Fundamentals of Statics

- Vector Addition (Analytical and Graphical Method)
- Moment of a Force
- Principle of Moments (Varignon’s Theorem)
- Couple and Moment of a Couple
- Resolution of a Force into a Force and a Couple acting at another point
Vector Addition (Analytical and Graphical Method)

Calculate the horizontal and vertical components of the force, $F$

$$F_x = \frac{F \cdot \cos(\theta)}{\sqrt{x^2 + y^2}}$$

$$F_y = \frac{F \cdot \sin(\theta)}{\sqrt{x^2 + y^2}}$$
Calculate the Resultant Force with the direction.

**Analytical Method:**

<table>
<thead>
<tr>
<th>Force</th>
<th>X-component</th>
<th>Y-component</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1 = 5k</td>
<td>$5(1)/\sqrt{2} = 3.5355$</td>
<td>$-5(1)/\sqrt{2} = -3.5355$</td>
</tr>
<tr>
<td>F2 = 30k</td>
<td>$30(1)/\sqrt{5} = 13.4164$</td>
<td>$30(2)/\sqrt{5} = 26.8328$</td>
</tr>
<tr>
<td>F3 = 20k</td>
<td>$20(2)/\sqrt{5} = 17.8885$</td>
<td>$-20(1)/\sqrt{5} = -8.94427$</td>
</tr>
<tr>
<td>F4 = 10k</td>
<td>-10</td>
<td>0</td>
</tr>
<tr>
<td>Resultant, R</td>
<td>24.8404</td>
<td>14.35301</td>
</tr>
</tbody>
</table>

Resultant Force, $R = \sqrt{(24.8404^2 + 14.35301^2)} = 28.689k$

Resultant Angle, $\theta = \tan^{-1}(14.35301/24.8404) = 30.02^0$
Graphical Method:

Scale: ½ inch = 5 k force
Moment of a Force

The moment of a force with respect to a reference point is equal to the product of the force and the perpendicular distance of the force from the reference point. Directions for moments about the reference point are either clockwise or counterclockwise rotations.

A commonly used sign convention: Counterclockwise rotation as positive (+), and clockwise rotation as negative (-).

Moment at A, \( M_A = -5 \times 10 = -50 \text{ k-ft (clockwise)} \)

Moment at A, \( M_A = -5 \times 6 = -30 \text{ k-ft (clockwise)} \)

Perpendicular distance, \( X = \left(\frac{10}{5}\right) \times 4 = 8 \text{ ft} \)

Moment at A, \( M_A = -5 \times 8 = -40 \text{ k-ft (clockwise)} \)
Principle of Moments (Varignon’s Theorem)

The moment of a force about a point is equal to the algebraic sum of the moments of the components of the force with respect to the same point.

Example:

\[
M_A = - (4 \times 10) + (3 \times 6) = -22 \text{ k-ft (Clockwise)}
\]
Couple and Moment of a Couple

A Couple is defined as two forces having equal magnitude, parallel in lines of action, but opposite in directions. Couples produce rotational effects on a body with no capacity of movement of the body in any direction.

The Moment of a couple, M is computed as the product of the force (F) times the perpendicular distance (d) between the two equal and opposite forces.

\[ M_{\text{couple}} = F \times d \]

Example: Determine the Moment at A, and at B (Assume Moment is positive for counterclockwise rotation)

Moment at A, \( M_A = -(5 \times 10) + (5 \times 6) = -20 \text{ k-ft (clockwise)} \)
Moment at B, \( M_B = -(5 \times 4) = -20 \text{ k-ft (clockwise)} \)

Moment at any location between A & B is \(-20 \text{ k-ft clockwise} \).
Resolution of a Force into a Force and a Couple acting at another point

Example - 1:
Calculate Moment at A

Moment at A: $M_A = 50 \text{ k-ft}$

(Clockwise)
Example - 2:

Calculate Moment at A

\[ \text{M}_A = 20 \text{ k-ft} \]  
(counterclockwise)

\[ \text{F}_A = 11 \text{ k} \]