A Bayesian Framework for Robust Kalman Filtering Under Uncertain Noise Statistics

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Abstract—In this paper, we propose a Bayesian framework for robust Kalman filtering when noise statistics are unknown. The proposed intrinsically Bayesian robust Kalman filter is robust in the Bayesian sense meaning that it guarantees the best average performance relative to the prior distribution governing unknown noise parameters. The basics of Kalman filtering such as the projection theorem and the innovation process are revisited and extended to their Bayesian counterparts. These enable us to design the intrinsically Bayesian robust Kalman filter in a similar way that one can find the classical Kalman filter for a known model.

Index Terms—Intrinsically Bayesian robust, Kalman filter, Bayesian innovation process, Bayesian projection theorem.

I. INTRODUCTION

Originally, the underlying models for signal estimation problems were assumed to be perfectly known; however, in many real-world applications, we often encounter imperfect models embracing uncertainty due to various factors such as lack of sufficient data, complexity, and randomness [1]. For these models, the objective changes from designing an optimal filter for a known model to a robust filter for a collection of models compatible with the partial prior knowledge, being called the uncertainty class.

Designing robust operators for a model with uncertainty has been viewed from two different perspectives: minimax robustness and Bayesian robustness. Minimax robustness [2] takes a conservative approach in which the robust filter is the one that has the best worst-case performance over all possible models in the uncertainty class. The performance of the robust filter in this approach is highly sensitive to extreme models with low prior probability. Bayesian robustness [3] addresses this issue by assuming a prior distribution for the uncertainty class. This robustness approach has been used in various engineering applications such as filtering [3]–[5], classification [6]–[8], signal compression [9], and blind image deconvolution [10]. Under this criterion, the aim is to design a filter with the best performance on average relative to the prior distribution. This filter is called an intrinsically Bayesian robust (IBR) filter. Assume that we wish to estimate random process $x(k)$ based upon observing another random process $y(k)$. Let $\theta$ be a parameter vector composed of the unknown parameters in the joint distribution of the two random processes. The uncertainty class $\Theta$ is composed of all possible values for $\theta$ and the IBR filter $\psi_{\text{IBR}}(\Theta)$ is given by

$$\psi_{\text{IBR}}(\Theta) = \arg \min_{\psi \in \Psi} \mathbb{E}_{\Theta}[\xi_{\theta}(x(k), \psi(y(k))], \quad (1)$$

the cost function $\xi_{\theta}(\bullet)$ measures the error of estimating $x(k)$ by $\psi(y(k))$ relative to parameter $\theta$. $\Psi$ is the class of all filters to which the optimization is restricted, and the expectation is taken relative to the prior distribution $\pi(\theta)$ over $\Theta$. An IBR filter is the optimal option when dealing with an uncertainty class of models. The problem of IBR Wiener filtering has been solved in [5] by introducing the concept of effective characteristics. Characteristics are functions of the random processes under study that determine the filter, such as the auto- and cross-correlation functions. The IBR Wiener filter can be solved by finding the classical Wiener filter for the effective characteristics. However, the IBR Kalman filter has heretofore been an open problem. In this paper, we present a framework for IBR Kalman filtering.

The Kalman filter [11] is a well known filter which is suitable for linear systems described by a state-space model. This filter can provide optimal state estimation when noise statistical information is available a priori. However, lack of knowledge regarding the noise statistics deteriorates the performance of the Kalman filter significantly. Thus, we desire robust Kalman filters. Most of the works found for robust Kalman filtering in the literature try to simultaneously estimate noise parameters along with state estimation. These methods are called adaptive Kalman filtering [12]–[15]. In fact, in these methods the actual underlying Kalman filter is not robust. To the best of our knowledge, the only efforts that have been made to design an actual robust Kalman filter have been those related to minimax robust Kalman filtering [16]–[18].

In this paper, we address the problem of IBR Kalman filtering. The two pillars of the proposed methodology are the Bayesian projection theorem and the Bayesian innovation process. Utilizing these two concepts, we solve for the IBR Kalman filter by extending concepts such as noise covariance matrices, Kalman gain matrix, and the estimation error covariance matrix, which are used for the classical Kalman filtering, to their effective counterparts. Using these effective matrices, the problem of designing an IBR Kalman filter reduces to the problem of designing an ordinary Kalman filter for a given state-space model corresponding to the effective matrices. In this paper, we only outline main results for the proposed IBR
Kalman filtering framework. A detailed discussion along with proofs and the derivation of the equations are provided in [19].

II. KALMAN FILTERING

A discrete-time linear dynamical system is described by a state-space model consisting of the state equation

$$x(k+1) = \Phi(k)x(k) + \Gamma(k)u(k),$$

(2)

and the observation equation

$$y(k) = H(k)x(k) + v(k),$$

(3)

for $k = 0, 1, 2, \ldots$. In these two equations, $x(k)$ is an $n \times 1$ random vector called the state vector, $\Phi(k)$ is an $n \times n$ matrix called the state transition matrix, $u(k)$ is a $p \times 1$ random vector called the process noise vector, $\Gamma(k)$ is an $n \times p$ matrix called the process noise transition matrix, $y(k)$ is a random vector of size $m \times 1$ and called the observation vector, $H(k)$ is an $m \times n$ matrix called the observation transition matrix, and finally $v(k)$ is an $m \times 1$ random vector called the observation noise vector. We use $z(k)$ to denote $H(k)x(k)$. The process and observation noise are discrete white-noise processes with

$$E[u(k)u^T(l)] = Q(k)\delta_{kl}, \quad \forall k, l = 0, 1, 2, \ldots,$$

$$E[v(k)v^T(l)] = R(k)\delta_{kl}, \quad \forall k, l = 0, 1, 2, \ldots,$$

$$E[u(k)v^T(l)] = 0_{m \times n}, \quad \forall k, l = 0, 1, 2, \ldots,$$

$$E[u(k)y^T(l)] = 0_{p \times m}, \quad \forall k, l = 0, 1, 2, \ldots,$$

$$E[u(k)y^T(l)] = 0_{p \times m}, \quad 0 \leq l \leq k - 1$$

where $0_{m \times n}$ is a zero matrix of size $m \times n$. The aim of the Kalman filter is to find the least-squares estimate $\hat{x}(k)$, the optimal estimate for $x(k)$ in the sense of the mean-square-error (MSE), using observation vectors $y(l), l \leq k - 1$. The Kalman filter was first proposed in a seminal paper by Kalman [11]. Later, Kailath in [20] used the notion of innovation process for deriving the Kalman filter.

The equations required for the classical Kalman filter are summarized in the left column of Table I. The innovation process $\tilde{z}(k)$ obtained in the first row is a discrete white-noise process. It is used in the third row to update the least-squares estimate $\tilde{x}(k)$. The matrix $K(k)$ computed in the second row is called the Kalman gain matrix. The matrix $P^x(k)$ is the optimal estimation error covariance matrix at time $k$; i.e.,

$$P^x(k) = E[(x(k) - \tilde{x}(k))(x(k) - \tilde{x}(k))^T].$$

$P^x(k)$ in the second row is $H(k)P^x(k)H^T(k)$.

III. IBR KALMAN FILTERING

Now assume that the process and observation noise have unknown covariance matrices being parameterized by unknown parameters $\theta_1$ and $\theta_2$, respectively:

$$E[u_\theta_1(k)u_\theta_1^T(l)] = Q_\theta_1(k)\delta_{kl},$$

$$E[v_\theta_2(k)v_\theta_2^T(l)] = R_\theta_2(k)\delta_{kl},$$

(4)

where $[\theta_1, \theta_2] = \theta \in \Theta$. We assume that $\theta_1$ and $\theta_2$ are statistically independent; therefore, prior distribution $\pi(\theta) = \pi(\theta_1)\pi(\theta_2)$. With unknown parameters $\theta_1$ and $\theta_2$, the state-space model is parameterized as

$$x_{\theta_1}(k+1) = \Phi(k)x_{\theta_1}(k) + \Gamma(k)u_{\theta_1}(k),$$

$$y_{\theta}(k) = H(k)x_{\theta_1}(k) + v_{\theta_2}(k).$$

(6)

(7)

While the state vector $x_{\theta_1}(k)$ only depends on $\theta_1$, observation vector $y_{\theta}(k)$ depends on both $\theta_1$ and $\theta_2$. Because the Kalman filter is linear, we would like the IBR Kalman filter to have the following linear form:

$$\hat{x}_{\theta}(k) = \sum_{l \leq k-1} G_{k,l}^{\Theta}y_{\theta}(l),$$

(8)

where the weighting function $G_{k,l}^{\Theta}$ is a matrix of size $n \times m$ chosen such that

$$G_{k,l}^{\Theta} = \arg \min_{G_{k,l} \in \Theta} E_{\theta} \left[ E \left[ \left( x_{\theta_1}(k) - \sum_{l \leq k-1} G_{k,l}y_{\theta}(l) \right)^T \times \left( x_{\theta_1}(k) - \sum_{l \leq k-1} G_{k,l}y_{\theta}(l) \right) \right] \right].$$

(9)

$\hat{x}_{\theta}(k)$ obtained in (8) is called the Bayesian least-squares estimate at time $k$ for $x_{\theta}(k)$. In fact, in (9), we aim at minimizing the expected MSE between the linear estimate $\sum_{l \leq k-1} G_{k,l}y_{\theta}(l)$ and $x_{\theta_1}(k)$ relative to the prior distribution $\pi(\theta)$ governing $\Theta$. The IBR Kalman filter defined in (8) is the filter that attains this minimum. The following theorem plays a major role in our development of IBR Kalman filtering.

**Theorem 1: (Bayesian Projection Theorem)** A linear filter with weighting function $G_{k,l}^{\Theta}$ as defined in (8) satisfies (9) (having minimum expected MSE across the uncertainty class) if and only if

$$E_{\theta} \left[ E \left[ \left( x_{\theta_1}(k) - \tilde{x}_{\theta}(k) \right)y_{\theta}^T(l) \right] \right] = 0_{n \times m}, \quad \forall l < k.$$  

(10)

In [20], the notion of innovation process was proposed for deriving the classical Kalman equations. In this paper, we extend it to the Bayesian innovation process.

**Definition 1:** Let $\tilde{x}_{\theta}(k)$ be the Bayesian least-squares estimate at time $k$ for $x_{\theta_1}(k)$ obtained in (8). Then

$$\tilde{z}_{\theta}(k) = y_{\theta}(k) - H(k)\tilde{x}_{\theta}(k)$$

(11)

is a zero-mean process called the Bayesian innovation process.

The innovation process used for classical Kalman filtering is a white-noise process. It can be shown that the Bayesian innovation process defined in (11) has the following property:

$$E_{\theta} \left[ E \left[ \tilde{z}_{\theta}(k)\tilde{z}_{\theta}^T(l) \right] \right] = E_{\theta} \left[ P_{\theta}^{y}(k) + R_{\theta_2}(k) \right] \delta_{kl},$$

(12)

where $P_{\theta}^{y}(k) = H(k)P_{\theta}^{x}(k)H^T(k)$ such that

$$P_{\theta}^{x}(k) = E \left[ (x_{\theta_1}(k) - \tilde{x}_{\theta}(k))\left( x_{\theta_1}(k) - \tilde{x}_{\theta}(k) \right)^T \right].$$

(13)

is the Bayesian estimation error covariance matrix at time $k$ relative to parameter $\theta$.

Based on the following lemma, we can use the Bayesian innovation process to find the IBR Kalman filter equations.
where, analogous to classical Kalman filtering terminology, the least-squares estimate is given by (a detailed derivation of the equations is provided in [19]).

The proofs for the theorem and lemma can be found in [19].

Having laid out the theoretical foundation, the equations required for the IBR Kalman filtering can now be derived (a detailed derivation of the equations is provided in [19]). According to Lemma 1, the Bayesian least-squares estimate $\hat{x}_\theta(k)$ is of the following form:

$$\hat{x}_\theta(k) = \sum_{l=k-1}^{\infty} G_{0,l}^\theta \tilde{z}_\theta(l). \tag{14}$$

As $\tilde{z}_\theta(k)$ is the Bayesian least-squares estimate obtained using $\tilde{z}_\theta(l)$, based on the Bayesian projection theorem, for $l \leq k - 1$,

$$E_\theta [E \left[ (x_\theta(k) - \hat{x}_\theta(k)) \tilde{z}_\theta(l) \right]] = 0_{n \times m}. \tag{15}$$

We substitute (14) for $\hat{x}_\theta(k)$ in (15) and then use the property of the Bayesian innovation process in (12). Then we rearrange the resulting equation to solve for $G_{0,l}^\theta$. We can find the Bayesian least-squares estimate $\hat{x}_\theta(k)$ by plugging in the resulting relation found for $G_{0,l}^\theta$ in (14).

It can be shown that an update equation for the Bayesian least-squares estimate is given by

$$\hat{x}_\theta(k+1) = \Phi(k)\hat{x}_\theta(k) + K_\theta(k)z_\theta(k),$$

where, analogous to classical Kalman filtering terminology, $K_\theta(k)$ is called the effective Kalman gain matrix and is given by

$$K_\theta(k) = \Phi(k)E_\theta \left[ P_\theta^\theta(k) \right] H^T(k)E_\theta^{-1} \left[ P_\theta^\theta(k) + R_{\theta_2}(k) \right]. \tag{16}$$

To update $K_\theta(k)$, we also need to keep track of the average Bayesian estimation error covariance matrix $E_\theta \left[ P_\theta^\theta(k) \right]$, which is updated as

$$E_\theta \left[ P_\theta^\theta(k+1) \right] = (\Phi(k) - K_\theta(k)H(k))E_\theta \left[ P_\theta^\theta(k) \right] \Phi^T(k) + \Gamma(k)E_{\theta_1} \left[ Q_{\theta_1}(k) \right] \Gamma^T(k), \tag{17}$$

where $E_{\theta_1}[\cdot]$ is the expectation taken relative to the prior distribution $\pi(\theta_1)$ of parameter $\theta_1$. This completes all equations needed for IBR Kalman filtering.

For a better comparison between the equations of the IBR Kalman filter and those of the classical Kalman filter, we put them alongside each other in Table I. We can see that the structure of IBR Kalman filtering is similar to that of classical Kalman filtering with $K(k)$, $P^\theta(k)$, $Q(k)$, and $R(k)$ replaced by effective matrices $K_\theta(k)$, $E_\theta \left[ P_\theta^\theta(k) \right]$, $E_{\theta_1} \left[ Q_{\theta_1}(k) \right]$, and $E_{\theta_2}[R_{\theta_2}(k)]$, respectively. Therefore, one can find the IBR Kalman filter in the same way that a classical Kalman filter is found for a known model corresponding to the aforementioned effective matrices. Initial conditions for the IBR Kalman filter are set to $E_\theta \left[ P_\theta^\theta(0) \right] = cov(x(0))$, the covariance matrix of the state vector at time zero, and $z_\theta(0) = E[x(0)]$. Note that $x(0)$ does not depend on $\theta$ because it is independent from future process and observation noise.

**IV. SIMULATION RESULTS**

To analyze the effectiveness of the proposed IBR Kalman filtering framework, we also find the average performance of the model-specific Kalman filters. To do so, the first step is to find the performance of the $\theta'$-specific Kalman filter, designed with respect to parameter $\theta'$, when applied to a state-space model with parameter $\theta$. There is a mismatch between the parameter value assumed for designing the model-specific Kalman filter and the actual parameter value of the model to which the filter is applied.

Let $P_{\theta',\theta'}^\theta(k)$ denote the covariance matrix of the estimation error obtained by applying the $\theta'$-specific Kalman filter to a model with parameter $\theta$. Then the average estimation error covariance matrix $E_\theta \left[ P_{\theta',\theta'}^\theta(k) \right]$ for the $\theta'$-specific Kalman is updated as

$$E_\theta \left[ P_{\theta',\theta'}^\theta(k+1) \right] = (\Phi(k) - K_{\theta'}(k)H(k))E_\theta \left[ P_{\theta',\theta'}^\theta(k) \right] \Phi^T(k) + \Gamma(k)E_{\theta_1} \left[ Q_{\theta_1}(k) \right] \Gamma^T(k) + K_{\theta'}(k)E_{\theta_2}[R_{\theta_2}(k)]K_{\theta'}^T(k), \tag{18}$$

in which $K_{\theta'}(k)$ is the Kalman gain matrix for the $\theta'$-specific Kalman filter, whose update is given by

$$K_{\theta'}(k) = \Phi(k)P_{\theta'}^\theta(k;\theta')H^T(k) \times \left( H(k)P_{\theta'}^\theta(k;\theta')H^T(k) + R_{\theta_2}(k) \right)^{-1}. \tag{19}$$

<table>
<thead>
<tr>
<th>Classical Kalman Filter</th>
<th>IBR Kalman Filter</th>
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<tbody>
<tr>
<td>$\tilde{x}(k) = y(k) - H(k)\tilde{x}(k)$</td>
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<tr>
<td>$K(k) = \Phi(k)P^\theta(k)H^T(k)(P^\theta(k) + R(k))^{-1}$</td>
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<tr>
<td>$\hat{x}(k + 1) = \Phi(k)\hat{x}(k) + K(k)\tilde{x}(k)$</td>
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</tr>
<tr>
<td>$P^\theta(k + 1) = (\Phi(k) - K(k)H(k))P^\theta(k) \Phi^T(k)$</td>
<td>$P_\theta^\theta(k + 1) = (\Phi(k) - K_\theta(k)H(k))E_\theta \left[ P_\theta^\theta(k) \right] \Phi^T(k)$</td>
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<tr>
<td>$+ \Gamma(k)Q(k)\Gamma^T(k)$</td>
<td>$+ \Gamma(k)E_{\theta_1} \left[ Q_{\theta_1}(k) \right] \Gamma^T(k)$</td>
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**Lemma 1:** The Bayesian least-squares estimate for $x_\theta(k)$ obtained using the Bayesian innovation process $\tilde{z}_\theta(l)$, $l \leq k - 1$, is also the Bayesian least-squares estimate for $x_\theta(k)$ obtained based upon observations $y_\theta(l)$, $l \leq k - 1$.
Note that to update the average error covariance matrix in (18), one also needs to update $K_\theta(k)$ as shown in (19), which in turn requires the update of the optimal estimation error covariance matrix $P_\theta(k)$ for model $\theta'$, which can be done similarly to $P_\theta(k)$ in Table 1. Therefore, to find the average error covariance matrix for an arbitrary model-specific Kalman filter, one needs to keep track of two different covariance matrices simultaneously, $E_\theta[P_\theta_\theta(k)]$ and $P_\theta(k; \theta')$. We report the trace $\text{Tr}\{E_\theta[P_\theta_\theta(k)]\}$ of the average estimation error covariance matrix as the average MSE of the $\theta'$-specific Kalman filter.

As remarked earlier, another robust design approach is minimax robustness, the aim being to find the filter with the best worst-case performance. We find the corresponding parameter $\theta_{\text{mm}}$ for the minimax robust Kalman filter as

$$
\theta_{\text{mm}} = \arg \min_{\theta' \in \Theta} \max_{\theta} \lim_{k \to \infty} \text{Tr}\{P_{\theta',\theta_{\text{mm}}}(k)\}.
$$

The average MSE of the minimax robust Kalman filter is simply the average MSE of the $\theta_{\text{mm}}$-specific Kalman filter, i.e., $\text{Tr}\{E_\theta[P_{\theta,\theta_{\text{mm}}}(k)]\}$.

We analyze the performance of the proposed Kalman filtering framework by applying it to the problem of state estimation in sensor networks. In our simulations, we consider the model used for sensor scheduling in traffic monitoring and management [21]. The state $x$ of a moving vehicle in a 2-D space is determined by four components: $(p_x, p_y, v_x, v_y)^T$; $p_x$ (vehicle position in $x$), $p_y$ (vehicle position in $y$), $v_x$ (velocity in the $x$ direction), and $v_y$ (velocity in the $y$ direction). If two sensors are utilized to measure vehicle dynamics, the state-space model describing the vehicle dynamics has the following form:

$$
\Phi(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \Gamma(k) = \begin{bmatrix} h^2/2 & 0 & 0 & 0 \\ 0 & h^2/2 & 0 & 0 \\ 0 & 0 & h & 0 \\ 0 & 0 & 0 & h \end{bmatrix}, \quad H(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

In this model, $h$ denotes the discretization step size being set to 0.5 in our simulations. The covariance matrices for the process noise and the observation noise are

$$
Q(k) = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \quad R_{\theta}(k) = \begin{bmatrix} \theta & 0 \\ 0 & \theta \end{bmatrix}.
$$

The covariance matrix for the observation noise is assumed to be unknown, with unknown parameter $\theta \in [0.2, 6.2]$. The initial conditions are set to $\text{cov}(x(0)) = 12 \times I_4$, $I_4$ being the identity matrix of size 4, and $E[x(0)] = 0$.

We employ two different priors: uniform prior and beta prior $B(0.1, 0.9)$, scaled over the interval $[0.2, 6.2]$. Figure 1 presents the average steady-state MSE for the IBR, minimax, and model-specific Kalman filters, given by $\text{Tr}\{E_\theta[P_{\theta}(k)]\}$, $\text{Tr}\{E_\theta[P_{\theta,\theta_{\text{mm}}}(k)]\}$, and $\text{Tr}\{E_\theta[P_{\theta,\theta_{\text{mm}}}(k)]\}$, respectively. To reach the steady-state, $k$ should be large enough (approximately $k \geq 10$). Using (20), the minimax robust Kalman filter corresponds to $\theta_{\text{mm}} = 6.2$. Note that as the minimax approach does not take into account the prior distribution, it does not change for different priors. As can be observed from the figure, the minimum average steady-state MSE is achieved by applying the IBR Kalman filter for both priors. It has been understood that the minimax approach performs poorly for nonuniform priors with low probability for extreme states. This is the case here where the difference between the performances of the minimax and IBR Kalman filters is larger for the beta prior in Figure 1 (b).

In Figure 2, we show the average MSE obtained for the minimax and IBR Kalman filters over time. It can be seen that the average MSE for the IBR Kalman filter is always lower than the one obtained by the minimax robust Kalman filter regardless of the number of observations used for estimation.

Figure 3 analyzes the steady-state performance of various Kalman filters over the uncertainty class for each prior. Figure 3 (b) shows the MSE surface when the observation noise variance $\theta'$ assumed to design the model-specific Kalman filter differs from the true value $\theta$. Figures 3 (c) and (d) illustrate performance of the IBR and minimax robust Kalman filters over the uncertainty class of observation noise variance for uniform and beta priors shown in Figure 3 (a). The lower bound for MSE is determined by the $\theta$-specific Kalman filter. Note that the IBR Kalman filter does not perform optimally for all $\theta$ values and sometimes even the minimax filter outperforms it. This observation is normal because the IBR filter is optimal relative to the prior and performs well in the regions with large prior probability mass. For example, as can be seen in Figure 3 (d), when $\theta < 2$, which has most of the prior probability concentration, the IBR Kalman filter performs close to the $\theta$-specific Kalman filter and much better than the minimax robust Kalman filter.
Figure 3: Performance analysis of different Kalman filter designs over the uncertainty class. (a) Different priors considered for the unknown observation noise variance $\theta$. (b) MSE surface that shows the performance of the $\theta'$-specific Kalman filter when it is applied to the model with parameter $\theta$. (c) MSE performance of various Kalman filters over the uncertainty class when the prior is uniform. (d) MSE performance of various Kalman filters over the uncertainty class for the beta prior.

V. Conclusion

In this paper, we have proposed a general framework for finding intrinsically Bayesian robust (IBR) Kalman filters. Introducing the notion of Bayesian innovation process, the problem of finding an IBR Kalman filter can be treated similarly to the problem of finding a model-specific Kalman filter. The proposed IBR Kalman filtering is guaranteed to achieve the best average performance relative to the prior and has reliable performance in the regions of the uncertainty class with high prior probability. Finally, it is worth mentioning that using IBR Kalman filter, one can develop an objective-based experimental design framework to guide experiments aimed at reducing model uncertainty pertinent to objective, herein filtering performance. A similar objective-based approach has been taken for experimental design in gene regulatory networks [22], [23], where the network topology is incomplete and the objective is to reduce the undesirable steady-state mass of the network.

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