A Simply supported beam with a concentrated load at mid-span:

**Loading Stages**

1. **Elastic**
   - \( F_b < F_y \)
2. **Elastic**
   - \( F_b = F_y \)
3. **Elastoplastic**
   - \( F_b = F_y \)
4. **Plastic**
   - \( F_b = F_y \)

**Plastic Hinge**
Bending Stress and Stain: ELASTIC

Strain < Yield Strain

\[ \varepsilon < \varepsilon_y \]

\[ f_b = \frac{(M_c)}{I} = \frac{M}{S} \]

Stress < Yield Stress

\[ f_b < F_y \]

\[ f_y = \frac{(M_y)}{I} \]
Bending Stress and Stain: YIELD

Strain = Yield Strain

\[ \varepsilon = \varepsilon_y \]

Stress = Yield Stress

\[ f_b = F_y \]

\[ f_b = \frac{(Mc)}{I} = M/S \]

\[ f_y = \frac{(My)}{I} \]
Bending Stress and Stain: Elasto-Plastic (Partially Plastic)

Strain > Yield Strain

\[ \varepsilon > \varepsilon_y \]

Stress = Yield Stress

\[ f_b = F_y \]
Bending Stress and Stain: Plastic (Fully Plastic)

Strain > Yield Strain

\[ \epsilon > \epsilon_y \]

Stress = Yield Stress

\[ f_b = F_y \]

Plastic Moment: \[ M_p = F_y Z \]

Where

\[ Z = \text{Plastic Section Modulus} \]
**EXAMPLE 1:**

Compute the plastic moment, \( M_p \) for a W16 X 57 of A992 Steel.

**Solution:**
From AISC Table, for W16 X 57,
\[ A = 16.8 \text{ in}^2; \quad Z_x = 105 \text{ in}^3 \]

For A992 Steel, \( F_y = 50 \text{ ksi} \)

\[ M_p = F_y Z = (50)(105) = 5250 \text{ k-in} = 5250/12 = 437.5 \text{ k-ft}. \]
AISC Classification of Shapes as (1) Compact (2) Non-compact, and (3) Slender depending on width to thickness ratios. (See AISC Table B 4.1)

**Flange Criterion:**

If the flange is continuously connected to the web, and

\[
\frac{b_f}{2 \ t_f} \leq 0.38 \sqrt{\frac{E}{F_y}}
\]

the shape is COMPACT

The shape is NONCOMPACT

\[
0.38 \sqrt{\frac{E}{F_y}} < \frac{b_f}{2 \ t_f} < 1.0 \sqrt{\frac{E}{F_y}}
\]

The shape is SLENDER

\[
\frac{b_f}{2 \ t_f} > 1.0 \sqrt{\frac{E}{F_y}}
\]
Web Criterion:

The shape is COMPACT

\[
\frac{h}{t_w} \leq 3.76 \sqrt{\frac{E}{F_y}}
\]

The shape is NONCOMPACT

\[
3.76 \sqrt{\frac{E}{F_y}} < \frac{h}{t_w} < 5.70 \sqrt{\frac{E}{F_y}}
\]

The shape is SLENDER

\[
\frac{h}{t_w} > 5.70 \sqrt{\frac{E}{F_y}}
\]

NOTES:

The web criterion is met by all standard I- and C- shapes listed in the AISC Manual, so only flange criterion needs to be checked.
EXAMPLE 2: Check compactness for W16 X 57 A992 Steel.

A 992 Steel, $F_y = 50$ ksi

AISC Table W16 X 57

$b_f/(2t_f) = 4.98$;

$0.38 \sqrt{(29000/50)} = 9.15 > 4.98$, Hence, the flange is COMPACT

$h/t_w = d/t_w = 33.0$; $3.76 \sqrt{(29000/50)} = 90.55 > 33.0$, Hence, considering web, section is COMPACT [NOTE for all shapes in the AISC Manual, section is compact considering web.

Therefore, **W16 X 57 is COMPACT**.
**LRFD: Bending Member**

\[ M_u \leq \phi M_n \]

Where
\[ \Phi = \text{Resistance factor} = 0.90 \text{ (LRFD)} \]
\[ M_u = \text{Factored Moment} \]
\[ \phi M_n = \text{Design Moment Moment Strength} \]

**LATERALLY SUPPORTED COMPACT BEAMS**

\[ M_n = M_p \]

Where
\[ M_p = F_y Z \leq 1.5 M_y \]

The limit of 1.5 \( M_y \) is to prevent excessive working load deformations, and is satisfied when
\[ F_y Z \leq 1.5 F_y S \]

or \[ Z/S \leq 1.5 \]

**NOTE:**
For C-, I and H shapes bent about the strong axis (x-x), \( Z/S \) will always be \( \leq 1.5 \).
For I and H shapes bent about the weak axis (y-y), \( Z/S \) will never be \( \leq 1.5 \).
EXAMPLE 3: The beam shown in figure is a W16 X 57 of A992 steel. It supports a reinforced concrete slab that provides a continuous lateral support of the compression flange. Is the beam adequate?

SOLUTION:

\[ \text{Wu} = 1.2(0.45 + 0.057) + 1.6(0.55) = 1.4884 \text{ k/ft} \]

\[ \text{Mu} = \text{wu}L^2/8 = 1.4884 \times (30)^2 / 8 = 167.445 \text{ k-ft.} \]

Check compactness for W16 X 57 A992 Steel.

A 992 Steel, \( F_y = 50 \text{ ksi} \)

AISC Table W16 X 57

\[ \frac{bf}{(2tf)} = 4.98; \]

\[ 0.38 \sqrt{29000/50} = 9.15 > 4.98 \text{ Hence, the flange is COMPACT} \]

\[ \frac{h}{tw} = \frac{d}{tw} = 33.0; \ 3.76 \sqrt{29000/50} = 90.55 > 33.0, \text{ Hence, considering web, section is COMPACT} \text{ [NOTE for all shapes in the AISC Manual, section is compact considering web.}] \]

Therefore, W16 X 57 is COMPACT.

Because the beam is compact and laterally supported, \( \text{Mn} = \text{Mp} = F_yZ_x = 50(105) = 5250 \text{ k-in/12} = 437.5 \text{ k-ft.} \)

Check for \( \text{Mp} \leq \text{My} \)

\[ \frac{Z_x}{S_x} = \frac{105}{92.2} = 1.1388 < 1.5 \text{ OK} \]

\[ \Phi \text{ Mn} = 0.9 \times 437.5 = 393.75 \text{ k-ft} > 167.445 \text{ k-ft \ OK} \]

The design moment is greater than the factored load moment. So W16 X 57 IS ADEQUATE
EXAMPLE 4: Select the least-weight wide-flange A992 steel beam section. It supports a reinforced concrete slab that provides a continuous lateral support of the compression flange.

Service DL= 450 lb/ft
LL = 550 lb/ft

SOLUTION:

Since, at this point beam W section’s weight is unknown, calculate Mu without the beam self weight

\[ W_u = 1.2(0.45) + 1.6(0.55) = 1.42 \text{ k/ft} \]

\[ M_u = w_u L^2/8 = 1.42 (30)^2 / 8 = 159.75 \text{ k-ft} \]

\[ M_n = M_p = F_y Z \]

\[ \phi M_n = \phi F_y Z \]

Considering \( M_u \)

\[ Z_{req} = M_u / (\phi F_y) = 159.75 \text{ (k-ft)} \times 12 / (0.9 \times 50) = 42.6 \text{ in}^3 \]

From AISC Table, \( W_{12 \times 30} \), \( Z_x= 43.1 \)

Check: \( \phi M_n = \phi F_y Z = 0.9(50)(43.1) = 1939.5 \text{ k-in} = 161.625 \text{ k-ft} \)

\[ W_u = 1.42 + 1.2(0.03)= 1.456 \text{ k/ft} \]

\[ M_u = 1.456 (30)^2 / 8 = 163.8 \text{ k-ft} > 161.625 \text{ NG} \]

Select \( W_{12 \times 35} \), \( Z_x=51.2 \)

Check: \( \phi M_n = \phi F_y Z = 0.9(50)(51.2) = 2304 \text{ k-in} = 192 \text{ k-ft} \)

\[ W_u = 1.42 + 1.2(0.035)= 1.462 \text{ k/ft} \]

\[ M_u = 1.462 (30)^2 / 8 = 164.475 \text{ k-ft} < 192 \text{ OK} \]

ANSWER: \( W_{12 \times 35} \)