Math 166 - Exam 3 Review

NOTE: For reviews of the other sections on Exam 3, refer to the first page of WIR #7 and #8.

Section 8.2 - Expected Value

- Average, or Mean The average, or mean, of the *n* numbers x_1, x_2, \dots, x_n is \bar{x} , where $\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$.
- Expected Value of a Random Variable X Let X denote a random variable that assumes the values x_1, x_2, \dots, x_n with associated probabilities p_1, p_2, \dots, p_n , respectively. Then the expected value of X, written E(X), is given by the following formula: $E(X) = x_1p_1 + x_2p_2 + \dots + x_np_n$.
- If P(E) is the probability of an event E occurring, then the odds in favor of E occurring are $\frac{P(E)}{P(E^c)}$, and the odds against E occurring are $\frac{P(E^c)}{P(E)}$.
- Whenever possible, odds are expressed as ratios of whole numbers. If the odds in favor of E are $\frac{a}{b}$, we say the odds in favor of E are a to b (or a:b). If the odds against E occurring are $\frac{b}{a}$, we say the odds against E are b to a (or b:a).
- If the odds in favor of an event E occurring are a to b, then the probability of E occurring is $P(E) = \frac{a}{a+b}$.
- Median The median is the middle value in a set of data arranged in increasing or decreasing order (when there
 is an odd number of entries). If there is an even number of entries, the median is the average of the two middle
 numbers.
- Mode The mode is the value that occurs most frequently in the set of data.

Section 8.3 - Variance and Standard Deviation

- Variance of a Random Variable X The variance of a random variable X is one measure of dispersion (spread) of a probability distribution about its mean. The units of variance are the <u>square</u> of the units of the random variable.
- Standard Deviation of a Random Variable X The standard deviation of a random variable X is another measure of dispersion (spread) of a probability distribution about its mean. The units of standard deviation are the same as the units of the random variable.
- Chebychev's Inequality Let X be a random variable with expected value μ and standard deviation σ . Then the probability that a randomly chosen outcome of the experiment lies between $\mu k\sigma$ and $\mu + k\sigma$ is at least $1 \frac{1}{k^2}$; that is, $P(\text{outcome is within } k \text{ standard deviations of } \mu) \ge 1 \frac{1}{k^2}$.

Section 8.3 - Variance and Standard Deviation

- A binomial experiment has the following properties:
- 1. The number of trials in the experiment is fixed.
- 2. There are two outcomes of the experiment: "success" and "failure."
- 3. The probability of success in each trial is the same.
- 4. The trials are independent of each other.
- Notation: In a binomial experiment it is customary to denote the probability of a success by the letter p and the
 probability of failure by the letter q.
- Computation of Probabilities in Bernoulli Trials In a binomial experiment in which the probability of success in any trial is p, the probability of exactly r successes in n independent trials is given by $C(n,r)p^rq^{n-r}$.
- If X is a binomial random variable associated with a binomial experiment consisting of n trials with probability of success p and probability of failure q, then the mean (expected value), variance, and standard deviation of X are

$$\mu = E(x) = np$$

$$Var(X) = \sigma_x^2 = npq$$

$$\sigma_x = \sqrt{npq}$$

1. Is the following statement correct? "The probability that Kurt spends less than \$15 on a new DVD is 0.4. Therefore the probability that Kurt spends more than \$15 on a new DVD is 0.6."

No. We cannot exclude the possibility that he spends exactly \$15.

The correction of this statement would read, "The probability that Kurt spends less than \$15 on a new DVD is . 4, so the probability that he spends \$15 or more on a new DVD is . 6.

(The complement of less than 15 is greater than or equal to 15.)

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2. A first-added die and a fair Solded die are rolled. What is the probability that

(a) the sum of the dice is
$$40^{\circ}$$
 8?

A. Sum is 4 (1,1)(1,2)(1,3)(1,4)(1,5)

B. Shmis 8 (2,1)(2,2)(2,3)(2,4)(2,5)

P(A UB) = P(A) + P(B) - P(A)(B) (3,1)(3,2)(3,3)(3,4)(3,5)

$$= \frac{3}{20} + \frac{3}{20} - 0 + \frac{3}{20} + \frac{3}{20} - 0 + \frac{3}{20} + \frac{3}{20} - \frac{3}{20} + \frac{3}{20}$$

Title: Nov 13-6:00 PM (3 of 22)

3. Are mutually exclusive events and independent events the same thing? Mutually Excl - the events cannot occur at the same time (no outcomes in common) -> P(A \ B) = 0 Indep events do not affect each offer P(ANB) = P(A)P(B) Also for Independent events, P(A1B) = P(A) and P(B(A) = P(B). 4. Jack and Jill are two weather forcasters in Gonzales. The probability that Jack accurately predicts the weather on any given day is 0.68, and the probability that Jill accurately predicts the weather on any given day is 0.72. If the probability that at least one of them is correct on any given day is 0.89, are Jack and Jill making their weather predictions independently?) \leftarrow Test for independence $P(A \cap B) \stackrel{!}{=} P(A)P(B)$ P(AUB) = P(A) + P(B) - P(ANB).89 = .68 + .72 - P(ANB) (Pranb) = .5

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5. Madison has 5 red, 7 yellow, and 4 blue crayons in her desk drawer. If she selects two at random, what is the probability that she will get two of the same color?

E-event that 2 crayons of the same color are drawn.

ch0036

 $P(E) = \frac{n(E)}{n(S)} = \frac{((S,2)+(t_1,2)+(t_1,2)+(t_1,2)}{(((16,2))}$

n(F): 2 red or 2 yellow or 2 blue (C(5,2) + C(1,2) + C(4,2)

Title: Nov 13-6:13 PM (5 of 22)

- 6. A local business employs 12 cashiers, 3 shift managers, and 5 stockers. Two employees are selected at random to attend a workshop.
 - (a) What is the probability that the first employee selected is a cashier?



(b) Assuming that the first employee selected is a cashier, what is the probability that the second employee selected is a cashier?

(c) What is the probability that neither the first nor the second employee selected is a C,-went the 1st employee selected is a cashier. cashier?

Title: Nov 13-6:13 PM (6 of 22)

- 7. A manufacturer of automobiles receives 500 car radios from each of three different suppliers. The shipment from supplier A contains 5 defective radios, the shipment from supplier B contains 7 defective radios, and the shipment from supplier C contains 2 defective radios. As a means of quality control, one radio is selected at random from each of the shipments. What is the probability that
- (a) all of the radios selected are working properly?

 Photographic properly?

 Photographic properly?

 Properly?

 Properly?

 Properly?

 Properly?

 Properly?

 Indep? yes!!!

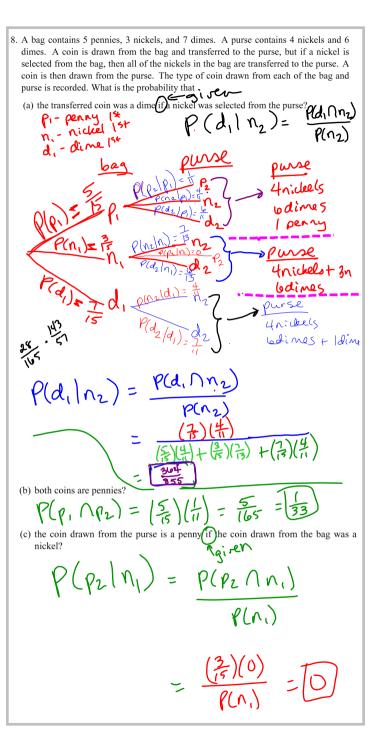
 Properly?

 Indep? yes!!!
- (b) at least one of the radios selected is defective?

P(at least Idefective) = |-P(0 defectives)| = |-.9722 = |.0278|

(c) exactly one of the selected radios is defective?

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Title: Nov 13-6:00 PM (8 of 22)

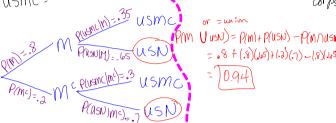
- 9. A naval academy has a student body that is 80% male. 35% of the males and 30% of the females plan to seek a commission in the United States Marine Corps, and all other students plan to seek a commission in the United States Navy.
 - (a) What is the probability that a student at this academy is male of plans to seek a commission in the Navy?

M - event the student is male

USN - event the student plans to join the US Navy.

ISMC - - - - - - - - US Marine

corps.



(b) What is the probability that a student who plans to seek a commission in the Marine Corps is female?

$$P(M^{c} | usmc) = \frac{P(M^{c} \cap usmc)}{P(usmc)}$$

$$= \frac{(.2)(.3)}{(.8)(.35) + (.2)(.3)}$$

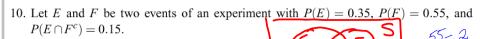
$$= \frac{3}{17}$$

(c) What is the percentage of students at this academy who plan to join the Marine Corps?

$$P(USMC) = (.8)(.35) + (.2)(.3)$$

= .34

34% plan to join the USMC.



(a) Find $P(E \cap F)$.

- (b) What is the probability that exactly one of these two events occurs? 15 + .35 = 1.5
- (c) Are E and F mutually exclusive?

(c) Are
$$E$$
 and F mutually exclusive?
NO! It is possible for E and F to occur
(d) Are E and F independent? at the same time
 $(P(E \cap F) \neq D)$

Test ?
$$P(E \cap F) \stackrel{?}{=} P(E) P(F)$$

$$2 \stackrel{?}{=} (.35)(.55)$$

$$2 \not= .1925 \text{ (not indep.)}$$

- (e) Find P(E|F). $P(E \cap F) = P(E \cap F)$ $P(F) = \frac{2}{55} = \boxed{4}$
- (f) Find the probability that at least one of the two events occurs.

$$P(EVF) = P(E) + P(F) - P(ENF)$$

= .35 + .55 - .2
= $\frac{1}{2}$

| 11. | Classify each of the following random variables and give the possible value | es they | each |
|-----|---|---------|------|
| | may assume. | | |

(a) X = the number of times a coin is flipped until tails appears.

X=1,2,3,4,... Infinite discrete

(b) X = the number of cards drawn (without replacement) from a standard deck of 52 playing cards until a red card is drawn.

X=1,2,3,...,27 finite discrete

(c) X = the weight of a newborn baby.

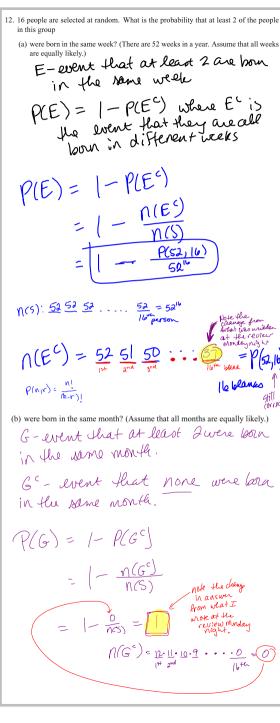
X70 Continuous

(d) X = the number of hours my cat Mouse sleeps in one day.

05×624 continuous.

(e) Cards are drawn one at a time with replacement from a well-shuffled deck of 52 playing cards until a club is drawn. Let X = the number cards drawn in the experiment.

X=1,2,3,...The discrete



Title: Nov 13-6:02 PM (12 of 22)

| 13. | The odds against | it snowing in College Station next winter | are | 2 17 | to 2. | What is 1 | the |
|-----|---------------------|---|-----|----------------|---------|--------------|-------|
| | probability that it | will snow in College Station next winter? | one | wa | y to wi | The His prob | nlen: |
| | | | | | | | |

Another
$$\frac{17}{17+2} = \frac{17}{19} = \text{probab the t it does not}$$

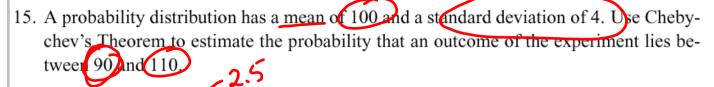
 Now : $\text{Polab it does} = 1 - \frac{17}{19} = \frac{2}{19}$

14. Two ards are drawn at random from a standard deck of 52 playing cards. What are the odds that the second card drawn is a king given that the first card drawn was a queen

odds in favor of this:
$$\frac{4}{51} = \frac{4}{47} \rightarrow \boxed{4 + 0.47}$$

4:47

Title: Nov 13-6:02 PM (13 of 22)



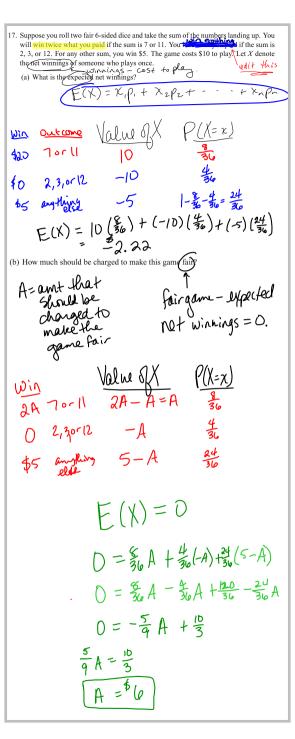
$$\mu = 100$$
 100
 -100
 -100
 -100
 $10 = 4k$
 -100
 $10 = 4k$
 -100
 $10 = 4k$

Title: Nov 13-6:02 PM (14 of 22)

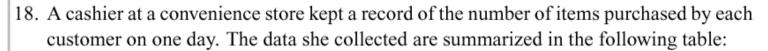
16. Fred wants to purchase a 10-year term life insurance policy that will pay his beneficiary \$100,000 in the event that Fred does not survive the next 10 years. Using life insurance tables, he determines that the probability that he will live another 10 years (s 0.97) What A=min aut. Min premium is the premium he can expert that makes the insurance to pay for company's expected gain = 0.

The premium. is the minimum amount that he can expect to pay for his premium? X= the insurance company's gain this problem is similar to #16 and #18 in Section

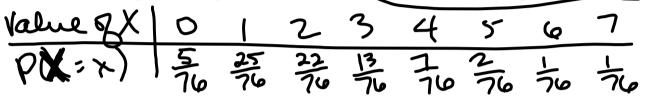
Title: Nov 13-6:02 PM (15 of 22)



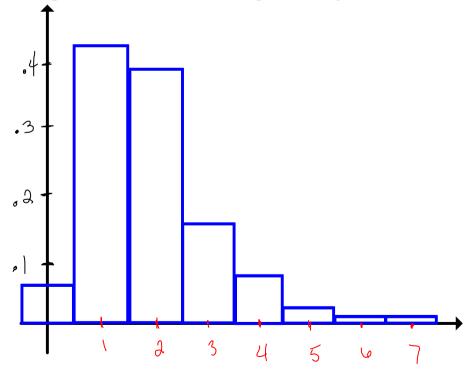
Title: Nov 13-6:03 PM (16 of 22)



(a) Write the probability distribution of the number of items purchased



(b) Draw a histogram associated with the probability distribution found in part (a).



Title: Nov 13-6:03 PM (17 of 22)

(c) Find
$$P(2 \le X \le 4)$$
. = $P(X=2) + P(X=3) + P(X=4)$
= $\frac{22}{76} + \frac{13}{76} + \frac{7}{76} = \boxed{42}$

(d) Find
$$P(X < 5)$$
.
 $1 - (76 + 76 + 76) = 72$

(e) How many items could a customer on that day be expected to buy?

$$E(Y) = X = 2.0921$$

[var stats 4, L2

(f) Compute the mean median, mode, standard deviation, and variance for the fre quency chart. Be sure to label all answers.

assuming this is a sample,

$$\overline{\chi} = 2.0921$$

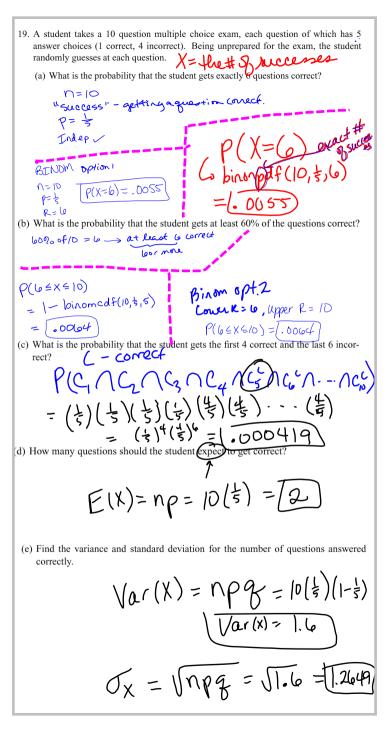
median = 2
mode = 1

Sample 5x = 1. 3873 = Standard devication sample 5x² = 1.9247 = variance

Note: Some instructors ALWAYS use Ox for standard deviation;

$$(population) \sigma_x = 1.3782 = Standard deviation$$

(population) $\sigma_{x}^{2} = 1.5994 = variance$



Title: Nov 13-6:03 PM (19 of 22)

20. A company manufactures one product. For quality control, a random sample of 6 items is selected from a each lot of products made by this company before the lot is shipped. If any defective items are found in the sample, the entire lot is rejected. If 2.3% of the items produced by this company are defective, what is the probability that a lot will be shipped? E- the lot is shipped lie the sample has ND defectives) P(E) = P(no dejectives) Approximate with binomial probab. 11 Success" (What you're noteingfor) - getting a defective p=.023 Judep-approximately P(X=D) = binompdf(6,023,0)n=6230ption1 =1.8697)
R=0

Title: Nov 13-6:04 PM (20 of 22)

21. A psychological study has determined that 4.8% of all kindergarteners have Attention Deficit Disorder (ADD). In an elementary school with 115 kindergarteners, find the probability that more than 30% have ADD.

more than 30% of 15: (.3)(1/5) = 34.5

Title: Nov 13-6:04 PM (21 of 22)

22. The police department of a certain town estimates that 23% of all drivers in their town do not wear their seatbelts. If 60 cars are stopped at random, what is the probability that more than 90% of the drivers are wearing their seatbelts?

n=60 15viceso" - wearing seatbelt p=1-.23=.77 Indep.√

more than 90% of 60: (.9)(60) = 54 more than 54 means > 55

 $P(55 \le X \le 60) = 1 - binomed s(60, .77, 54)$ = [.0028]

BINOM program! N = 60 Option 2 P = .77 Lower R = 55upper R = 60 $P(55 \le X \le 60) = [.0028]$