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Establish Effective Lower Bounds of Watershed Slope for Traditional Hydrologic Methods

Theodore G. Cleveland, David B. Thompson, Xing Fang, Ming-Han Li

Texas Tech University
College of Engineering
Box 41023
Lubbock, Texas 79409-1023

Texas Department of Transportation
Research and Technology Implementation Office
P. O. Box 5080
Austin, TX 78763-5080


This study was conducted in cooperation with the Texas Department of Transportation and the Federal Highway Administration.

Equations to estimate timing parameters for a watershed contain watershed slope as a principal parameter and estimates are usually inversely proportional to topographic slope. Hence as slope vanishes, the estimates approach infinity. The research identified, from literature, data, modeling, and physical experiments, the dimensionless topographic slope where “low-slope” behavior should be assumed.

Alternate timing equations based from parametric studies using the Diffusion Hydrodynamic Model (DHM) were developed. A slope-offset approach was explored and is suggested as an adaptation of current technology. The offset value suggested is 0.0005 and an illustrative example is presented in this report. The dimensionless slope suggested for consideration of alternate approaches is 0.003.

The researchers found that current streamgaging techniques are not well suited to low-slope hydrologic studies and different measuring methods are suggested.
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Acknowledgements

A project of the magnitude of TxDOT Project 0–6382 cannot be completed without the contributions of a large number of individuals beyond the principal investigators. The service of TxDOT engineers David Stolpa and George “Rudy” Herrmann began early in the conceptual stages of the project before a problem statement was even developed. Their contributions were supplemented by other TxDOT advisors, including Jaime Villena-Morales (Project Director), Amy Ronnfeldt, Elie Alkhoury, Kathy Dyer, Mark Mikulenka, and David Zwernemann.

The contributions of the co-researchers were substantial. The authors thank William H. Asquith of the U.S. Geological Survey for substantial software development as well as conceptual and editorial support. His efforts were supplemented by Meghan Roussel and Joe Vrabel also of the USGS. Xing Fang and Nirajan Dhakal at Auburn University have an uncanny ability to find obscure literature that is critical to project progress. David Thompson provided his insight and direction, as well as his writing and typesetting skills.
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1. Introduction

Texas Department of Transportation (TxDOT) analysts design roadways and proximal infrastructure. A substantial fraction of transportation dollars, and hence design efforts, are spent on stormwater drainage and conveyance infrastructure.

Methods currently and widely used by analysts to estimate the characteristic time or “watershed timing parameter” contain watershed slope as a principal parameter such that the watershed timing parameter is inversely proportional to the characteristic slope. Therefore, as topographic slope decreases, the watershed timing parameter increases. Within hydrologic models, the decrease in topographic slope produces a rightward shift in the time to equilibrium (steady state) and/or peak discharge and a reduced value for that peak discharge. Furthermore, conservation of mass requires that the reduced discharge rate be accommodated by storage (that is watershed runoff is stored by the system for a longer time). Effectively, this storage requirement increases the time base of runoff (stretches the hydrograph).

Thus, for some finite topographic slope, the hydraulic behavior approximated by a rainfall-runoff model deviates from the assumptions used in the development of the method and unrealistic estimates of watershed time response result. As a result, designers of projects in all districts and area offices where low-relief terrain is prevalent, such as the Gulf Coast and the High Plains of Texas, are challenged to account for low-slope hydraulic behavior using a combination of judgement and current estimation tools.

The ability to appropriately estimate watershed travel time for low-slope conditions will reduce uncertainty in predicting design discharges, resulting in improved decisions on structure size and corresponding cost. The net result is improved use of financial resources, reduced risk of costly overestimation of design discharge (size of structure), and reduced risk of underestimation (size of structure) and the potential resulting damage to highway infrastructure and adjacent real property.

The purposes of this research project were:

1. Identify from literature, data, modeling, and experiments, the dimensionless topographic slope where conventional hydrologic methods become suspect and alternate approaches should be considered;

2. Identify, develop, and document such alternate approaches and provide procedures and guidelines for when and how to use them;

3. Develop a research plan (guidance, instrumentation selection and/or invention) for more intensive study of low-slope watershed response should further work and supporting data be deemed necessary by TxDOT.
2. Watershed Slope

The parameters most important for understanding overland flow are topography, surface roughness, soil infiltration characteristics (abstractions), and the distribution, duration, and intensity of precipitation. Of these components, the focus of the research reported herein is the impact of topographic slope on the hydraulics of overland and watershed channel flow.

2.1. Why Does Slope Matter?

Topographic slope appears in watershed time equations as a consequence of simplification of the momentum equation for practical application in most topographic regimes. Figure 1 is a sketch of an impervious, sloped surface with a constant rainfall intensity applied. The slope of the surface affects the magnitude of the discharge and depth of flow at any fixed distance downstream.

![Figure 1: Sketch for momentum equation. Adapted from Yu and McNown (1964).](image)

Slope also affects direction of flow. Figure 2, upper panel, is a conceptual diagram that contains a surface with a finite-length intensity rainfall field similar to that used by Yu and McNown (1964). Precipitation (runoff) accumulates on the plane and produces a unidirectional flow in the downslope direction. The lower panel is a depiction of a surface with zero slope. Flow moves away from the centroid of the runoff-intensity field, either left or right depending on the location of the point of interest relative to the intensity field.

Slope also impacts infiltration and storage abstractions — a low slope system provides more “ponding” time for infiltration. Using Figure 2 as representative of the slope effect, the lower panel suggests greater ponding depth (driving force for infiltration) as well as lower mean velocity in the flow field, which extends the time for infiltration to occur at a location. Similarly, small irregularities in the surface are less important for watersheds with substantial topographic slope. However, such irregularities may constitute substantial storage in the zero-slope case.
2.2. What is “Low Slope?”

2.2.1. Fundamental Matters

Equation 1 is a one-dimensional equation of motion for runoff on an impervious surface\(^1\) with reference to Figure 1 and adapted from Yu and McNown (1964).

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = g \left( S_0 - \frac{\tau}{\rho gh} - \frac{\partial h}{\partial x} \right) - \frac{u \sigma}{h},
\]

where \(\tau\) is the boundary stress (friction), \(\sigma\) is the rainfall intensity, \(h\) is the water depth, \(S_0\) is the topographic slope, and \(u\) is the section average velocity.

The left-hand side of Equation 1 contains the convective and local acceleration terms, whereas the right-hand side are the net forces on a unit mass resulting from gravity \((S_0)\), boundary shear \((\tau)\), and an additional retarding term to represent rainfall \((\sigma)\) impact on the free surface. At equilibrium, the local acceleration term becomes zero and the convective acceleration, depth taper \(\frac{\partial h}{\partial x}\), and rainfall retarding effect are relatively small compared to the gravitational term, \(g S_0\). The time to reach this equilibrium condition is related to the time-of-concentration and other characteristic response times.

\(^1\)This development does not consider infiltration losses.
Yu and McNown (1964), following fluid mechanics convention, equate the shear stress to a friction factor (based on limited experimental data circa 1964) and substituting $\tau = \frac{f u^2}{\rho}$, develop for equilibrium conditions an equation of motion, Equation 2,

$$u = \sqrt{\frac{8g h}{f} \sqrt{S_0}}.$$  

Equation 2 is a simplified equation of motion structurally similar to Manning’s equation of motion — there is a constant associated with friction ($f$), a variable associated with the depth of flow ($h$), and a driving term associated with local topographic slope ($S_0$). If travel distance is considered, the relation to time is recovered from $x = ut$, as in Equation 3,

$$t = \frac{x}{\sqrt{\frac{8gh}{f} \sqrt{S_0}}}.$$  

The consequence of this simplification is that topographic slope is a fundamental component of the equilibrium equation of motion and appears in the denominator of time equations. Later researchers, the authors included, continued this simplification.

The analysis changes for relatively small slopes because the convective terms and the depth taper are no longer negligible compared to the remaining terms (mainly because the gravitational term completely vanishes in the limit). Furthermore, at $x = 0$ in the illustration, zero depth cannot be achieved, thus the axis of symmetry will by necessity have non-zero depth, although the flow will become bidirectional in such a case.

Therefore, “low slope” is postulated to begin where topographic slope is such that the convective terms are no longer negligible. The remainder of this section is a review of literature where low-slope behavior is examined with a goal to determine, from literature review, the dimensionless slope at which the convective terms and depth taper are no longer negligible.

### 2.2.2. Literature-Derived Estimates of “Low-Slope” Behavior Initiation

When the topographic slope is less than 0.5% on an impermeable plane surface using a rainfall simulator, de Lima and Torfs (1990) demonstrated that water depth at the upper boundary is non-zero. This literature finding is one indicator of where “low-slope” begins — it is a topographic slope where water depth must be non-zero (at the axis of symmetry) to sustain equilibrium flow.

Based on numerical backwater computations, van der Molen and others (1995) confirmed the findings reported by de Lima and Torfs (1990). These results are depicted in Figure 3. The non-zero depth accumulated at slopes of less than 0.2% (0.002) in these two studies is the first source of a value for dimensionless slope where “low-slope” behavior begins.

Sheridan and Mills (1985) conducted hydrologic research on relatively low slope coastal plain watersheds in the Southeast United States (near Tifton, Georgia). Their primary study area is the 129-square mile Little River Watersheds (LRW) operated by the Southeast Watershed Research Laboratory (SEWRL) of the USDA-Agricultural Research Service (ARS). Drainage areas
Figure 3: Water depth at the top of the field as a function of slope and calculated water depth over distance for several low slopes (≤ 0.5%), from van der Molen and others (1995, Figure 4).

for subwatersheds within LRW range from 1.0–44 square miles and main channel slopes from 0.10–0.81% (Sheridan, 2002).

Sheridan and Mills (1985) concluded that low-slope watersheds are damped hydrologic systems and flows in the design storm range are spread across broad, flat, heavily vegetated floodplains. Because energy and topographic gradients are relatively low, flow velocities in floodplain drainage networks are quite low compared to traditional channel flow situations. Channel and watershed slopes did not appear substantially important for time estimation, and they recommended the following simple relations for estimating time of concentration and time to peak of hydrograph, based on stepwise multiple regression analyses:

\[
\begin{align*}
    t_c &= 2.20 MCL^{0.92}, \quad \text{and} \\
    t_p &= 1.33 MCL^{0.89},
\end{align*}
\]

where \( MCL \) is the main channel length (kilometers) and \( t_c \) and \( t_p \) have units of hours. In the context of the time relation in Equation 3, the denominator is essentially constant (depending on what particular characteristic time is sought, and that time’s relation to the characteristic length, \( MCL \)).

The lack of importance of slope as an explanatory variable in Sheridan and Mills (1985), combined with reported topographic slopes in Sheridan (2002) at less than 0.81% is a second source of a value
for dimensionless slope where “low-slope” behavior begins.

Su and Fang (2004a) developed a numerical model based on the two-dimensional (2D), depth-integrated, shallow wave equations to simulate overland flow on the idealized rectangular geometry depicted in Figure 4 — with the acceleration terms preserved. They simulated the behavior of a 10-meter by 35-meter watershed with no-flow boundaries (dashed lines in the figure) on three sides and a variable width rectangular outlet on the fourth side. This “numerical” watershed could be configured with different friction behavior (Manning’s $n$), different longitudinal slope, and different outlet widths.

![Diagram of rectangular simulation domain](image)

Figure 4: Rectangular simulation domain. Adapted from Su and Fang (2004a). The outlet is open in this figure, width of opening was variable in their study — smallest opening can be visualized as a slot on the end of the box.

The Su and Fang model was validated using experimental data from Izzard (1946) and field data from Emmett (1970). Su and Fang used the model to compute the time to equilibrium discharge. The time to equilibrium is analogous to the conventional meaning of $T_c$ for full watershed contribution. Su and Fang performed parametric studies wherein rainfall intensity, watershed slope, the friction terms, and the outlet restriction were varied. In cases where the slopes used in the simulation corresponded to those in the experimental and field data, their results agreed within 10% of those of other researchers. The Su and Fang findings were presented as a set of dimensional charts and tables specific to their geometry. Based on those results, some general comments are:

1. $T_c$ increases as slope is decreased. The rate of increase is roughly one doubling of $T_c$ for every 10-fold decrease in slope, with negligible change in $T_c$ at slopes less than 0.0005;

2. At zero slope, $T_c$ increases with increasing friction (Manning’s $n$) — the increase is superlinear but not quadratic; roughly every doubling of the friction term increases $T_c$ by a factor close to $\sqrt{2}$;

3. $T_c$ increases with increasing outlet restriction thus having the effect of reducing peak discharge rate.

Finally Su and Fang (2004a) proposed Equation 6 for estimating $T_c$ on watersheds of slopes less than 0.0005.

$$T_c = C_{outlet} \times \frac{n^{0.43} L^{0.33}}{i^{0.4}}$$  \hspace{1cm} (6)

$^2$A boundary specification of no-flow is the glass wall assumption, which is common in 2D practice.
where $C_{\text{outlet}}$ is an outlet factor, $i$ is the rainfall intensity, $n$ is Manning’s $n$, and $L$ is the basin longitudinal length (35 meters in their study). Of particular note in this equation is the absence of slope, similar to the findings of Sheridan and Mills (1985), and the dependence of $T_c$ on roughness and input intensity — a dependence that appears in other literature. The same model was later applied to a practical case of drainage modeling in Beaumont, Texas (Fang and Su, 2006). The results compared well to high-water marks from Tropical Storm Allison in 2001.

The relatively negligible change in $T_c$ at slopes less than 0.05% in Su and Fang (2004a) is a third source of a value for dimensionless slope where “low-slope” behavior begins.

Cleveland and others (2008) used a particle tracking model to compute an ensemble of travel times based on an equation of motion structurally identical to Equation 2. They interpreted these ensembles using a statistical-mechanical analog to generate watershed hydrographs. They found that a significant increase in hydrograph prediction uncertainty occurred when watersheds with slopes in the range 0.0002–0.002 were included. Cleveland and others speculated that use of the dimensionless topographic slope in Equation 2 was invalid for slopes in this range. Although they did not set out to examine low-slope behavior, this study represents a fourth source for a value for dimensionless slope (0.002) where “low-slope” behavior begins.

Li and others (2005) and Li and Chibber (2008) developed regression equations for $T_c$ time of concentration from laboratory data collected from terrains with slope ranging from 0.24% to 0.48%. They included terrain slope as an independent variable in power-law models, but later concluded that the exponent of the slope parameter is nearly one order of magnitude less than those of existing models (based on larger scale studies) and therefore the importance of slope as an explanatory variable is minimal. This result is similar to the conclusion described by Sheridan and Mills (1985). Li and others and Li and Chibber ultimately reported that the contribution of slope to time parameter estimation is negligible for laboratory terrains with slope less than 0.5%.

Riggs (1976) suggested a “simplified” slope-area method for estimation of high-magnitude peak streamflow in natural channels. The slope-area method is commonly used by USGS personnel and others for post-flood estimation of peak streamflow based on high-water marks. The high-water marks are used to determine the water-surface slope. The estimates of water-surface slope are combined with multiple estimates of cross-sectional area and other hydraulic properties to develop multiple estimates of peak streamflow, from which is selected a representative estimate. Riggs (1976) sought a quick, reproducible, and inexpensive alternative or complement to the slope-area method.

Figure 5 is a plot of the relation between roughness coefficient and dimensionless water-surface slope, using values extracted from Riggs (1976, Figure 1). Although these data are attributed to Barnes (1967), they were augmented by Riggs. The datapoints were digitized by W. H. Asquith from a scanned image of the Riggs figure. The relation is reasonably well-defined, and Riggs concludes that “a relation of this type might be used to modify an estimated $n$ [roughness] according to slope; or one might conclude that the two variables are so highly related that only one of the two [slope or roughness] is needed [for] computing [peak streamflow].”

The Riggs quote triggered author interest in a regression equation between dimensionless water-surface slope and Manning’s $n$ as another means to explore the onset of “low-slope” behavior as
a function of slope. Because the channel topographic slope is substituted for water-surface slope for normal flow (negligible depth taper), the regression equation could subsequently be substituted for slope in watershed-timing equations. Such a regression line is depicted on Figure 5 as the solid curve passing among the markers in the plot. Based on this curve, “steeper” watersheds require relatively large values of roughness to agree with observations. Similarly (and important for the
research), at relatively small values of slope the roughness term rate of change is comparatively small for a logarithmic change in slope.

Using an estimate of Manning’s $n$ of 0.030 as a representative value for a natural system, the corresponding anticipated slope is 0.003 (0.3%). Furthermore, a reduction of the slope by an order of magnitude results only in a decrease of estimated roughness by one-third (and into numerical values associated with engineered channel surfaces).

Although examination of low-slope behavior was not an objective of Riggs (1976), the comparatively independent analysis produces a fifth source of a value for dimensionless slope (0.003) for which watersheds with slopes less than this value exhibit “low-slope” behavior.

The results of the various estimates of topographic slope that define the onset of “low-slope” behavior are listed in Table 1. These values constitute literature-based estimates of the threshold for onset of low-slope behavior in computations of watershed behavior. A significant part of the remainder of this report comprises extension of these estimates through examination of results from experimental watersheds (both field plots and laboratory test beds) and through use of a numerical hydrodynamic model to extend experimental results. This work is documented in subsequent sections of this report.

Table 1: Dimensionless Slope ($S_0$) where “Low-Slope” Behavior is in Effect.

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<th>Slope ($S_0$)</th>
<th>Methods</th>
<th>Reference(s)</th>
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<tr>
<td>0.0005</td>
<td>Numerical Model Experiments</td>
<td>Su and Fang (2004a)</td>
</tr>
<tr>
<td>0.002</td>
<td>Numerical Model Experiments</td>
<td>van der Molen and others (1995)</td>
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<tr>
<td>0.002</td>
<td>Numerical Model Experiments</td>
<td>Cleveland and others (2008)</td>
</tr>
<tr>
<td>0.003</td>
<td>Observed Data Analysis</td>
<td>Current authors using Riggs (1976)</td>
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<tr>
<td>0.005</td>
<td>Physical Model Experiments</td>
<td>Li and others (2005) and Li and Chibber (2008)</td>
</tr>
<tr>
<td>0.005</td>
<td>Physical Model Experiments</td>
<td>de Lima and Torfs (1990)</td>
</tr>
<tr>
<td>0.008</td>
<td>Observed Data Analysis</td>
<td>Sheridan and Mills (1985)</td>
</tr>
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</table>
3. Developing Alternate Models for Low-Slope Conditions

The second component of this research was to identify, develop, and document alternate methods for use in low-slope conditions. For this component of the research, experiments were conducted at Texas A&M University. The experiments used two small field plots and the Texas A&M laboratory rainfall simulator. Experimental conditions were relatively controlled. The intent was to accomplish two tasks, (1) to extend the existing rainfall-runoff database the research team assembled in previous studies and (2) to develop data for calibration of a two-dimensional hydrodynamic model.

The hydrodynamic model was used to conduct parametric studies of the relation between slope, rainfall rate, travel distance, and losses to identify alternate hydrologic methods for use in low-slope situations. The hydrodynamic model was used to identify values that are appropriate for an equation structurally similar to Equation 7,

\[ T_c = \beta_0 n^{\beta_1} L^{\beta_2} i^{\beta_3} (S_{LB} + S_0)^{\beta_4}, \]

where \( n \) is Manning’s \( n \), \( \beta_0, \beta_1, \beta_2, \beta_3, \) and \( \beta_4 \) are parameters, \( L \) is the characteristic length (of either the main channel or overland flow), \( i \) is the rainfall intensity, \( S_{LB} \) is a lower bound or threshold of slope, and \( S_0 \) is a measure of watershed slope.

3.1. Experiment Description

Two types of experiments were conducted — the first comprised passive field studies of small field plots (reflective of how one would collect data on an actual outdoor watershed) and the second was use of a controlled indoor simulator (such that both slope and rainfall application rate could be controlled).

3.1.1. Small-Plot Field Studies

Texas A&M University personnel instrumented two small field plots with recording raingages and flow measurement equipment. The pair of research watersheds are located at the Texas A&M University Riverside campus. The distance between the field plots was a few hundred feet. The surface of the first plat was concrete and the other was an open field of prairie.

The concrete-surface field plot is a portion of an abandoned airstrip taxiway. The plot area is a rectangular plot with dimensions of 100 ft by 50 ft. The outlet was located in one corner of the rectangle. The 5,000 ft\(^2\) area is bordered by a 7 inch tall soil berm. The slope along the main diagonal is 0.0025. Figure 6 is an image of the concrete plot looking upstream from the outlet towards the far corner. The H-flume used to measure discharge from the watershed and the tipping-bucket raingage are both evident in Figure 6.

The grass field plot is also located at the Texas A&M University Riverside campus about 500 ft from the concrete plot. The grass watershed is also a 100 ft by 50 ft rectangle with the outlet located in
one corner of the rectangle. Similar to the concrete plot, the grass 5,000 ft$^2$ plot is bordered by soil berms. Site surface was maintained by mowing. The slope along the main diagonal is 0.0025. Figure 7 is an image of the research watershed looking upstream from the outlet towards the far corner. The H-flume that is used to measure discharge from the watershed and the tipping bucket raingage are also shown on Figure 7.

The instrumentation for both sites comprised an ISCO tipping-bucket rain gage and an ISCO sampler equipped with a bubbler flow module located in a 0.75 ft H-flume. The raingage records accumulated rainfall depths in 0.01 in resolution every minute. The bubbler flow module records instantaneous depth in the H-flume at 0.001 ft resolution every minute. The instrumentation is manually connected and powered ahead of each forecasted rainfall event. The recorder is set to trigger (store data) when rainfall intensity exceeds 0.01 inches per hour or the flume discharge depth is over 0.003 ft.

The recorded raw data were adjusted because instrument zero was difficult to maintain. For example, the depth reading by the bubbler should be zero in dry conditions but often recorded a substantial departure. The Texas A&M team treated these initial readings as offsets that should be subtracted from flow depths measured later during a recording event. The raw data are adjusted using the following procedure:

1. Remove the discharge depth offset values from the recorded depth time series.
2. Calculate the instantaneous discharge from the watershed using the adjusted H-flume$^3$ depth

$^3$Equation 8 was provided by the flume manufacturer. $D$ is the measured depth in feet
using Equation 8;

\[ Q_{cfs}(t) = 35.3146667 \times 10^{0.0351+2.6434\log_{10}(0.3048D)+0.2243\log_{10}(0.3048D)^2} \] (8)

3. Calculate total volumes of rainfall and discharge expressed as watershed depth for data QA/QC.

During the research project 26 events were recorded for the concrete study watershed and 4 events were recorded for the grass watershed. Figure 8 is an annotated plot of the adjusted data recorded during the field studies. The rainfall input is the thick stair-step trace on the plot\(^4\) that in nearly all cases defines the upper boundary for the plot, that is, the maximum value of the rainfall exceeds all other values on the plot. The trace that will appear to have variable shading is the observed runoff\(^5\). The solid trace that tends to be smoother than others is a unit hydrograph model used in data analysis. The unit hydrograph model represents an exponential hydrograph (a Gamma model with shape parameter of unity) and was used to compute characteristic response times for later analysis.

\(^4\)If this report is printed in color, the rainfall input will be the blue trace
\(^5\)If this report is printed in color, the observed runoff will be the black-grey trace.
Figure 8: An example rainfall-runoff time-series plot from the field studies. Figure 16 in the next section was constructed from the time values extracted from these time series.
3.1.2. Rainfall Simulator Studies

Texas A&M University personnel conducted a series of controlled simulator studies on an indoor artificial watershed. Water was supplied by a rainfall simulator (controlled intensity) on a bare soil bed (controlled slope). Five longitudinal slopes were examined. They were 0.00, 0.001, 0.002, 0.005, and 0.01.

The artificial watershed is a steel-framed bed 30 ft long by 6 ft wide. The bed is filled with soil to a depth of about 1 ft. Figure 9 is an image of the test-bed in the rainfall simulator enclosure. The rainfall simulator is a nozzle-type design suspended above the test bed. (The rainfall simulator is not shown in the image.) Unique to the watershed design is that the nozzle-array tilts with the bed so the distance from nozzle to the soil bed is constant regardless of the slope. Such an arrangement means that the impact velocity of a water drop onto the test bed is the same regardless of slope. The Texas A&M team filled the bed with clay and then compacted soil with a lawn roller. The bed was stored outdoors for more than a month before simulator tests were conducted. The rainfall simulator can produce a sustained rainfall intensity up to 4.5 inches per hour.

Figure 9: Photograph looking upstream from the outlet for the rainfall-simulator watershed. Longitudinal dimension from the outlet (foreground) to the end of the watershed is 30 ft.

Two bubbler flow modules were used to record discharge depth and surface runoff depth near the outlet with a resolution of 0.001 ft. Values are recorded at one-minute intervals. The discharge is measured using a 22.5° V-notch weir box. Rainfall intensity is measured using an inline flowmeter connected to the rainfall simulator. A recording tipping-bucket raingage was used as an independent check of supply rate.

The experimental procedure used for the simulator experiments is as follows:

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6The month-long storage was a consequence of scheduling time in the simulator rather than any research forethought.
1. Saturate soil until surface runoff appears;
2. Drain the surface runoff by raising the slope up to 20 percent. The resulting condition is initial saturation;
3. Set the bed to the intended slope;
4. Start rainfall by choosing a flow rate. Maintain a constant flow rate (and constant intensity);
5. Measure the discharge rate in the weir box;
6. Stop rainfall application after maximum discharge rate is observed for a short interval (at least a few minutes);
7. Continue recording the discharge rate until system is drained.

The raw data in these experiments are adjusted in a fashion similar to the field experiments. The adjustment procedure for the simulator experiments is:

1. Remove the discharge depth offset values from the recorded depth time series;
2. Calculate the instantaneous discharge from the watershed using the adjusted V-notch weir box\(^7\) depth using Equation 9,
   \[
   Q_{cis}(t) = 0.4979D^{2.5}
   \]  
   where \(D\) is the measured depth in feet.
3. Calculate total volumes of rainfall and discharge expressed as watershed depth for data QA/QC.

A total of 30 experiments were conducted at the five different slopes with two different rainfall intensities. Results were similar to those from the field studies. The collective results (from both field plots and artificial watershed) were used to fit a hydrograph similar to that depicted in Figure 8 for each experimental time series. These fitted results were combined with prior work to produce Figure 16, which is described later in the report.

### 3.2. Modeling Description

Modeling was conducted to examine the interaction of multiple parameters (slope, cover, intensity, and so forth) as a parametric study to develop alternative methods for application in low-slope settings. Two types of modeling were conducted — the first was a parametric study using a hydrodynamic model, and the second was a statistical analysis of collected data. The hydrodynamic model used the DHM code (Hromadka and Yen, 1986) to first calibrate to field plot and simulator studies collected as part of the research reported herein, then to explore an alternative approach for low-slope settings. The data analysis used unit-hydrograph-type interpretation to search for a behavioral trend explainable by slope and other conventional variables.

\(^7\)Equation 9 was provided by the weir-box manufacturer.
3.2.1. Diffusion Hydrodynamic Model (DHM)

The diffusion hydrodynamic model (DHM), developed by Hromadka and Yen (1986), is a two-dimensional numerical model based on the diffusion equation to approximate the horizontal momentum of vertically-averaged free-surface flow. The governing equations for DHM include the continuity equation and two equations of motion.

Equation 10 is the continuity equation at the centroid of a computational grid cell. The variables $q_x$ and $q_y$ are the discharge per unit width in the $x$ and $y$ directions and $H$ is the water-surface elevation,

$$\frac{\partial H}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0. \quad (10)$$

Equation 11 is the $x$-component of the momentum equation on the computational grid. The variables $h$ and $g$ are the flow depth and gravitational acceleration,

$$\frac{\partial q_x}{\partial t} + \frac{\partial}{\partial x} \left( \frac{q_x^2}{h} \right) + \frac{\partial}{\partial y} \left( \frac{q_x q_y}{h} \right) + gh \left( S_{fx} + \frac{\partial H}{\partial x} \right) = 0. \quad (11)$$

Equation 12 is the $y$-component of the momentum equation on the computational grid,

$$\frac{\partial q_y}{\partial t} + \frac{\partial}{\partial y} \left( \frac{q_y^2}{h} \right) + \frac{\partial}{\partial x} \left( \frac{q_x q_y}{h} \right) + gh \left( S_{fy} + \frac{\partial H}{\partial y} \right) = 0. \quad (12)$$

The resulting set of coupled equations (Equations 10–12) is solved by integrated finite-difference methods on a regular grid (Hromadka and Yen, 1986).

The friction slope component in the previous two equations is approximated using Manning’s equation and assuming quasi-steady flow (steady within a computational time step). The precipitation input depth is converted into a volume by a product of the input depth over a time interval and the computational cell area. The rainfall volume is then added into the continuity equation as an internal source term at each computational cell. DHM was used by its original authors to model overland flow hydrographs, develop synthetic unit hydrographs in five test catchments and was compared with runoff hydrographs developed using SCS dimensionless unit hydrographs, hence the tool was suitable and has been previously used for the kind of examination in this research.

DHM on impervious surfaces

The Auburn University research team developed a utility program to construct DHM input files for different watersheds. DHM was tested using measured rainfall and runoff data collected from two lab testing impervious surfaces by Yu and McNown (1964). The slopes of impervious surfaces were 2% (500 ft long watershed) and 0.5% (252 ft long watershed). Simulated discharge hydrographs from DHM agreed well with observed discharge hydrographs for constant and variable rainfall inputs. DHM was then used to simulate discharge hydrographs from the Texas A&M field experiments. Rainfall hyetographs and runoff hydrographs were observed and recorded for 26 rainfall events between April 2009 and March 2010.

Figure 10 is a typical result of the DHM application. The simulated and observed discharges are in reasonable agreement. This result is important because DHM is characterized by watershed
geometry, slope, cover type, and a grid-spacing specification and in that context constitutes a testbed for the more detailed parametric study.

Figure 10: Typical result for DHM model applied to Texas A&M impervious (runway) experimental conditions.

Figure 11 is a plot of cumulative precipitation and discharge. The impact of different outlet boundary conditions and grid spacing are displayed. Based on results presented in Figure 11, the effect of grid spacing is minor as is the effect of outlet boundary condition.

The Nash-Sutcliffe efficiency, $E$, is a statistical measure of goodness of fit used in evaluation of hydrologic models (Legates and McCabe, Jr., 1999). For hydrograph simulations, a good agreement between simulated results and the measured data is considered to be obtained when the $E$ exceeds a value of 0.7 (Bennis and Crobeddu, 2007). The average $E$ for hydrographs simulated using DHM was 0.74 for the 26 rainfall events. Therefore, simulated discharges are considered to have reasonable agreement with observed discharges for the rainfall events recorded on the outdoor concrete surface.

One option in DHM is a kinematic-wave model\(^8\) for the momentum equation. The Auburn team attempted use of the kinematic-wave simplification because it is often used in hydrologic modeling. They learned that the kinematic model would not work at zero slope. However, the diffusion form

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\(^8\)The kinematic-wave equation is a simplification of the momentum equation in which conservation of momentum occurs when the friction slope is equal to the bed topographic slope.
of the momentum equation (Equations 11 and 12) produce results where the topographic-slope of the solution domain is zero. Differences between the kinematic wave and DHM simulated results increased as watershed topographic slope decreased.\footnote{At zero slope, no solution exists for the kinematic-wave equation because depth grows without bound as slope approaches zero.}

A set of numerical experiments was conducted using DHM to estimate the watershed time of concentration $T_c$. The practical determination of such a time from experimental data and numerical modeling is varied (Ben-Zvi, 1984). In this study, $T_c$ is selected as the time when the discharge reaches 98 percent of the equilibrium flow. In the numerical experiments conducted for this study, the slope is varied and the $T_c$ is recorded. Of particular note is that at zero slope $T_c$ is finite (the DHM flow depth is surely reflective of the zero topographic slope, but the depth taper transports water out of the model). Figure 12 is a depiction of one set of numerical experiments.

In other studies (Izzard, 1946; Ben-Zvi, 1984; Su and Fang, 2004b; Wong, 2005), a variety of threshold values of discharge were used to identify the characteristic time ($T_c$). Examples are 80, 95, and 97 percent of the equilibrium flow. In each approach, a particular value was used to accommodate the asymptotic nature of the arithmetic in the model studies.

Figure 11: Typical result for DHM model applied to Texas A&M impervious (runway) experimental conditions, cumulative values. Variable computational grid size is examined in the figure — grid spacing and discharge condition have minor effect in this simulation.
Results from the numerical experiments were used as input data for exploration using regression methods. The objective of the regression exploration was to choose a threshold topographic slope where low-slope (asymptotic) behavior begins. Figure 13 is a depiction of results from the analysis. In Figure 13, the asymptotic value for the simulated system is 100 minutes. The plot of $T_c$ versus slope intersects this value at a slope of about 0.0001. That is, from the numerical experiments, the threshold value for defining “low-slope” is about 0.0001. However, the estimate of the threshold derived from the literature is approximately one order of magnitude greater.

On Figure 13, the regressions were developed based on the assumption that low-slope behavior is exhibited for watersheds slopes of 0.0005 and 0.003, respectively. Differences between the two lines are relatively minor and both intersect the asymptotic value at the same order of magnitude. Therefore, the researchers chose a dimensionless slope of 0.003 as the inception of low-slope behavior (largely based on the literature examination and the departure of the DHM values from a straight line in Figure 13 at that value). Based on these results, the researchers suggest that low-slope behavior is likely for watersheds with slopes less than about 0.003 and should be assumed for those watersheds with slopes less than 0.0005.

DHM was used to generate a suite of estimates of $T_c$ for 90 different combinations of slope, distance, surface conditions, excess precipitation intensity, and outlet size. These values were used to develop Equation 13 to estimate $T_c$ for functionally impervious terrains with topographic slope ranging from zero to 10 percent.
\[ T_c = C n^{0.6175} L^{0.551} i^{0.388} \left( S_{LB} + S \right)^{0.436} \]  

where \( C \) is the outlet factor, \( n \) is Manning’s \( n \), \( L \) is the characteristic length (of either the main channel or overland flow), \( i \) is the rainfall intensity, \( S_{LB} \) is a low slope threshold, and \( S \) is the topographic slope.

Equation 14 is an equation developed in the same analysis for estimating the outlet factor, \( C \), described in Su and Fang (2004b) and appearing in Equation 13. To obtain the outlet factor, critical depth at the watershed outlet was assumed. The researchers varied flow lengths and found that while there was some sensitivity to length, the topographic slope itself had far more impact on the numerical value of the factor.

\[ C = 1.612 - 1.196 \frac{S}{S_{LB} + S} \]  

where \( C \) is the outlet factor, \( S_{LB} \) is a low slope threshold, and \( S \) is topographic slope.

Equations 13 and 14 constitute an alternative model for estimating \( T_c \) in low-slope situations. These equations are developed without regard for functionally impervious versus pervious conditions.

**DHM on pervious surfaces**

The analytical approach used for impervious surfaces was also applied to results from the field and laboratory experiments conducted by Texas A&M personnel (the grass plot and the rainfall simulator studies). Figures 14 and 15 are typical results from DHM application to the grass plot studies. Only a few hydrologic events produced runoff from the grass plots and most of these...
produced little runoff. This was an unanticipated result. The poor agreement of DHM results and field observations is attributed to the low volume of runoff from the grass plots. Because little runoff occurred, obtaining a reasonable estimate of rainfall losses was practically impossible.

Figure 14: Typical incremental result for DHM model applied to Texas A&M grass-plot experimental conditions.

The Auburn team added a Green-Ampt model to DHM to allow estimation of surface infiltration (prior to this addition, losses were external to the program). The modified program was used to reexamine the grass-plot experiments and the rainfall simulator studies.

When DHM was applied to results from the rainfall simulator experiments, which used soil similar to the grass plot studies, results were comparable to the impervious plot studies. Therefore, the validated DHM model and the rainfall simulator results were used to develop parametric studies for low-slope alternative timing models applicable to pervious surfaces.

For development of a dataset for use in developing the alternative timing model for pervious surfaces, soil properties were grouped into three categories — sand, loam, and clay. Each category was examined using DHM and adjusting other variables in the timing model to develop a relation between soil properties, slope, and $T_c$. Ranges in the parameters used in DHM are listed in Table 2.
Saturated hydraulic conductivity \( (K_{sat}) \) and the soil type assigned to the particular range of values are included in Table 2. The remaining infiltration parameters were adjusted to be appropriate for the soil type as per values reported in Rawls and Brakensiek (1982). Including non-pervious components, over 1,000 combinations of roughness (Manning’s \( n \)), slope, rainfall intensity, plot length, saturated hydraulic conductivity, suction pressure, and moisture deficit were examined.

Unlike the result from DHM modeling of watersheds with impermeable surfaces, no single structure model was found. Therefore, a time adjustment (an additive offset to \( T_c \) from the low-slope time response model for impervious surfaces) for each soil type was constructed. The general model structure is comprised of an impervious component and a pervious additive component. Equation 15 is the general structure of the model, with a temporal offset \( (\Delta T_{pervious}) \) computed using Equations 16, 17, or 18,

\[
T_c = 1.82 n^{0.61} L^{0.57} i^{0.39} (S_{LB} + S)^{0.32} + \Delta T_{pervious},
\]  

(15)

where the \( \Delta T_{pervious} \) for the three different soil groups computed from

\[
\Delta T_{pervious-sand} = 82.2 n^{0.09} L^{0.13} K_{sat}^{2.29} \psi^{1.06} D^{0.70},
\]

(16)
Table 2: Parameter values used for pervious surface, low-slope parametric modeling.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Low Value</th>
<th>High Value</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>$S_0$</td>
<td>0.000</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Length (ft)</td>
<td>$L$</td>
<td>65</td>
<td>820</td>
<td></td>
</tr>
<tr>
<td>Hyd. Conductivity (in/hr)</td>
<td>$K_{sat}$</td>
<td>0.01</td>
<td>0.06</td>
<td>Clay Group</td>
</tr>
<tr>
<td>Hyd. Conductivity (in/hr)</td>
<td>$K_{sat}$</td>
<td>0.24</td>
<td>0.50</td>
<td>Loam Group</td>
</tr>
<tr>
<td>Hyd. Conductivity (in/hr)</td>
<td>$K_{sat}$</td>
<td>1.00</td>
<td>5.00</td>
<td>Sand Group</td>
</tr>
<tr>
<td>Manning’s $n$</td>
<td>$n$</td>
<td>0.085</td>
<td>0.80</td>
<td>Appropriate for Soil Type</td>
</tr>
<tr>
<td>Intensity (in/hr)</td>
<td>$i$</td>
<td>0.32</td>
<td>10.0</td>
<td>Varied for Soil Type</td>
</tr>
<tr>
<td>Soil Suction (in)</td>
<td>$\psi$</td>
<td>1.8</td>
<td>12.6</td>
<td>Appropriate for Soil Type</td>
</tr>
<tr>
<td>Moisture Deficit</td>
<td>$D$</td>
<td>0.01</td>
<td>0.45</td>
<td></td>
</tr>
</tbody>
</table>

\[
\Delta T_{\text{pervious-loam}} = 53.6 \frac{n^{0.13} L^{0.22} K_{sat}^{0.83} \psi^{0.58} D^{0.74}}{i^{1.66} (S_{LB} + S)^{0.05}}, \quad \text{or} \quad (17)
\]

\[
\Delta T_{\text{pervious-clay}} = 22.0 \frac{n^{0.21} L^{0.18} K_{sat}^{0.46} \psi^{0.28} D^{0.68}}{i^{1.41} (S_{LB} + S)^{0.19}}. \quad (18)
\]

### 3.3. Summary

Field experiments were conducted to develop measured data to calibrate and validate a hydrodynamic model. Use of a calibrated hydrodynamic model allows geometric and material parameters to be varied to examine the effect of slope on runoff production. These parametric studies were conducted using DHM, which was modified to incorporate a rainfall loss model. The result was a dataset from which a set of regression models were developed to estimate watershed response time for low-slope conditions.

The resulting regression models are appropriate for use where the topographic slope is less than about 0.003. A slope offset is used in the denominator of the resulting equations that has a value of $S_{LB} = 0.0005$ to ensure that the denominator of the timing models does not vanish. Although the value chosen (0.0005) is somewhat arbitrary, its magnitude is supported by both the hydrodynamic modeling of impervious surfaces and published literature.

Two sets of timing parameter (regression) equations were developed. The first is intended for use with watersheds that have functionally impervious surfaces and the second for use with watersheds with pervious surfaces that can be described by soil textural description and Green-Ampt parameters. The mathematical structure of the two sets are similar. The equations are presented to document the research but are not suggested for practical use because of a circular dependence of intensity and time. Instead an adaptation of the existing Kerby-Kirpich approach is suggested and discussed later in the report.
4. Data Acquisition in Low-Slope Watersheds

Figure 16 is a plot of observed time of peak discharge \( (T_p) \) for more than 100 Texas watersheds. Based on prior research, \( T_p \) and \( T_c \) are highly correlated and convey similar information. Data from various studies of Texas watersheds conducted by the Texas Department of Transportation (0–4193, 0–4194, 0–4696) were plotted along with data collected from the Texas A&M field plots and rainfall simulator.

In general, results from the extended research database and those from the Texas A&M laboratory watershed fall near the regression line superimposed on Figure 16. The exception, which was unanticipated by the research team, are results from the two field-scale plots used by the Texas A&M team. The researchers are of the opinion that the unusual behavior of these two watersheds is partly an instrumentation issue. The approach used by A&M personnel was sound, the tools used are certainly state-of-the-practice, yet the results are unanticipated.

The earlier work of Li and others (2005), and Li and Chibber (2008) produced findings similar to those reported herein. That is, the characteristic time of low-slope watersheds was much greater than anticipated. The work reported by Li and others (2005) and Li and Chibber (2008) used an experimental approach and instrumentation similar to that used by the Texas A&M team.
However, based on what was learned by the research team, use of H-flumes and other conventional critical-depth devices is limited for low-slope watersheds. The primary reason is that a low-slope system (by definition) has insufficient drop at the measurement location for the devices to work properly. That is, standard instrumentation requires sufficient energy (head) to develop critical depth. In a low-slope environment, the result is that the “pond” formed upstream from the device results in sufficient storage to affect the measurement. Therefore, the unusual results from the Texas A&M field sites are likely a consequence of the H-flume requirement to be at grade with the watershed itself and subtle, unmeasurable, adverse alignments of the device itself. The results are further complicated because precipitation depths from many events were insufficient to satisfy the hydrologic abstractions on the grass plot for events that occurred during the course of this research project.

Therefore, a component of this research project was to examine alternative data acquisition concepts in anticipation of the needs of future researchers and engineers to estimate discharge in low-slope systems. The likely approach in low-slope systems is to leverage existing culvert (cross drainage) structures and use depth differences across the culvert structure to estimate discharge from a low-slope contributing area. The measurement technologies could be conventional, but the researchers have explored emergent technologies that might enhance understanding of low-slope, shallow-flow behavior.

4.1. Data Acquisition in Low-Slope Watersheds Using Culvert Hydraulics

Task 3B of the project, which is titled “Design of a Low-Slope Data Acquisition Program,” involves research commentary regarding “technical challenges” of monitoring streamflow in low-slope environments. In this section of the report, the researchers address findings related to streamflow monitoring using culvert-based hydraulic computations. The authors suggest that there are two strongly interrelated purposes of culvert-based hydraulic computations: (1) the design of culverts based on discharge criteria with possible addition of either headwater or tailwater criteria, and (2) the forensic estimation of discharge (usually peak discharge) given observed headwater and tailwater data. Infrastructure engineers, such as many TxDOT designers, perform the former; whereas, hydrology monitoring agencies, such as the USGS, perform the later. Readers of this section are expected to be well versed in the basics of culvert hydraulics and the discussion is written with this assumption.

The design of culverts can be based on the HY8 program of the Federal Highway Administration\(^\text{10}\) based on methods described by Normann and others (1985). In contrast, computation of peak discharge based on culvert hydraulics can be completed using the USGS CAP\(^\text{11}\). For the specific needs of TxDOT designers, Charbeneau and others (2002, 2006) provide potential refinements to culvert design in Texas; however, although informative, for the purposes of this section the findings of Charbeneau and others are not directly applicable.

\(^\text{10}\)HY8 is available from http://www.fhwa.dot.gov/engineering/ hydraulics/software/hy8/ at the time of this writing.

\(^\text{11}\)The CAP program is available from http://water.usgs.gov/software/CAP/ at the time of this writing.
The authors acknowledge that either of the previously referenced software programs (and possibly others) can be used for either design or forensic discharge determination. Furthermore, the literature and engineering textbooks abound with refinements. Perhaps the most accessible sources on culvert hydraulics for the purposes of Task 3B of the project are the techniques monograph by Bodhaine (1968) and the CAP user manual (Fulford, 1998).

Recently established USGS streamflow-gaging stations located on several low-slope watersheds in the Texas Panhandle (funded by a cooperative program between TxDOT and the USGS) resulted in the availability of some data. The authors acquired culvert geometry, approach section geometry (often at the edge of right-of-way), and other relevant data from the lead hydrologist on the project (Glenn Harwell, USGS, Fort Worth). Mr. Harwell reported in fall of 2009 considerable difficulty and lack of success in performing computations using the CAP software for several culvert systems having what are believed to be prevailing conditions of low slope. Further evaluation by one of the authors (Asquith) using other computational means yielded the same result.

After conducting research into potential problems with lack of computable results and some heuristic arguments, the authors concluded that a discharge solution is only possible if the velocity head in the approach section was ignored and pure ponding conditions on the inlet prevailed. A blanket assumption of ponding conditions is not philosophically attractive and the need for such an assumption is a problem.

The source of the problem is the small geometric conditions (focus on cross sectional area) at the approach section relative to the flow cross section in the culvert system. The small geometric conditions of the approach appear in small low-sloped watersheds of the Texas High Plains and presumably similar settings because the thalweg or “main channel” (if even definable) can have such small cross sectional area that computationally the approach flow appears supercritical. Because velocity head scales with the square of velocity, the addition of the approach velocity head on the headwater specific energy causes the culvert computations to iteratively grow a discharge to infinity—that is, there is no possibility of numerical convergence and no discharge results.

Occurrence of supercritical flow in an approach section in a low-slope watershed appears inconsistent with the general hydraulic conditions of the setting. So what can be done for the objective of discharge data acquisition in low-slope watersheds using culvert hydraulics? Before providing an answer, the authors acknowledge that a water-surface control point could exist between a surveyed approach section and the culvert inlet in which resulting culvert computations would be unreliable. However, the authors suggest that this should seldom be a problem unless the borrow ditch in the right-of-way is relatively deep near the stream crossing. Assuming therefore that the approach section was surveyed in a reliable location and the hydraulics are sufficiently characterized to make a discharge measurement using culvert hydraulic computations, the authors suggest that three watershed areas (“the subarea triplet”) be determined: (1) the watershed area sourcing to the left borrow ditch and arriving along the borrow ditch, (2) the watershed area sourcing to the right borrow ditch and arriving along the borrow ditch, and (3) the watershed area sourcing the main channel and the approach section.

---

12Heuristic—Computing proceeding to a solution by trial and error or by rules that are only loosely defined, adapted from Dictionary.app from www.apple.com.
Figure 17 is a sketch of the triplet concept. The sketch shows a plan view of a stream crossing running vertical in the sketch (the stream could be quite small). The roadway runs horizontal in the sketch, and a culvert system is depicted in the middle of the sketch. The borrow ditches are the two small streams running parallel to the roadway and intersecting the main stream a short distance from the culvert system. The approach section and inlet section relative positions on the upstream side of the culvert are also shown on the sketch. The implication is that the approach section is located beyond where the borrow ditches drain into the culvert system. The three arrows on the inlet side of the sketch depict the approach flow (running downward in the sketch), and the left and right borrow ditch contributions. The three watershed areas as described above are depicted on the sketch and would be determined by conventional delineation techniques, or by engineering judgement where the topographic relief is too ambiguous.

Watershed areas (1) and (2) provide water perpendicular to the direction of flow through the culvert and hence do not provide additional energy in the form of velocity. The authors suggest that because the watersheds in question are small that the relative timing of the arrival of the flood wave along each of these three areas is reasonably contemporaneous. Therefore, the percent of flow to “route” through the approach section for purposes of computing velocity head at the culvert inlet and frictional losses between the approach and the culvert inlet should be computed as:

\[
\eta = \frac{A_{\text{approach}}}{A_{\text{left}} + A_{\text{right}} + A_{\text{approach}}},
\]

where \(\eta\) is also known as \text{fraction.of.flow}\textsuperscript{13} and is an additional property of the approach section and the three \(A_i\)’s represent the three subareas or conceptual regions of the water that contribute flow to the culvert. The total contributing drainage area of the water is the sum of the three subareas, which are formally described in the previous paragraph.

For example, if about one third of the total watershed area resides in each of the three subareas \((\eta = 0.33)\), then the discharge to use in computation of approach velocity head and frictional losses between the approach and the culvert inlet is \(0.33Q_i\) for \(Q_i\) representing the total discharge through the culvert system for the \(i^{th}\) iteration of the computations. This concept is suggested as a way to reduce the velocity head when computing discharge from stage measurements around such culverts, without resorting to a ponding assumption.

Numerical experimentation with selected culverts from the current (2010) USGS and TxDOT streamflow-gaging program in Texas demonstrates that approach flow can be held to subcritical and discharge convergence will occur for values of \(\eta\) that seem well within logical application of engineering judgement \((\eta \approx 0.3–0.8)\). Readers might grasp that the \(\eta\) correction also can be thought of as a correction for a channel or approach section that is skewed to the roadway. In essence, \(\eta\) behaves similarly to a vector projection or cosine correction. However, the uniqueness of the discussion in this section is that such a correction might be needed for low-slope watersheds although geometrically the main channel is not skewed to the flow axis through the culvert system. In conclusion, the use of the triplet subarea concept provides an objective method to apply to culvert

\textsuperscript{13}The authors show \text{fraction.of.flow} as a “variable” because one of the authors (Asquith) wrote a culvert analysis program using the R programming language in which such a variable is used, thereby implementing results from this research project.
hydraulic computations to accommodate more realistic hydraulic conditions when gaging small low-sloped watersheds using the principles of culvert flow.

4.2. Emergent Technologies

Culvert hydraulics and adaptations of computational methods previously presented in this research report stimulated some emergent technologies that will be of value in future low-slope environments. USGS researchers considered several technical modifications to typical streamflow-gaging techniques for monitoring low-slope hydraulics. Research into video capture technology with frame-by-frame velocimetry was initiated. Energy losses are potentially small over “gageable” reach lengths. Therefore, gage-height monitoring to better than 0.01 feet (the USGS standard) likely is required. Researchers conceptually developed variations on laser-based gage-height recording within comparatively short “stilling wells” with reflective floats. Lasers promise small footprints and greater and more reliable accuracy than pressure transducers. (Experience in recent years is showing that small footprint autonomous pressure loggers have difficulty operating in situations in which extended dry periods
are followed by submersion — data record can exhibit various degrees of unreliability.)

Further, out of concerns for differential movement of the ground surface during wet-up and dry-down periods in clay-dominated low-slope environments, lateral lasers offer a mean to continuous monitor differential movement of two or more stage sensing locations. Laser range-finding is a venerable technology but with limited adoption in surface-water hydrology. Many manufacturers exist and most “units” use serial communication. The USGS infrastructure is heavily invested, but not uniquely dependent, on the SDI-12 (serial digital interface) protocol. One or more manufacturers build inexpensive serial (RS232 interface) to SDI-12 converters. However, the promise of using the laser at high sampling rates likely will overwhelm the speed of the SDI-12 sensor bus and direct programming communication with the RS232 interface is required. Recent discussions with manufacturers conducted by USGS researchers indicates that conventional data loggers can be programmed to communicate with various laser-based systems.

The research team successfully experimented with laser technology, but was unable to examine its utility outside the research laboratory for this project. Software was developed to interpret results and was incorporated into the culvert hydraulics already discussed.

An imaging approach that uses low-power infrared LED arrays was discovered late in this project and is currently under study external to this project. This technology is one that could potentially leverage existing camera systems and is as promising as laser-based systems and anticipated to be lower cost.
5. Application

There are two principal findings from the research conducted for this project. The first is identification of the topographic slope at which “low-slope” effects are anticipated. The second is a hydrologic timing model for consideration for low-slope conditions. The purpose of this chapter is to provide guidance for application of the findings of this research project. A secondary purpose is to provide a few examples of application of the research results.

5.1. Guidance for Application of Research Results

The approaches for timing estimation on small slopes, runoff generation, and other calculations of interest explored in this research are:

1. Application of conventional techniques, but using a small offset to the topographic slope — the researchers suggest the offset should be 0.0005 (0.05 percent added slope). Such an approach is reasonably well justified from the impervious field site and its corresponding numerical model simulation. In the simulation the response time asymptotically approached a constant at a dimensionless slope of 0.0001–0.0005 (Figure 13). This is the simplest adaptation of existing techniques to low-slope conditions and is the suggested approach for low-slope conditions for hydrologic modeling. An illustration is provided later in this section of the report.

2. Application of the pervious surface model as described by Equations 15, 16, 17, and 18 for cases where small distances are involved over a pervious surface. Confounding the computations in this model is that the rainfall intensity (and hence some sense of duration) is part of the model. Therefore the result is implicit and the analyst is presented with a circular problem of needing to specify time to compute an intensity, then needing intensity to compute a time. The solution requires iteration but yields both time and intensity.

   The intensity appears in the denominator and the motivation of the study was to keep time from approaching huge values, so the researchers suggest that the smallest intensity that produced runoff (in their experiments) is a reasonable surrogate lower bound that provides an initial upper bound watershed response time for hydrologic modeling. The values from the parametric study that represent the lowest runoff producing intensities for sand, loam, and clay soil textures are 1.5, 0.32, and 0.2 inches per hour, respectively. This model combines a loss model with the timing estimation, and while such a situation is acknowledged to be realistic, the separation of slope effects from soil effects was not possible.

   The impervious surface model as described by Equation 13 is appropriate for cases where small distances are involved over a functionally impervious surface. It is a subset of the pervious models and has a similar issue with regards to intensity being part of the timing estimate.

For practical use the researchers suggest the adaptation of the conventional approach with a slope offset. The offset is suggested to be applied at dimensionless slopes less than 0.003 and the offset value is \( S_{LB} = 0.0005 \). The choice of when to use the alternate occurs at a slope where the asymptotic timing behavior becomes apparent in Figure 13. The offset value is chosen as described earlier.
5.2. Use of the Slope Offset for Estimation of Time of Concentration in Texas for Low-Slope Conditions

5.2.1. Introduction

Formal guidance for estimation of time of concentration (critical storm duration) for watersheds characterized as having limited topographic slope is provided in this supplement. Limited topographic slope watersheds are those watersheds having main-channel slopes less than about 0.05 percent or 0.0005 dimensionless. Main-channel slope $S_c$ is computed as the change in elevation from the watershed divide to the watershed outlet divided by the curvilinear distance of the main channel (primary flow path) between the watershed divide and the outlet.

The authors emphasize that there is ambiguity in the meaning of low-slope watersheds and the primal objective of this project was for objective mitigation for conditions in which the engineer computes $S \rightarrow 0$ (slope effectively vanishing) with the result that traditional methods for estimating watershed response time produce unreasonable results. The guidance described in this supplement is an adaptation and extension of the supplement in Roussel and others (2005, pp. 33–34). That supplement is extended here to watersheds with limited topographic slope. Such watersheds are predominant in the High Plains and Coastal Regions of Texas.

Roussel and others (2005) concluded that the watershed time estimation methods of Kerby (1959) and Kirpich (1940) should be considered for general application in Texas. These methods are combined as “the Kerby-Kirpich approach” for estimating watershed time of concentration $T_c$. The Kerby-Kirpich approach requires comparatively few input parameters, is straightforward to apply, and produces readily interpretable results. The Kerby-Kirpich approach produces $T_c$ estimates consistent with watershed time values independently derived from real-world storms and runoff hydrographs. Application of the Kerby-Kirpich is demonstrated in the context of a low-slope watershed in this supplement.

5.2.2. The Modified Kerby Method

For small watersheds where overland flow is an important component of overall travel time, the Kerby (1959) method modified for a base slope can be used. The modified Kerby equation is

$$T_c = K(LN)^{0.467}(S_{LB} + S_o)^{-0.235},$$

where $T_c$ is overland flow time of concentration, in minutes; $K$ is a units conversion coefficient, in which $K = 0.828$ for traditional units and $K = 1.44$ for SI units; $L$ is an overland-flow length, in feet or meters as dictated by $K$; $N$ is a dimensionless retardance coefficient; $S_{LB}$ is a base slope value (offset) that provides mitigation for vanishing topographic slope $S_o$; and $S_o$ is dimensionless slope of terrain conveying the overland flow. In the development of the Kerby equation, the length of overland flow was as much as about 1,200 feet (366 meters). Hence, this length is considered an upper limit and shorter values in practice generally are expected. The dimensionless retardance coefficient used is similar in concept to the well-known Manning’s roughness coefficient. However, for
Table 3: Typical values of retardance coefficient \( N \) of Kerby (1959) method for overland flow

<table>
<thead>
<tr>
<th>Generalized terrain description</th>
<th>Retardance coefficient (( N ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pavement</td>
<td>0.02</td>
</tr>
<tr>
<td>Smooth, bare, packed soil</td>
<td>0.10</td>
</tr>
<tr>
<td>Poor grass, cultivated row crops, or moderately rough packed surfaces</td>
<td>0.20</td>
</tr>
<tr>
<td>Pasture, average grass</td>
<td>0.40</td>
</tr>
<tr>
<td>Deciduous forest</td>
<td>0.60</td>
</tr>
<tr>
<td>Dense grass, coniferous forest, or deciduous forest with deep litter</td>
<td>0.80</td>
</tr>
</tbody>
</table>

For a given type of surface, the retardance coefficient for overland flow will be considerably larger than \( n \) for open-channel flow. Typical values for the retardance coefficient are listed in Table 3.

5.2.3. The Modified Kirpich Method

For channel-flow component of runoff, the modified Kirpich (1940) equation is

\[
T_c = KL^{0.770}(S_{LB} + S_c)^{-0.385}, \tag{21}
\]

where \( T_c \) is the time of concentration in minutes, \( K \) is a units conversion coefficient, in which \( K = 0.0078 \) for traditional units and \( K = 0.0195 \) for SI units, \( L \) is channel-flow length in feet or meters as dictated by \( K \), \( S_{LB} \) is a base slope value (offset) that provides mitigation for vanishing slope \( S_c \), and \( S_c \) is dimensionless main-channel slope as determined in Roussel and others (2005).

5.2.4. Example Application

An example application of the Kerby-Kirpich method for a watershed is informative. A schematic of the longitudinal profile and other properties of the example watershed is shown in Figure 18. For this example, suppose a hydraulic design is needed to convey runoff from a watershed with a drainage area of 0.5 square miles. On the basis of field examination and topographic maps, the length of the main channel from the watershed outlet (the design point) to the watershed divide is 5,280 feet. Elevation of the watershed at the outlet is about 3,400 ft. From a topographic map, elevation along the main channel at the watershed divide is estimated to be 3,401.1 feet. The analyst assumes that overland flow will have an appreciable contribution to the time of concentration for the watershed. The analyst estimates that the length of overland flow is about 500 ft. The area representing overland flow is average grass (\( N = 0.40 \)). The slope for the overland-flow component is 0.03 percent (\( S_o = 0.0003 \)).
5.2.5. Time of overland flow

For the overland-flow $T_c$, the analyst applies the Kerby equation

$$T_c = 0.828(500 \times 0.40)^{0.467} (0.0003)^{-0.235},$$

(22)

from which $T_c$ is about 73 minutes. In the analyst’s professional opinion, this result might be greater than what was expected. The overland flow slope is not very large, particularly when compared with a value typical of many applications (about 0.01). Therefore, the analyst believes a low-slope setting is possible and decides to reevaluate the overland flow $T_C$ given a low-slope assumption.

For the overland-flow $T_c$, the analyst applies the modified Kerby equation with a base slope suggested by the authors that represents a transition from gravitationally dominated flow to differential depth driven flow. Using the base slope of $S_{LB} = 0.0005$, the modified Kerby equation is

$$T_c = 0.828(500 \times 0.40)^{0.467}(0.0005 + 0.0003)^{-0.235},$$

(23)

from which $T_c$ is about 53 minutes, which is about 20 minutes less than the previous computation.

5.2.6. Time of main channel flow

For the channel $T_c$, the analyst applies the Kirpich equation, but first dimensionless main-channel slope is required

$$S = \frac{3401.1 - 3400}{5280} = 0.00021$$

(24)

or about 0.021 percent. For this application, the overland flow length is subtracted from the main channel length\(^\text{14}\), such that $L = 5280 - 500 = 4780$ ft. The value for slope and the channel length

\(^{14}\)Depending on the particular watershed, it is not always the case that the characteristic overland flow length for estimating the overland flow $T_c$ is colinear with the main channel. In some cases, the characteristic overland flow component is not aligned with the main channel. One of the authors (Thompson) chooses several candidate locations on a watershed to estimate the characteristic (representative) overland flow $T_c$, estimates $T_c$ for those locations, and then chooses a reasonable value.
are used in the Kirpich equation
\begin{equation}
T_c = 0.0078(5280 - 500)^{0.770}(0.00021)^{-0.385},
\end{equation}
from which \(T_c\) is about 138 minutes. Similar to the result from the overland flow analysis, the analyst believes a low-slope condition has been encountered and decides to reevaluate the main channel \(T_c\) under that assumption.

For the channel \(T_c\), the analyst applies the modified Kirpich equation with a base slope suggested by the authors that represents a transition from gravitationally dominated flow to differential depth driven flow. Using the base slope of \(S_{LB} = 0.0005\), the modified Kirpich equation is
\begin{equation}
T_c = 0.0078(5280 - 500)^{0.770}(0.0005 + 0.00021)^{-0.385},
\end{equation}
from which \(T_c\) is about 87 minutes, which is 51 minutes shorter than the previous computation.

5.2.7. Consideration of the watershed time values

The values for watershed \(T_c\) are:
- Adding the overland flow and channel flow components of \(T_c\) from the Kerby-Kirpich approach gives a watershed \(T_c\) of about 211 minutes \((73 + 138)\).
- Adding the overland flow and channel flow components of \(T_c\) from the modified Kerby-Kirpich approach gives a watershed \(T_c\) of about 140 minutes \((53 + 87)\).

For design purposes, the \(T_c = 140\) minutes is preferred. The \(T_c = 211\) minutes is too long.

Finally, as a quick check, the analyst can evaluate the \(T_c\) by using an ad hoc method representing \(T_c\), in hours, as the square root of drainage area, in square miles. For the example, the square root of the drainage area yields a \(T_c\) estimate of about 0.71 hour or about 42 minutes, which is expected to be considerably less than the previous two estimates of watershed \(T_c\) because the estimate is based on community engineering experience in Texas with normal (not low-slope) watersheds.
6. Summary

Methods currently used to estimate the timing parameter for a watershed contain some form of watershed slope as a principal parameter. That time response (travel time or time of concentration) is usually inversely proportional to topographic slope.

The research team identified, from literature, data, modeling, and physical experiments, the dimensionless topographic slope where alternate hydrologic approaches should be considered. The dimensionless slope suggested for consideration of alternate approaches is 0.003.

The project team examined several alternate approaches from parametric studies using the Diffusion Hydrodynamic Model (DHM). Inclusion of a rainfall loss model in DHM was required to address results from field experiments and rainfall simulator experiments. Initial results retained rainfall intensity as part of the timing estimate creating an implicit estimation problem. A simple slope offset was also explored and is suggested as an adaptation of current technology. The offset value suggested is 0.0005 and an illustrative example is presented in this report.

The project team developed a research plan for more intensive study of low-slope hydrology should further work and supporting data be needed. The principal findings are that current streamgaging techniques are not well suited to low-slope hydrologic studies and different measuring methods are suggested. Several promising emerging technologies are presented as worthy of further exploration.

6.1. Recommended Future Research

The parametric studies combined rainfall loss issues with timing issues. While watershed loss has been studied before in Texas (and elsewhere) the effect in low-slope conditions is poorly understood in both a scientific and an engineering sense. Therefore, the integrated loss/low-slope hydraulics is an area worthy of further study. Low-slope hydraulics is amenable to existing tools (HEC-RAS, SWMM, etc.) but what fraction of the incoming precipitation actually becomes discharge is still poorly understood. The initial-abstraction, constant loss-rate approach (Asquith and Roussel, 2007) should be re-examined for applicability in low-slope conditions in the context as a tool for use in integrated hydrology/hydraulics modeling.

The research teams noticed something unusual about low-slope response based on results from the field plots. It is the researchers’ opinion that the problem is probably an instrumentation issue, but increased rainfall losses because of relatively long detention time is relevant. The researchers suggest the following list of future research topics to consider in low-slope as need and funding emerge.

1. Identify and instrument a comparatively large “low-slope” watershed near transportation infrastructure that is drained by culverts or similar structures. The instrumentation should include the emerging technologies described in this report in parallel with more conventional technologies to explore both the instrumentation issue and the timing/loss issue at a meaningful

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15The choice of “hydraulics” is intentional!
physical scale. The watersheds should be selected in several parts of the state — at least the coastal plains and the Panhandle region. The goal of these studies would be to further refine the timing model suggested in this research or discover a way to directly model hydraulics and loss behavior.

2. Develop documentation for how to model low-slope watersheds using existing tools. A particular question is how to manage the integrated loss and hydraulics components in a practical sense with the goal to develop a guidance document (or a modeler-training manual).

3. The parametric studies required unusual values of Manning’s $n$ to produce results the researchers believed to be meaningful. This finding is not reported in detail, but occurs elsewhere in the literature. A study to determine if the unusual values arise because of the low topographic slope or because of the relatively shallow flows encountered in the experiments is worth pursuing.

4. The effect of slope on flow direction is mentioned in this report, but this research did not attempt to address the direction effect. A study of how to establish meaningful potential flow directions in low-slope regimes would be a worthy future research topic. A caveat is that in certain regimes, direction could change with inundation so this research would be necessarily multi-dimensional model dependent.
References


Charbeneau, R., A. Henderson, R. Murdock, and L. Sherman (2002). Hydraulics of channel expansion leading to low-head culverts. Texas department of transportation research report FHWA/TX–03–2109–1, Center for Transportation Research, University of Texas, Austin, Texas.


