Shear Forces and Bending Moments in Beams

REATIONS
SHEAR FORCE DIAGRAMS
MOMENT DIAGRAMS

[EXAMPLES]

• Equilibrium Method for V and M Diagrams
• Semi-graphical Method for V and M Diagrams
Equilibrium Method for V and M Diagrams

Q1:

\[ P = 6k \]

\[ R_A = \frac{6 \cdot (12 - 3)}{12} = 4.5k \]

\[ R_B = \frac{6 \cdot 3}{12} = 1.5k \]

\[ 4.5 \times 3 = 13.5k \text{-ft} \]

\[ -1.5k \]
Q2:

\[ R_A = \frac{5(10+2)}{10} = 6k \]
\[ R_B = \frac{5(2)}{10} = 1k \]

\[ M = -5 \times 2 = -10 \text{ k-ft} \]
Q3: Find reactions, Shear Force, Location of zero shear forced, Maximum Moment, Mid-span moment.

Take a moment about C and Find Reaction at A

\[ R_A = \frac{20 \times 15}{20} = 15 \text{ kips} \]

Take a moment about A and Find Reaction at C

\[ R_c = \frac{20 \times 5}{20} = 5 \text{ kips} \]

[CHECK: Sum of all the forces Upward = sum of the all the forces downward
\[ 15 + 5 = 20 \text{ OK} \]
A weight of 2 kips per foot (w = 2 k/ft) is applied to a beam. The reactions at the supports are given as:

- At A: $R_A = 15$ kips
- At B: $V = 0$ kips
- At C: $R_C = 5$ kips

The shear force diagram indicates:

- Maximum shear force occurs at $x = 15/2 = 7.5$ ft, given by $V = 0$ kips.
- The shear force changes from positive to negative at point B.

The bending moment diagram shows:

- Maximum bending moment ($M_{max}$) at $x = 7.5$ ft, calculated as $M_{max} = 0.5(15 \times 7.5) = 56.25$ k-ft.
- Bending moment at point B is $M = 5 \times 10 = 50$ k-ft.

The diagram illustrates the shear force and bending moment for the given beam conditions.
Q4: Find reactions, Support Moment; Draw Shear Force and Moment diagrams.

\[ M_b = (4 \times 2) + (2 \times 5) + (0.25 \times 10 \times 5) = 30.5 \text{k-ft} \text{ (Anti-clockwise)} \]

\[ R_{bh} = 4 \text{k} \text{ (towards left)} \]

\[ R_{bv} = 2 + (0.25 \times 10) = 4.5 \text{k} \text{ (up)} \]

\[ 4.5 - (0.25 \times 5) = 3.25 \]

\[ 3.25 - 2 = 1.25 \]

\[ 1.25 - (0.25 \times 5) = 0 \]

\[ 8 + (0.5 \times 1.25 \times 5) = 11.125 \text{k-ft} \]

\[ 4 \times 2 = 8 \text{k-ft} \]

\[ R_A = \frac{12 \times 8 + (4 \times 12 \times 6)}{12} = 32k \]

\[ R_B = \frac{12 \times 4 + (4 \times 12 \times 6)}{12} = 28k \]

\[ V = 0 \quad \text{at} \quad (12-5) = 7 \text{ ft} \]

\[ X = \frac{4}{4} = 1 \text{ ft} \]

\[ M_{\text{max}} = 28(12-5) - 4(12-5)(12-5)/2 = 98 \text{ k-ft} \]
Q6: A simply supported beam with a trianularly distributed downward load is shown in Fig. Calculate reaction; draw shear force diagram; find location of \( V = 0 \); calculate maximum moment, and draw the moment diagram.

\[
\begin{align*}
F &= (0.5 \times 6 \times 9) = 27k \\
x &= (2/3)(9) = 6 \text{ ft}
\end{align*}
\]

\[
\begin{align*}
R_A &= (27k)(9-6)/9 = 9k \\
R_B &= (27k)(6)/9 = 18k
\end{align*}
\]

\[
\begin{align*}
X &= 5.2' \\
V &= 0 \\
M_{\text{max}} &= 31.2 \text{ k-ft}
\end{align*}
\]

Find \( X = ? \) (Location at \( V = 0 \))

\[
(6/9)(X)(X/2) = 9 \\
X = \sqrt{(9 \times 18/6)} = 5.2 \text{ ft}
\]

\[
M_{\text{max}} = 9(5.2) - [0.5(6/9)(5.2)(5.2)](5.2/3) = 31.2 \text{ k-ft}
\]
Q7:

Calculate Reactions

\[ R_B = (4 \times 20) + 16 - 70 = 26k \]

\[ R_B = [(4 \times 20) \times 10 + (16 \times 20)]/16 = 70k \]
X = 26/4 = 6.5'

\[ M = (26 \times 6.5) - (4 \times 6.5 \times 6.5/2) = 84.5 \text{ k-ft} \]

\[ M = (16 \times 4) - (4 \times 4 \times 2) = 32 \text{ k-ft} \]

\[ M = (26 \times 16) - (4 \times 16 \times 16/2) = -96 \text{ k-ft} \]
Semi-graphical Method for V and M Diagrams

Q8:

- **P = 6k**
- **3 ft**
- **R_A** = \( \frac{6(12 - 3)}{12} = 4.5k \)
- **R_B** = \( \frac{6 \times 3}{12} = 1.5k \)
- **A** = 6 ft
- **B** = 3 ft

**V Diagram:**
- **4.5k**
- **4.5 \times 3 = 13.5k-ft**
- **Area**
- **(4.5 \times 3) - (1.5 \times 9) = 0 k-ft**

**M Diagram:**
- **-1.5k**
Q9:

\[ R_A = \frac{[12 \times 8 + (4 \times 12 \times 6)]}{12} = 32k \]
\[ R_B = \frac{[12 \times 4 + (4 \times 12 \times 6)]}{12} = 28k \]

Shear Force Diagram:

- At A: \[ V = 0 \]
- At B: \[ V = 12 - 5 = 7 \text{ ft} \]

- Maximum shear force: \[ V_{\text{max}} = 96 + 0.5 \times 4 \times 1 = 96 + 2 = 98 \text{ k-ft} \]

- Bending Moment Diagram:
  - At A: \[ M = 0.5(32 + 16)(4) = 96 \text{ k-ft} \]
  - Maximum bending moment: \[ M_{\text{max}} = 96 + 0.5 \times 4 \times 1 = 96 + 2 = 98 \text{ k-ft} \]
Thank you.