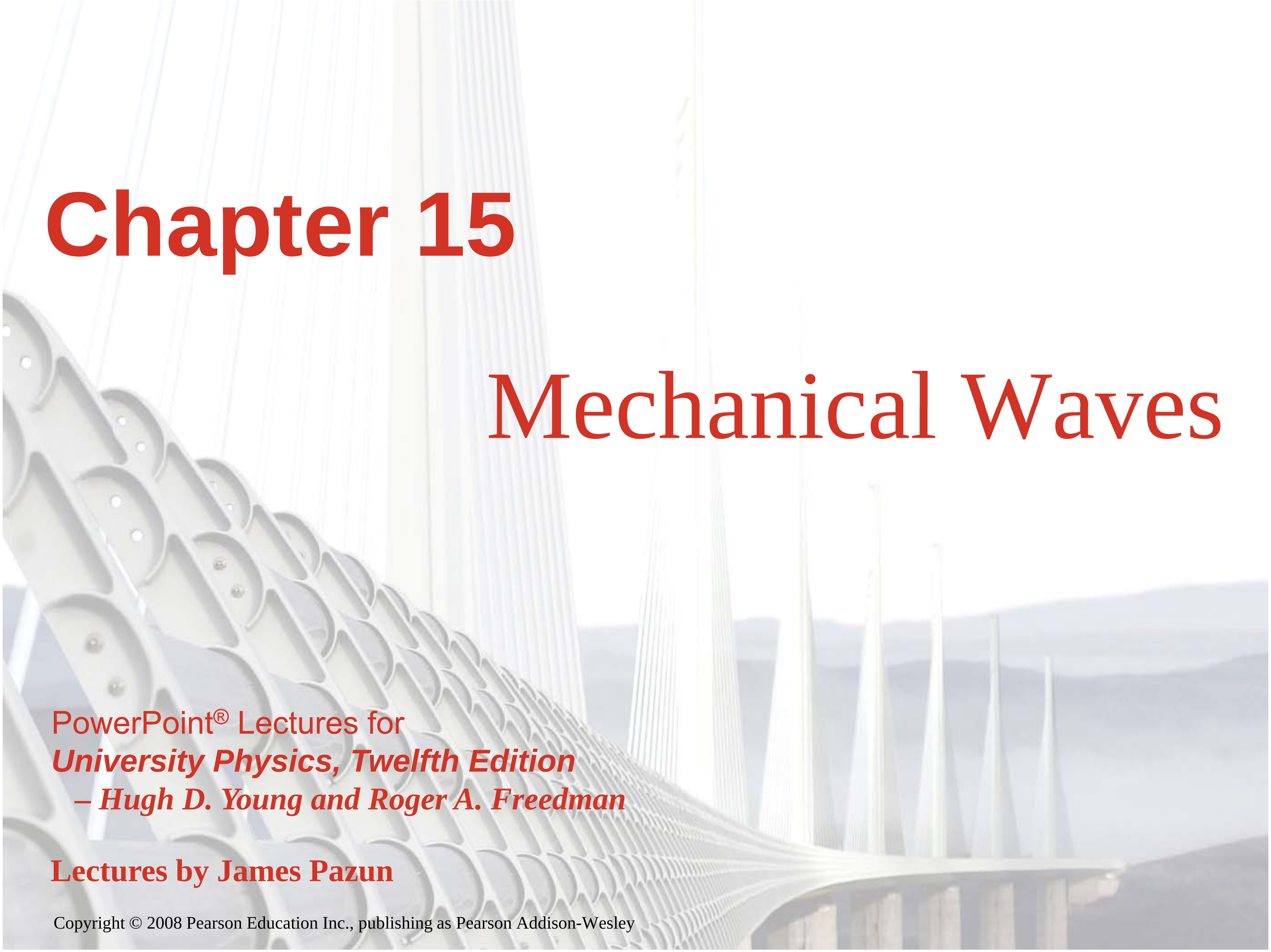


Chapter 15

Mechanical Waves

PowerPoint® Lectures for
University Physics, Twelfth Edition
– Hugh D. Young and Roger A. Freedman

Lectures by James Pazun

Goals for Chapter 15

- To study waves and their properties
- To consider wave functions and wave dynamics
- To calculate the power in a wave
- To consider wave superposition
- To study standing waves on a string

Introduction

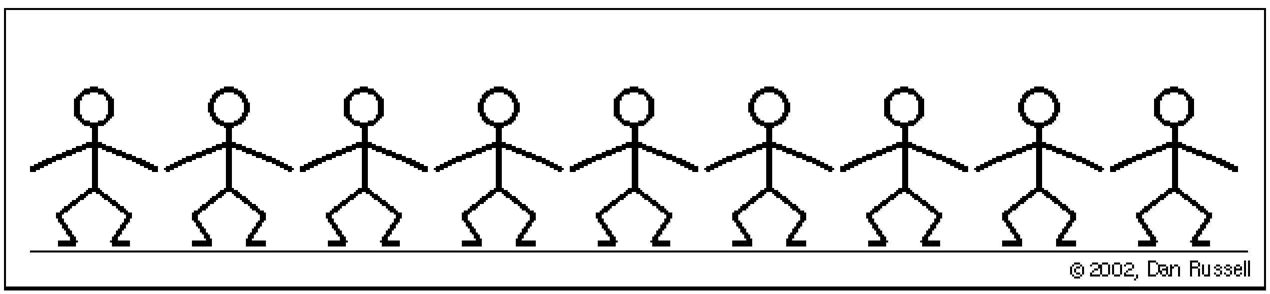
- At right, you'll see the piles of rubble from a highway that absorbed just a little of the energy from a wave propagating through the earth in California. In this chapter, we'll focus on ripples of disturbance moving through various media.

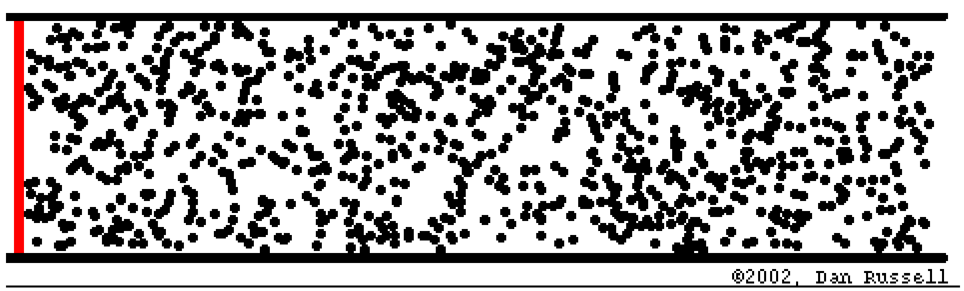
Waves: functions of space AND time

Definition of a Wave

Wave (from webster): "a disturbance or variation that transfers energy progressively from point to point in a medium and that may take the form of an elastic deformation or of a variation of pressure, electric or magnetic intensity, electric potential, or temperature."

A wave is a **disturbance** or **variation** which travels through a medium. The medium through which the wave travels may experience some local oscillations as the wave passes, but the particles in the medium to **not** travel with the wave. The disturbance may take any of a number of shapes, from a finite width pulse to an infinitely long sine wave.

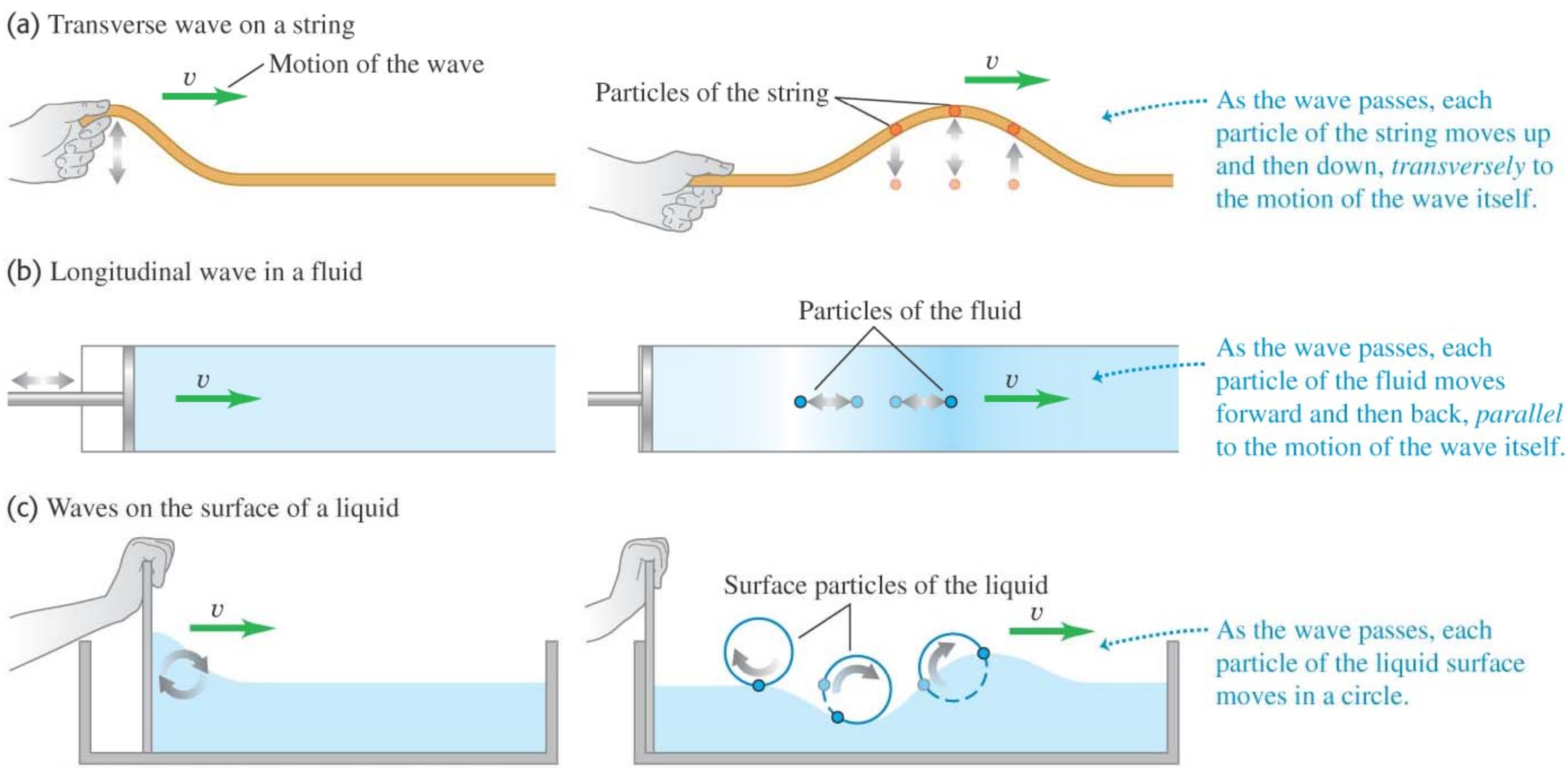




Useful website: http://www.kettering.edu/~drussell/demos.html

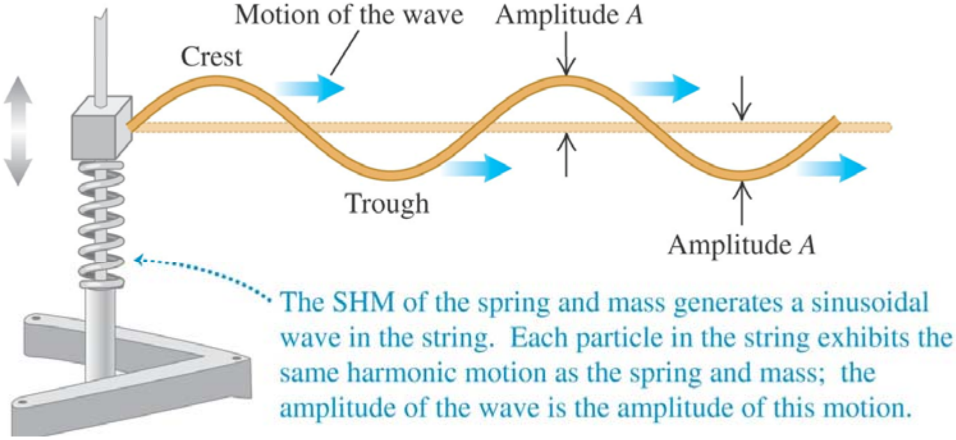
Types of mechanical waves

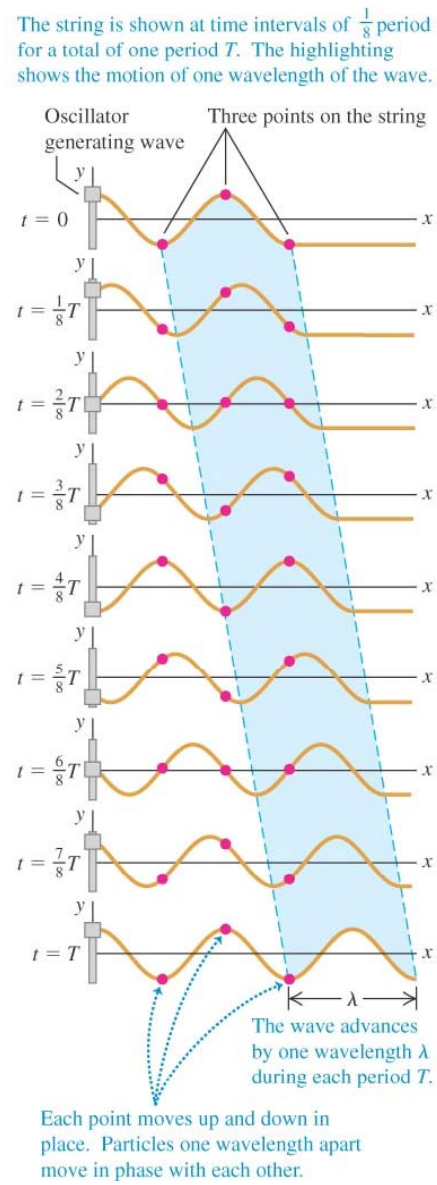
- Waves that have compressions and rarefactions parallel to the direction of wave propagation are longitudinal.
- Waves that have compressions and rarefactions perpendicular to the direction of propagation.



Periodic waves

- A detailed look at periodic transverse waves will allow us to extract parameters.





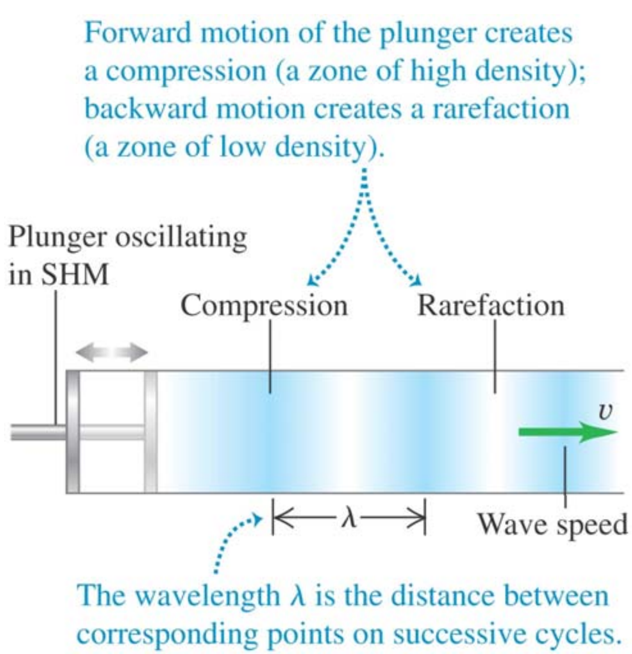
In Ch. 13 we oscillatory motion had only a frequency (period) associated with it. For a wave you have a frequency AND a wavelength (λ). The relation between them is

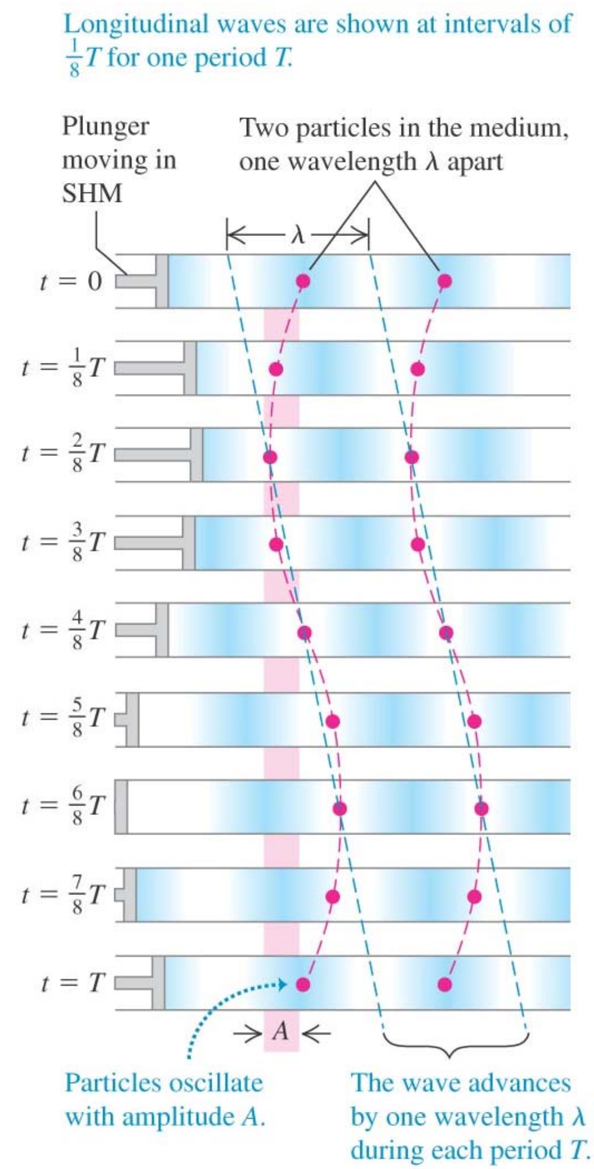
$$f = \frac{v}{\lambda}$$

The speed of the wave, v, DOES NOT depend on f or λ, only on the type of medium they travel on (e.g. air, water, ground, etc.)

Periodic waves II

- A detailed look at periodic longitudinal waves will allow us to extract parameters just as we did with transverse waves.





Mathematical description of a wave

- When the description of the wave needs to be more complete, we can generate a wave function with *y*(*x*,*t*).

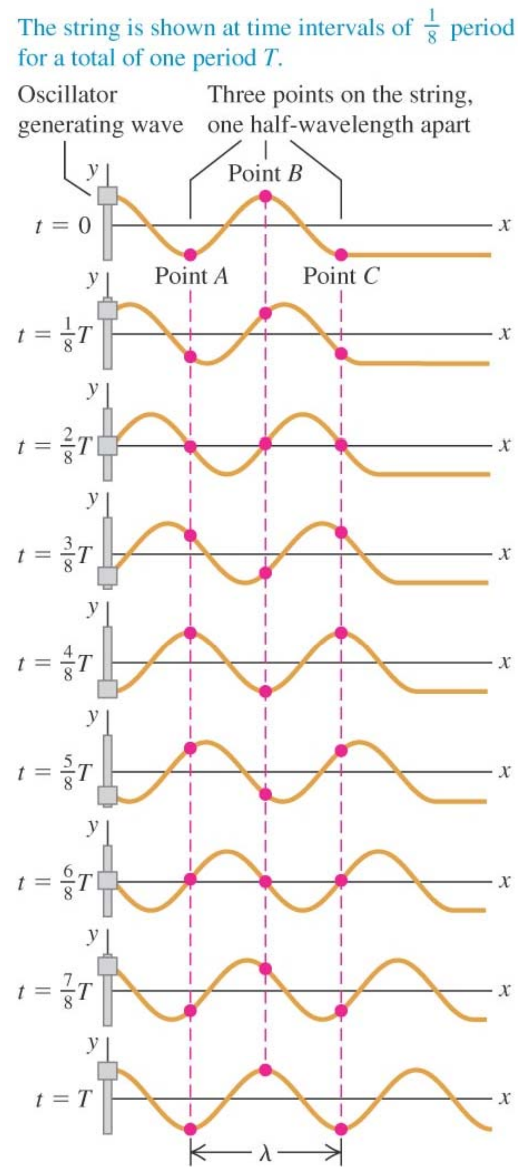
$$y(x=0,t) = A\cos(\omega t)$$

$$y(x,t) = A\cos\left(\omega(t - \frac{x}{v})\right)$$

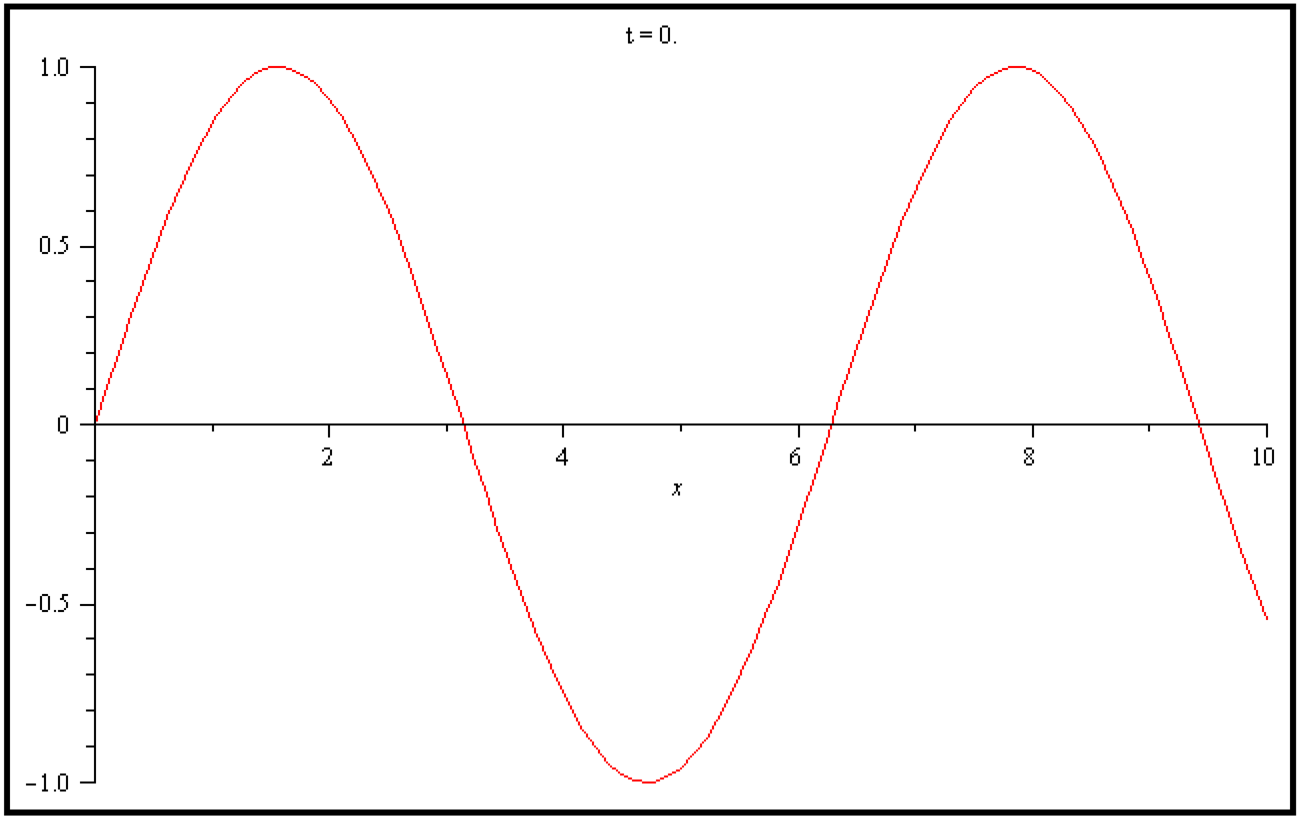
$$y(x,t) = A\cos(\omega t - kx))$$

Wave number $$k = \frac{2\pi}{\lambda}$$

$$f = \frac{v}{\lambda}$$ or $$v = \frac{\omega}{k}$$



$$y(x,t) = A\cos\left(2\pi(ft - \frac{x}{\lambda})\right)$$

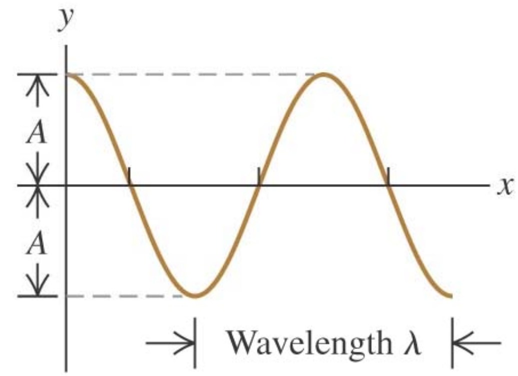


For this example $\frac{2\pi}{\lambda} = 1, \omega = 1$

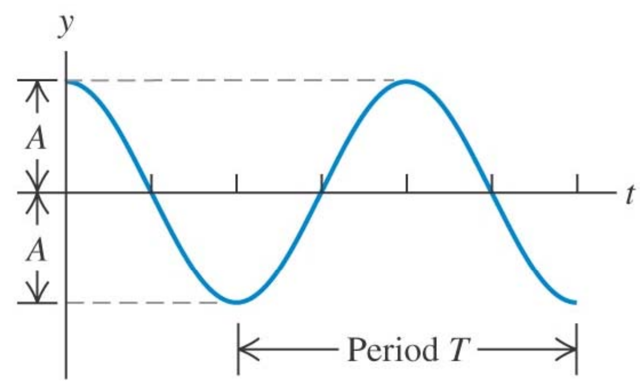
Graphing wave functions

- Informative graphic presentation of a wave function is often either *y*-displacement versus *x*-position or *y*-displacement versus *x*-time.

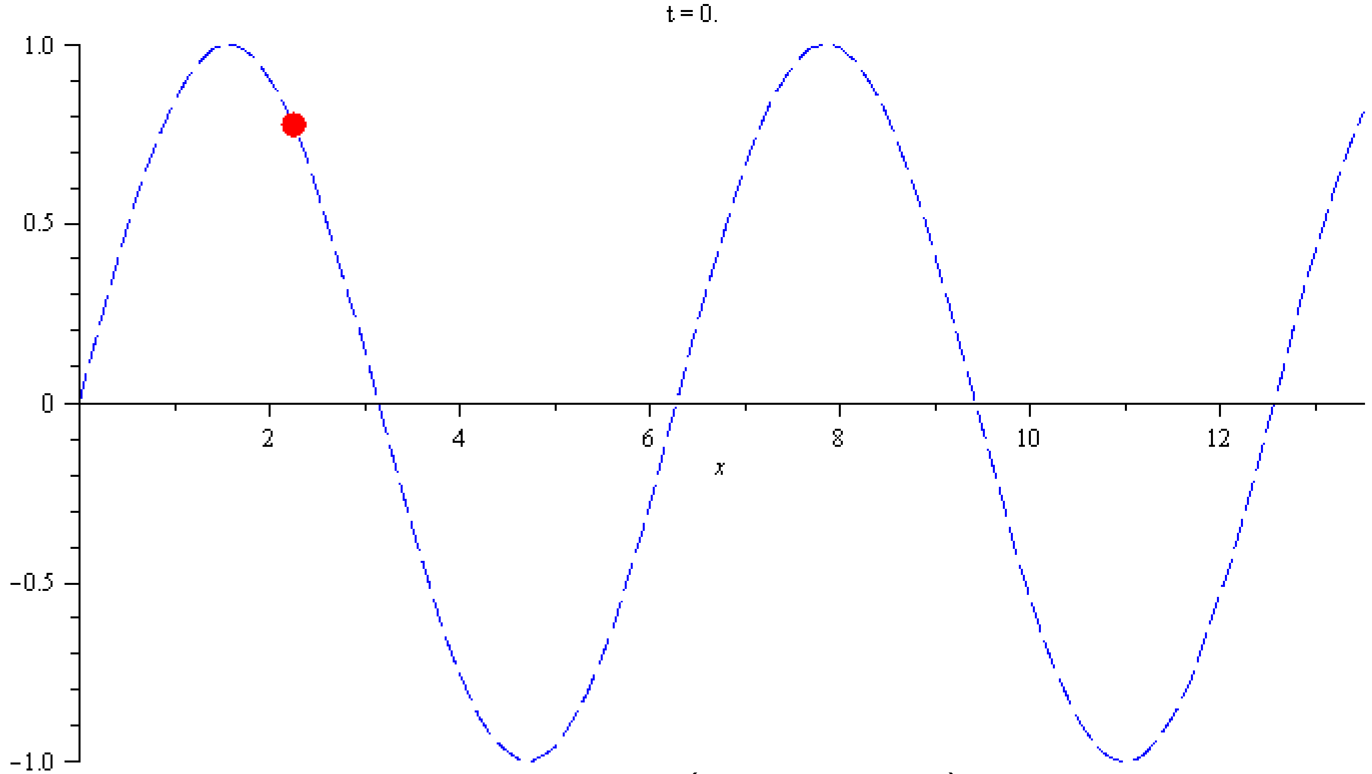
(a) If we use Eq. (15.7) to plot y as a function of x for time $t = 0$, the curve shows the *shape* of the string at $t = 0$.



(b) If we use Eq. (15.7) to plot y as a function of t for position $x = 0$, the curve shows the *displacement* y of the particle at $x = 0$ as a function of time.



Focus on the red circles (a particular phase)



$$y(x,t) = A\cos\left(2\pi\left(ft - \frac{x}{\lambda}\right)\right)$$

phase velocity, $v_{phase} = \lambda/T$

Another velocity - material velocity*

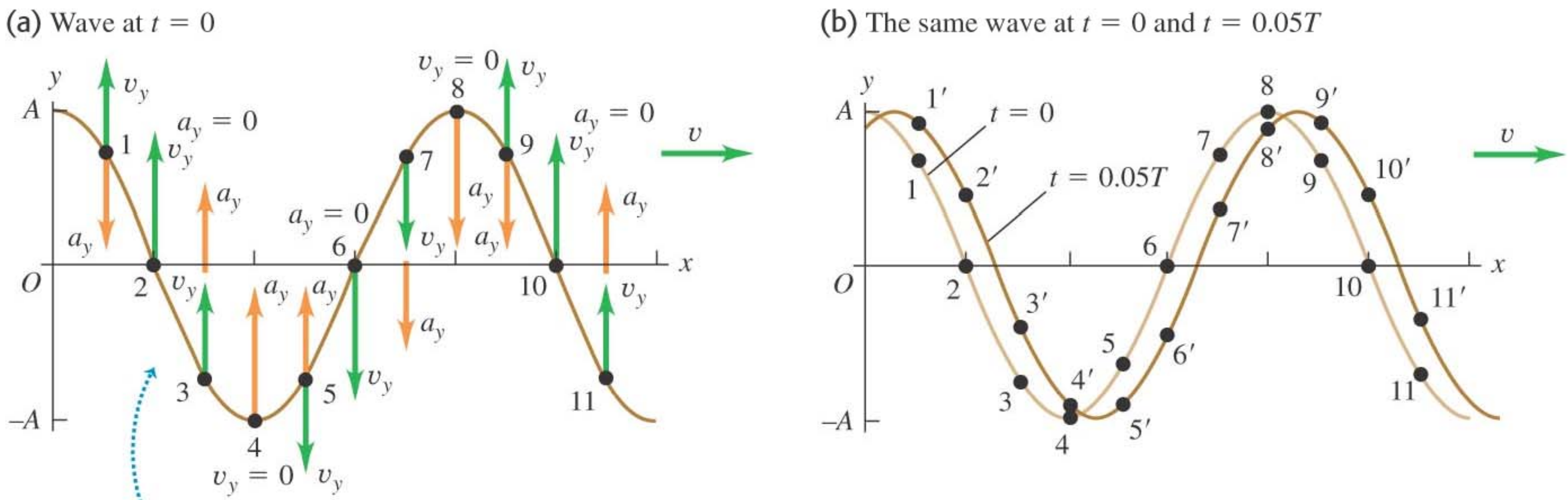
When a wave or disturbance propagates, the particles of the MEDIUM do not propagate, but they move a little bit from their equilibrium positions

material velocity, v_{mat}=dy/dt,

Material velocity is speed of the particles in the medium. If ψ has dimensions of length, this is a velocity as we normally think of it. *It's a very bad name because waves don't necessarily need a medium to propagate and ψ might well represent something more abstract like an electric field. But it will do for now.

Particle velocity and acceleration in a sinusoidal wave

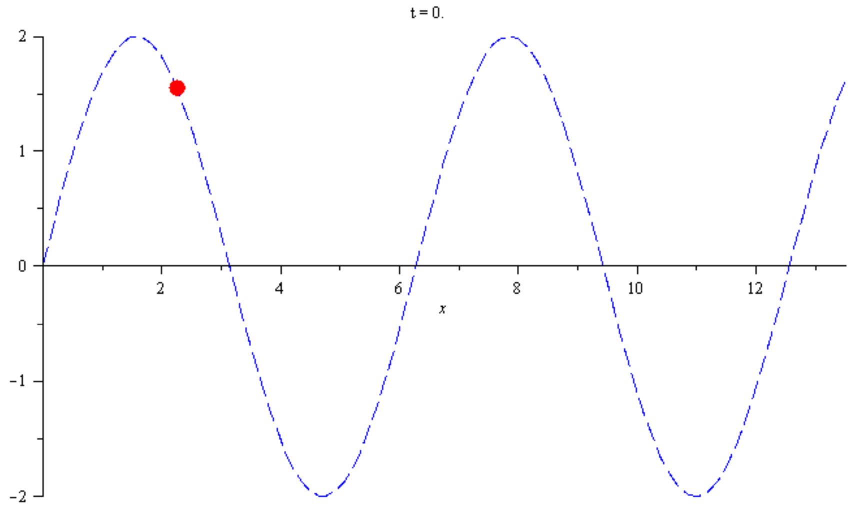
- From the wave function, we have an expression for the kinematics of a particle at any point on the wave.



- Acceleration a_y at each point on the string is proportional to displacement y at that point.
- Acceleration is upward where string curves upward, downward where string curves downward.

Focus on the red circle that marks a particular x

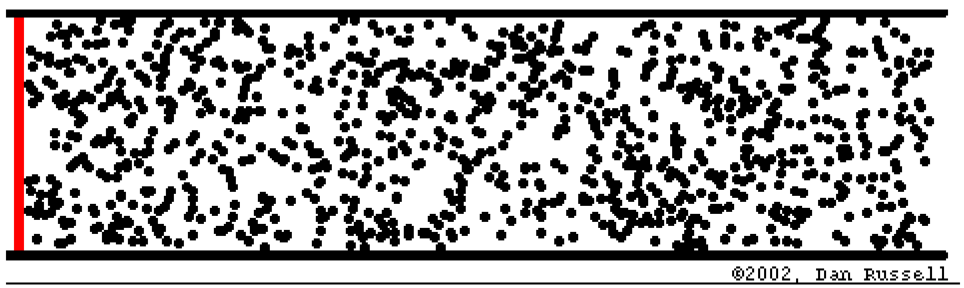
If the material velocity is perpendicular to the phase velocity, the wave is “**transverse**”.



$$y(x,t) = A\cos\left(\omega t \pm \frac{2\pi}{\lambda}x\right)$$

$$\frac{dy}{dt}(x,t) = -\omega A\sin\left(\omega t \pm \frac{2\pi}{\lambda}x\right)$$

If the material velocity is parallel to the phase velocity, the wave is “**longitudinal**”.



material velocity, v_{mat}=dy/dt

How do these functions arise?

PROVIDED $\omega/k = v$, a constant, they are solutions to the differential equation:

$$\frac{\partial^2 y(x,t)}{\partial^2 x} = -k^2 A\cos(\omega t - kx) \qquad \frac{\partial^2 y(x,t)}{\partial^2 t} = -\omega^2 A\cos(\omega t - kx)$$

$$\boxed{\mathrm{v}^2 \frac{\partial^2}{\partial x^2} y(x,t) = \frac{\partial^2}{\partial t^2} y(x,t)}$$

This equation results when:

- Newton's law is applied to a string under tension
- Kirchoff's law is applied to a coaxial cable
- Maxwell's equations are applied to source-free media … and many other cases …

Speed of waves

There is a tricky part of waves that you must understand. Even though we write all the time that $v=k\omega=f\lambda$, v DOES not depend on f or λ!! It only depends on the type of medium the wave travels in.

We can derive (slowly) that in general the speed of a wave is

$$v = \sqrt{\frac{\text{(restoring force returning the system to equilibrium)}}{\text{(inertia resisting the retun to equilibrium)}}}$$

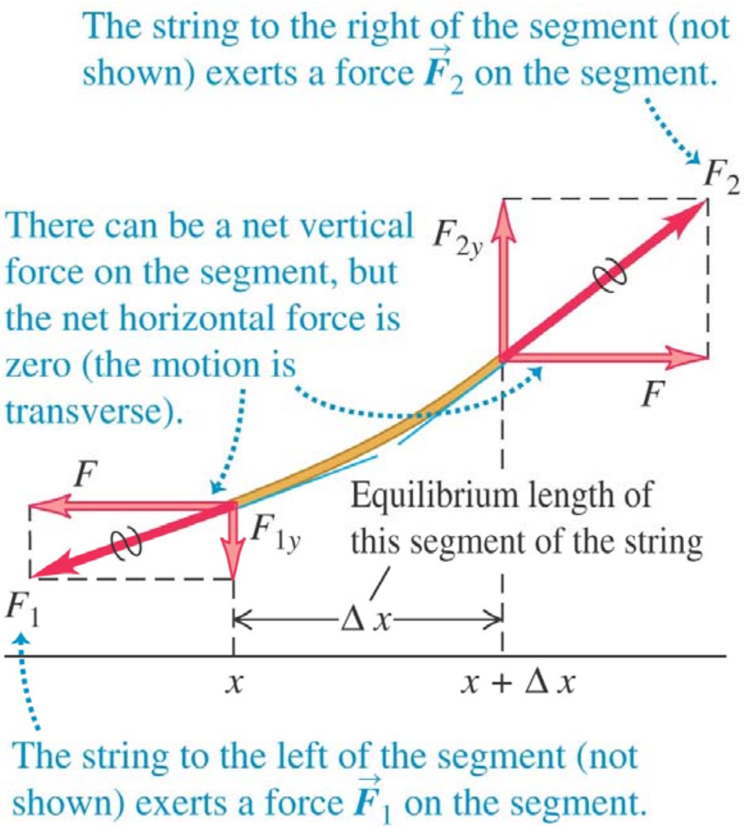
A key examples is a string

$$v = \sqrt{\frac{F_T}{\mu}}$$

$\mu=M/L=$linear density

The speed of a transverse wave II

- We can take a second glance at the speed of a transverse wave on a string. How does one get it?



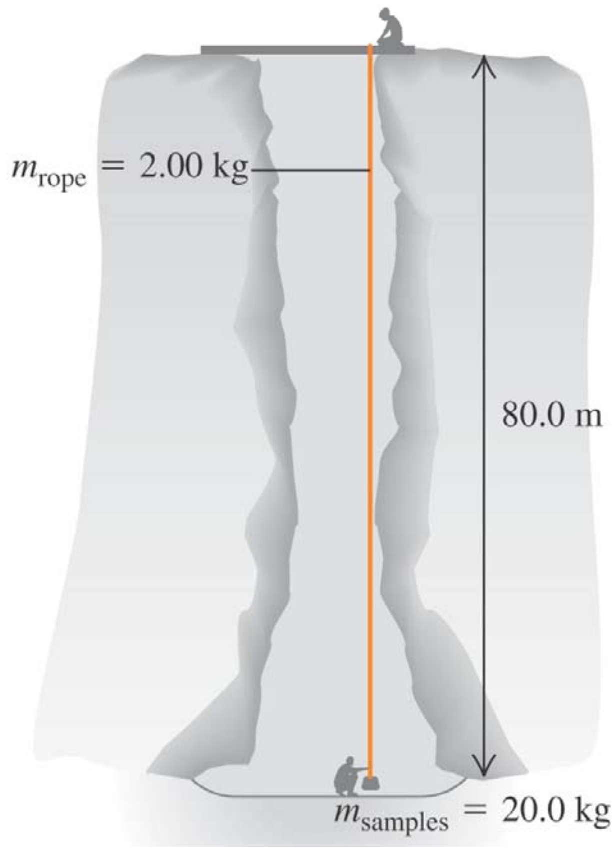
To derive it one looks at a small piece of the medium (mass $\Delta m=\mu\Delta x$) and writes Newton's 2nd law. The right hand side is the second derivative with respect to time and one can show (Sec. 4 in book) that the left hand side (sum of forces in the y-direction) yields the second derivative with respect to position. The constant that one obtains are equated to the velocity (previous slide).

Example

In the figure shown below, what is the speed of a wave in the string?
If it is shaken at 2 Hz, how many wavelengths fit in the string?

$$F_T = m_{samples}g = 196\ \text{N}$$

$$\Rightarrow \text{v} = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{F_T L}{M_{string}}} = 88.5\ \text{m/s}$$



From this velocity one gets the wavelength

$$\lambda = \frac{\text{v}}{f} = \frac{88.5\ \text{m/s}}{2.00\ \text{s}^{-1}} = 44.3$$

$$\Rightarrow \frac{80}{44.3} = 1.81\ \text{cycles}$$

Wave intensity and Power

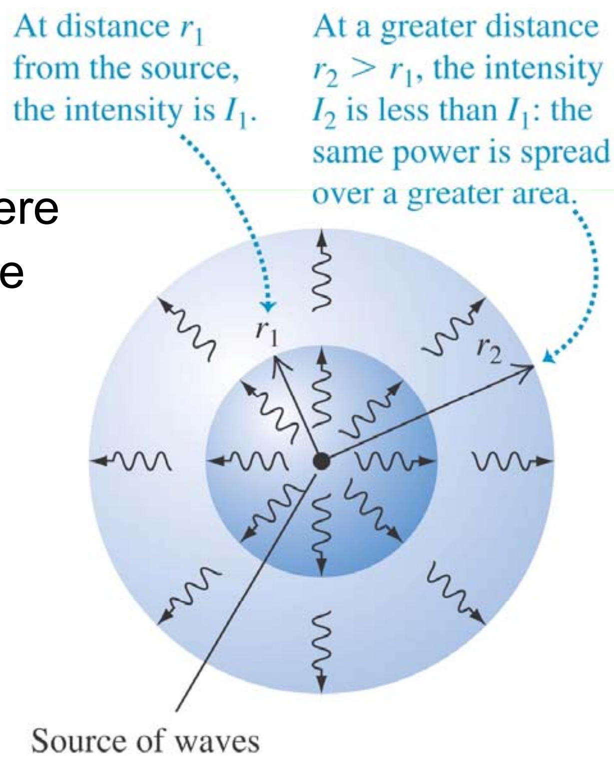
Average power of a wave on a string: $P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$

As the wave propagates outward from its source, the power generated is spread out over the area it covers. This is the intensity of a wave. If it is a point source (speaker, etc.) then the intensity of a wave goes down as $1/r^2$

Power at position r, away from a point source

$$I(r) = \frac{P}{4\pi r^2}$$

Area of a sphere

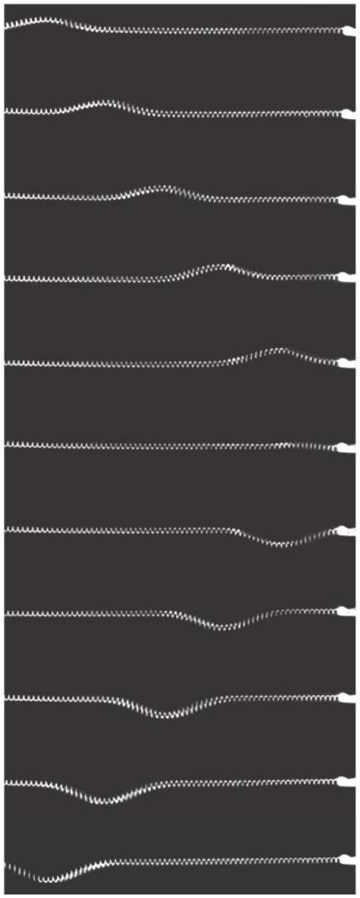


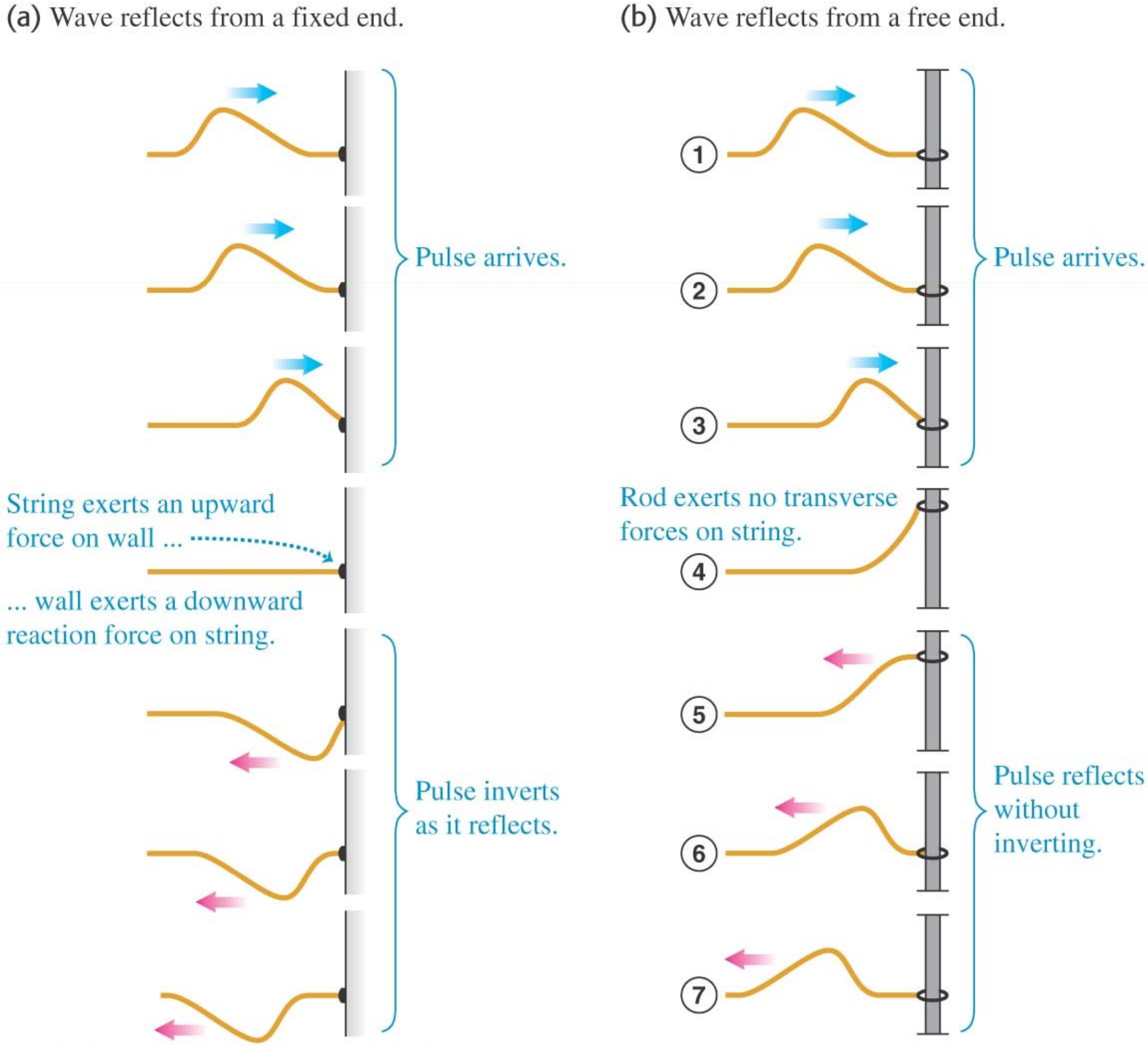
Example: a tornado warning siren on top of a pole radiates sound in all directions. If the intensity is 0.250 W/m^2 at a distance of 15.0 meters, At what distance is the intensity 0.01 W/m^2?

$$\frac{I_1}{I_2} = \frac{r_1^2}{r_2^2} \Rightarrow r_1 = \sqrt{\frac{I_1}{I_2}}r_2 = 75.0\text{ m}$$

Effects of boundaries on wave reflection

Waves in motion from one boundary (the source) to another boundary (the endpoint) will travel and reflect.

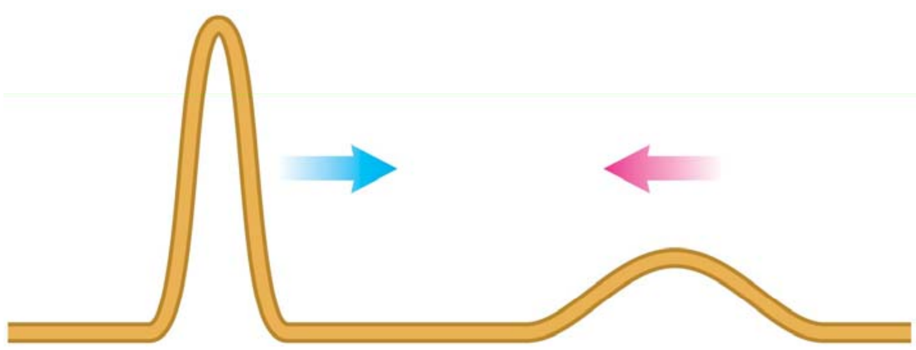


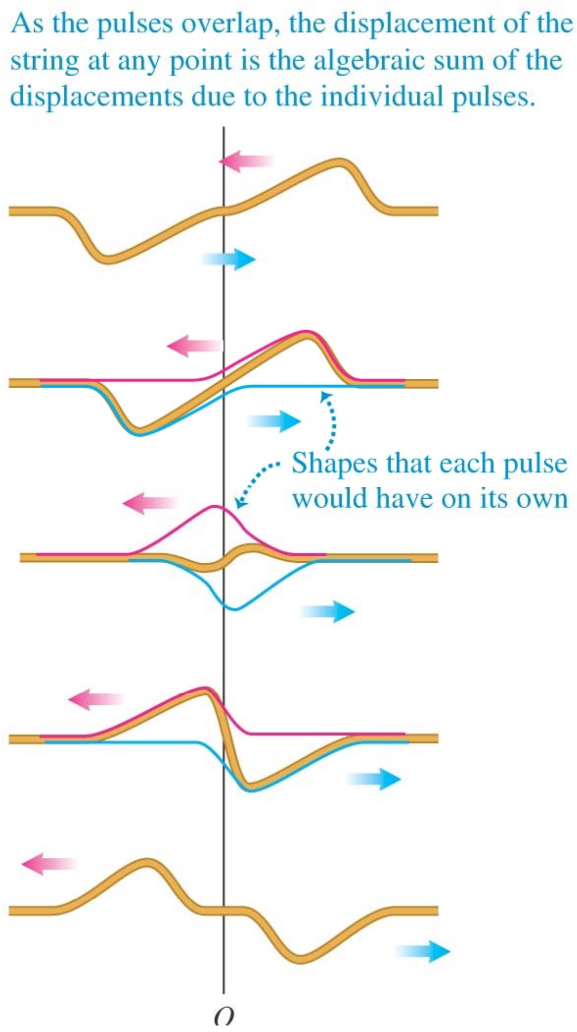


Wave superpostion

- When waves “collide” their amplitudes add (if one is negative and the other positive they can add to zero)

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

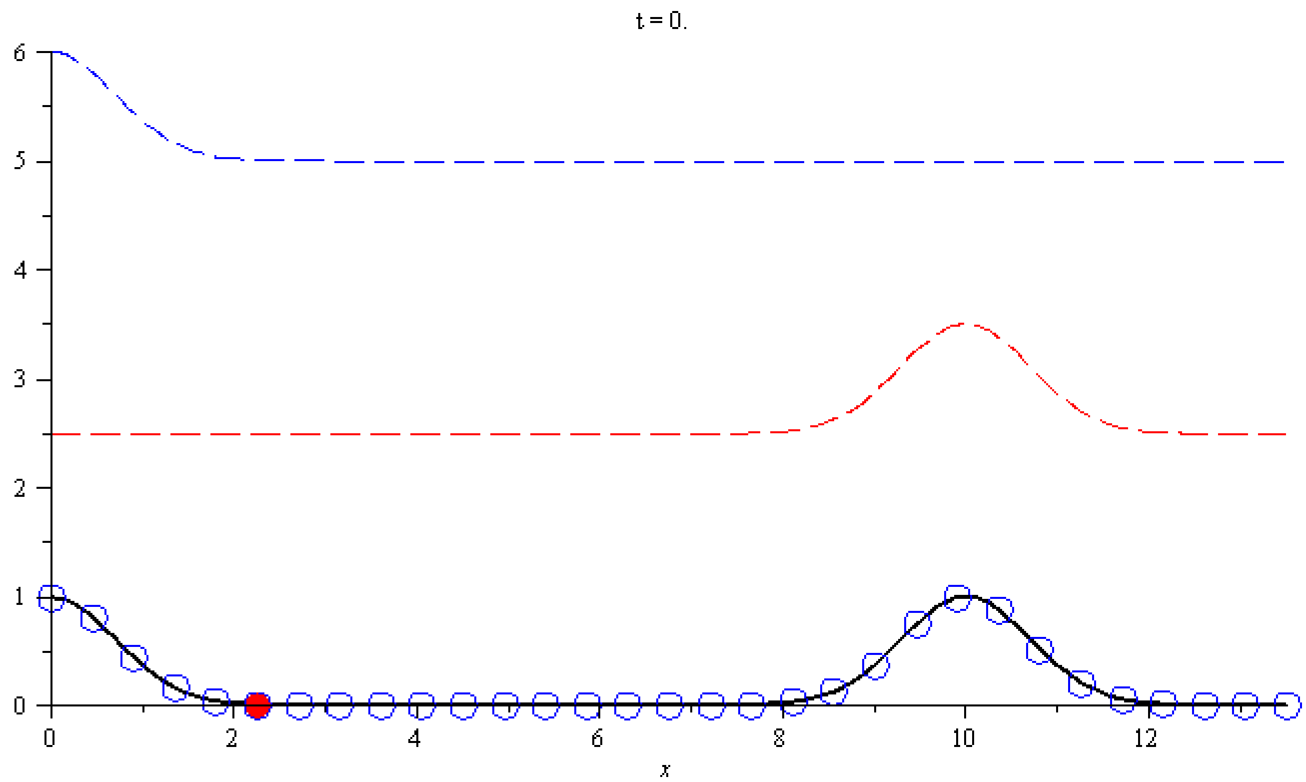




Adding waves

The disturbance amplitude add and it results in a total disturbance for each position and time.

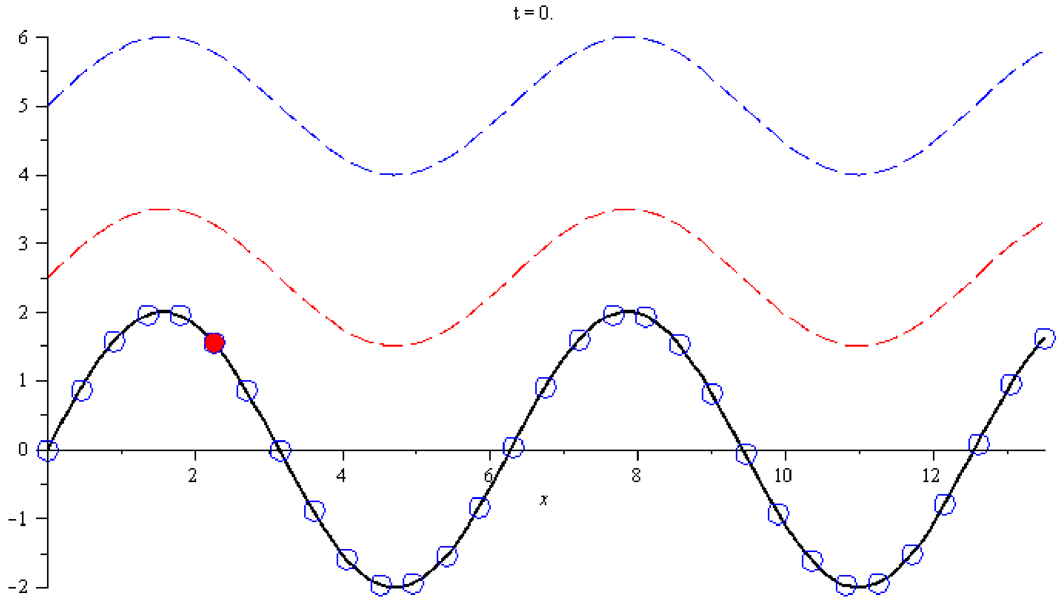
$$y(x,t) = e^{-(kx-\omega t)^2} + e^{-(kx-10k+\omega t)^2}$$



Adding two waves that propagate in opposite directions and have the same amplitude: standing waves

moves to the right   moves to the left

$$y(x,t) = A\cos(kx - \omega t + \phi_1) + A\cos(kx + \omega t + \phi_2)$$



Standing waves - functions of x (only) multiplied by functions of t (only)

$$y(x,t) = 2A\cos\left(\frac{1}{2}(kx - \omega t + \phi_1 + kx + \omega t + \phi_2)\right)\cos\left(\frac{1}{2}(kx - \omega t + \phi_1 - kx - \omega t - \phi_2)\right)$$

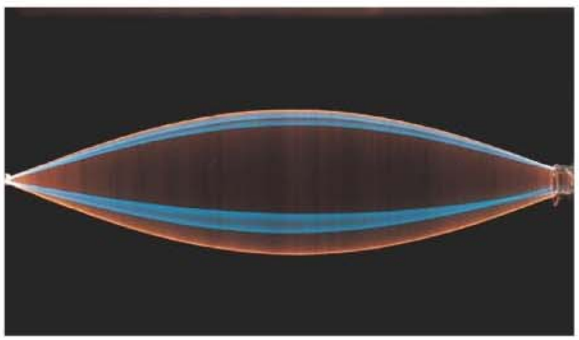
$$y(x,t) = 2A\cos\left(kx + \frac{(\phi_1 + \phi_2)}{2}\right)\cos\left(-\omega t + \frac{(\phi_1 - \phi_2)}{2}\right)$$

Standing waves are superpositions of traveling waves

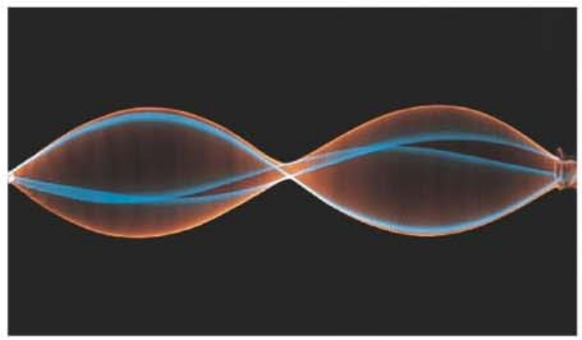
Standing waves on a string

- Fixed at both ends, the resonator was have waveforms that match. In this case, the standing waveform must have nodes at both ends. Differences arise only from increased energy in the waveform.

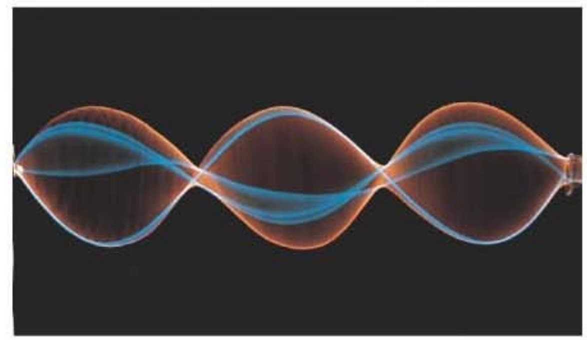
(a) String is one-half wavelength long.



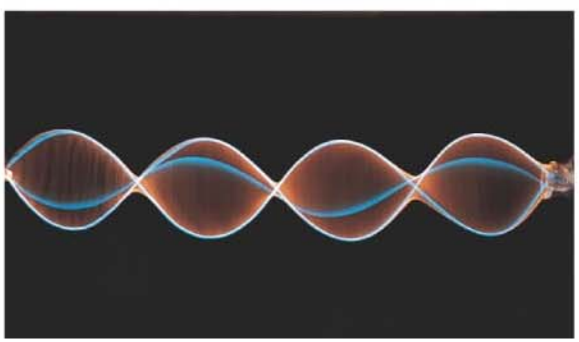
(b) String is one wavelength long.



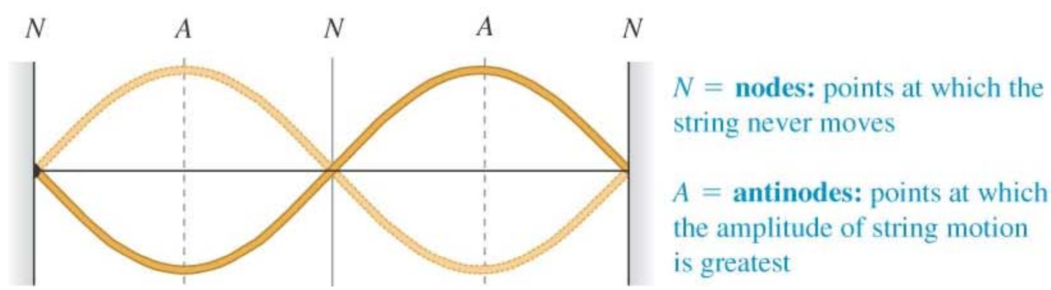
(c) String is one and a half wavelengths long.



(d) String is two wavelengths long.



(e) The shape of the string in (b) at two different instants



http://www.youtube.com/watch?v=s9GBf8y0IY0&feature=related

Normal modes (harmonics) on a string

The length of the string determines the allowed wavelengths:

$$L = n\frac{\lambda_n}{2} \Rightarrow \lambda_n = \frac{2L}{n} \qquad \text{where n=1,2,3,….}$$

Since the velocity is fixed by the tension and the density of the string the frequency as also discrete allowed values. These are called the harmonics of the string (normal modes)

$$f_n = \frac{v}{\lambda_n} \Rightarrow f_n = n\frac{v}{2L} \qquad \text{where n=1,2,3,… and} \qquad v = \sqrt{\frac{F_T}{\mu}}$$