Chapter 14

Periodic Motion

PowerPoint® Lectures for
University Physics, 14th Edition
– Hugh D. Young and Roger A. Freedman

Lectures by Jason Harlow

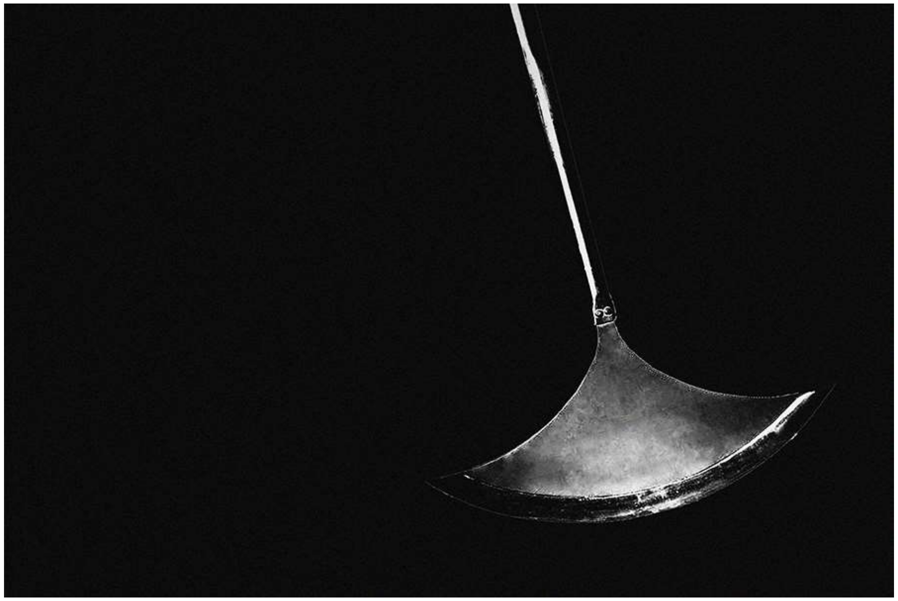
Learning Goals for Chapter 14

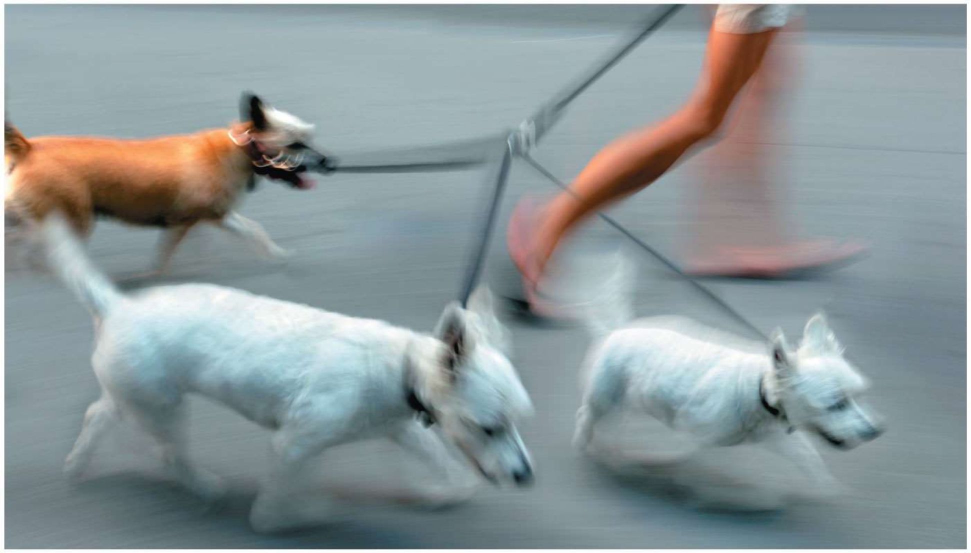
Looking forward at …

- how to describe oscillations in terms of amplitude, period, frequency, and angular frequency.
- how to apply the ideas of **simple harmonic motion** to different physical situations.
- how to analyze the motions of a pendulum.
- what determines how rapidly an oscillation dies out.
- how a driving force applied to an oscillator at a particular frequency can cause a very large response, or **resonance**.

Introduction

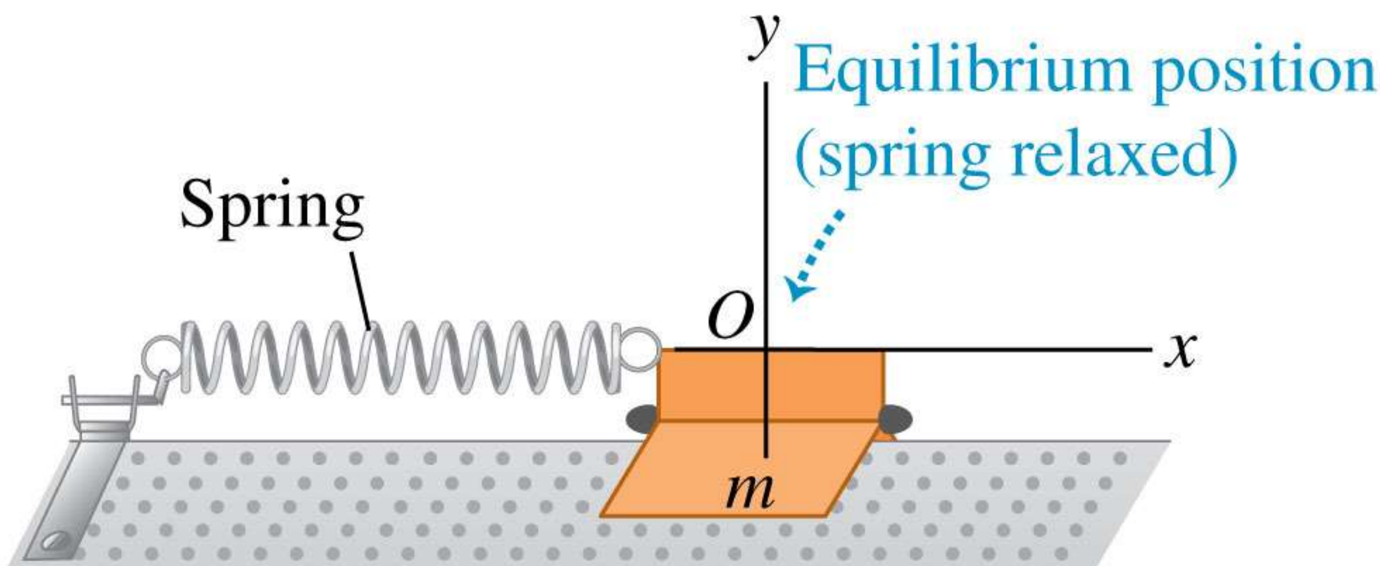
- What would happen if you doubled a pendulum's mass?
- Why do dogs walk faster than humans? Does it have anything to do with the characteristics of their legs?
- Many kinds of motion (such as a pendulum, musical vibrations, and pistons in car engines) repeat themselves. We call such behavior **periodic motion** or **oscillation**.



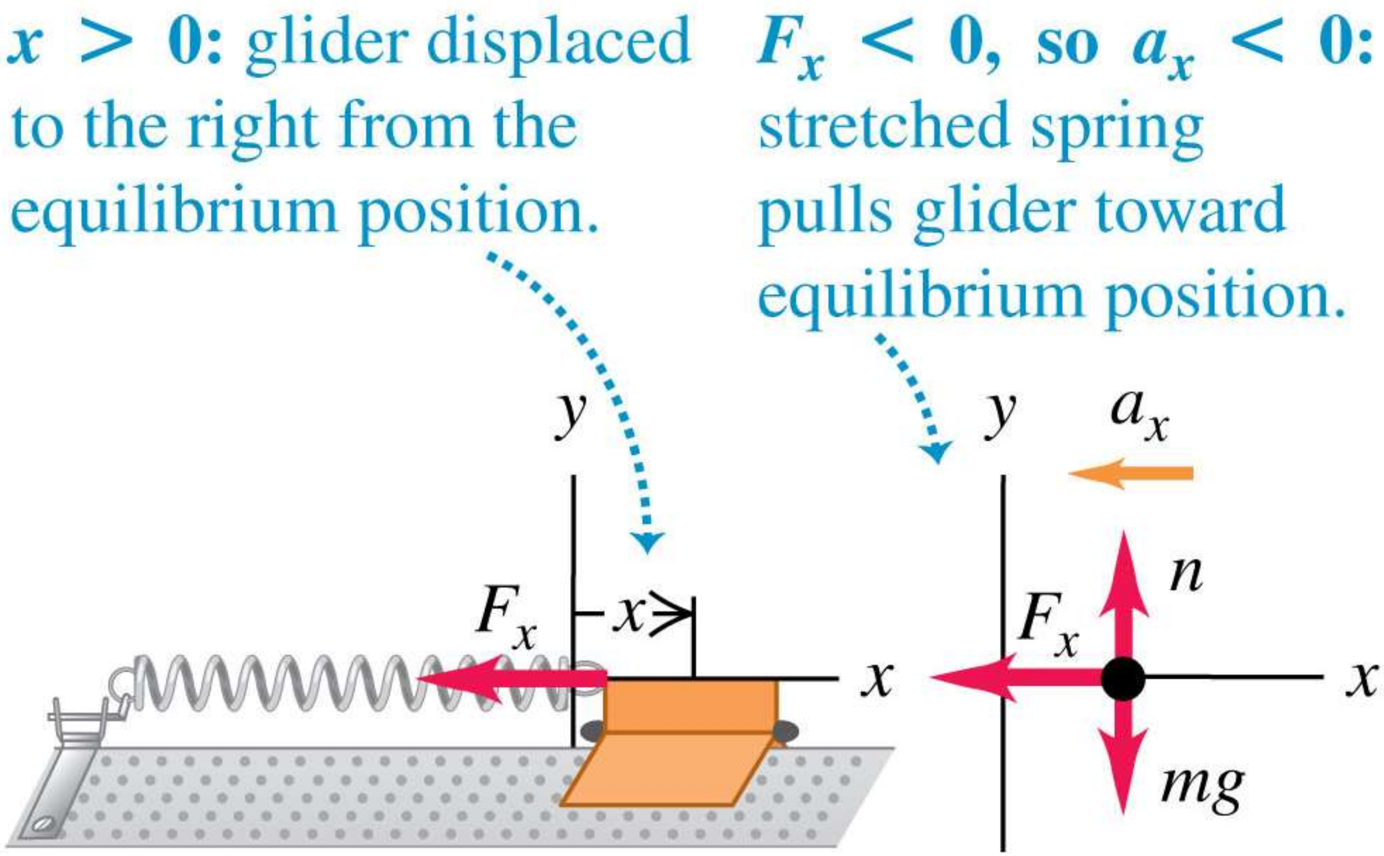


What causes periodic motion?

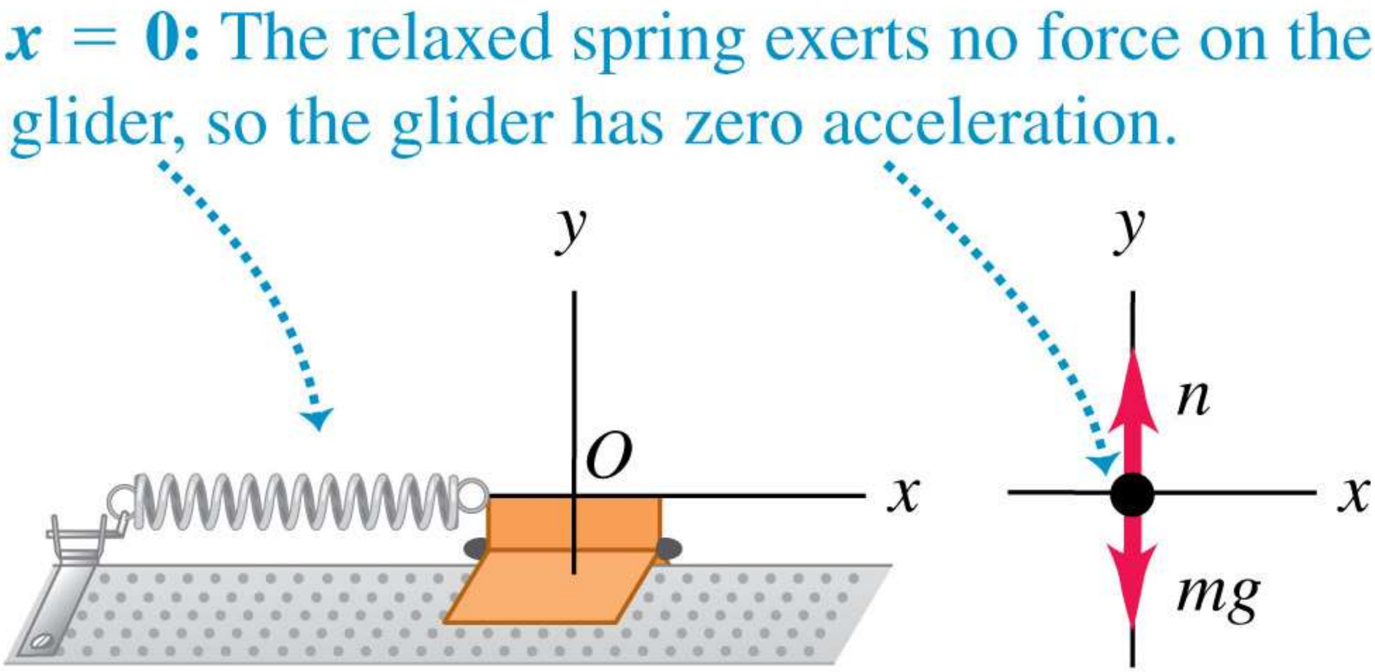
- If a body attached to a spring is displaced from its equilibrium position, the spring exerts a **restoring force** on it, which tends to restore the object to the equilibrium position.
- This force causes **oscillation** of the system, or **periodic motion**.



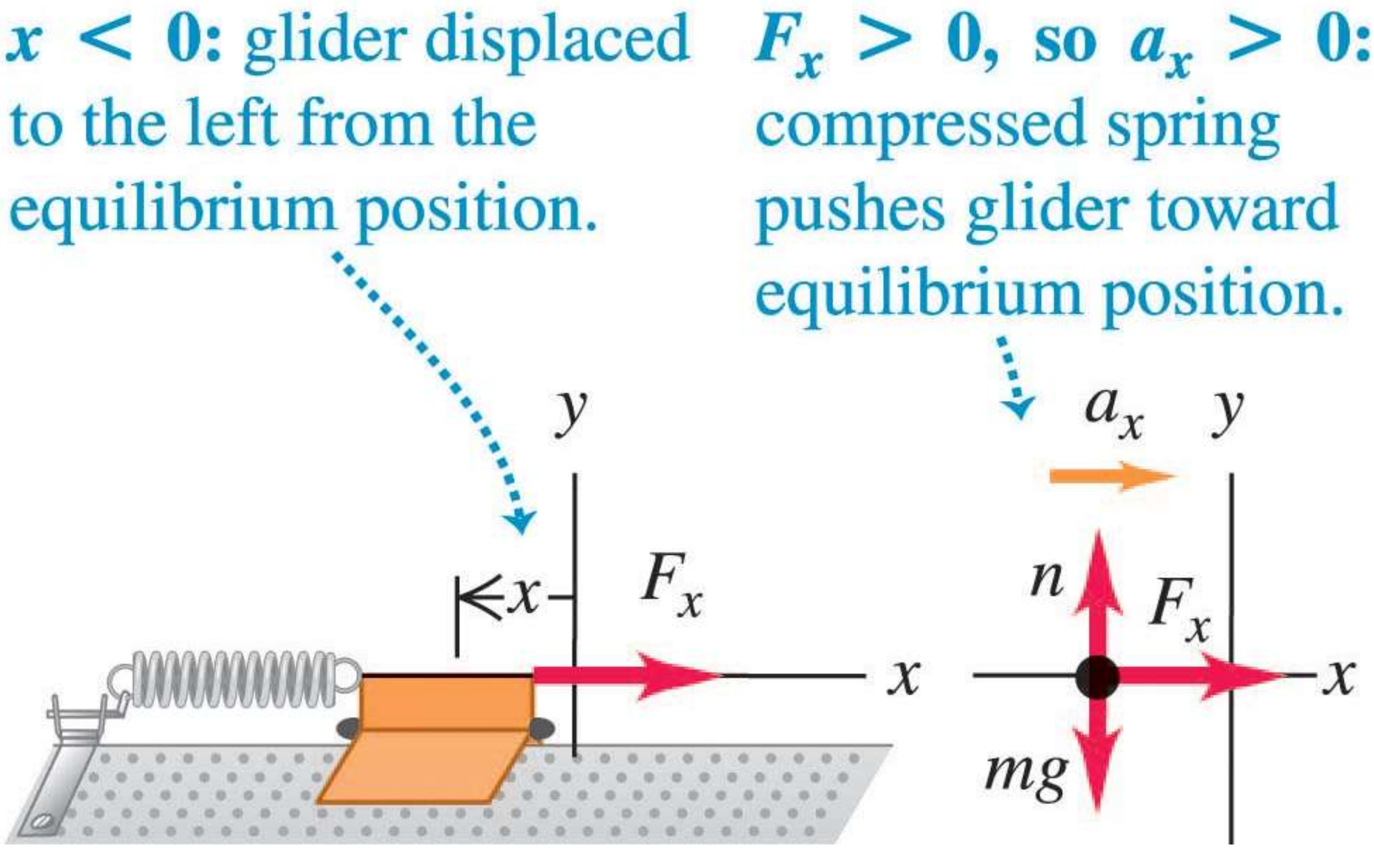
What causes periodic motion?



What causes periodic motion?



What causes periodic motion?

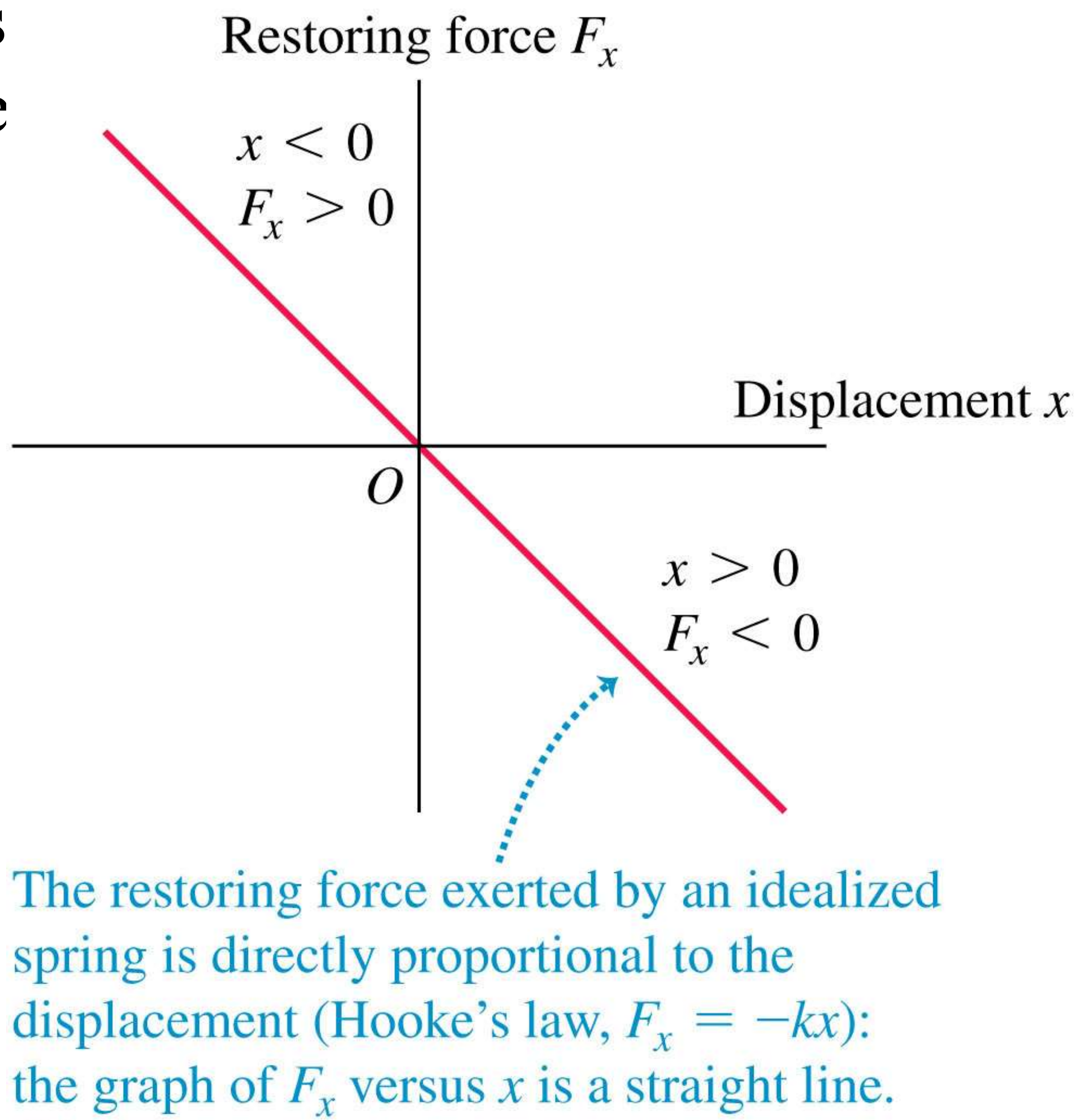


Characteristics of periodic motion

- The **amplitude**, A, is the maximum magnitude of displacement from equilibrium.
- The **period**, T, is the time for one cycle.
- The **frequency**, f, is the number of cycles per unit time.
- The **angular frequency**, ω, is 2π times the frequency: $\omega = 2\pi f$.
- The frequency and period are reciprocals of each other: $f = 1/T$ and $T = 1/f$.

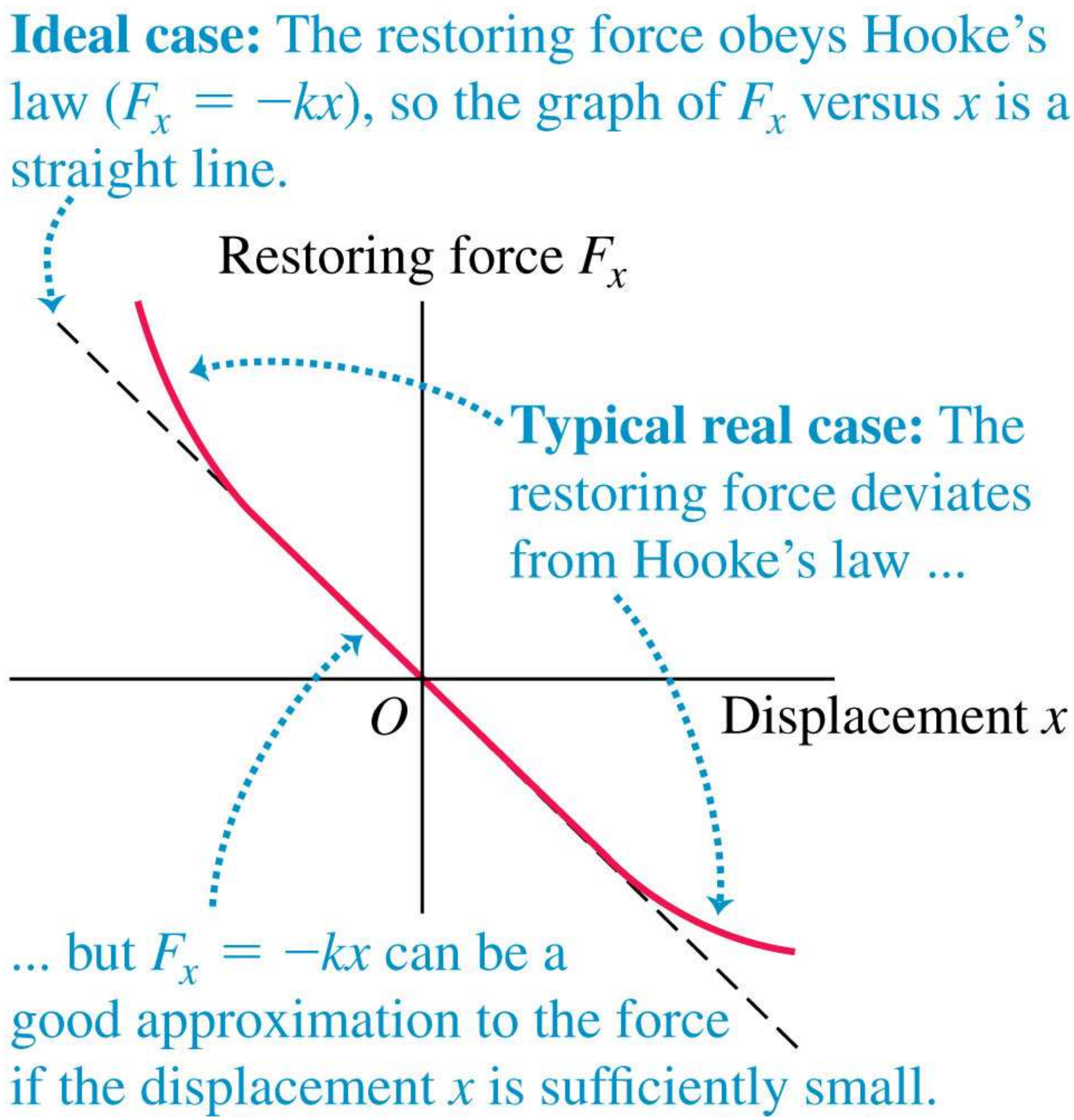
Simple harmonic motion (SHM)

- When the restoring force is *directly proportional* to the displacement from equilibrium, the resulting motion is called **simple harmonic motion** (SHM).

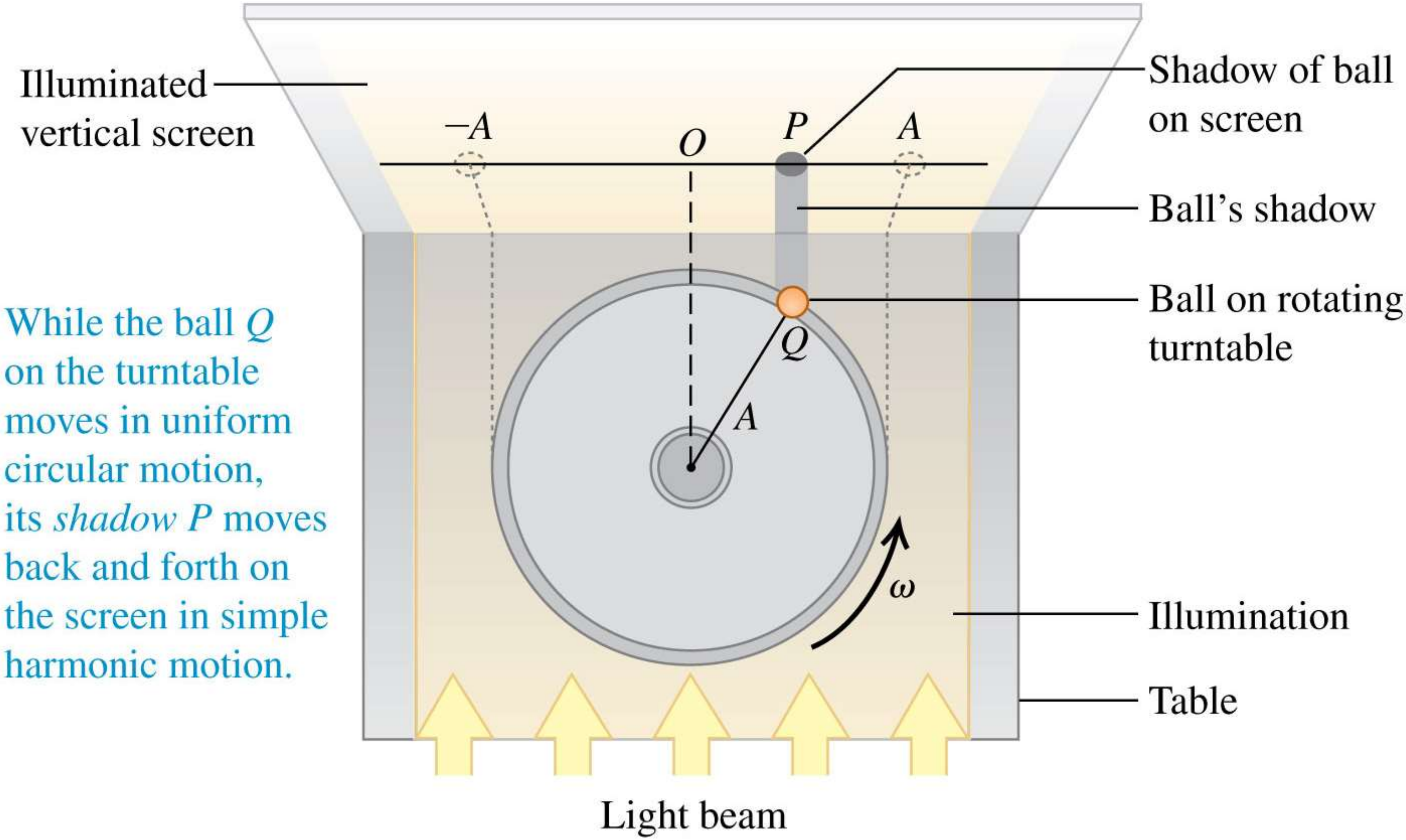


Simple harmonic motion (SHM)

- In many systems the restoring force is approximately proportional to displacement if the displacement is sufficiently small.
- That is, if the amplitude is small enough, the oscillations are approximately simple harmonic.



Simple harmonic motion viewed as a projection

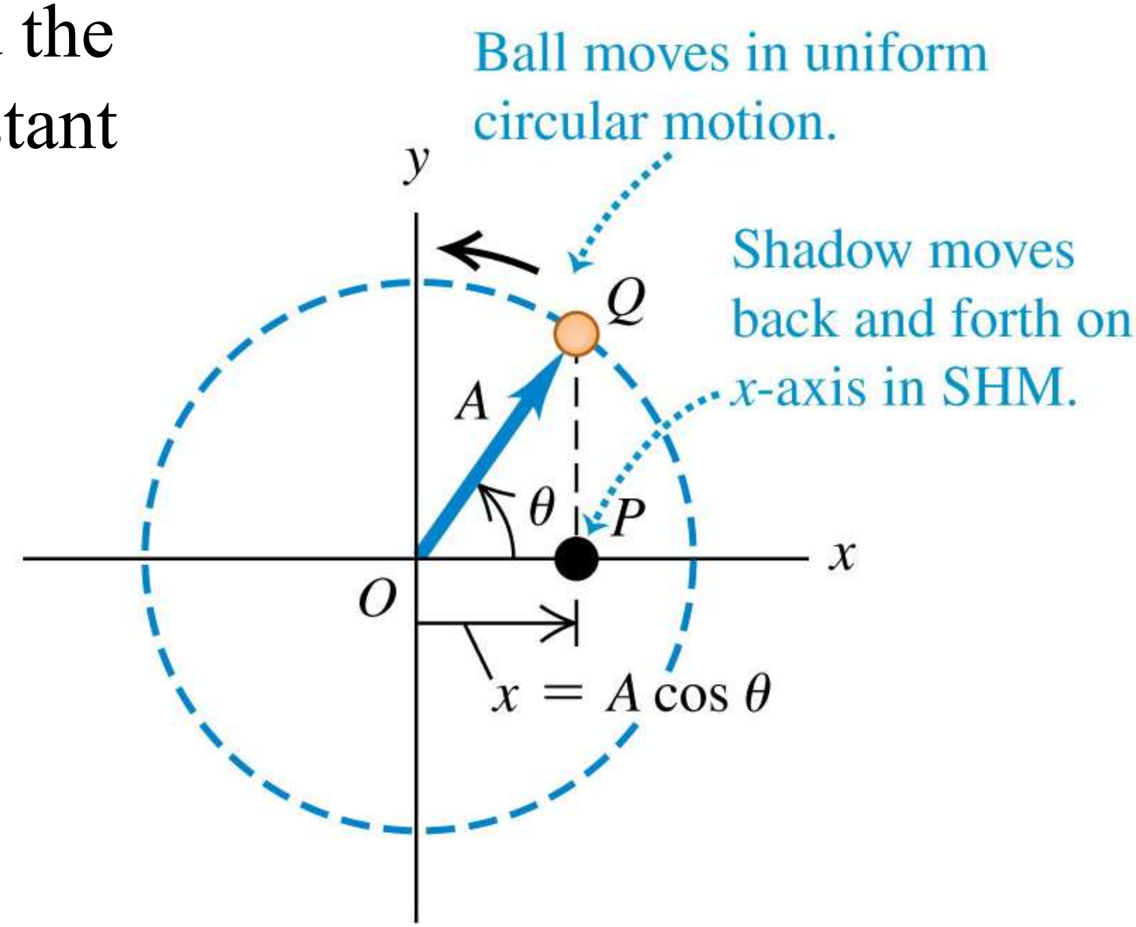


Simple harmonic motion viewed as a projection

- The circle in which the ball moves so that its projection matches the motion of the oscillating body is called the **reference circle**.

- As point *Q* moves around the reference circle with constant angular speed, vector *OQ* rotates with the same angular speed.

- Such a rotating vector is called a **phasor**.



Characteristics of SHM

- For a body of mass *m* vibrating by an ideal spring with a force constant *k*:

Angular frequency for simple harmonic motion

$$\omega = \sqrt{\frac{k}{m}}$$

k: Force constant of restoring force
m: Mass of object

Frequency for simple harmonic motion

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

ω: Angular frequency
k: Force constant of restoring force
m: Mass of object

Period for simple harmonic motion

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

f: Frequency
ω: Angular frequency
m: Mass of object
k: Force constant of restoring force

Characteristics of SHM

- The greater the mass m in a tuning fork's tines, the lower the frequency of oscillation, and the lower the pitch of the sound that the tuning fork produces.

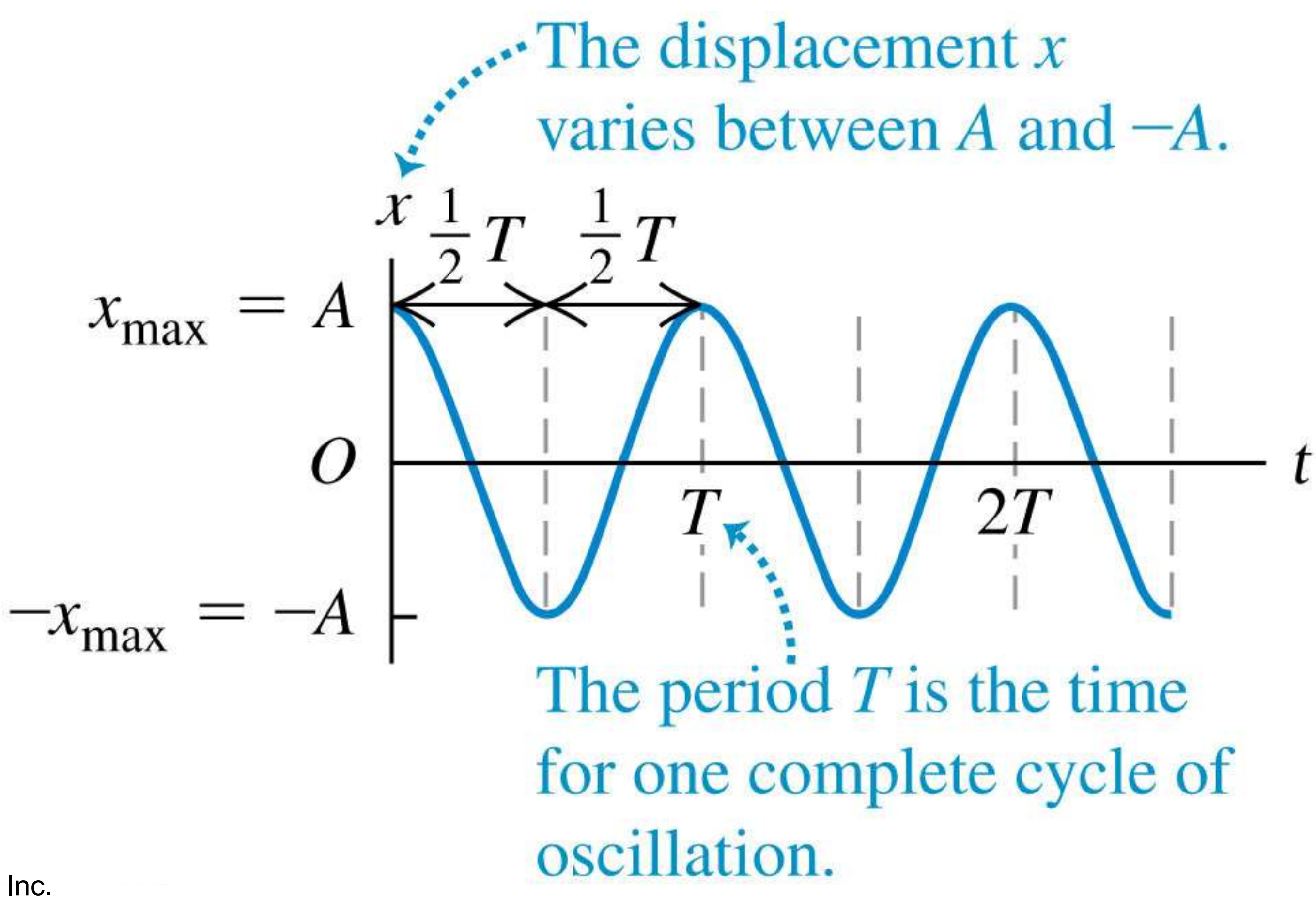
Displacement as a function of time in SHM

- The displacement as a function of time for SHM is:

Displacement in simple harmonic motion as a function of time:

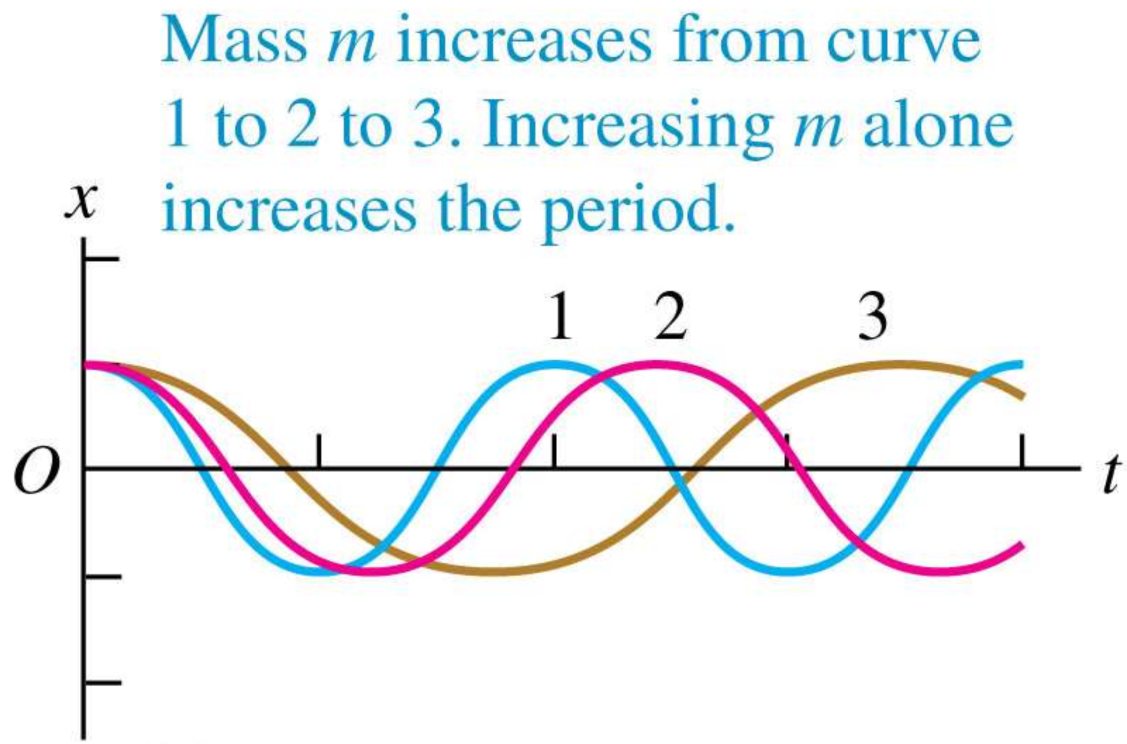
$$x = A\cos(\omega t + \phi)$$

- Amplitude: A
- Time: t
- Phase angle: ϕ
- Angular frequency $= \sqrt{k/m}$



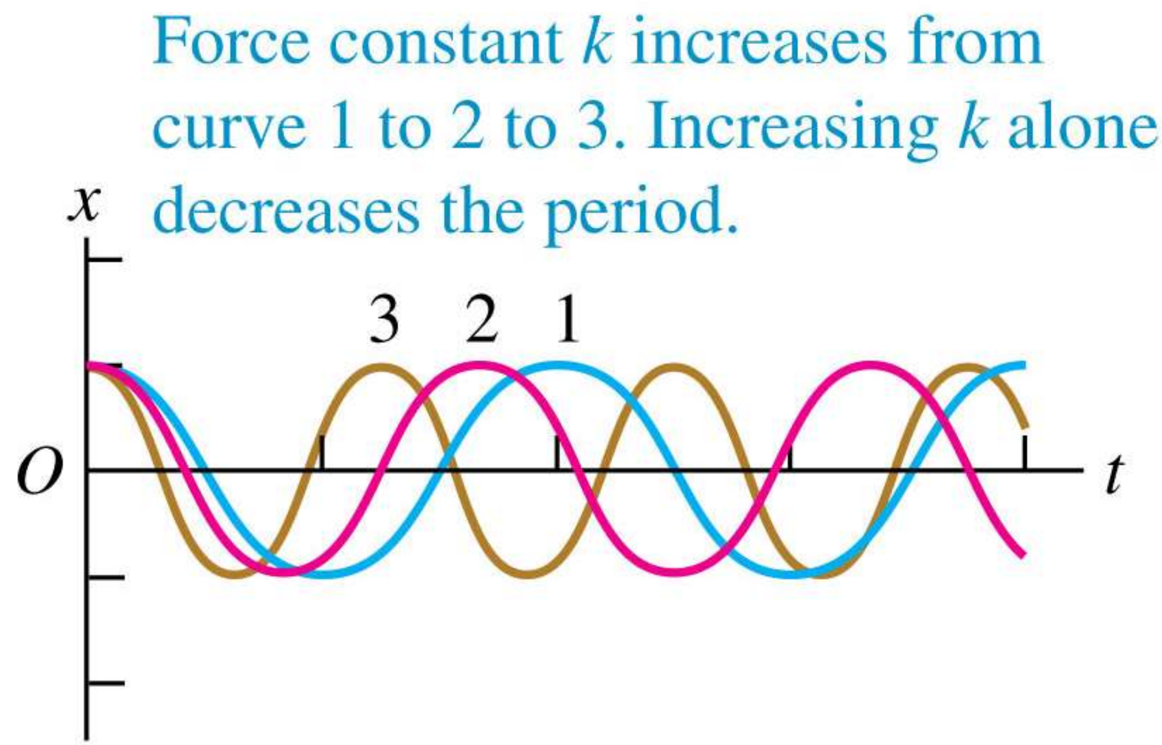
Displacement as a function of time in SHM

- Increasing *m* with the same *A* and *k* increases the period of the displacement vs time graph.



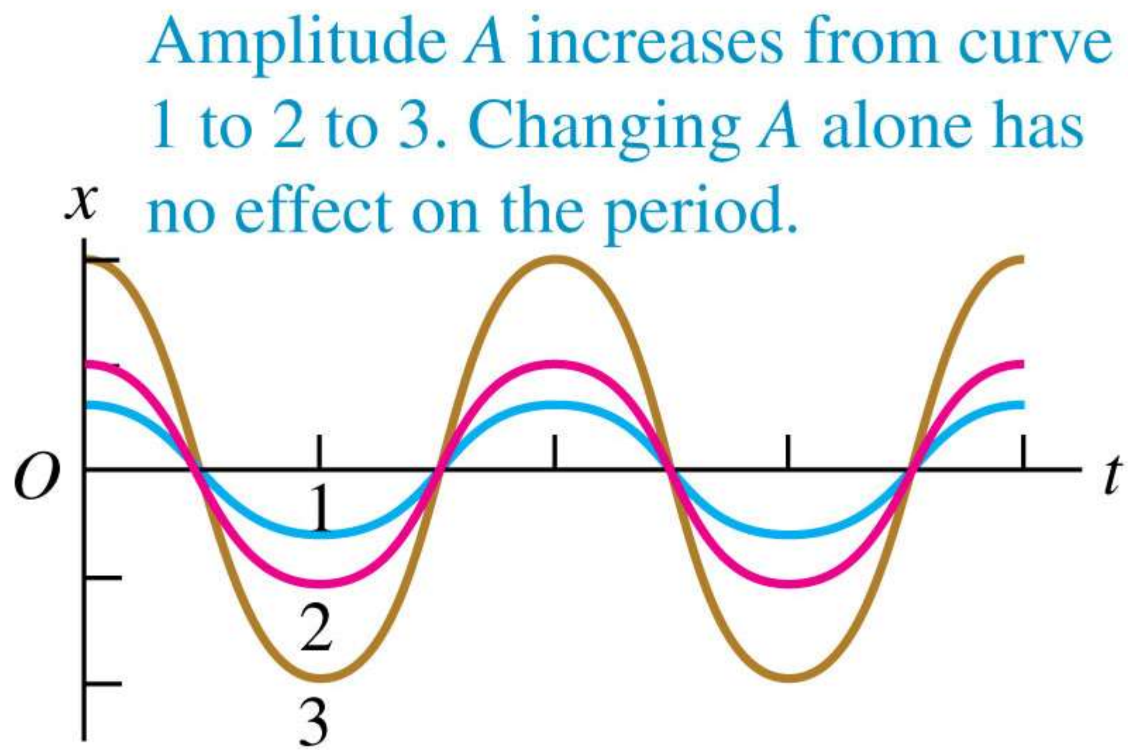
Displacement as a function of time in SHM

- Increasing *k* with the same *A* and *m* decreases the period of the displacement vs time graph.



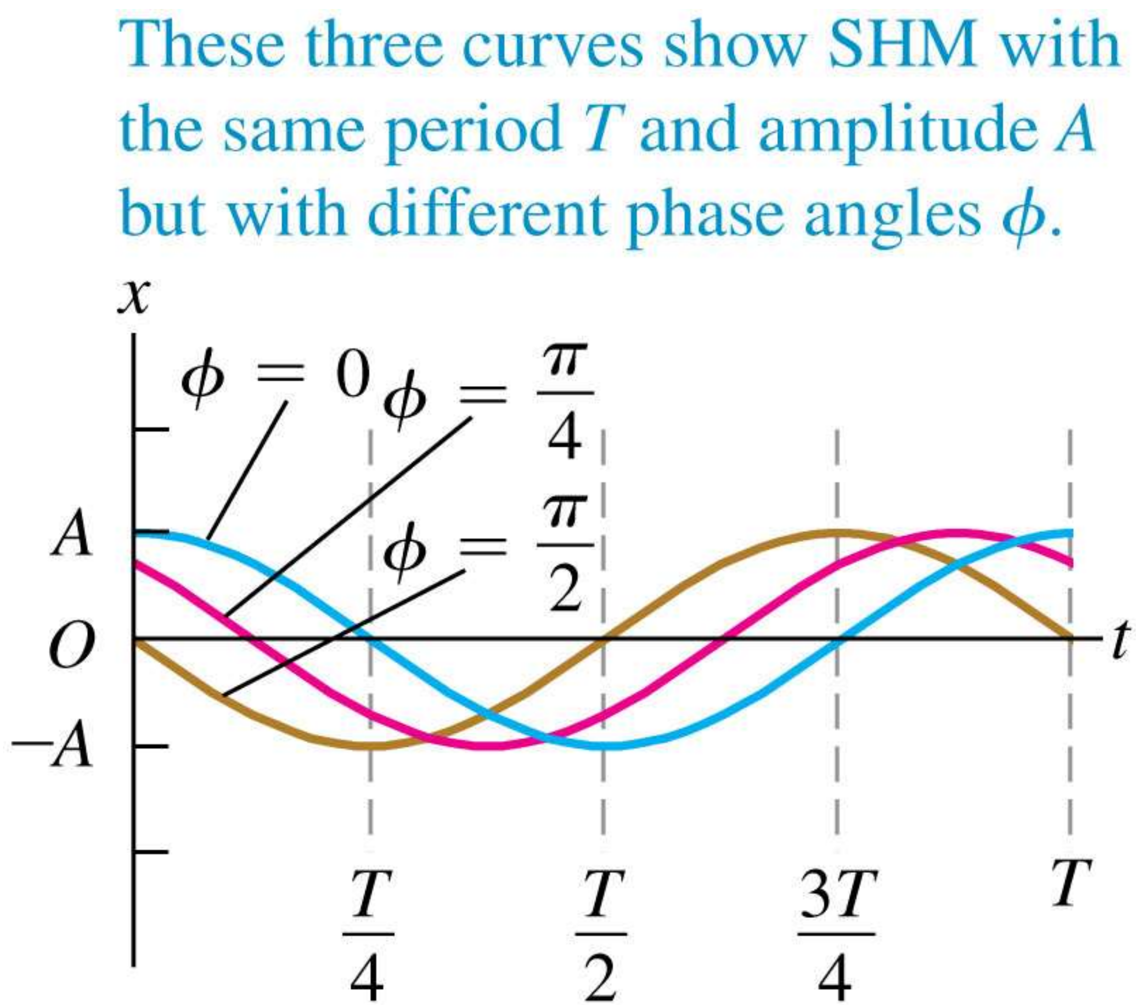
Displacement as a function of time in SHM

- Increasing A with the same m and k does not change the period of the displacement vs time graph.

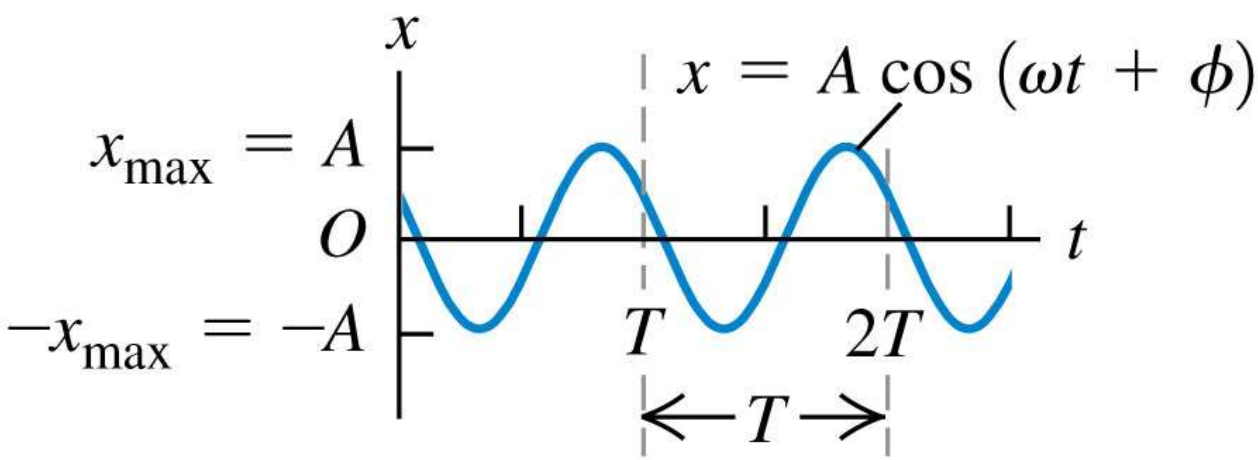


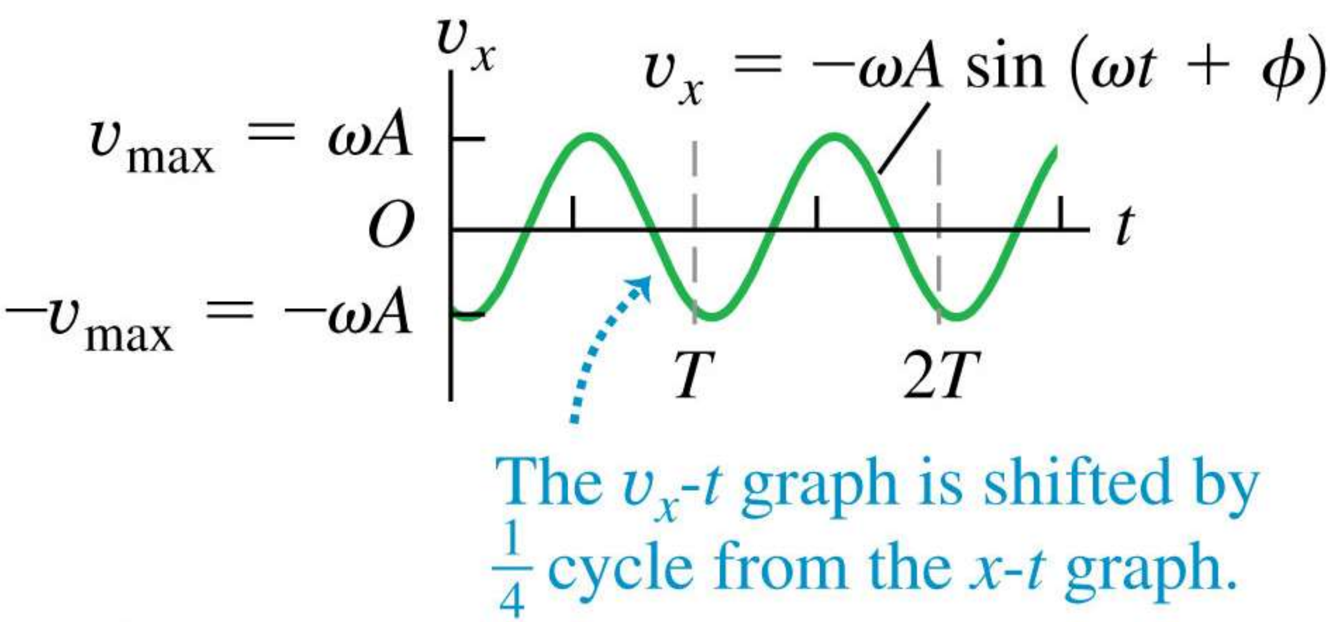
Displacement as a function of time in SHM

- Increasing ϕ with the same A, m, and k only shifts the displacement vs time graph to the left.

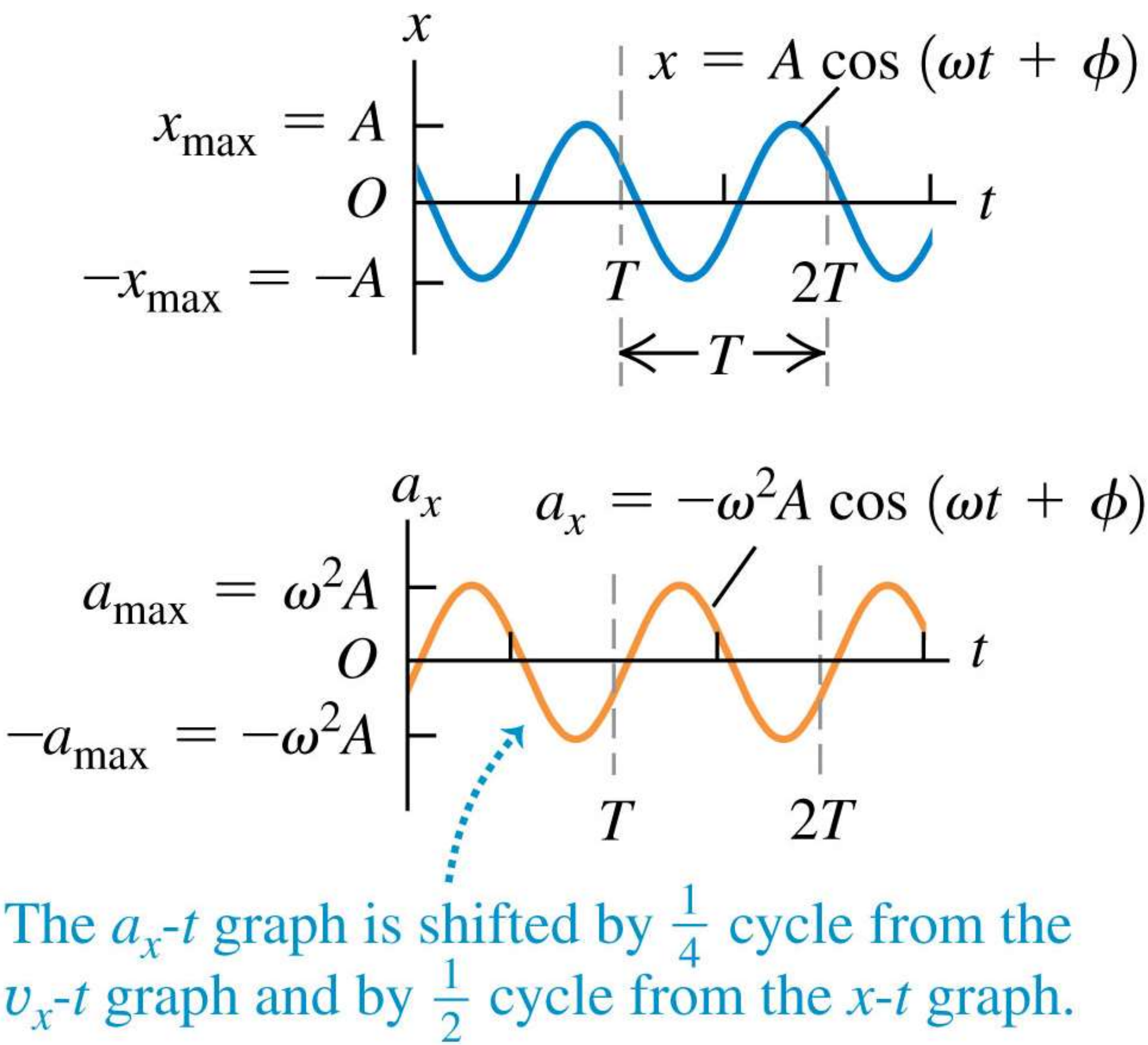


Graphs of displacement and velocity for SHM





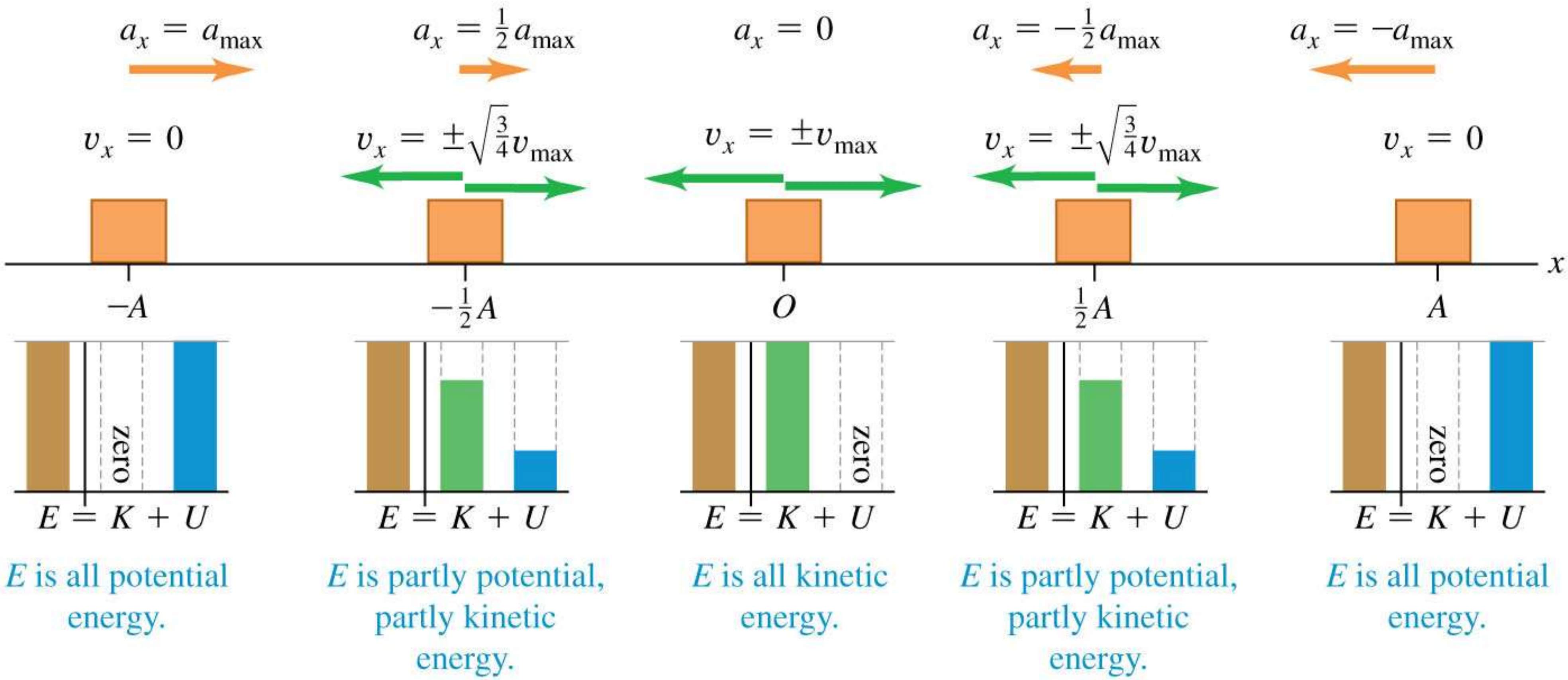
Graphs of displacement and acceleration for SHM



Energy in SHM

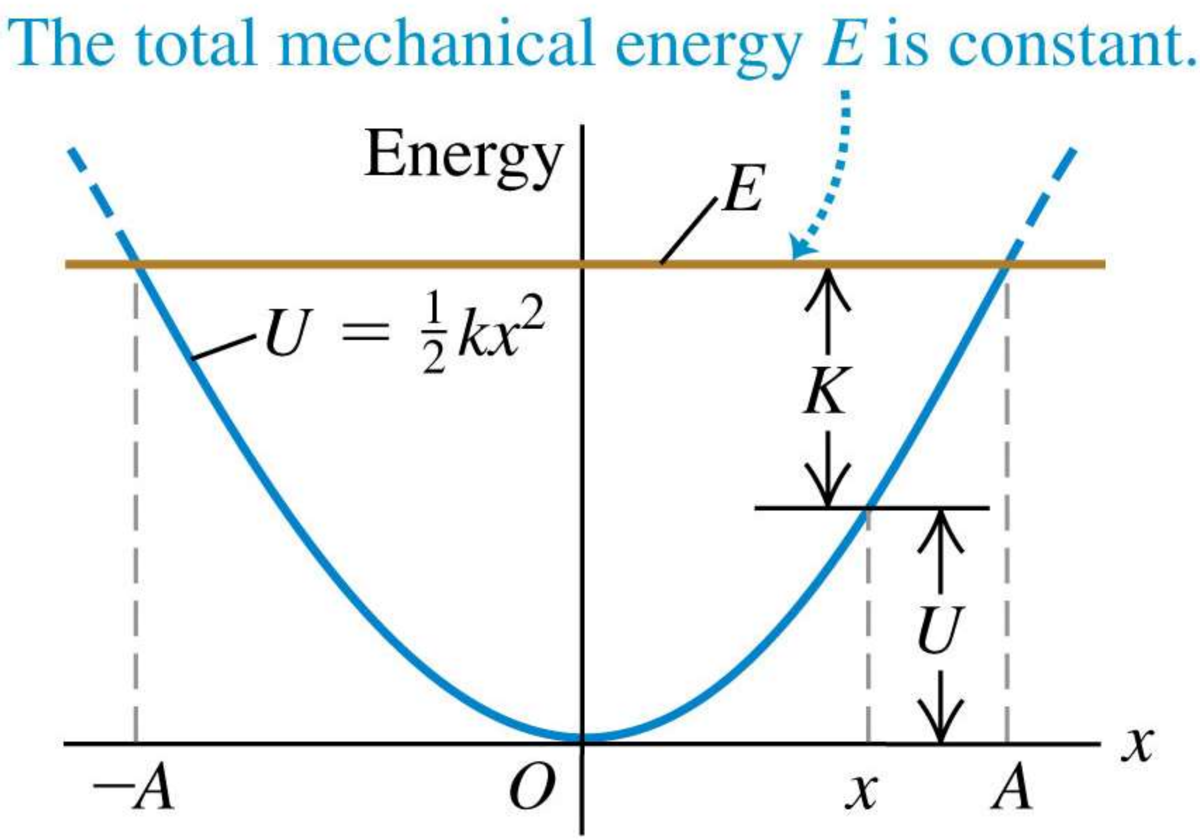
- The total mechanical energy $E = K + U$ is conserved in SHM:

$$E = 1/2\ mv_x^2 + 1/2\ kx^2 = 1/2\ kA^2 = \text{constant}$$



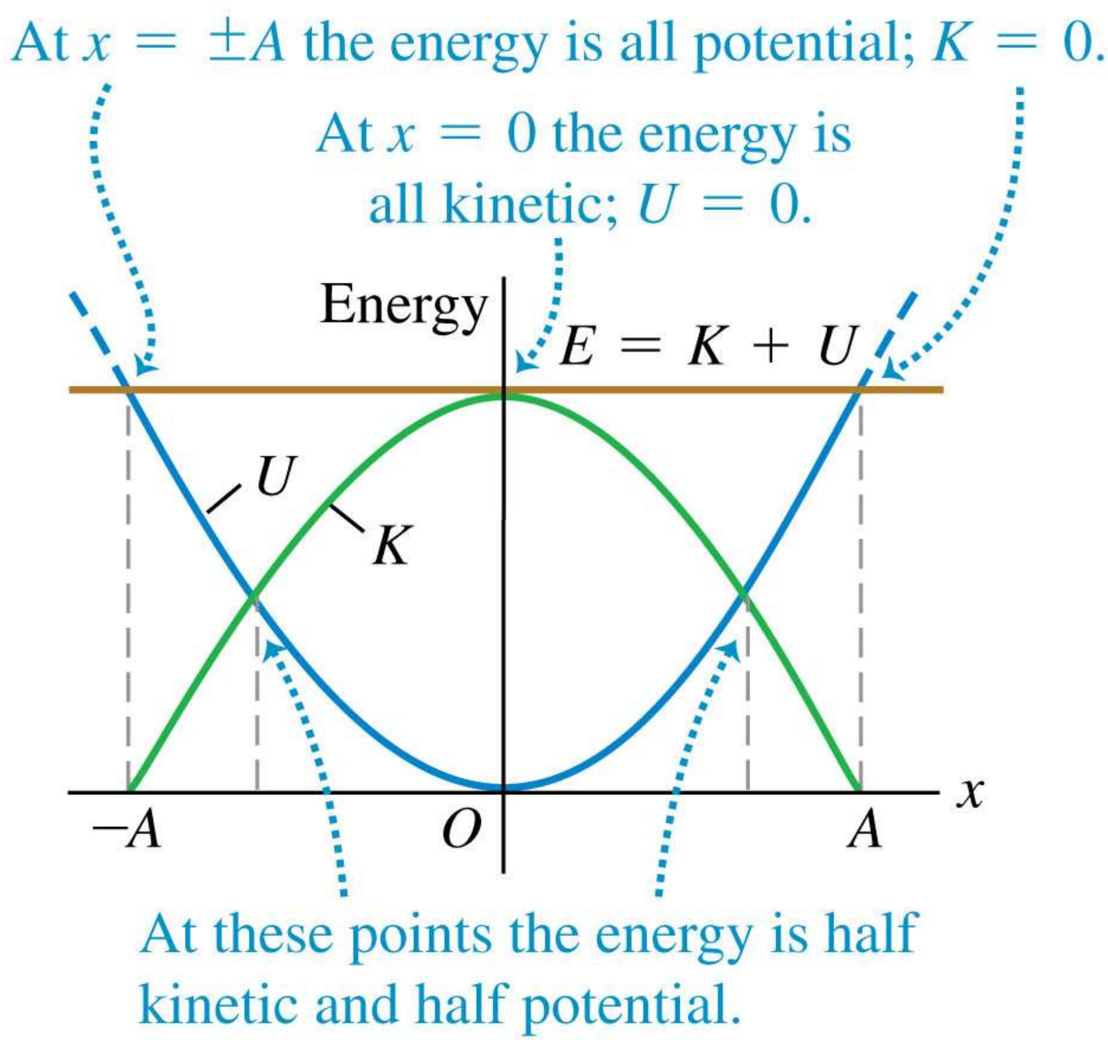
Energy diagrams for SHM

- The potential energy U and total mechanical energy E for a body in SHM as a function of displacement x.



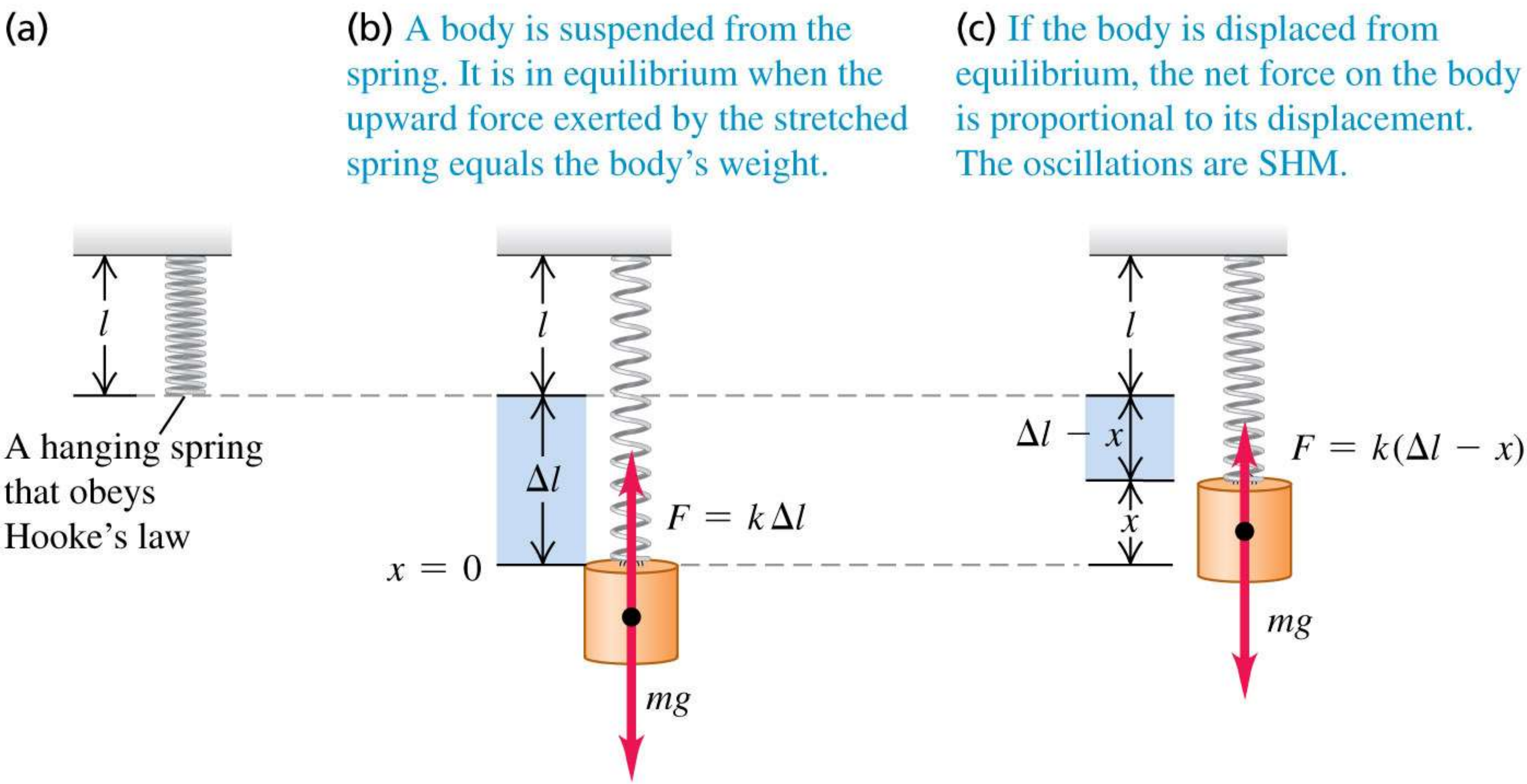
Energy diagrams for SHM

- The potential energy U, kinetic energy K, and total mechanical energy E for a body in SHM as a function of displacement x.



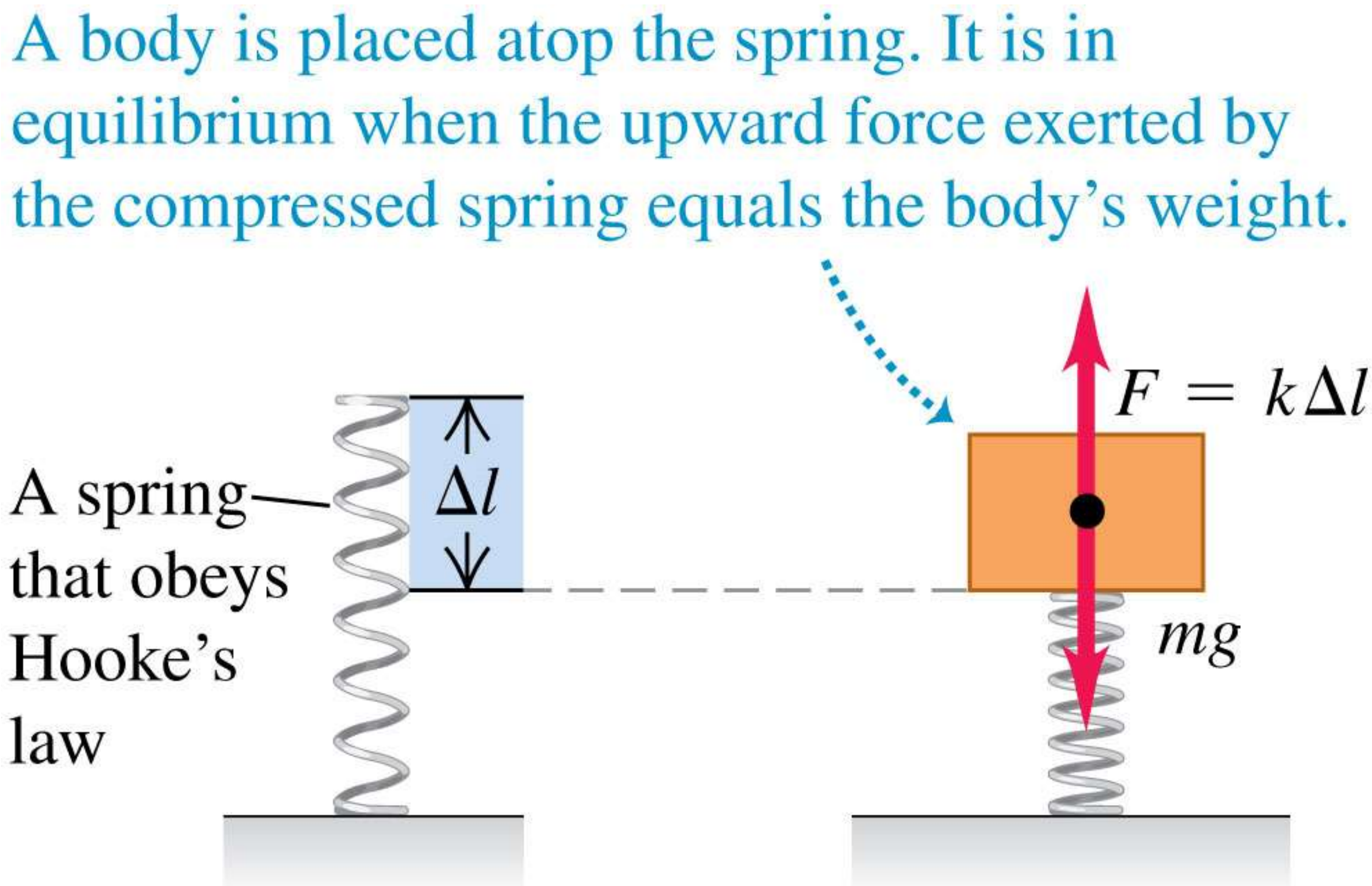
Vertical SHM

- If a body oscillates vertically from a spring, the restoring force has magnitude kx. Therefore the vertical motion is SHM.



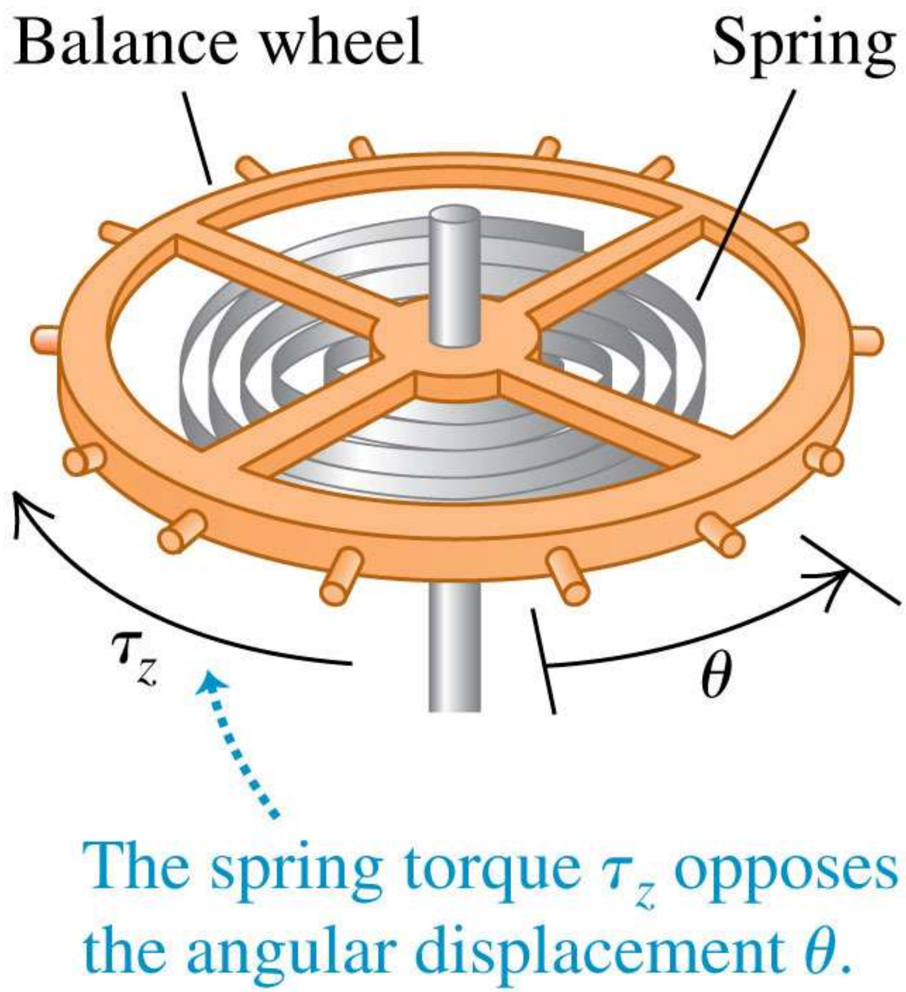
Vertical SHM

- If the weight mg compresses the spring a distance Δl, the force constant is $k = mg/\Delta l$.

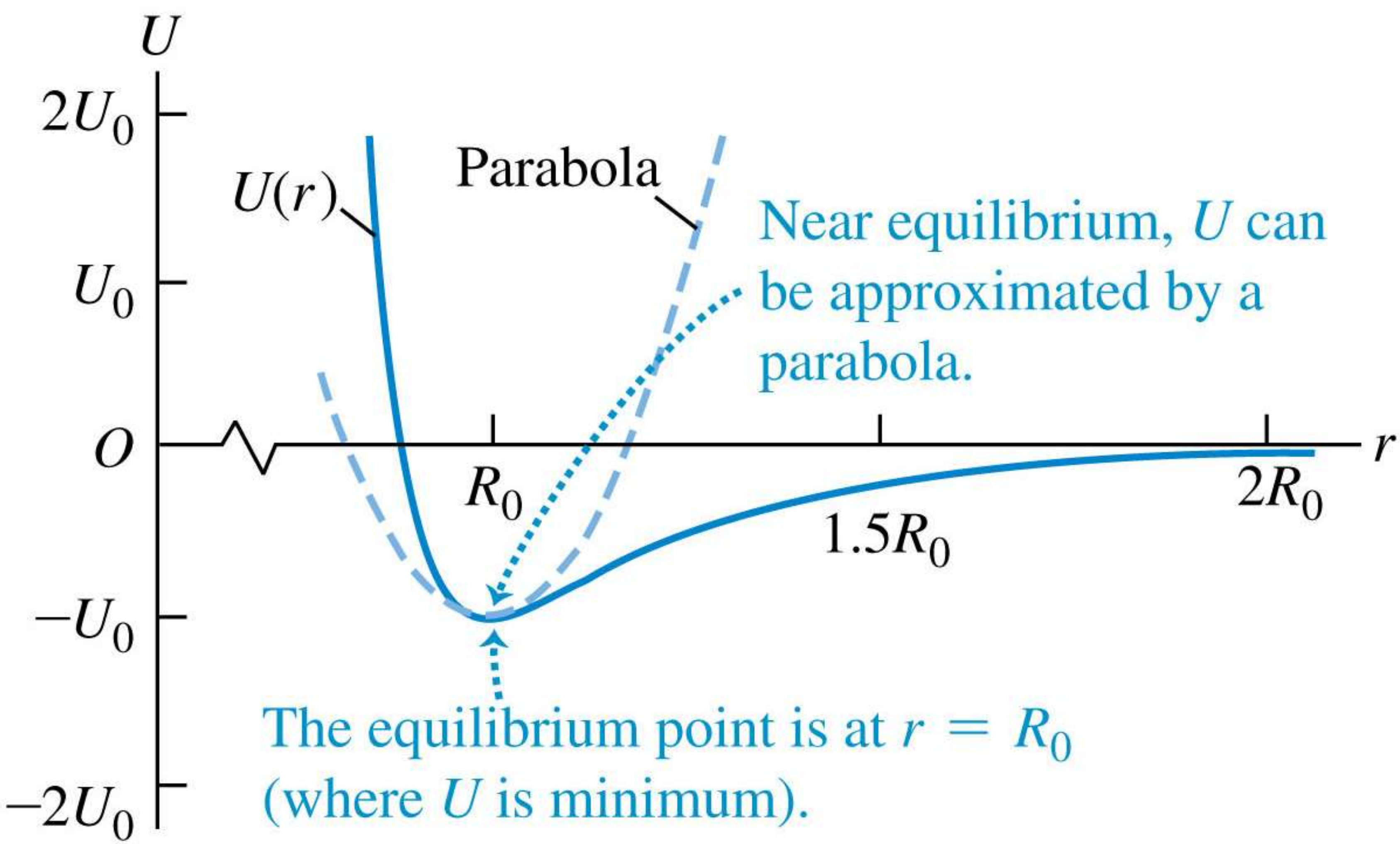


Angular SHM

- A coil spring exerts a restoring torque $\tau_z = -\kappa\theta$, where κ is called the **torsion constant** of the spring.
- The result is *angular* simple harmonic motion.



Potential energy of a two atom system

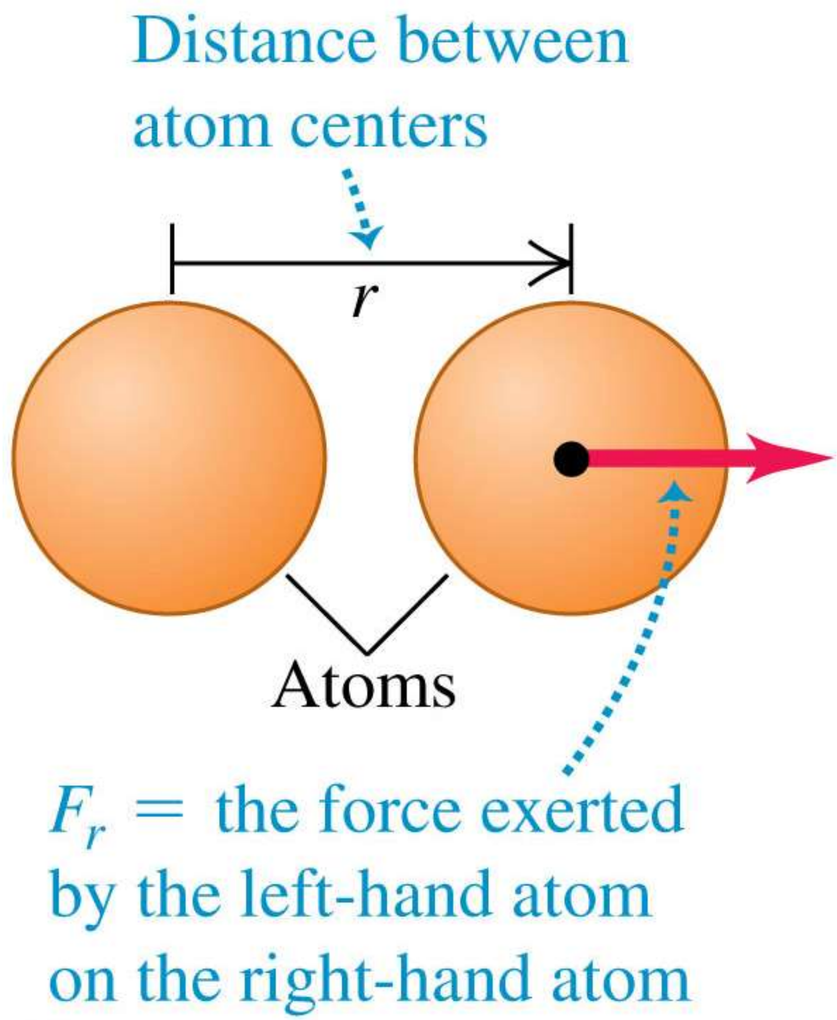


Vibrations of molecules

- Shown are two atoms having centers a distance r apart, with the equilibrium point at $r = R_0$.
- If they are displaced a small distance x from equilibrium, the restoring force is approximately

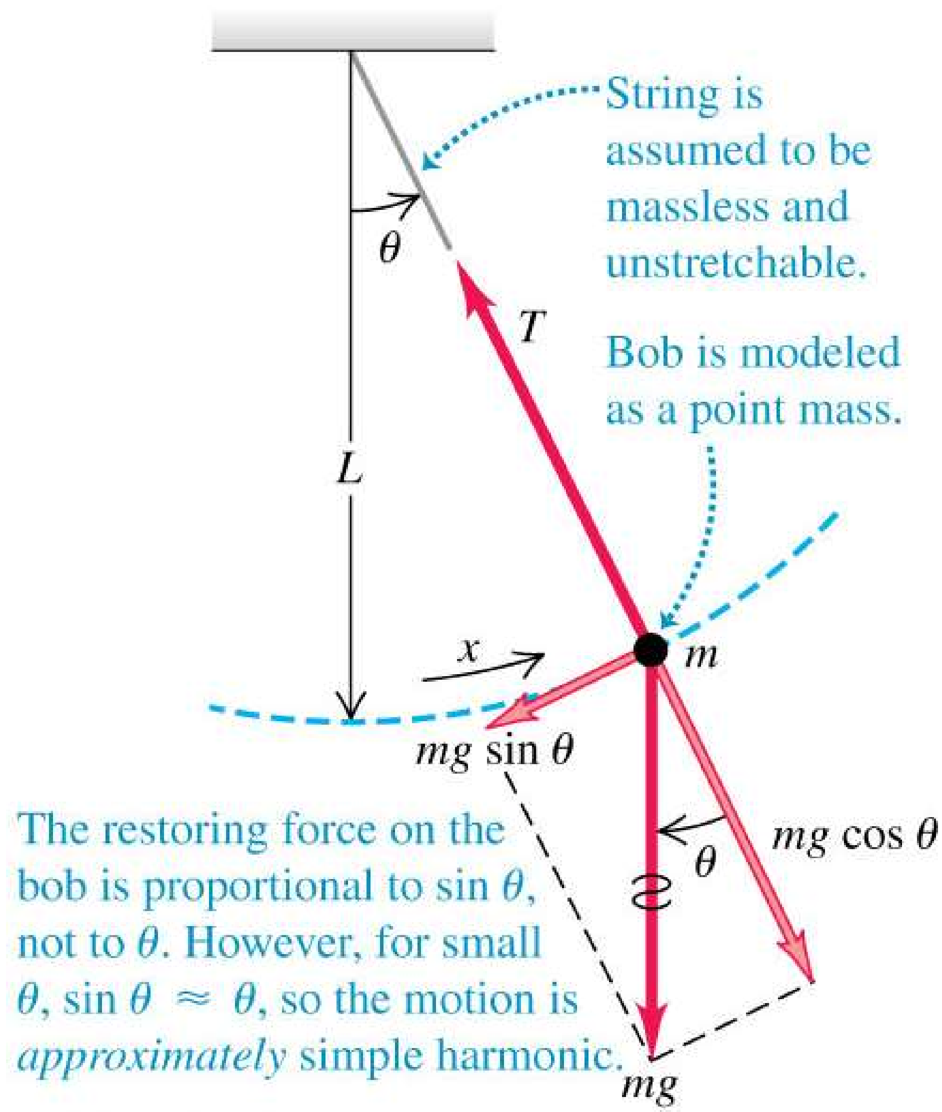
$$F_r = -(72U_0/R_0^2)x$$

- So $k = 72U_0/R_0^2$, and the motion is SHM.



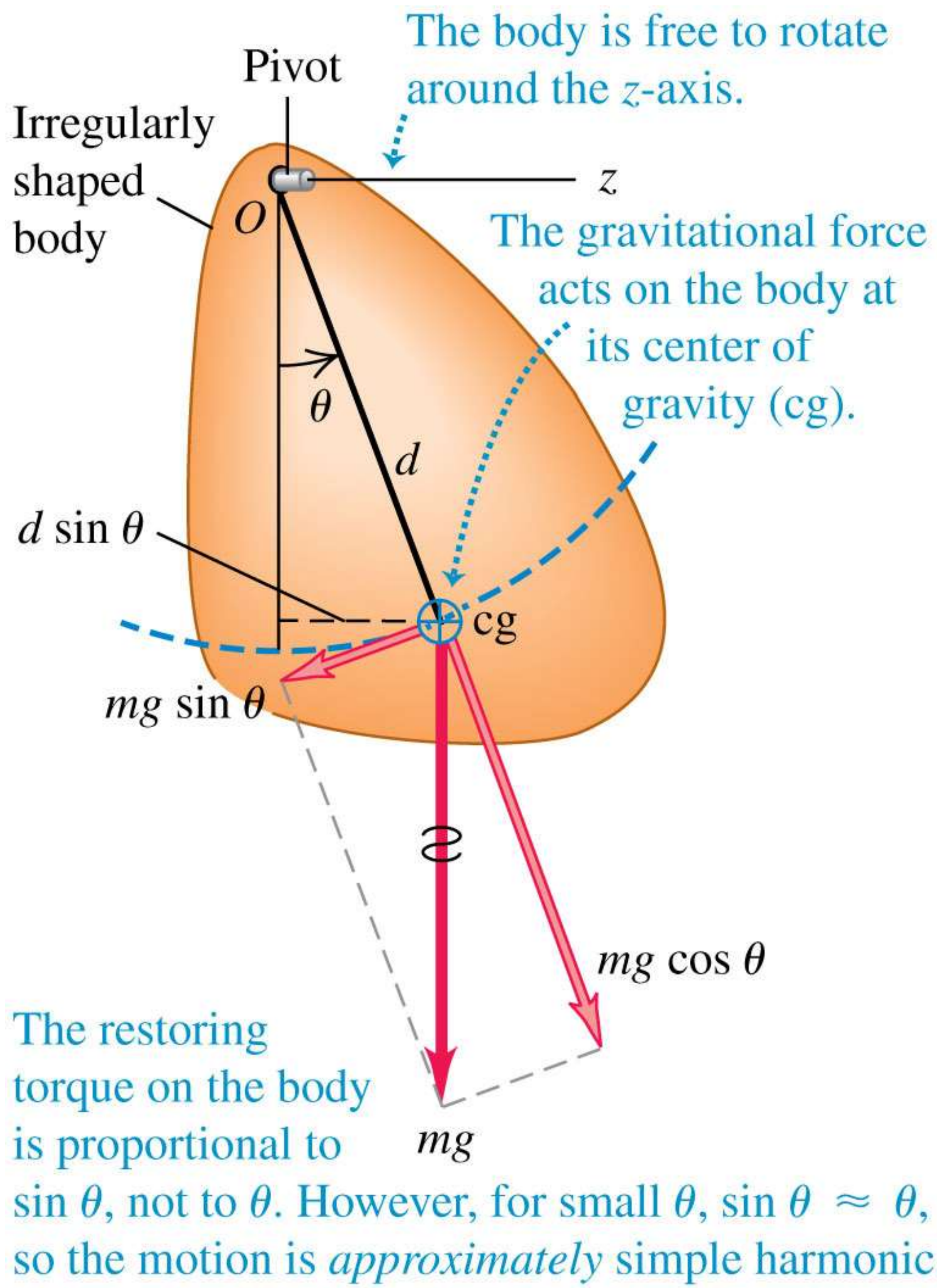
The simple pendulum

- A **simple pendulum** consists of a point mass (the bob) suspended by a massless, unstretchable string.
- If the pendulum swings with a small amplitude θ with the vertical, its motion is simple harmonic.



The physical pendulum

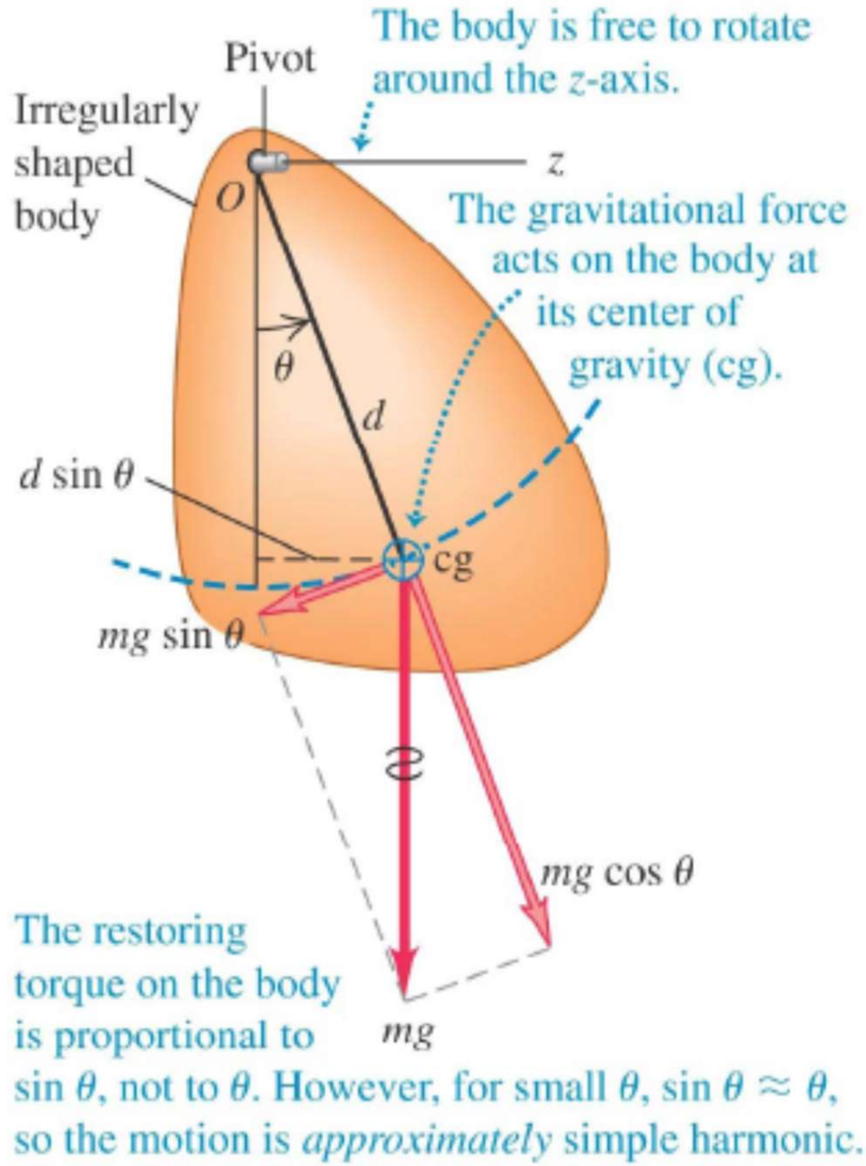
- A **physical pendulum** is any real pendulum that uses an extended body instead of a point-mass bob.
- For small amplitudes, its motion is simple harmonic.



The physical pendulum

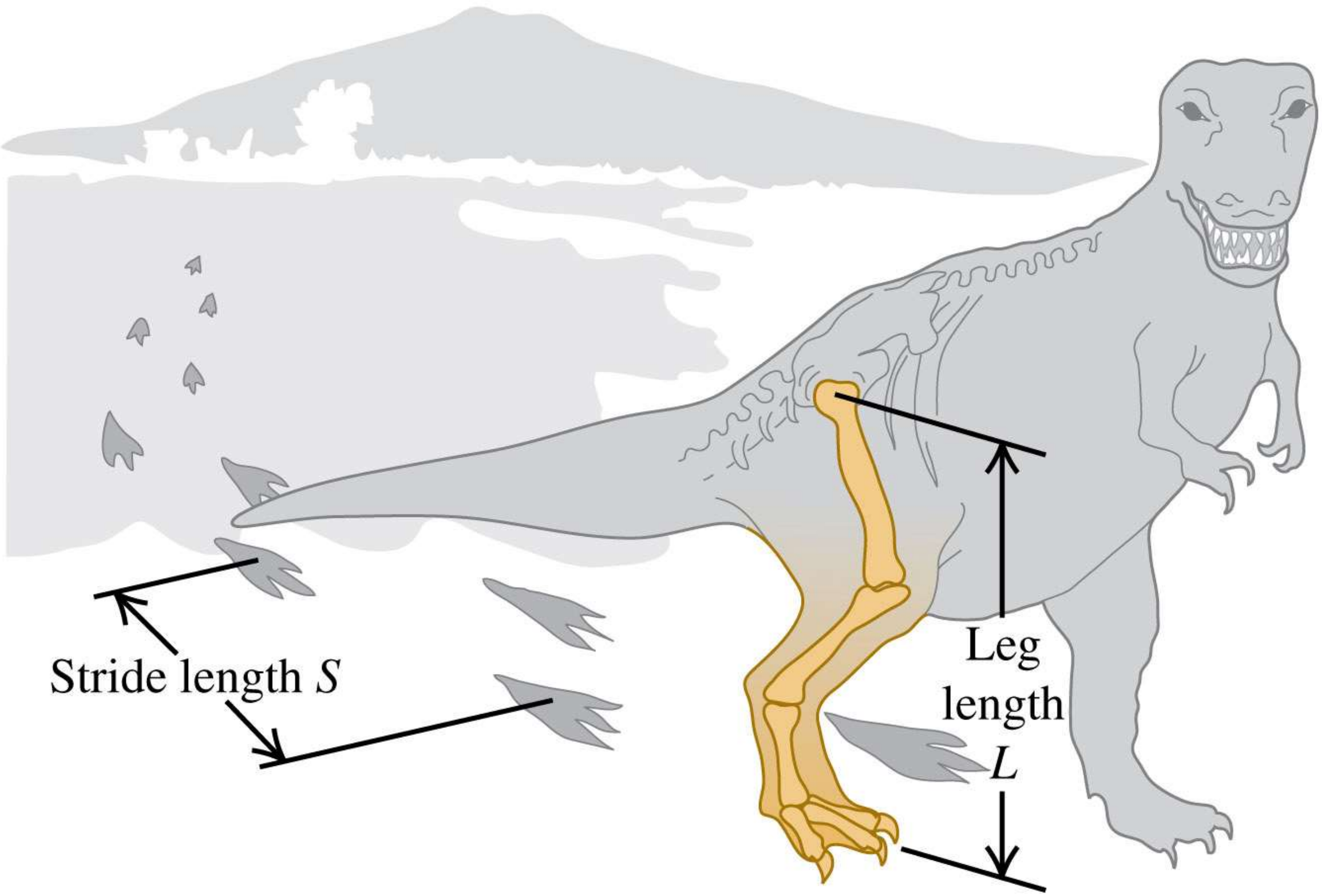
- A physical pendulum is any real pendulum that uses an extended body in motion. This illustrates a physical pendulum.

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$



Tyrannosaurus rex and the physical pendulum

- We can model the leg of *Tyrannosaurus rex* as a physical pendulum.



Dinosaurs, long tails, and the physical pendulum

All walking animals have a natural walking pace. This is the number of steps per minute that is more comfortable than a faster or slower pace. One can take this pace from considering the leg as a physical pendulum. (I for a rod pivoted at one end is $ML^2/3$)

(a) Estimate the natural pace of a human.

(b) Tyrannosaurus rex had a leg of length L=3.1 m and a stride of S=4.0m. Estimate the walking speed of Tyrannousaurus rex.