Math 166 - Week in Review #2

Section 2.4 - Matrices

- The size of a matrix is always *number of rows* × *number of columns*.

- c_{ij} represents the entry of the matrix C in row i and column j.

- To add and subtract matrices, they must be the same size.

- When adding or subtracting matrices, add or subtract corresponding entries.

- A scalar product is computed by multiplying each entry of a matrix by a scalar (a number).

- Transpose - The rows of the matrix A become the columns of A^T.

Section 2.5 - Multiplication of Matrices

- The matrix product AB can be computed only if the number of columns of A equals the number of rows of B.

- If $C = AB$, then c_{ij} is computed by multiplying the ith row of A by the jth column of B.

- Identity Matrix - Denoted by I_n, the identity matrix is the $n \times n$ matrix with 1's down the main diagonal (from upper left corner to lower right corner) and 0's for all other entries.

- If A is $m \times n$, then $AI_n = A$ and $I_m A = A$.

Section 2.6 - The Inverse of a Square Matrix

- Only square matrices can have inverses, *but not all square matrices have inverses.*

- A square matrix that does not have an inverse is called a **singular matrix**.

- The inverse of A, denoted A^{-1}, is the square $n \times n$ matrix such that $AA^{-1} = A^{-1}A = I_n$.

- Systems of equations can be represented as a matrix equation of the form $AX = B$ where A is the coefficient matrix, X is a column vector containing the variables, and B is a column vector containing constants.

- If A has an inverse, the solution to the matrix equation is $X = A^{-1}B$.

- If A does not have an inverse (i.e., if A is singular), this does NOT imply the system has no solution. It simply means that you must use another method to solve the system.

Section 2.7 - Leontief Input-Output Model

- The input-output model is used to study the relationship between industrial production and consumer demand.

- If A is the input-output matrix and X represents the total output of all industries, then AX represents the internal consumption for that economy.

- Subtracting the internal consumption from the total output gives the net output of goods and services for consumer demand, D:
$$X - AX = D$$

- To find the amount of goods and services that must be produced to satisfy consumer demand, solve for X in the above equation to obtain
$$X = (I - A)^{-1}D$$

1. Let $A = \begin{bmatrix} 4 & x \\ 3 & -7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix}$, $C = \begin{bmatrix} 4 & -5 \\ 0 & b \\ 7 & -10 \end{bmatrix}$, and $D = \begin{bmatrix} 5 & -1 & 3a \\ 2 & 6 & 4 \end{bmatrix}$. Compute each of the following:

   (a) $B + 3D$

   (b) $2C + B$

   (c) $4D - 3C^T$

   (d) $4a_{21} - 2c_{32} + 7d_{13}$

   (e) DB

   (f) $B^T DA$

   (g) CD^T

   (h) BB^T

   (i) A^2

2. Using matrix algebra, solve for the matrix D:
$$D = AD + B$$

3. Solve for x and y:
$$3 \begin{bmatrix} 2 & x \\ 5y & -1 \end{bmatrix} - \begin{bmatrix} -6 & 1 \\ 3y & -5 \end{bmatrix}^T = \begin{bmatrix} 12 & -7 \\ -2x & 2 \end{bmatrix}$$

4. The times (in minutes) required for assembling, testing, and packaging large and small capacity food processors are shown in the following table:

	Assembling	Testing	Packaging
Large	45	15	10
Small	30	10	5

   (a) Define a matrix T that summarizes the above data.

   (b) Let $M = \begin{bmatrix} 100 & 200 \end{bmatrix}$ represent the number of large and small food processors ordered, respectively. Find MT and explain the meaning of its entries.

   (c) If assembling costs \$3 per minute, testing costs \$1 per minute, and packaging costs \$2 per minute, find a matrix C that, when multiplied with T, gives the total cost for making each size of food processor.

5. If $A = \begin{bmatrix} -3 & 7 \\ 8 & 10 \end{bmatrix}$, find A^{-1}.

6. If $B = \begin{bmatrix} -3 & 5 \\ 6 & -10 \end{bmatrix}$, find B^{-1}.

7. Solve the following system of equations using matrix inverses.
$$\begin{aligned} 3x + 2y &= z + 2 \\ -3y + 2 &= -2x \\ x &= y + z + 4 \end{aligned}$$

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8. A small village has two major industries: steel and electronics. For each unit of steel produced, 0.02 units of steel and 0.15 units of electronices are used by the village. For each unit of electronics produced, 0.1 units of steel and 0.01 units of electronics are used by the village. The remaining steel and electronics products are then available to export to a local city. If that city demands 500 units of steel and 800 units of electronics, how many units of steel and electronics pruducts should be produced by the village to meet its own needs and those of the city?

9. Consider three sectors of the US economy: crude petroleum (*crude*), petroleum-refining and related industries (*refining*), and chemical production (*chemical*). The following table gives the number of units of crude, refining, and chemical products consumed in the production of one unit of crude product, one unit of refining product, and one unit of chemical product.

	To	**Crude**	**Refining**	**Chemical**
From	**Crude**	0.31	0.42	0.050
	Refining	0.0086	0.11	0.13
	Chemical	0.010	0.47	0.38

(a) What is the input-output matrix for this problem?

(b) Explain the meaning of the entries in row 1 of this matrix.

(c) How many units of refining products are consumed in the production of 7,500 units of crude product?

(d) How many units of chemical products are required to produce 500 units of each sector in this economy?

(e) If a neighboring city demands 5,500 units of crude products, 6,750 units of refining products, and 1,250 units of chemical products, how much should this economy produce to satisfy internal consumption and meet the city's demand?

(f) Referring to (e), how many units of each product are consumed internally in meeting the other city's demands?