Week-in-Review #2

1) Let $A = \begin{bmatrix} 4 & x \\ 3 & -7 \end{bmatrix}$ 2x2, $B = \begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix}$ 2x3, $C = \begin{bmatrix} 4 & -5 \\ 0 & b \\ 7 & -10 \end{bmatrix}$ 3x2,

and $D = \begin{bmatrix} 5 & -1 & 3a \\ 2 & 6 & 4 \end{bmatrix}$ 2x3. Compute each of the following.

(a) $B + 3D$

2x3 + 2x3

$$= \begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix} + 3\begin{bmatrix} 5 & -1 & 3a \\ 2 & 6 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix} + \begin{bmatrix} 15 & -3 & 9a \\ 6 & 18 & 12 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 17 & -6 & 9a \\ 14 & k+18 & 6 \end{bmatrix}}$$

(b) $2C + B$

3x2 + 2x3

We cannot add 2C and B because they are not the same size.

(c) $4D - 3C^T$

2x3 − 2x3

$$= 4\begin{bmatrix} 5 & -1 & 3a \\ 2 & 6 & 4 \end{bmatrix} - 3\begin{bmatrix} 4 & 0 & 7 \\ -5 & b & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & -4 & 12a \\ 8 & 24 & 16 \end{bmatrix} + \begin{bmatrix} -12 & 0 & -21 \\ 15 & -3b & 30 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 8 & -4 & 12a-21 \\ 23 & 24-3b & 46 \end{bmatrix}}$$

(d) $4a_{21} - 2c_{32} + 7d_{13}$

$$= 4(3) - 2(-10) + 7(3a)$$

$$= 12 + 20 + 21a$$

$$= \boxed{32 + 21a}$$

(e) DB ← not possible because the # of columns of D does not equal the # of rows of B.

Size of D	Size of B
2×3	2×3

$\neq$

(f) B^TDA not possible because the # of columns of D ≠ to the # of rows of A.

Size of B^T	Size of D	Size of A
3×2	2×3	(2×2)

$=$ $\neq$

(3×3) $\neq$

(g) CD^T not possible

Size of C	Size of D^T
3×2	3×2

$\neq$

(h) BB^T

Size of B | Size of B^T

2×3 ✓ 3×2

=

BB^T is 2×2

$$C = BB^T = \begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix}\begin{bmatrix} 2 & 8 \\ -3 & k \\ 0 & -6 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 13 & 16-3k \\ 16-3k & k^2+100 \end{bmatrix}$$

c_{11} = row 1 of B * col 1 of B^T

$c_{11} = 4+9+0 = 13$

c_{12} = row 1 of B * col 2 of B^T

$= 16-3k+0$

$= 16-3k$

$c_{21} = 16-3k+0$

$c_{22} = 64+k^2+36 = k^2+100$

(i) $A^2 = A \cdot A$

Size of A | Size of A

2×2 ✓ 2×2

=

2×2

$$A\cdot A = \begin{bmatrix} 4 & x \\ 3 & -7 \end{bmatrix} * \begin{bmatrix} 4 & x \\ 3 & -7 \end{bmatrix} = \begin{bmatrix} 16+3x & -3x \\ -9 & 3x+49 \end{bmatrix}$$

$4x-7x$

$12-21$

$AB \neq BA$

2) Using matrix algebra, solve for the matrix D:

$$D = AD + B$$

$\frac{3}{2}$ $\frac{2}{3}$ $\times$ $\frac{3}{2}5$

$$D - AD = B$$

$$(I_n - A)^{-1}(I_n - A)D = (I_n - A)^{-1}B$$

$$I_n D = (I_n - A)^{-1}B$$

$$\boxed{D = (I_n - A)^{-1}B}$$

3) Solve for x and y:

$$3\begin{bmatrix} 2 & x \\ 5y & -1 \end{bmatrix} - \begin{bmatrix} -6 & 1 \\ 3y & -5 \end{bmatrix}^T = \begin{bmatrix} 12 & -7 \\ -2x & 2 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 3x \\ 15y & -3 \end{bmatrix} - \begin{bmatrix} -6 & 3y \\ 1 & -5 \end{bmatrix} = \begin{bmatrix} 12 & -7 \\ -2x & 2 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 3x-3y \\ 15y-1 & 2 \end{bmatrix} = \begin{bmatrix} 12 & -7 \\ -2x & 2 \end{bmatrix}$$

$$3x-3y=-7 \qquad \longrightarrow \qquad 3x-3y=-7$$
$$15y-1=-2x \qquad\qquad 2x+15y=1$$

$$\begin{array}{cc} x & y \\ \end{array}$$
$$\left[\begin{array}{cc|c} 3 & -3 & -7 \\ 2 & 15 & 1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 1/3 \end{array}\right]$$

$$\boxed{\begin{aligned} x &= -2 \\ y &= 1/3 \end{aligned}}$$

4) The times (in minutes) required for assembling, testing, and packaging large and small capacity food processors are shown in the following table:

	Assembling	Testing	Packaging
Large	45	15	10
Small	30	10	5

(a) Define a matrix T that summarizes the data above.

$$T = \begin{bmatrix} 45 & 15 & 10 \\ 30 & 10 & 5 \end{bmatrix}$$

(b) Let $M = [100 \quad 200]$ represent the number of large and small food processors ordered, respectively. Find MT and explain the meaning of its entries.

$$MT = [100 \quad 200] \begin{bmatrix} 45 & 15 & 10 \\ 30 & 10 & 5 \end{bmatrix}$$

$1 \times 2 \quad 2 \times 3$

1×3

$$= [100 \cdot 45 + 200 \cdot 30 \qquad 100 \cdot 15 + 200 \cdot 10 \qquad 100 \cdot 10 + 200 \cdot 5]$$

$$= \boxed{[10500 \quad 3500 \quad 2000]}$$

The total # of minutes required for assembling all large + small food processors.

The total # of testing minutes

Total # of minutes for packaging the large + small processors

(c) If assembling costs \$3 per minute, testing costs \$1 per minute, and packaging costs \$2 per minute, find a matrix C that, when multiplied with T, gives the total cost for making each size of food processor.

$$C = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

To make a large processor

$45*3 + 15*1 + 10*2$

$$T = \begin{bmatrix} 45 & 15 & 10 \\ 30 & 10 & 5 \end{bmatrix}$$

small

$30*3 + 10\times1 + 5*2$

$$\boxed{TC}$$

5) If $A = \begin{bmatrix} -3 & 7 \\ 8 & 10 \end{bmatrix}$, find A^{-1}.

$$A^{-1} = \begin{bmatrix} -5/43 & 7/86 \\ 4/43 & 3/86 \end{bmatrix}$$

6) If $B = \begin{bmatrix} -3 & 5 \\ 6 & -10 \end{bmatrix}$, find B^{-1}.

B has no inverse.

B is singular.

7) Solve the following system of equations using matrix inverses.

$$3x+2y=z+2$$
$$-3y+2=-2x$$
$$x=y+z+4$$

Use the matrix equation $AX=B$.

$$\left.\begin{array}{l} 3x+2y-z=2 \\ 2x-3y=-2 \\ x-y-z=4 \end{array}\right\}$$ equivalent to $AX=B$

$$A=\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 3 & 2 & -1 \\ 2 & -3 & 0 \\ 1 & -1 & -1 \end{bmatrix} \end{array}, \quad X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B=\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$$

(coefficient matrix) variable column constants column

$$A^{-1}AX=A^{-1}B$$

$$X=A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}=X=\begin{bmatrix} -1 \\ 0 \\ -5 \end{bmatrix}$$

$x=-1$
$y=0$
$z=-5$

8. steel, electronics

City demands $\begin{bmatrix} 500 \\ 800 \end{bmatrix}$ units of steel
- - - electronics.

How many units of steel and electronics products should be produced by the village to meet its own needs and those of the nearby city?

Let x_1 = the number of units of steel that should be produced.

Let x_2 = - - - - - - - - electronics -.

Input-output matrix A:

$$\begin{array}{cc|cc} & & \text{output (to)} & \\ & & \text{steel} & \text{elec.} \\ \text{input} & \text{steel} & .02 & .1 \\ \text{(from)} & \text{electronics} & .15 & .01 \end{array} = A$$

Find the demand matrix D:

$$D = \begin{bmatrix} 500 \\ 800 \end{bmatrix}$$

Let $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$X = (I_2 - A)^{-1} D$$

$$X = \begin{bmatrix} 601.9682 \\ 899.2881 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$x_1 = 601.9682$ units of steel should be produced

$x_2 = 899.2881$ units of electronics should be produced.

9) crude petroleum (crude)
petroleum refining & related industries (refining)
chemical production (chemical)

		to		
		Crude	Refining	chemical
from	Crude	0.31	.42	.05
	Refining	.0086	.11	.13
	Chemical	.01	.47	.38

(a) input-output matrix A is

$$A = \begin{bmatrix} .31 & .42 & .05 \\ .0086 & .11 & .13 \\ .01 & .47 & .38 \end{bmatrix}$$

(b) Explain the meaning of the entries in row 1 of this matrix.

$a_{11} = .31$ = the # of units of crude used in producing one unit of crude product.

$a_{12} = .42$ = the # of units of crude used in producing one unit of refining product.

$a_{13} = .05$ = the # of units of crude used in producing one unit of chemical product.

(c) How many units of refining products are consumed in the production of 7500 units of crude product?

$(.0086) * 7500 =$ $\boxed{\text{64.5 units of refining product}}$

(d) How many units of chemical products are required to produce 500 units of each sector in this economy?

$.01 * 500 + .47 * 500 + .38 * 500$

$= (.01 + .47 + .38) 500$

$= \boxed{\text{430 units of chemical}}$

(e) If a neighbouring city demands 5500 units of crude, 6,750 units of refining, and 1,250 units of chemical products, how much should this economy produce to satisfy internal consumption and meet the demands of the city?

Let x_1 = the # of units of crude product that should be produced

Let x_2 = - - - - refining - - - -

Let x_3 = - - - - chemical - - - -

Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $D = \begin{bmatrix} 5500 \\ 6750 \\ 1250 \end{bmatrix}$.

$$X = (I - A)^{-1} D$$

$$X = \begin{bmatrix} 14140.0671 \\ 9,050.9006 \\ 9,105.3612 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The economy should produce 14,140.0671 units of crude product, 9050.9006 units of refining product, and 9,105.3612 units of chemical product.

(f) Referring to (e), how many units of each product are consumed internally in meeting the other city's demands?

Total production − demand = internal consumption.

$$\begin{bmatrix} 14140.0671 \\ 9,050.9006 \\ 9,105.3612 \end{bmatrix} - \begin{bmatrix} 5500 \\ 6750 \\ 1250 \end{bmatrix} = \begin{bmatrix} 8640.0671 \\ 2300.9006 \\ 7855.3612 \end{bmatrix}$$

8640.0671 units of crude product, 2300.9006 units of refining product, and 7855.3612 units of chemical products are used internally.

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