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From: Nick Dolan & Jacob Alford, Students  
Date: April 24, 2007  
Subject: Lab 6 Tech Memo

The purpose of this technical memo, is to explore 2 different methods of calculating the spring constant of a spring. One such method involves hanging weights of varying mass from the spring, and measuring the vertical displacement of the string, while the other method involves measuring the oscillation period of a spring when a weight is oscillating on the end of it. After the spring constant is calculated in both manners, we will then explore the nature of the effective spring constant, when the two springs are attached end to end.

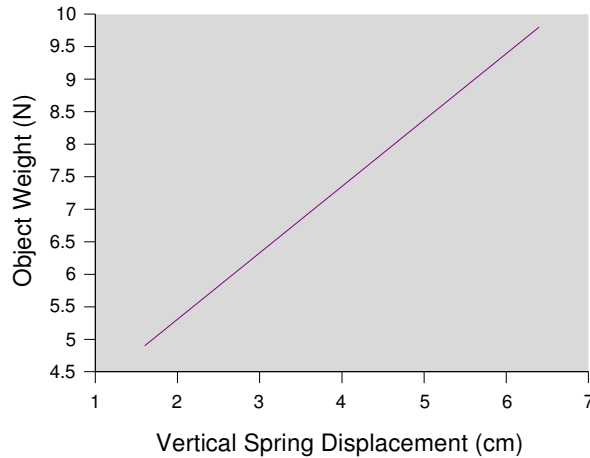
## Predictions

The two mathematical methods for calculating the spring constant of a spring will now be explained. The first method involves hanging weights of varying mass from the end of the string and measuring vertical displacement. With springs, we know Hooke's Law, which states  $F = k \cdot x$ . Since the only force acting on the spring in these cases is the weight pulling it downward, we can substitute  $m \cdot g$  for  $F$ , making the equation  $m \cdot g = k \cdot x$ . Now, we simply solve for  $k$  to get an expression for the first method of solving for spring constants,  $k = (m \cdot g) / x$ . The next method involves knowing the oscillation period of the spring, as a weight of known mass oscillates attached to the bottom of the spring. It is known that  $\omega = (2\pi) / T$ , where  $T$  is the oscillation period of the oscillating object. It is also known that  $\omega = \sqrt{k/m}$ , where  $k$  is the spring constant, and  $m$  is the mass. Setting these two expressions for omega equal, we get  $(2\pi) / T = \sqrt{k/m}$ . Now, we simply solve for  $k$  to get the expression for the second method of solving for the spring constant using oscillation periods,  $k = (4\pi^2 \cdot m) / (T^2)$ . If one wanted to calculate the spring constant for two springs attached together, they would use  $1/k_{\text{total}} = 1/k_1 + 1/k_2$ .

## Analysis

Our group attached a spring to the end of a hanging string, and hung weights of varying masses from the end of this spring. We measured both the unstretched length of the spring, and the length of the spring when each mass was hung from it, and subtracted these values to find the total displacement of the spring when the mass was hung from it. The springs were 9.4 cm long when they were unstretched, and when a .5 kg mass was hung from the end, there was 1.6 cm of total displacement, while when a 1 kg mass was hung from the end, there was 6.4 cm of total displacement. Using this data, we can make a simple linear graph of vertical displacement vs. hanging mass.

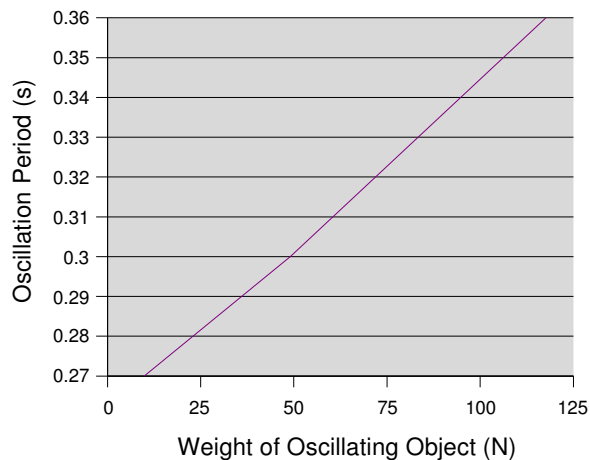
### Weight vs. Spring Displacement



Converting cm to m, and solving for k using the expression for the first method ( $k=mg/x$ ), we find k to be 306.5 with the 4.9 N weight, and 153.1 with the 9.8 N weight. If we calculate k using simply the slope of the above graph, we get k to be 102.1. There is a lot of uncertainty with these methods, as we only measured the displacement of 2 different weights, so our sample size was very small.

Next, our group attached weights of varying mass to the end of springs suspended from a string, and made the weights oscillate, as we measured the period of oscillation for each weight. Setting a 1 kg mass to oscillate, we found the period of oscillation to be approximately .27 s, with a 5 kg mass, the period was found to be .30 s, and with a 12 kg mass, the calculated period was .36 s. Making a graph of oscillation period, to object weight we get the following:

### Object Weight vs. Oscillation Period



If we use the expression for the second method of calculating a spring constant described in the Predictions section, we find that k for the 1 kg mass is 541.5, k for the 5 kg mass is 2193.2, and k for the 12 kg mass is 3655.4. If we calculate k using simply the slope of the graph, we find k to be 122.2. The extreme variance in the calculated k's suggests a data sample size that is too small, and measurements that are not accurate enough.

Next, our group attached the two springs together, and made the same measurements before, except now the two springs were attached to the suspended string, and hanging end to end. Unstretched the two springs together were 18.8 cm. A hanging mass of .5 kg caused a displacement of about .7 cm, while a hanging mass of 1 kg caused a spring displacement of about 1.7 cm. The  $k$  calculated for the .5 kg mass is about 700, while the  $k$  calculated for the 1 kg mass turns out to be about 576.5. Again, there is a large variance here which suggests that our sample data size is too small, or our measurements are not accurate enough. If we calculated the spring constant of the attached springs using the method described in predictions, and the average  $k$  values of the springs reported above (which can be given little to no credit due to the wild variance in  $k$ 's calculated with different methods), we get  $k$  to be 505.3, which is close to some of the calculated values, but still offers little confidence, and could just be a coincidence.

## **Conclusion**

Our group calculated the  $k$  for a spring, in several different ways. Each way gave wildly different values for  $k$ , which suggests some sort of mistake in recording data, a sample data size that was too small, or extremely inaccurate measurements. All 3 problems could have been present. Since we only measured the displacement of 2 different masses with the first method, and the oscillation of 3 different masses with the second method, our range of data was very small, and any inaccuracies would have made a huge impact, especially given the diminutive nature of the displacements and periods recorded. It is hard to draw any real conclusions from this experiment, other than saying our lab group needs to be more accurate and careful while performing the experiment, and we need to use a larger sample data size (test more masses).