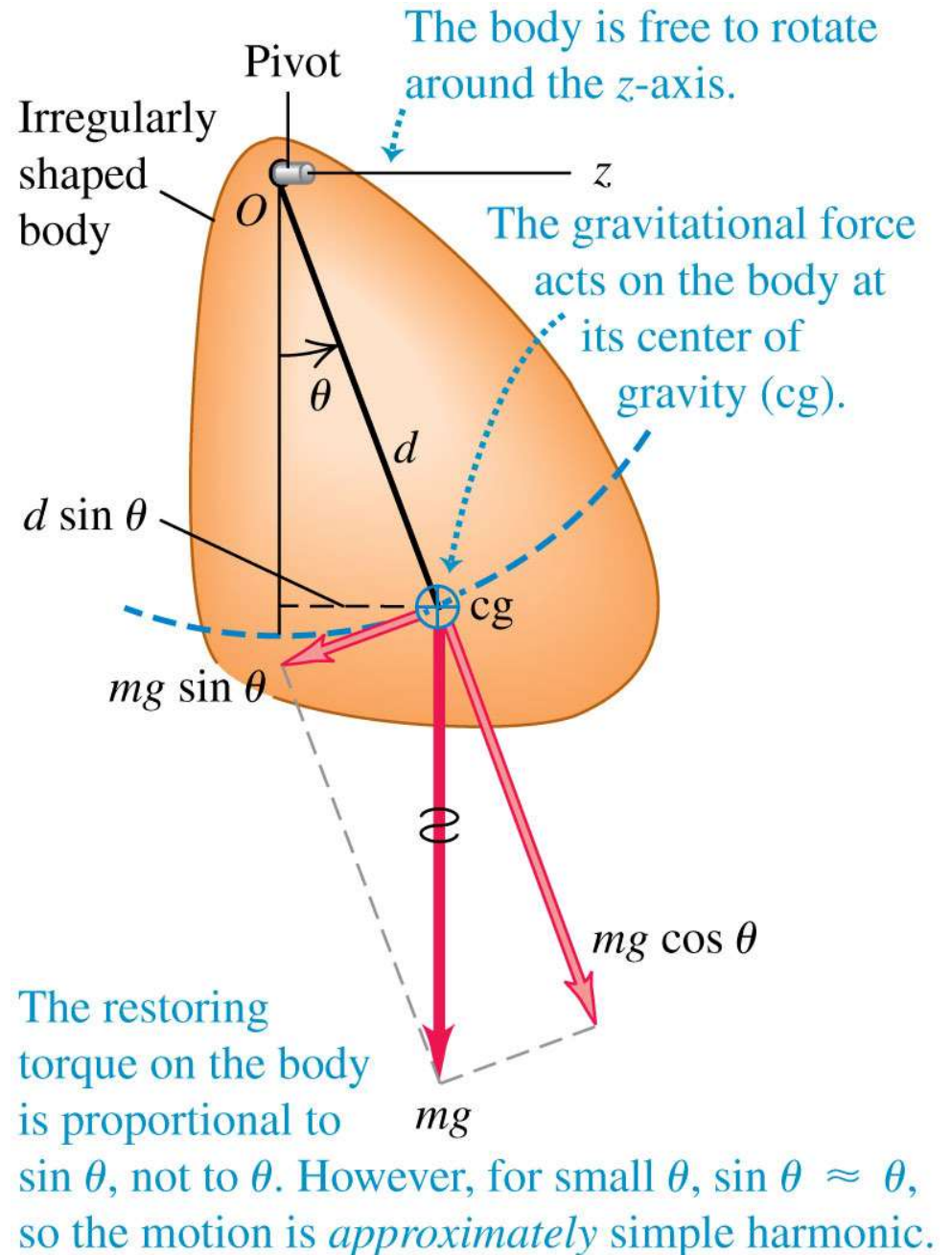
The simple pendulum

- A **simple pendulum** consists of a point mass (the bob) suspended by a massless, unstretchable string.
- If the pendulum swings with a small amplitude θ with the vertical, its motion is simple harmonic.



The physical pendulum

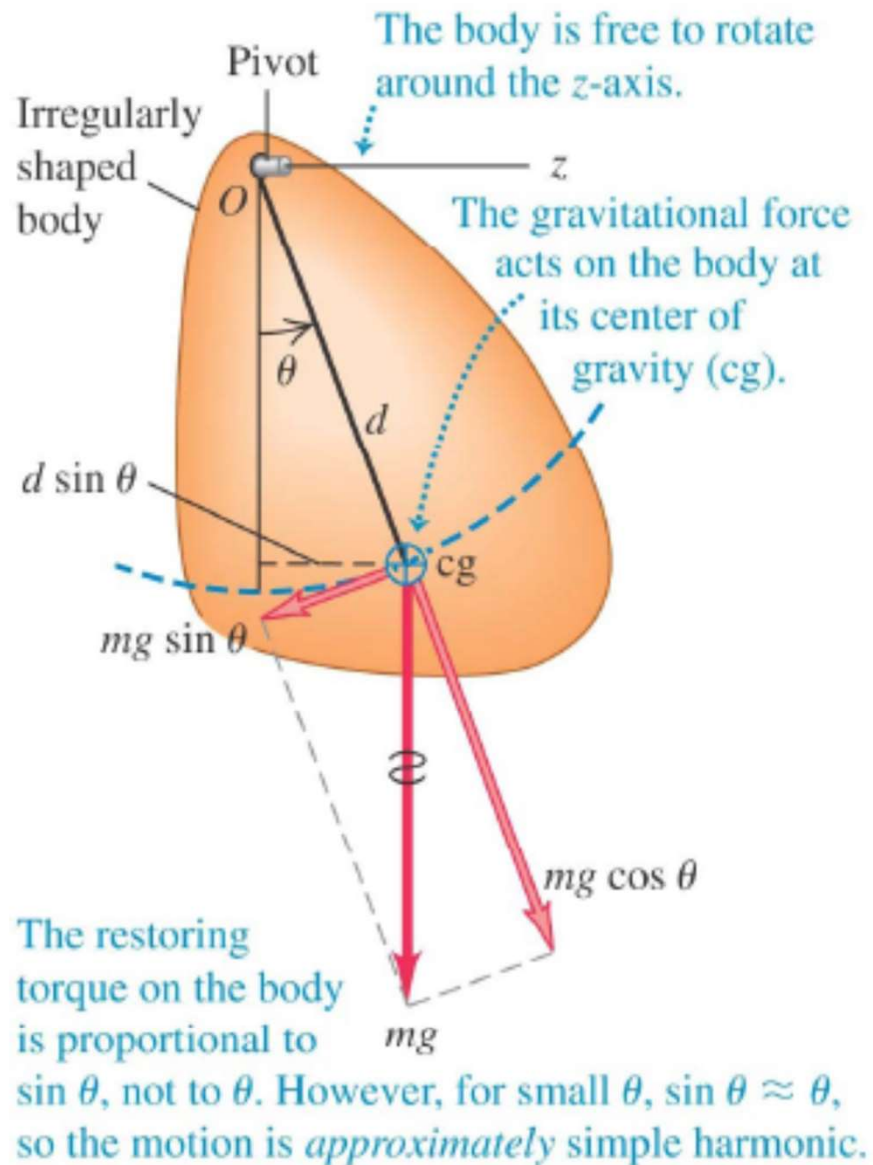
- A **physical pendulum** is any real pendulum that uses an extended body instead of a point-mass bob.
- For small amplitudes, its motion is simple harmonic.



The physical pendulum

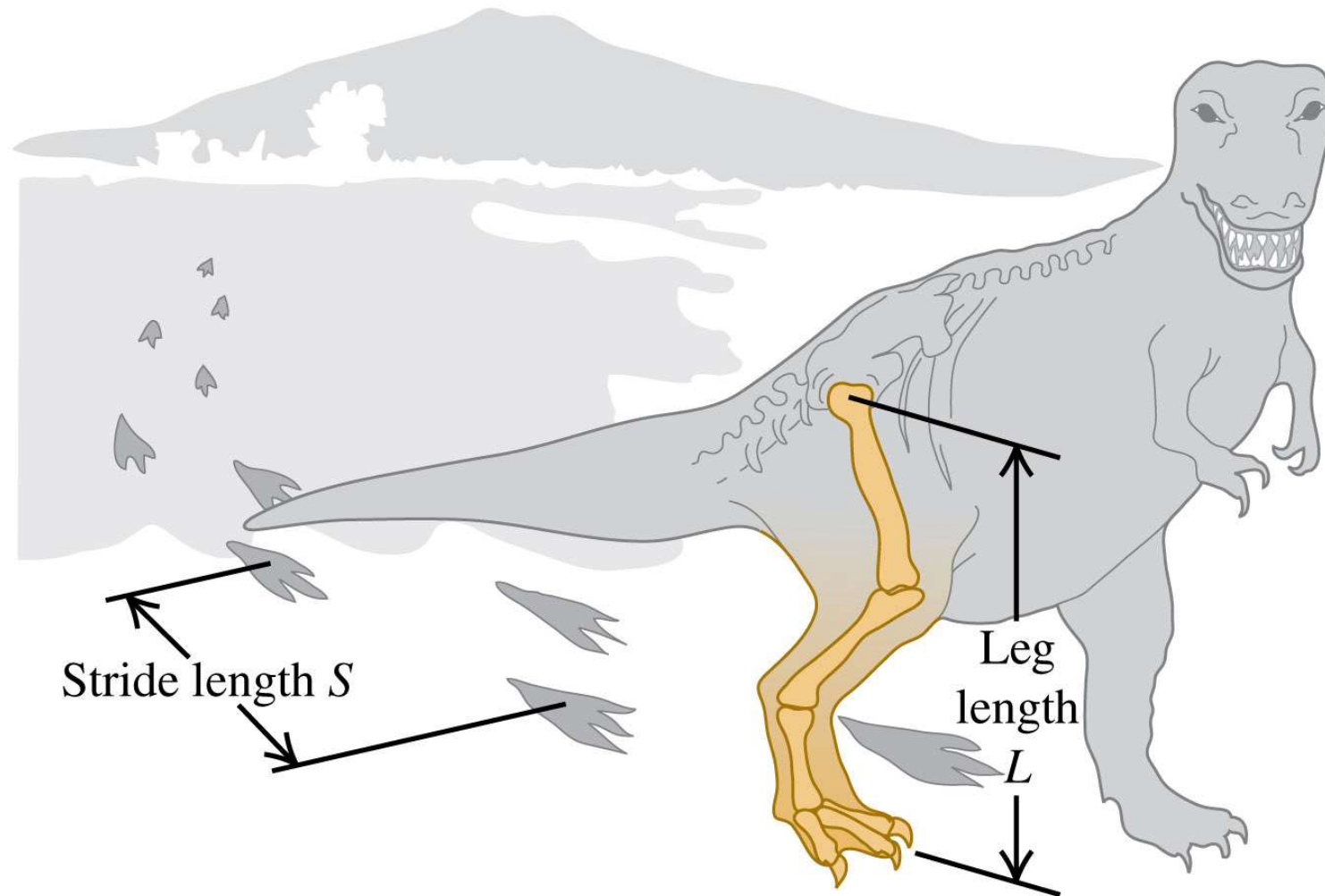
- A physical pendulum is any real pendulum that uses an extended body in motion. This illustrates a physical pendulum.

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$



Tyrannosaurus rex and the physical pendulum

- We can model the leg of *Tyrannosaurus rex* as a physical pendulum.



Dinosaurs, long tails, and the physical pendulum

All walking animals have a natural walking pace. This is the number of steps per minute that is more comfortable than a faster or slower pace. One can take this pace from considering the leg as a physical pendulum. (I for a rod pivoted at one end is $ML^2/3$)

- (a) Estimate the natural pace of a human.
- (b) Tyrannosaurus rex had a leg of length $L=3.1$ m and a stride of $S=4.0$ m. Estimate the walking speed of Tyrannosaurus rex.

