Q1: Design stirrup for the simply-supported beam as shown in Figure. Use #4 U stirrups. The supports are 12" wide, and the loads shown are service loads. Beam cross-section is shown in SECTION A-A. Use $f_c' = 3000$ psi, and $f_{yt} = 40000$ psi.

Service DL = 2.5 k/ft
Service LL = 3.0 k/ft

$L = 24' - 0" + 6" + 6" = 25' - 0"

$d = 30" - (1.5 + 4/8 + 10/16) = 27.375"

SECTION A-A
Solution:

Fig. 5 Shear Reinforcement requirements
1. Draw Shear, $V_u$ Diagram (Fig. 5)

   Bean self weight, $SW = [(30 \times 16)/144] 0.15 = 0.5 \text{k/ft}$

   $W_u = 1.2(D + SW) + 1.6 (L) = 1.2(2.5+0.5) + 1.6(3.0) = 8.4 \text{k/ft}$

   Reactions, $R_A = R_B = W_u \frac{L}{2} = 8.4 \times 25 /2 = 105 \text{ kips}$

2. Calculate $V_u$ at a distance $d$ from the face of support; $V_u @d = 105 – 8.4(27.375+6)/12 = 81.64 \text{kips}$

3. On the $V_u$ diagram, identify locations where (1) Shear Reinforcement required, (2) where shear reinforcement not required, (3) where shear carried by stirrups, $\varphi V_s$ , and (4) where minimum shear reinforcement required (Shear carried by concrete, $\varphi V_c$ ). [Note: SEE Fig. 5]

4. Calculate $\varphi V_c = 2 \lambda \varphi \sqrt{f_{c}^{'}} (b_w x d) = 2(1)(0.75) \sqrt{3000 \times (16 \times 27.375)} /1000 = 35.99 \text{kips}$

   $\varphi = 0.75$;
   $\lambda = 1$ for normal weight concrete; 0.85 for sand-lightweight concrete; 0.75 for all lightweight concrete.

5. Calculate $\varphi V_s = [V_u - \varphi V_c] = 81.64 – 35.99 = 45.65 \text{kips}$

   **Check:** If $8\varphi \sqrt{f_{c}^{'}}. b_w d < [\varphi V_s]$, then SECTION SHOULD BE ENLARGED [STOP AT THIS STEP]

   $8\varphi \sqrt{f_{c}^{'}}. b_w d < [\varphi V_s] = 8(0.75) \sqrt{3000 \times (16 \times 27.375)} /1000 = 143.94 \text{kips} > \varphi V_s$

6. No Stirrups are needed if $V_u < 0.5 \varphi V_c$. $81.64 > (0.5 \times 35.99)$ YES, STIRRUPS REQUIRED
DESIGN STIRRUPS

7. Determine required spacing of vertical U stirrups based on $\phi V_s$

Calculate theoretical stirrup spacing, $S = \phi A_v x f_y x d / [V_u - \phi V_c]$

Use #4 U Stirrups:

$$S = 0.75 (2x0.2)(40)(27.5)/[45.65] = 7.2 \text{ in (CONTROLS)}$$

S must satisfy

$$S \leq d/2 \leq 24 \text{ inch}$$

$$S = d/2 = 27.375/2 = 13.69 \text{ in}$$

$$S \leq 24 \text{ in}$$

If $[V_u - \phi V_c] > 4\phi \sqrt{f'c' \cdot b w \cdot d}$ Then $S \leq d/4$

$\{4\phi \sqrt{f'c' \cdot b w \cdot d}\} = 144.6 /2 = 71.97 \text{ kips} > 45.65 \text{ kips}$. Therefore, $S = d/4$ Not Controls.

**USE #4 U @ 7 in**

8. Determine spacing of vertical U stirrups based on minimum shear reinforcement.

S is smaller of the two:

For #4U Stirrups

$$S = A_v f_y /[50 b_w] = (2 x 0.2)(40000)/[50x 16] = 20 \text{ in}$$

$$S = A_v f_y /[0.75\sqrt{f'c'} \cdot b_w] = (2x0.2)(40000)[0.75 x \sqrt{3000} \times 16] = 24.34 \text{ in}$$

S must satisfy

$$S \leq d/2 \leq 24 \text{ inch}$$

$$S = d/2 = 27.375/2 = 13.69 \text{ in (CONTROLS)}$$ **USE #4 U @ 13.5 in O.C.**
\[ S = 24 \text{ in} \]

Shear reinforcement **NOT required** from the C.L. of the beam to a distance of \( (0.5 \times 36.35)/8.4 = 2.15 \text{ ft} \)

Shear reinforcement required from support to a distance \( = 12.5 - 2.15 = 10.35 \text{ ft} \)

Distance “X” from the support beyond which #4 @ 13.5 in O.C. stirrup spacing can be used.

First find out for what \( V_u \) value will require #4 U @ 13.5 in

\[
V_u = [\varphi A_v x f_y x d / S ] + \varphi V_c
\]

\( V_u = 0.75 (2 \times 0.2)(40)(27.375)/(13.5) + 36.35 = 60.32 \text{ kips} \)

\( X = (R_a - V_u)/W_u = (105 - 60.32)/(8.4) = 5.32 \text{ in} \) **(Say 5.5 ft)**