Math 166 - Exam 3 Review

NOTE: For reviews of the other sections on Exam 3, refer to the first page of WIR #7 and #8.

Section 8.2 - Expected Value

- Average, or Mean - The average, or mean, of the n numbers $x_1, x_2, \ldots, x_n$ is $\bar{x}$, where $\bar{x} = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$.
- Expected Value of a Random Variable X - Let X denote a random variable that assumes the values $x_1, x_2, \ldots, x_n$ with associated probabilities $p_1, p_2, \ldots, p_n$, respectively. Then the *expected value* of X, written $E(X)$, is given by the following formula: $E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$.
- If $P(E)$ is the probability of an event E occurring, then the odds in favor of E occurring are $\dfrac{P(E)}{P(E^c)}$, and the odds against E occurring are $\dfrac{P(E^c)}{P(E)}$.
- Whenever possible, odds are expressed as ratios of whole numbers. If the odds in favor of E are $\dfrac{a}{b}$, we say the odds in favor of E are a to b (or $a : b$). If the odds against E occurring are $\dfrac{b}{a}$, we say the odds against E are b to a (or $b : a$).
- If the odds in favor of an event E occurring are a to b, then the probability of E occurring is $P(E) = \dfrac{a}{a+b}$.
- **Median** - The median is the middle value in a set of data arranged in increasing or decreasing order (when there is an odd number of entries). If there is an even number of entries, the median is the average of the two middle numbers.
- **Mode** - The mode is the value that occurs most frequently in the set of data.

Section 8.3 - Variance and Standard Deviation

- **Variance of a Random Variable** X - The variance of a random variable X is one measure of dispersion (spread) of a probability distribution about its mean. The units of variance are the square of the units of the random variable.
- Standard Deviation of a Random Variable X - The standard deviation of a random variable X is another measure of dispersion (spread) of a probability distribution about its mean. The units of standard deviation are the same as the units of the random variable.
- Chebychev's Inequality - Let X be a random variable with expected value μ and standard deviation σ. Then the probability that a randomly chosen outcome of the experiment lies between $\mu - k\sigma$ and $\mu + k\sigma$ is at least $1 - \dfrac{1}{k^2}$; that is, $P(\text{outcome is within } k \text{ standard deviations of } \mu) \geq 1 - \frac{1}{k^2}$.

Section 8.4 - Binomial Distribution

- A *binomial experiment* has the following properties:
  1. The number of trials in the experiment is fixed.
  2. There are two outcomes of the experiment: "success" and "failure."
  3. The probability of success in each trial is the same.
  4. The trials are independent of each other.
- **Notation:** In a binomial experiment it is customary to denote the probability of a success by the letter p and the probability of failure by the letter q.

- Computation of Probabilities in Bernoulli Trials - In a binomial experiment in which the probability of success in any trial is p, the probability of exactly r successes in n independent trials is given by $C(n,r)p^r q^{n-r}$.
- If X is a binomial random variable associated with a binomial experiment consisting of n trials with probability of success p and probability of failure q, then the **mean** (expected value), **variance**, and **standard deviation** of X are

$$\begin{aligned} \mu &= E(x) = np \\ \text{Var}(X) &= \sigma_x{}^2 = npq \\ \sigma_x &= \sqrt{npq} \end{aligned}$$

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1. Is the following statement correct? "The probability that Kurt spends less than \$15 on a new DVD is 0.4. Therefore the probability that Kurt spends more than \$15 on a new DVD is 0.6."

2. A fair 4-sided die and a fair 5-sided die are rolled. What is the probability that

   (a) the sum of the dice is 4 or 8?

   (b) the sum of the dice is 8 or at least one 3 is showing?

   (c) the sum of the dice is 6 if the 5-sided die shows an even number?

   (d) the sum of the dice is 6 provided that exactly one 2 is showing?

3. Are mutually exclusive events and independent events the same thing?

4. Jack and Jill are two weather forcasters in Gonzales. The probability that Jack accurately predicts the weather on any given day is 0.68, and the probability that Jill accurately predicts the weather on any given day is 0.72. If the probability that at least one of them is correct on any given day is 0.89, are Jack and Jill making their weather predictions independently?

5. Madison has 5 red, 7 yellow, and 4 blue crayons in her desk drawer. If she selects two at random, what is the probability that she will get two of the same color?

6. A local business employs 12 cashiers, 3 shift managers, and 5 stockers. Two employees are selected at random to attend a workshop.

   (a) What is the probability that the first employee selected is a cashier?

   (b) Assuming that the first employee selected is a cashier, what is the probabilty that the second employee selected is a cashier?

   (c) What is the probability that neither the first nor the second employee selected is a cashier?

7. A manufacturer of automobiles receives 500 car radios from each of three different suppliers. The shipment from supplier A contains 5 defective radios, the shipment from supplier B contains 7 defective radios, and the shipment from supplier C contains 2 defective radios. As a means of quality control, one radio is selected at random from each of the shipments. What is the probability that

   (a) all of the radios selected are working properly?

   (b) at least one of the radios selected is defective?

   (c) exactly one of the selected radios is defective?

8. A bag contains 5 pennies, 3 nickels, and 7 dimes. A purse contains 4 nickels and 6 dimes. A coin is drawn from the bag and transferred to the purse, but if a nickel is selected from the bag, then all of the nickels in the bag are transferred to the purse. A coin is then drawn from the purse. The type of coin drawn from each of the bag and purse is recorded. What is the probability that

   (a) the transferred coin was a dime if a nickel was selected from the purse?

   (b) both coins are pennies?

   (c) the coin drawn from the purse is a penny if the coin drawn from the bag was a nickel?

9. A naval acadamy has a student body that is 80% male. 35% of the males and 30% of the females plan to seek a commission in the United States Marine Corps, and all other students plan to seek a commission in the United States Navy.

   (a) What is the probability that a student at this academy is male or plans to seek a commission in the Navy?

   (b) What is the probability that a student who plans to seek a commission in the Marine Corps is female?

   (c) What is the percentage of students at this academy who plan to join the Marine Corps?

10. Let E and F be two events of an experiment with $P(E) = 0.35$, $P(F) = 0.55$, and $P(E \cap F^c) = 0.15$.

   (a) Find $P(E \cap F)$.

   (b) What is the probability that exactly one of these two events occurs?

   (c) Are E and F mutually exclusive?

   (d) Are E and F independent?

   (e) Find $P(E|F)$.

   (f) Find the probability that at least one of the two events occurs.

11. Classify each of the following random variables and give the possible values they each may assume.

(a) X = the number of times a coin is flipped until tails appears.

(b) X = the number of cards drawn (without replacement) from a standard deck of 52 playing cards until a red card is drawn.

(c) X = the weight of a newborn baby.

(d) X = the number of hours my cat Mouse sleeps in one day.

(e) Cards are drawn one at a time with replacement from a well-shuffled deck of 52 playing cards until a club is drawn. Let X = the number cards drawn in the experiment.

12. 16 people are selected at random. What is the probability that at least 2 of the people in this group

(a) were born in the same week? (There are 52 weeks in a year. Assume that all weeks are equally likely.)

(b) were born in the same month? (Assume that all months are equally likely.)

13. The odds against it snowing in College Station next winter are 17 to 2. What is the probability that it will snow in College Station next winter?

14. Two cards are drawn at random from a standard deck of 52 playing cards. What are the odds that the second card drawn is a king given that the first card drawn was a queen?

15. A probability distribution has a mean of 100 and a standard deviation of 4. Use Chebychev's Theorem to estimate the probability that an outcome of the experiment lies between 90 and 110.

16. Fred wants to purchase a 10-year term life insurance policy that will pay his beneficiary $100,000 in the event that Fred does not survive the next 10 years. Using life insurance tables, he determines that the probability that he will live another 10 years is 0.97. What is the minimum amount that he can expect to pay for his premium?

17. Suppose you roll two fair 6-sided dice and take the sum of the numbers landing up. You will win twice what you paid if the sum is 7 or 11. You lose what you paid if the sum is 2, 3, or 12. For any other sum, you win \$5. The game costs \$10 to play. Let X denote the net winnings of someone who plays once.

    (a) What is the expected net winnings?

    (b) How much should be charged to make this game fair?

18. A cashier at a convenience store kept a record of the number of items purchased by each customer on one day. The data she collected are summarized in the following table:

Number of customers	5	25	22	13	7	2	1	1
Number of items purchased	0	1	2	3	4	5	6	7

(a) Write the probability distribution of the number of items purchased.

(b) Draw a histogram associated with the probability distribution found in part (a).

(c) Find $P(2 \leq X \leq 4)$.

(d) Find $P(X < 5)$.

(e) How many items could a customer on that day be expected to buy?

(f) Compute the mean, median, mode, standard deviation, and variance for the frequency chart. Be sure to label all answers.

19. A student takes a 10 question multiple choice exam, each question of which has 5 answer choices (1 correct, 4 incorrect). Being unprepared for the exam, the student randomly guesses at each question.

(a) What is the probability that the student gets exactly 6 questions correct?

(b) What is the probability that the student gets at least 60% of the questions correct?

(c) What is the probability that the student gets the first 4 correct and the last 6 incorrect?

(d) How many questions should the student expect to get correct?

(e) Find the variance and standard deviation for the number of questions answered correctly.

20. A company manufactures one product. For quality control, a random sample of 6 items is selected from a each lot of products made by this company before the lot is shipped. If any defective items are found in the sample, the entire lot is rejected. If 2.3% of the items produced by this company are defective, what is the probability that a lot will be shipped?

21. A psychological study has determined that 4.8% of all kindergarteners have Attention Deficit Disorder (ADD). In an elementary school with 115 kindergarteners, find the probability that more than 30% have ADD.

22. The police department of a certain town estimates that 23% of all drivers in their town do not wear their seatbelts. If 60 cars are stopped at random, what is the probability that more than 90% of the drivers are wearing their seatbelts?