A Bullet Is Fired into a Wooden Block

Description: Conceptual: A bullet embeds in a stationary, frictionless block: type of collision? what is conserved? v_final?

A bullet of mass $m_b$ is fired horizontally with speed $v_i$ at a wooden block of mass $m_w$ resting on a frictionless table. The bullet hits the block and becomes completely embedded within it. After the bullet has come to rest within the block, the block, with the bullet in it, is traveling at speed $v_f$.

Part A

Which of the following best describes this collision?

Hint 1. Types of collisions

An inelastic collision is a collision in which kinetic energy is not conserved. In a partially inelastic collision, kinetic energy is lost, but the objects colliding do not stick together. From this information, you can infer what completely inelastic and elastic collisions are.

ANSWER:

- perfectly elastic
- partially inelastic
- perfectly inelastic
Part B
Which of the following quantities, if any, are conserved during this collision?

**Hint 1. When is kinetic energy conserved?**
Kinetic energy is conserved only in perfectly elastic collisions.

ANSWER:
- kinetic energy only
- momentum only
- kinetic energy and momentum
- neither momentum nor kinetic energy

Part C
What is the speed of the block/bullet system after the collision?

Express your answer in terms of $v_i$, $m_w$, and $m_b$.

**Hint 1. Find the momentum after the collision**
What is the total momentum $p_{\text{total}}$ of the block/bullet system after the collision?

Express your answer in terms of $v_f$ and other given quantities.

ANSWER:
\[ p_{\text{total}} = (m_w + m_b) v_f \]

**Hint 2. Use conservation of momentum**
The momentum of the block/bullet system is conserved. Therefore, the momentum before the collision is the same as the momentum after the collision. Find a second expression for $p_{\text{total}}$, this time expressed as the total momentum of the system before the collision.

Express your answer in terms of $v_i$ and other given quantities.

ANSWER:
\[ p_{\text{total}} = m_b v_i \]

ANSWER:
\[ v_f = \frac{m_b v_i}{m_w + m_b} \]
± Catching a Ball on Ice

**Description:** ± Includes Math Remediation. Find the final momentum of a person who catches a ball on a frictionless surface. Find the final momentum of a person on a frictionless surface off of whom a ball bounces.

Olaf is standing on a sheet of ice that covers the football stadium parking lot in Buffalo, New York; there is negligible friction between his feet and the ice. A friend throws Olaf a ball of mass 0.400 kg that is traveling horizontally at 10.5 m/s. Olaf's mass is 68.6 kg.

**Part A**

If Olaf catches the ball, with what speed \( v_f \) do Olaf and the ball move afterward?

**Express your answer numerically in meters per second.**

**Hint 1. How to approach the problem**

Using conservation of momentum and the fact that Olaf's initial momentum is zero, set the initial momentum of the ball equal to the final momentum of Olaf and the ball, then solve for the final velocity.

**Hint 2. Find the ball's initial momentum**

What is \( p_i \), the initial momentum of the ball?

**Express your answer numerically in kilogram meters per second.**

**ANSWER:**

\[
p_i = m_b v_i = 4.20 \text{ kg} \cdot \text{m/s}
\]

**ANSWER:**

\[
v_f = \frac{m_b v_i}{m + m_b} = 6.09 \times 10^{-2} \text{ m/s}
\]

**Part B**

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If the ball hits Olaf and bounces off his chest horizontally at 7.70 m/s in the opposite direction, what is his speed $v_f$ after the collision?

Express your answer numerically in meters per second.

**Hint 1. How to approach the problem**

The initial momentum of the ball is the same as in Part A. Apply conservation of momentum, keeping in mind that both Olaf and the ball have a nonzero final momentum.

**Hint 2. Find the ball’s final momentum**

Taking the direction in which the ball was initially traveling to be positive, what is $\vec{p}_{ball,f}$, the ball’s final momentum?

Express your answer numerically in kilogram meters per second.

**ANSWER:**

$$\vec{p}_{ball,f} = -v_2 m_b = -3.08 \text{ kg m/s}$$

**ANSWER:**

$$v_f = \frac{m_b (v_1 + v_2)}{m} = 0.106 \text{ m/s}$$

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**Collision at an Angle**

**Description:** Straightforward application of momentum conservation in an inelastic collision.

Two cars, both of mass $m$, collide and stick together. Prior to the collision, one car had been traveling north at speed $2v$, while the second was traveling at speed $v$ at an angle $\phi$ south of east (as indicated in the figure). After the collision, the two-car system travels at speed $v_{final}$ at an angle $\theta$ east of north.

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**Part A**

Find the speed $v_{final}$ of the joined cars after the collision.
Express your answer in terms of $v$ and $\phi$.

**Hint 1. Determine the conserved quantities**

Which of the following statements is true for the collision described?

**ANSWER:**

- Momentum is conserved but kinetic energy is not conserved.
- Kinetic energy is conserved but momentum is not conserved.
- Both kinetic energy and momentum are conserved.
- Neither kinetic energy nor momentum is conserved.

Apply conservation of momentum:

\[ \vec{p}_{initial} = \vec{p}_{final} \]

Find both components (north and east) of the initial momentum $\vec{p}_{initial}$ using the information in the problem introduction. The magnitude of $\vec{p}_{initial}$ is equal to the magnitude of the momentum vector for the two-car system after the collision: $p_{final} = (2m)v_{final}$.

**Hint 2. The component of the final velocity in the east-west direction**

Find the component of $\vec{v}_{final}$ in the east-west direction.

Express your answer in terms of $v$ and $\phi$.

**Hint 1. Find the east-west component of the initial momentum**

What is $p_e$, the magnitude of the total momentum $\vec{p}_e$ of the two cars in the east-west direction? (Take eastward to be positive, westward negative.)

**ANSWER:**

\[ p_e = mv \cos(\phi) \]

Now use the conservation of momentum equation to find $v_{final}$.

**ANSWER:**

\[ v_{final \ (east-west)} = \frac{v \cos(\phi)}{2} \]

**Hint 3. Find the north-south component of the final momentum**

Find the component of $\vec{v}_{final}$ in the north-south direction.

Express your answer in terms of $v$ and $\phi$. 
Hint 1. Find the north-south component of the initial momentum

What is the magnitude \( p_n \) of the total momentum \( \vec{p}_n \) of the two cars in the north-south direction? (Take northward to be positive, southward negative).

**Express your answer in terms of \( m, v, \) and \( \phi \).**

**ANSWER:**

\[
p_n = 2mv - mv \sin(\phi)
\]

**hint 4. Math help**

Let \( v_{\text{final},x} \) be the east-west component of \( v_{\text{final}} \), and \( v_{\text{final},y} \) the north-south component. Then

\[
v_{\text{final}} = \sqrt{v_{\text{final},x}^2 + v_{\text{final},y}^2}.
\]

You will also need to use the following trigonometric identity when you evaluate the right-hand side of the above equation in terms of \( v \) and \( \phi \):

\[
\cos^2 \phi + \sin^2 \phi = 1.
\]

**ANSWER:**

\[
v_{\text{final}} = \frac{v}{2} \sqrt{5 - 4 \sin(\phi)}
\]

**Part B**

What is the angle \( \theta \) with respect to north made by the velocity vector of the two cars after the collision?

**Express your answer in terms of \( \phi \). Your answer should contain an inverse trigonometric function using the notation \( \text{asin} \), \( \text{atan} \) etc. and not \( \text{arcsin} \), \( \text{arctan} \) etc.**

**Hint 1. A formula for \( \tan \theta \)**

Let \( v_{\text{final},x} \) be the east-west component of \( \vec{v}_{\text{final}} \), and \( v_{\text{final},y} \) the north-south component. Then

\[
\tan \theta = \frac{v_{\text{final},x}}{v_{\text{final},y}}
\]

since the angle asked for is the angle east of north.

**ANSWER:**

Exercise 8.4

Description: Two vehicles are approaching an intersection. One is a m1 pickup traveling at v1 from east to west (the \(-x\)-direction), and the other is a m2 sedan going from south to north (the \(+y\)-direction at v2). (a) Find the \(x\)-component of the net...

Two vehicles are approaching an intersection. One is a 2600 kg pickup traveling at 18.0 m/s from east to west (the \(-x\)-direction), and the other is a 1600 kg sedan going from south to north (the \(+y\)-direction at 23.0 m/s).

Part A

Find the \(x\)-component of the net momentum of this system.

ANSWER:

\[ p_x = -m_1v_1 = -4.68 \times 10^4 \text{ kg} \cdot \text{m/s} \]

Part B

Find the \(y\)-component of the net momentum of this system.

ANSWER:

\[ p_y = m_2v_2 = 3.68 \times 10^4 \text{ kg} \cdot \text{m/s} \]

Part C

What is the magnitude of the net momentum?

ANSWER:

\[ p = \sqrt{m_1^2v_1^2 + m_2^2v_2^2} = 5.95 \times 10^4 \text{ kg} \cdot \text{m/s} \]

Part D

What is the direction of the net momentum?

ANSWER:
Exercise 8.36

Description: A m1 sports car is moving westbound at v1 on a level road when it collides with a m2 truck driving east on the same road at v2. The two vehicles remain locked together after the collision. (a) What is the velocity (magnitude) of the two vehicles just ...

A 1050 kg sports car is moving westbound at 15.0 m/s on a level road when it collides with a 6320 kg truck driving east on the same road at 11.0 m/s. The two vehicles remain locked together after the collision.

Part A

What is the velocity (magnitude) of the two vehicles just after the collision?

ANSWER:

\[ v = \frac{m_2v_2 - m_1v_1}{m_1 + m_2} = 7.30 \text{ m/s} \]

Part B

What is the direction of the velocity of the two vehicles just after the collision?

ANSWER:

- [ ] to the west
- [x] to the east

Part C

At what speed should the truck have been moving so that it and car are both stopped in the collision?

ANSWER:

\[ v = \frac{m_1v_1}{m_2} = 2.49 \text{ m/s} \]

Part D

Find the change in kinetic energy of the system of two vehicles for the situations of part A.

ANSWER:

\[ \theta = \frac{\tan^{-1} \left( \frac{m_1}{m_2} \right) \cdot 180}{\pi} = 51.8^\circ \text{ west of north.} \]
ΔK = \frac{(m_2v_2 - m_1v_1)^2}{2m_1m_2} - \frac{m_1v_1^2}{2} - \frac{m_2v_2^2}{2} = -3.04 \times 10^5 \text{ J}

**Part E**

Find the change in kinetic energy of the system of two vehicles for the situations of part C.

**ANSWER:**

ΔK = -\frac{m_1v_1^2}{2} - \frac{m_2v_2^2}{m_2} = -1.38 \times 10^5 \text{ J}

**Exercise 8.40**

**Description:** To protect their young in the nest, peregrine falcons will fly into birds of prey (such as ravens) at high speed. In one such episode, a m1 falcon flying at v1 hit a m2 raven flying at v2. The falcon hit the raven at right angles to its original...

To protect their young in the nest, peregrine falcons will fly into birds of prey (such as ravens) at high speed. In one such episode, a 610 g falcon flying at 20.0 m/s hit a 1.50 kg raven flying at 8.0 m/s. The falcon hit the raven at right angles to its original path and bounced back at 5.0 m/s. (These figures were estimated by the author as he watched this attack occur in northern New Mexico.)

**Part A**

By what angle did the falcon change the raven's direction of motion?

**Express your answer using two significant figures.**

**ANSWER:**

θ = \frac{\arctan \left( \frac{m_1(v + v_1)}{m_2 v_2} \right)}{\pi} \cdot 180 = 52 ^\circ

**Part B**

What was the raven's speed right after the collision?

**Express your answer using two significant figures.**

**ANSWER:**

v = \sqrt{\left( \frac{m_1}{m_2} (v + v_1) \right)^2 + v_2^2} = 13 \text{ m/s}
Exercise 8.42

**Description:** A m-g bullet is fired horizontally into a 1.20-kg wooden block resting on a horizontal surface. The coefficient of kinetic friction between block and surface is 0.20. The bullet remains embedded in the block, which is observed to slide along the surface before stopping.

A 6.00-g bullet is fired horizontally into a 1.20-kg wooden block resting on a horizontal surface. The coefficient of kinetic friction between block and surface is 0.20. The bullet remains embedded in the block, which is observed to slide 0.390 m along the surface before stopping.

**Part A**

What was the initial speed of the bullet?

**Express your answer with the appropriate units.**

**ANSWER:**

\[ v = \frac{m + 1.2}{m} \sqrt{2 \cdot 0.2 \cdot 9.8 s} = 250 \text{ m/s} \]

Also accepted: \[ \frac{m + 1.2}{m} \sqrt{2 \cdot 0.2 \cdot 9.8 s} = 249 \text{ m/s} \]

Exercise 8.44

**Description:** A ## kg block is attached to a very light horizontal spring of force constant ## N/m and is resting on a smooth horizontal table. (See the figure below.) Suddenly it is struck by a ## kg stone traveling horizontally at ## m/s to the right, whereupon...

A 15.0 kg block is attached to a very light horizontal spring of force constant 475 N/m and is resting on a smooth horizontal table. (See the figure below.) Suddenly it is struck by a 3.00 kg stone traveling horizontally at 8.00 m/s to the right, whereupon the stone rebounds at 2.00 m/s horizontally to the left.

**Part A**

Find the maximum distance that the block will compress the spring after the collision. *(Hint: Break this problem into two parts - the collision and the behavior after the collision - and apply the appropriate conservation law to each part.)*

**Enter your answer using three significant figures.**

**ANSWER:**

\[ x = \sqrt{\frac{m_1}{k} \left( \frac{m_2 (v_1 + v_2)}{m_1} \right)^2} = 0.355 \text{ m} \]

Exercise 8.50

**Description:** You are at the controls of a particle accelerator, sending a beam of v_1 protons (mass m) at a gas target of an unknown element. Your detector tells you that some protons bounce straight back after a collision with one of the...
You are at the controls of a particle accelerator, sending a beam of 4.20×10⁷ m/s protons (mass m) at a gas target of an unknown element. Your detector tells you that some protons bounce straight back after a collision with one of the nuclei of the unknown element. All such protons rebound with a speed of 3.90×10⁷ m/s. Assume that the initial speed of the target nucleus is negligible and the collision is elastic.

**Part A**

Find the mass of one nucleus of the unknown element. Express your answer in terms of the proton mass m.

**ANSWER:**

\[
\frac{v_1 + v_2}{v_1 - v_2} = 27.0 \text{ m}
\]

**Part B**

What is the speed of the unknown nucleus immediately after such a collision?

**ANSWER:**

\[
\frac{v_1 - 2}{1 + \frac{m_{nuc}}{m}} = 3.00 \times 10^6 \text{ m/s}
\]

**Exercise 8.54**

**Description:** A 1200-kg SUV is moving along a straight highway at 12.0 m/s. Another car, with mass 1800 kg and speed 20.0 m/s, has its center of mass 40.0 m ahead of the center of mass of the SUV. (a) Find the position of the center of mass of the...

A 1200-kg SUV is moving along a straight highway at 12.0 m/s. Another car, with mass 1800 kg and speed 20.0 m/s, has its center of mass 40.0 m ahead of the center of mass of the SUV.

**Part A**

Find the position of the center of mass of the system consisting of the two cars.
Express your answer using three significant figures.

ANSWER:

\[ x_{\text{cm}} = 16.0 \text{ m behind the leading car} \]

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**Part B**

Find the magnitude of the system’s total momentum, by using the given data.

**Express your answer to three significant figures and include the appropriate units.**

ANSWER:

\[ P_1 = 5.04 \times 10^4 \text{kg} \cdot \left( \frac{\text{m}}{\text{s}} \right) \]

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**Part C**

Find the speed of the system’s center of mass.

**Express your answer to three significant figures and include the appropriate units.**

ANSWER:

\[ v_{\text{cm, x}} = 16.8 \frac{\text{m}}{\text{s}} \]

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**Part D**

Find the system’s total momentum, by using the speed of the center of mass.

**Express your answer to three significant figures and include the appropriate units.**

ANSWER:

\[ P_2 = 5.04 \times 10^4 \text{kg} \cdot \left( \frac{\text{m}}{\text{s}} \right) \]

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**Part E**

Compare your results of parts B and D.

ANSWER:

- \( P_1 > P_2 \)
- \( P_1 < P_2 \)
- \( P_1 = P_2 \)
Exercise 8.56

Description: At one instant, the center of mass of a system of two particles is located on the x-axis at \( x = 2.0 \text{ m} \) and has a velocity of \( (5.0 \text{ m/s}) \hat{i} \). One of the particles is at the origin. The other particle has a mass of 0.10 kg and is at rest on the...

At one instant, the center of mass of a system of two particles is located on the x-axis at \( x = 2.0 \text{ m} \) and has a velocity of \( (5.0 \text{ m/s}) \hat{i} \). One of the particles is at the origin. The other particle has a mass of 0.10 kg and is at rest on the x-axis at \( x = 8.0 \text{ m} \).

Part A

What is the mass of the particle at the origin?

Express your answer using two significant figures.

ANSWER:

\[ m_1 = 0.30 \text{ kg} \]

Part B

Calculate the total momentum of this system.

Express your answer using two significant figures.

ANSWER:

\[ \hat{P} = 2.0 \text{ kg} \cdot \text{m/s} \hat{i} \]

Part C

What is the velocity of the particle at the origin?

Express your answer using two significant figures.

ANSWER:

\[ v_1 = 6.7 \text{ m/s} \hat{i} \]

Problem 8.72

Description: A small wooden block with mass \( m_1 \) is suspended from the lower end of a light cord that is \( l \) long. The block is initially at rest. A bullet with mass \( m_2 \) is fired at the block with a horizontal velocity \( v_0 \). The bullet strikes the block ...

A small wooden block with mass 0.725 kg is suspended from the lower end of a light cord that is 1.68 m long. The block is initially at rest. A bullet with mass 0.0138 kg is fired at the block with a horizontal velocity \( v_0 \). The bullet strikes the block and becomes embedded in it. After the collision the combined object swings on the end of the cord. When the block has risen a vertical height of 0.700 m, the tension in the cord is 4.94 N.

Part A

What was the initial speed \( v_0 \) of the bullet?

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Express your answer with the appropriate units.

ANSWER:

\[
v_0 = \frac{m_1 + m_2}{m_2} \sqrt{\frac{lT}{m_1 + m_2} + 9.81 (3h - l)} = 210 \text{ m/s}
\]

Problem 8.80

Description: \( m \_S \) stone rests on a frictionless, horizontal surface. A bullet of mass \( m \_B \), traveling horizontally at \( v \_1 \), strikes the stone and rebounds horizontally at right angles to its original direction with a speed of \( v \_2 \). (a) Compute the magnitude of the...

0.150 kg stone rests on a frictionless, horizontal surface. A bullet of mass 9.00 g , traveling horizontally at 390 m/s , strikes the stone and rebounds horizontally at right angles to its original direction with a speed of 250 m/s.

Part A

Compute the magnitude of the velocity of the stone after it is struck.

ANSWER:

\[
V = \frac{m_B \cdot 10^{-3}}{m_S} \sqrt{v_1^2 + v_2^2} = 27.8 \text{ m/s}
\]

Part B

Compute the direction of the velocity of the stone after it is struck.

ANSWER:

\[
\theta = \frac{\tan^{-1}(\frac{32}{1}) \cdot 180}{\pi} = 32.7^\circ \text{ from the initial direction of the bullet}
\]

Part C

Is the collision perfectly elastic?

ANSWER:

- yes
- no

Problem 8.90
Jonathan and Jane are sitting in a sleigh that is at rest on frictionless ice. Jonathan's weight is 800 N, Jane's weight is 600 N, and that of the sleigh is 1000 N. They see a poisonous spider on the floor of the sleigh and immediately jump off...

Part A

Calculate the magnitude of the sleigh's horizontal velocity after they jump out.

ANSWER:

0.105 m/s

Part B

What is the direction of the sleigh's horizontal velocity after they jump out.

ANSWER:

- [ ] to the left
- [x] to the right

Problem 8.92

Description: A 45.0-kg woman stands up in a 60.0-kg canoe 5.00 m long. She walks from a point 1.00 m from one end to a point 1.00 m from the other end (the figure ). (a) If you ignore resistance to motion of the canoe in the water, how far does the canoe move...

A 45.0-kg woman stands up in a 60.0-kg canoe 5.00 m long. She walks from a point 1.00 m from one end to a point 1.00 m from the other end (the figure ).
If you ignore resistance to motion of the canoe in the water, how far does the canoe move during this process?

ANSWER:

\[ d_{\text{canoe}} = 1.29 \text{ m to the left} \]