Digital Piracy: A Competitive Analysis

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In recent years, the issue of copyright protection for intellectual properties such as computer software, music CDs, and videos has become increasingly important. It is often claimed that illegal copying of intellectual property costs companies billions of dollars in lost revenues and reduces firms’ incentives to innovate. Some researchers have shown that copying can be beneficial to firms when there are strong network effects and copying expands the market. In this paper, we first examine the impact of illegal copying of software and other similar intellectual properties on firms’ prices, profits, and quality choices, even when there are no network effects and the market is saturated. We show that contrary to the claims of manufacturers, there are conditions under which copying can increase firms’ profits, lead to better quality products, and increase social welfare. This is because weaker copyright protection enables firms to reduce price competition by allowing price-sensitive consumers to copy. Thus, weaker copyright protection can serve as a coordination device to reduce price competition. We also examine how equilibrium copyright enforcement is affected by network externalities. In contrast to previous research, we show that strong network effects can sometimes lead to a firm choosing higher levels of copyright protection. Our results show that in the presence of strong network effects, stronger copyright enforcement by one firm can serve as a coordinating device to reduce price competition.

Key words: piracy; pricing; innovation; network effects; game theory

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1. Introduction

Illegal reproduction of intellectual properties such as computer software, books, music CDs, videos, etc. is increasingly becoming a major concern for both industry practitioners and the government. The 2005 Global Software Piracy Report commissioned by the Business Software Alliance estimates that 35% of software is pirated, which leads to an estimated loss of more than $31 billion to firms. Software piracy rates are as high as 92% in Vietnam and 90% in China. In the United States, piracy rates are estimated to be 21%. The study concludes that software piracy is one of the industry’s worst problems. However, copying of intellectual property is not limited to software. In fact, piracy of music and movies is a major concern to the entertainment industry, and some researchers have suggested that it will lead to radical changes in the industry (Moul 2006). The losses because of music piracy alone are estimated to be more than $10 billion (Murphy 2003). With the growth of the Internet, piracy is becoming even more prevalent because copying of intellectual properties is becoming easier and more difficult to prevent. For example, peer-to-peer networks such as Kazaa, BitTorrent, and others can enable consumers to illegally download a variety of software, music, and movies for free.

Many industry analysts see piracy as one of the key threats to profitability and innovation. They claim that piracy leads to higher prices for legitimate users, lower profits for the firms, reduced new product innovation, and is generally harmful to society (see, for example, BSA 2004). A 2003 survey of Business Software Alliance company’s CEOs reveals that they believe that piracy is the number one hindrance to software innovation. To deter piracy, industry associations such as the Recording Industry Association of America are beginning to aggressively prosecute pirates. Some companies have also tried to combat piracy by making their products more difficult to copy using digital rights management software. For example, Intuit incorporated a feature in its 2003 TurboTax such that the software could only be installed on one computer and Windows XP has an activation code.

Previous research has examined the role of copying and its impact on profits. This research has tried to identify conditions under which copying can lead to higher profits for the firm and the optimal level of copyright protection (e.g., Conner and Rumelt 1991, Givon et al. 1995, Hui and Png 2003). The general conclusion of prior research is that piracy could be beneficial if there are strong network effects, and some consumers who pirate would not have bought the product in any case. The intuition for the result is that network effects can significantly increase firm’s market size and the consumers who copy the product therefore can benefit the firm by increasing the...
size of the potential market. Although direct network effects may be important for many software products, as markets for digital products become more mature, the potential for piracy to increase market size will be limited (see also Li 2005).

In this paper, we reexamine conventional wisdom, that piracy is necessarily harmful in situations where network effects are weak and all pirates would buy absent the ability to copy. We also reexamine the intuition that strong network effects lead to weaker incentives for firms to impose copyright protection. Furthermore, we also study the impact of copying on the innovation process.\(^1\) Intuition would suggest that piracy reduces the incentives for innovation and would necessarily lead to lower quality products. Our research attempts to formally address how piracy impacts the level of innovation by firms and how firms should choose copyright protection levels taking these into account. The answers to these questions are important from both managerial and public policy perspective. To the extent that firms’ policies can impact piracy levels, it is important for the firms to understand how it should manage copyright protection. Furthermore, our results can provide insights into the pricing and R&D choices that firms should make. From a public policy perspective, understanding the impact of piracy on social welfare is critical for the government to develop sound copyright enforcement policies.

To address these issues, we develop a game-theoretic model in which there are two firms in the market. The firms are on the opposite ends of the Hotelling line. The market consists of two types of consumers: the first type of consumers do not copy the product because they place much lower value on copies either because of the unavailability of support for pirated versions or the fact that they find copying morally unacceptable, and therefore do not consider pirated versions to be substitutes for the originals. The second group of consumers consists of those who might copy the product. These consumers consider pirated versions to be equal substitutes for the originals. However, their ability to copy would depend on the firm’s copyright protection policy. In particular, we assume that firm \(i\) can choose copyright protection level \(a_i\) such that only \(a_i\) proportion of these consumers can copy its product. The parameter \(a_i\) represents the level of piracy for firm \(i\)’s product.

Our results show that piracy can change the composition of the market and alter strategic interaction between firms. In particular, we show that piracy can lead to reduced price competition. This reduction in price competition can sometimes lead to higher profits for the firm when piracy increases. Our analysis of the impact of piracy on innovation strategy shows that piracy can sometimes lead to higher levels of innovation and improved social welfare. We show that even with absent network effects, firms can choose weak copyright protection in equilibrium. This is because weak copyright enforcement enables firms to coordinate to charge higher prices from the segment of consumers that does not copy. We then examine the impact of network effects on incentives to choose copyright protection. In contrast to the prior literature, we show that in mature markets when network externalities are strong, copyright enforcement can help reduce price competition. In particular, we show that as network externalities become stronger, one firm begins to more strongly enforce copyright protection. These results therefore show that strong network externalities are neither necessary nor sufficient for firms to choose weak copyright protection. Our results suggest that the role of copyright protection depends on the strength of network externalities. Absent network externalities, weak copyright protection, can lead to reduced price competition while the opposite may hold in the case of strong network externalities.

The rest of the paper is organized as follows. In §2, we discuss the relevant literature. In §3, we describe our model and examine the implications of copying on firm’s prices, innovation levels, and profits. In §§4 and 5, we extend the model to consider various cases to see whether our model is robust to changes in assumptions. Section 6 concludes the paper with a discussion of the implications of the main results and directions for future research.

### 2. Related Literature

Previous researchers have examined how piracy can affect firms’ profits. In a monopoly setting, Conner and Rumelt (1991) examine the impact of copying software on firms’ profits. Their models assume that copying by individuals confers a network externality that benefits users of the product. In other words, as the number of users (which includes people who copy the product) increases, the value that consumers place on the product increases (see Katz and Shapiro 1985 for a discussion of network externalities). Thus, some consumers are willing to pay more for the product when the product is copied. Their results suggest that if network effects are large, copying can increase firms’ profits by increasing the market size. Takeyama (1994) also shows that copying can lead to higher profits and social welfare in the presence of network externalities, but only when copying increases the market size.

\(^1\) In this paper, we use the terms piracy and copying interchangeably.
Shy and Thisse (1999) extend the monopoly results of Conner and Rumelt (1991) and Takeyama (1994) to a duopoly framework. In their model, firms sell to high-valuation consumers who do not copy and low-valuation consumers who potentially can copy. They consider a scenario in which the low-valuation consumers have sufficiently low reservation prices such that only a fraction of these consumers enter the market. In their Hotelling model, this implies that in their formulation, firms do not compete for the low-valuation consumers because the marginal consumer is indifferent between buying the software or not adopting the software at all (see also Peitz 2004). Thus, in their framework, firms allowing piracy increases the market size. Their results show that firms can benefit by not protecting their software (thereby enabling piracy) if network effects are strong. We also consider a duopoly framework and find that their results can be significantly altered if the low-valuation consumers are more price sensitive, but have sufficiently high valuations such that firms compete to sell to these consumers. We show that when the market is saturated and there is limited opportunity for market growth, higher network effects can actually lead to higher levels of copyright protection. This is because when firms compete for the low-valuation consumers, as they do in our framework, allowing piracy by both firms can intensify price competition. Therefore, in such situations, strict copyright enforcement by one firm can serve as a coordination device to reduce price competition.

Givon et al. (1995) empirically demonstrate that piracy can help diffusion of legal copies. By using data for spreadsheets and word processing software sales in the United Kingdom, they find that piracy helped the diffusion of software and 80% of new software purchases were influenced by pirates. Hui and Png (2003) show that if we consider the positive network effects of piracy, then industry estimates of lost profits because of piracy more than doubled the actual losses.2

In an independent research, Gu and Mahajan (2005) also examine how piracy affects firms’ profits in the presence of competition. Their result shows that piracy can lead to reduced price competition, and can be beneficial to firms when markets have high wealth gaps. In our paper, we also examine the competitive effect, but our analysis is different than theirs in many significant ways. First, unlike their paper, we endogenize copyright protection. Second, we allow firms to set innovation levels, while they do not. Finally, we also consider the presence of network effects and how equilibrium copyright protection is affected by network effects. We show that the presence of network effects can critically impact the role of copyright protection in reducing price competition.

Our work on the impact of copying on innovation is also related to the vast literature on intellectual property rights (see, for example, Arrow 1962, Novos and Waldman 1984, Varian 2005). This literature attempts to determine the optimal breadth and length of patents to ensure that firms have sufficient incentives to invest in innovation. For example, Novos and Waldman (1984) show that lower profits because of copying can reduce the incentive to provide quality. Johnson (1985) shows that copying can reduce firms’ incentives to provide variety.

There is also research that examines copying by firms. Conner (1995) shows that in the presence of network effects, a monopolist might benefit from entry by a clone. Using a diffusion-based model, Xie and Sirbu (1995) show that the presence of network effects can encourage firms to share technology, so that products diffuse faster and a monopolist may therefore prefer entry by a compatible competitor. Purohit (1994) shows that the presence of clones can sometimes lead to higher quality products. Sun et al. (2004) show that the presence of strong network effects can make it optimal for a firm to freely license its product.

Our work differs from the previous literature in significant ways. First, unlike most of the prior literature on copying by consumers, we develop a duopoly model and study how the presence of competition can affect firms’ preferences for copyright enforcement. Modeling competition in the context that we study is important because most digital products compete with other products. For example, Microsoft Money competes with Intuit’s Quicken, McAfee’s virus protection program competes with Symantec’s Norton Antivirus, Word competes with WordPerfect, etc. Even for entertainment software, movies, and music, firms compete to the extent that consumers have limited time and money, and usually choose among the available alternatives. Our research also extends previous research in that we allow firms to not only set prices but also to determine innovation levels in the presence of copying. Our research shows that the presence of competition can significantly alter

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2 Another stream of research shows that if there are no network effects but firms are able to price discriminate, then copying can lead to higher profits (Liebowitz 1985, Besen and Kirby 1989). For example, publishers of books can charge different subscription rates from individuals and libraries. The producer can then charge the libraries an amount proportional to the amount of copying that occurs in the library. Thus the producer can indirectly appropriate revenues from users who copy. In the context that we study, firms have little ability to appropriate revenues from the consumers who purchase the original product and are the source of the copies. Furthermore, this research assumes that copies cannot be made from copies, which is clearly not true in the case of digital products. Thus, this research is only tangentially related to the context that we study.
results from prior research. In particular, we show that piracy can benefit firms even without strong network effects, and both the firms and society may be better off with higher levels of piracy. We also show that in some situations, strong network effects can lead to a more stringent copyright enforcement by some firms.

3. Model

We first consider a monopolist selling to differentiated consumers. This analysis serves as a benchmark for the subsequent duopoly analysis. We assume that the monopolist (firm 1) is at one end of the Hotelling line, in particular, at 0.5

We assume that there are two segments of consumers. The first segment of consumers places a low value on the pirated version of the software, and therefore do not copy the product. This segment could potentially consist of large organizations and professionals. For these consumers, the availability of manuals and support (which is only available for nonpirated versions) is valuable (see, for example, Shy and Thisse 1999). Alternatively, these consumers may consider pirating to be morally unacceptable, and therefore do not find copies to be acceptable substitutes for legal software (see, for example, Swinyard et al. 1990, Gopal and Sanders 1997, Bagchi et al. 2006). The other segment of consumers view the pirated version to be the same as the original, and therefore might copy the product. For these consumers, support may be less of an issue and they also do not find copying to be as morally objectionable. However, their ability to copy would depend on the copyright enforcement policies of the firm.

3.1. Noncopiers

We assume that noncopiers have heterogenous preferences. In particular, these consumers are distributed on the Hotelling line according to a continuously differentiable distribution \( f(\cdot) \). The reservation price that a segment 1 consumer has for the monopolist’s product is given by

\[ R_1^n(\theta) = v + q_1 - \theta, \]

where \( \theta \) is the location of the consumer, \( v \) is the base quality, and \( q_1 \) is the quality improvement offered (over the base) by the firm. Note that \( q_1 \) can be interpreted as the quality of an upgrade over the initial product that the firm had when it entered the market.4 We assume that consumers are aware of the quality of the product.5

\[ \theta \] Consumer location.
\[ f(\cdot) \] The pdf for the distribution of \( \theta \).
\[ F(\cdot) \] The cdf for the distribution of \( \theta \).
\[ v \] Base valuation without additional R&D.
\[ q_1 \] Quality chosen by firm 1.
\[ \beta \] The level of the potential copier segment.
\[ \alpha_i \] The level of piracy permitted by firm \( i \).
\[ p_i \] Price charged by firm \( i \).
\[ k \] The R&D cost parameter.
\[ D_i(\cdot) \] Demand for firm \( i \) in the potential copier segment.
\[ D_{nc}(\cdot) \] Demand for firm \( i \) in the noncopier segment.
\[ \Pi_{i1}(\cdot) \] Profit function for firm 1 at the second stage where firms decide prices (given \( q_1 \) and \( \alpha_i \)).
\[ \Pi_{i2}(\cdot) \] Profit function for firm 1 at the third stage where firms decide quality (given \( q_1 \)).
\[ \Pi_{i1}(\cdot) \] Profit function for firm 1 at the second stage where firms decide quality (given \( q_1 \)).
\[ \Pi_{i1}(\cdot) \] Profit function for firm 1 at the first stage where firms decide prices (given \( q_1 \) and \( \alpha_i \)).
\[ \gamma \] The network externality parameter.
\[ z_i^n \] The expected network size for firm \( i \).
\[ \eta \] Index of market competitiveness.

\[ \text{The consumer surplus that a noncopier consumer at } \theta \text{ gets from buying the product 1 is given by } \]
\[ CS_1^n(p_1) = R_1^n(\theta) - p_1, \]

where \( p_1 \) is the price for product 1. We assume that a consumer buys the product as long as \( CS_1^n(p_1) \geq 0 \). Furthermore, we assume that each consumer only purchases one product. It is easy to see that the sales of firm 1 in this segment is given by

\[ x_1^n = \begin{cases} 
1 & \text{if } p_1 \leq v + q_1 - 1 \\
F(v + q_1 - p_1) & \text{otherwise.}
\end{cases} \]

3.2. Potential Copiers

We assume that there is another segment of consumers who could copy. These consumers find the quality of the original and the copy to be the same. We assume that the size of this segment is given by \( \beta.6 \) Note that in our formulation, each consumer buys one unit. Thus the parameter \( \beta \) should not be interpreted in terms of the number of customers but the size of the sale. Therefore, \( \beta \) may be smaller or larger than 1.

We assume that while all potential copiers could copy, their ability to do so would depend on the company’s efforts to protect their product. In particular, we assume that the company can affect the level

\[ \begin{align*}
3 & \text{The result for a monopoly does not, however, depend on the location of firm 1.}
4 & \text{Table 1 summarizes the model notation.}
5 & \text{In many cases, consumers may be unaware of the quality and use the pirated version of products such as movies and music to judge}
6 & \text{The parameter } \beta \text{ could be affected by the exogenous factors such as government policies on copyright enforcement.}
\end{align*} \]
of piracy by using preventive measures such as the use of nonstandard disks, hardware locks, software-based protection, and forced activation (Gopal and Sanders 1997). This assumption captures the empirical fact that some consumers copy one software purchase another software. In other words, while these consumers are predisposed to copying, a firm can affect their ability to do so, leading to the possibility that not all consumers in this segment copy. We assume that if firm i chooses a protection level \( \alpha_i \), then there is a probability \( \alpha_i \) that a consumer in this segment will have zero copying costs for copying a particular product, and probability \( 1 - \alpha_i \) that the costs will be so high that it will not be profitable to copy the product. The two-point cost distribution that we use is widely used in the literature in various contexts such as price promotions (see, for example, Shaffer and Zhang 1995), price matching refunds (see, for example, Png and Hirshleifer 1987, Jain and Srivastava 2000), advertising (see, for example, Zettelmeyer 2000), and others. A more general formulation would allow for a distribution of costs. This would substantially complicate the analysis without affecting the basic arguments in the paper. In Technical Appendix A (available at http://mktsci.pubs.informs.org), we develop a model in which consumers have a distribution of costs and show that the basic nature of our results remain true even in this more complicated scenario.

The reservation price for product 1 for a consumer at \( \theta \) is given by \( R_1^1(\theta) \). Prior empirical work on copying suggests that consumers who copy have, in general, a lower willingness to pay and are more price sensitive (see, for example, Cheng et al. 1997). To incorporate this in our model, we assume that

\[
R_1^1(\theta) = \delta R_1^1(\theta) = \delta(v + q_i - \theta),
\]

where \( 0 < \delta < 1 \). Note that the assumption that these consumers have lower transportation costs (and therefore are more price sensitive) can be justified by the fact that, in general, piracy is inversely related to income (Cheng et al. 1997, Bagchi et al. 2006). Also, note that we are assuming that these consumers discount \( v \) and the transportation costs \( \theta \) by the same factor \( \delta \). In an alternative formulation, we could have

\[
R_1^!1(\theta) = \delta_1(v + q_i) - \delta_2 \theta,
\]

where \( \delta_1 \) and \( \delta_2 \) are different. Our results would go through even with this alternate assumption. We make the assumption that \( \delta_1 = \delta_2 \) for notational simplicity. Note that the assumption \( R_1^1(\theta) < R_1^2(\theta) \) does not imply that all potential copiers have lower reservation prices than the noncopiers. For example, it is possible that \( R_1^1(0) > R_1^2(1) \). Thus the assumption only implies that on average noncopiers have higher reservation prices than the potential copiers.

As we discussed before, the parameter \( \alpha_i \) represents the level of piracy in the market. Using this formulation, the total sales for product 1 in this segment is given by

\[
x^i_1 = \begin{cases} 
\beta(1 - \alpha_i) & \text{if } p_1 \leq \delta(v + q_i) - \delta \\
\beta(1 - \alpha_i)\frac{(v + q_i) - p_1}{\delta} & \text{otherwise}.
\end{cases}
\]

Without any loss in generality, we assume that marginal costs for the product is zero. This assumption is also reasonable in the contexts that we are studying. The sequence of decision is as follows. In the first stage, the firm decides on the copyright enforcement policy. In the second stage, the firm decides the quality level \( q_i \). We assume that it costs firm a fixed amount \( k_{q_i}^2/2 \) to add quality improvement of \( q_i \), where \( k > 0 \) is sufficiently large, so that the profit function is concave. After choosing the quality level, the firm sets the price in the third stage after which consumers decide whether to buy, copy, or not to use the product.

Therefore, firm 1’s profit function in the third stage is given by\(^7\)

\[
\Pi_{13}(p_1; q_i) = (x^{i*}_1 + x^i_1)p_1.
\]

The firm selects the optimal \( p_1^* \) and then selects \( q_i^* \) by maximizing

\[
\Pi_{12} = \max_{q_i} \left\{ \Pi_{13}(q_i) - k_{q_i}^2/2 \right\}.
\]

Finally, the firm decides on the optimal copyright protection level by choosing \( \alpha_i \) to maximize \( \Pi_{11}(\alpha_i) \). In general, a firm would incur some costs to reduce piracy. This could lead to firms choosing less than perfect protection. However, in this paper, we are interested in examining whether a firm would prefer lower copy protection even absent such cost considerations. Therefore we assume that the cost of copyright protection is almost zero. In particular, a firm would prefer lower copyright protection if the profits are the same at different levels of \( \alpha_i \), absent copyright protection costs.\(^8\) The main insights of the paper continue to hold in the case where copyright protection costs are significant (see discussion on p. 18 and footnote 18).

**Proposition 1.** The monopolist chooses \( \alpha_i = 0 \). Furthermore, if \( v \) is sufficiently large, then the firm produces a higher quality product, and social welfare is higher under perfect copyright protection as opposed to no copyright protection.

\(^7\) In a monopoly, it does not matter whether the firm sequentially decides on quality and price. However, in a duopoly, the assumption does matter. The sequential structure is reasonable given that product quality is more difficult to change than the prices.

\(^8\) The assumption of almost zero costs is only used for equilibrium selection.
Proofs of all propositions are in the appendix. This benchmark proposition shows that under the setup that we have chosen, piracy is invariably bad for the firm and society. Consequently, the firm would prefer to impose perfect copyright protection and curb all piracy. Furthermore, if \( v \) is large enough so that all consumers are served, then an increase in \( \alpha_i \) leads to the firm investing less in R&D.\(^9\) Consequently, the firm produces a lower quality product under higher levels of piracy. Also, when \( \alpha \) is sufficiently large, a monopolist produces an inefficient level of quality (from a social welfare perspective). As the level of piracy increases, the problem of underprovision of quality is further exacerbated. Thus, social welfare is reduced as \( \alpha \) increases. Therefore, if the firm can control piracy, then this proposition shows that under no piracy, the firm produces a higher quality product, makes higher profits, and social welfare is higher. This proposition thus lends some credence to the advocates of strong copyright protection, in markets where network externalities are not strong. We now extend the analysis by considering a situation in which there are two firms in the market.

4. Duopoly

We now assume that there is a second firm in the market at the other end of the Hotelling line, i.e., at 1. The other assumptions are very similar to those of the monopoly case. Thus the reservation price for firm 2’s product from the noncopier segment is given by

\[
R_2^*(\theta) = v + q_2 - (1 - \theta),
\]

where \( q_2 \) is the quality improvement offered by firm 2. Analogously, the reservation price for firm 2’s product from the potential copier segment is

\[
R_2^*(\delta) = \delta R_2^*(\theta), \quad \text{where } 0 < \delta < 1.
\]

We assume that \( \delta \) is sufficiently high that the firm would prefer to cater to the potential copier segment. We also assume that \( v \) is sufficiently high and the competition is sufficiently intense that the market is fully covered in a duopoly.\(^{10}\) This assumption is standard in Hotelling models because it considerably simplifies the analysis (see, for example, Shaffer and Zhang 1995). However, in our context, this assumption is particularly useful because it replicates conditions under which a monopolist firm would never prefer piracy and would produce lower quality products under piracy (see Proposition 1). This enables us to highlight how the presence of competition might change the results. However, we show later that the qualitative nature of our results is valid even if the total market size is not fixed and some consumers do not buy.

We also impose some additional restrictions on the value distribution function \( f(\cdot) \). In particular, we assume that \( f(\cdot) \) is a symmetric differentiable function with support \((0, 1)\). In other words, we assume that \( f'(\cdot) \) exists \( \forall x \in (0, 1) \) and \( f(0.5 - x) = f(0.5 + x), \forall x \in (0, 0.5) \).

As before, each firm decides on the level of copyright protection \( \alpha_i, i = 1, 2 \). Thus \( \beta(1 - \alpha_1)(1 - \alpha_2) \) consumers in the copying segment cannot copy any product while \( \beta \alpha_2 \alpha_2 \) can make copies of either product, and will therefore copy a product. Furthermore, \( \beta \alpha_1(1 - \alpha_1) \) consumers can copy product \( i \), but not copy product \( j \). We start the analysis under the assumption that these consumers prefer to copy product \( i \) rather than purchase product \( j \).\(^{11}\)

With this setup, we can determine the demand for firms 1 and 2 for given prices. Focusing on interior solutions, the demand for firm 1 is given by

\[
x_1 = F(\theta_1^i) + \beta(1 - \alpha_1)(1 - \alpha_2)F(\theta_1^j),
\]

where \( \theta_1^i \) is the consumer in the noncopier segment who is indifferent between 1 and 2. \( \theta_1^j \) is analogously defined for the potential copier segment. These can be shown to be

\[
\theta_1^i = \frac{1 + q_1 - q_2 + p_2 - p_1}{2},
\]

\[
\theta_1^j = \frac{\delta + \delta(q_1 - q_2) + p_2 - p_1}{2\delta}.
\]

As in the monopoly case, in the first stage, the firms decide on the piracy levels \( \alpha_1, \alpha_2 \). In the second stage, the firms decide on quality improvement levels \( q_1 \) and \( q_2 \) at a fixed cost \( kq_1^2/2 \). In the third stage, the firms decide on prices given quality levels \( q_1 \) and \( q_2 \). Finally, consumers make their purchase or copying decisions. We will look for the symmetric subgame-perfect equilibrium of the game, and therefore first consider the third stage of the game. The third stage profit function for firm 1 (denoted by \( \Pi_{13} \)) is given by

\[
\Pi_{13} = p_1[F(\theta_1^i) + \beta(1 - \alpha_1)(1 - \alpha_2)F(\theta_1^j)],
\]

\(^9\) For example, consider the case when \( f(\cdot) \) is uniform with range \((0, 1)\) and \( \beta = 1, \alpha = 2, v = 2, \delta = 0.6 \), then the firm would produce all the consumers. In this case, the firm would produce a quality level of 0.6 and charge a price 0.96 without piracy and set quality level 0.3 and price 0.78 with \( \alpha = 1 \). Also, note that the full market coverage assumption is required only for the second part of the proposition.

\(^{10}\) If \( f(\cdot) \) is uniform with range \((0, 1)\) and \( \beta = 1 \), then a sufficient condition for these to hold is that \( \delta > -(5 + 4\sqrt{2})/7 \approx 0.098 \) and \( v > 0.5 + 1/\delta \).

\(^{11}\) In any symmetric equilibrium with \( q_1 = q_2 \), this would hold if \( \theta \sim U(0, 1) \) and \( \delta < 1 \), as we show in the appendix. In cases where we use a specific distribution for \( f(\cdot) \), we explicitly check this condition—see the proofs of Propositions 5–8 in the appendix.
where we assume that the marginal costs for both firms are zero. We consider the symmetric case when $\alpha_1 = \alpha_2 = \alpha$ and $q_1 = q_2 = q$. The relevant first-order condition for firm 1 is given by

$$\left[ -\frac{f(\theta_1^*)}{2} - \beta (1-\alpha_1)(1-\alpha_2) f(\theta_1^*) \right] p_1 + [F(\theta_1^*) + \beta (1-\alpha_1)(1-\alpha_2) F(\theta_1^*)] = 0. \quad (15)$$

Using symmetry, the prices are given by (see the appendix for details)

$$p_1^* = \frac{\delta [1 + \beta (1-\alpha)^2]}{f(1/2)[\delta + \beta (1-\alpha)^2]}.$$

Note that the optimal price is lower as $f(1/2)$ increases. Intuitively, as the distribution shifts more toward the center, price competition is increased and prices are reduced. Also, note that as $\delta$ decreases, prices decrease. This is also intuitive since as $\delta$ decreases, the potential copier segment becomes more price sensitive. The proposition below examines the impact of copying on prices and profits.

**PROPOSITION 2.** In the symmetric case, where $\alpha_1 = \alpha_2 = \alpha$ and $q_1 = q_2 = q$, then firms’ prices are higher as $\alpha$ increases. Furthermore, both firms make higher profits as $\alpha$ increases if

$$\alpha > 1 - \sqrt{\frac{1 - 2\delta}{\beta}}. \quad (17)$$

Firms’ profits are higher under no copyright protection ($\alpha = 1$) as opposed to the case when firms enforce copyright protection if

$$\delta < \frac{1}{2 + \beta}. \quad (18)$$

The proposition examines the situation in which both firms have chosen the same levels of copyright protection and quality. The results show that it is possible for both firms to be better off with no copyright protection even when there are no network effects.

To understand the intuition for the proposition, first note that as copyright protection level decreases, the number of price-sensitive consumers who are willing to purchase the product also decreases. Thus, firms are less motivated to reduce prices to attract the price-sensitive segment of the market. In other words, copying reduces price competition between the two firms. To understand the profit implications, first note that using the envelope theorem, we have

$$\frac{d\Pi_{13}}{d\alpha} = \frac{\partial \Pi_{13}}{\partial \alpha} + \frac{\partial \Pi_{13}}{\partial p_2} \cdot \frac{\partial p_2}{\partial \alpha}. \quad (19)$$

The first term represents the direct effect of copying on firm profits. This term does not take into account the effect of $\alpha$ on competition and only considers own direct effects. In other words, if there were no competitors, this term would determine the impact of copying on firm profits. We label this term as the monopoly effect. Note that from Proposition 1, we know that the monopoly effect must be negative. This is because copying reduces sales. To understand the complete picture, we also need to consider the strategic effect, which is captured in the second term in (19). As indicated in the first part of the proposition, copying also reduces price competition, i.e., $\partial p_2^* / \partial \alpha > 0$. Since, in general, $\partial \Pi_{13} / \partial p_2 > 0$, the second term in (19), i.e., the strategic effect is positive and increases profits. The total effect would, in general, depend on $\delta$, i.e., the price sensitivity of the copier segment and the level of copying, i.e., $\alpha$. It is also important to note that while the monopoly effect is centered on lost sales from the potential copier segment only, the strategic effect impacts profits from both the segments. This is because the overall price increase leads to higher profits from noncopiers and higher profits from the consumers (who actually purchase in the potential copier segment).

The proposition shows that under the conditions specified in (17), the strategic effect dominates the monopoly effect, and the firm’s profits are higher under increased levels of copying.\(^{12}\) The proposition also shows that if $\delta$ is sufficiently small, i.e., the copier segment is sufficiently price sensitive, firms are better off with no copyright protection as opposed to complete protection. Note that the condition in (18) implies that if there is a large noncopier segment, i.e., $\beta$ is large, then the monopoly effect would not be offset by the strategic effect and the firms are better off with complete copyright protection. On the other hand, if the copier segments are sufficiently price sensitive, we could have situations in which both firms are better off with loose copyright protection. Note, however, that this does not imply that firms will choose no copyright protection, because although $\alpha = 1$ is better for both firms, we could encounter a prisoner dilemma and firms might end up choosing some copyright protection. Of course, because firms choose the level of copyright protection in our framework, we will need to explore whether no copyright protection is indeed an equilibrium strategy under the condition specified in (18). We will explore this in Proposition 4.

It is also important to note that these profit implications provide only an incomplete picture in that they do not consider the impact of piracy on innovation levels. Nevertheless, there are many situations in which such an analysis is appropriate. For example, a company such as Symantec sells its software

\(^{12}\) Cabral and Villas-Boas (2005) refer to this as the Bertrand Supertrap. See their paper for other examples of this phenomenon.
in the global market and can charge different prices in different local markets. However, the innovation decisions are based on the global market and may not be influenced by piracy levels in small markets. For example, while Symantec could alter its prices in response to the 92% piracy rates in Vietnam, it is not clear that this will have a substantial impact on the R&D levels. Of course, the U.S. piracy rates of 21% may indeed affect Symantec’s R&D decisions.

Now, we consider the second stage of the game in which the firms independently decide on quality levels. The problem for firm 1 is therefore to choose $q_1(q_2)$, which is defined as

$$q_1(q_2) = \arg \max_{q_1} \left[ \Pi_{12}(q_1, q_2) - \frac{kq_1^2}{2} \right]. \tag{20}$$

Firm 2 analogously determines $q_2(q_1)$, and we look for the symmetric Nash equilibrium for the game. We first have the following result:

**Proposition 3.** If firms choose $\alpha_1 = \alpha_2 = \alpha$, then in the symmetric equilibrium firms produce a higher quality product and charge a higher price as $\alpha$ increases if and only if (17) holds. Furthermore, if $k$ is sufficiently large and (18) holds, then firms are better off with no copyright protection.

This result shows that firms may spend more on innovation and make higher profits under piracy, even when there are no network effects and the full market is served without copying. To understand this result, note that $\alpha$ affects the incentives to innovate in two different ways. First, an increase in $\alpha$ decreases the total number of buyers. This, in general, will tend to decrease the firms’ incentives to spend in R&D. However, an increase in $\alpha$ also changes the level of market competition. This effect tends to increase firms’ incentives to invest in R&D. The overall effect is such that firms produce a higher quality product as $\alpha$ increases if (17) holds. Recall, however, that under (17), the second-stage profits of the firm are higher when we ignore the R&D costs. However, when (17) holds, firms invest more in R&D and incur higher R&D costs, which tends to reduce profits. If $k$ is sufficiently large, then the negative impact of $\alpha$ through increased R&D is outweighed by the positive effect of $\alpha$ on reduction of price competition. Thus, for sufficiently large $k$, firms produce a higher quality product, charge higher prices, and make higher profits when (17) holds.

It is also useful to examine the case when (17) does not hold. In this situation, the firm always produces a lower quality product as $\alpha$ increases. However, even in this situation, it is possible for the firms to be better off with copying. To see this, recall that there are three forces that determine the final impact of piracy on profits. First, piracy reduces price competition. Second, piracy leads to lost sales, and finally piracy affects R&D expenditures. If (17) does not hold, then the negative effect of lost sales is greater than the positive effect of reduced price competition. However, once we consider R&D, if (17) does not hold, then as $\alpha$ increases, firms spend less on R&D and save developmental costs. Thus the overall impact is ambiguous. Nevertheless, once we consider R&D, it is possible that firms’ profits increase with $\alpha$ even if (17) does not hold.

Now, we analyze the first stage of the game in which both firms independently choose the level of copyright protection for their software. We have the following result:

**Proposition 4.** If (18) holds, then both firms choose no copyright protection, i.e., $\alpha_1 = \alpha_2 = 1$. Furthermore, firm profits and social welfare are higher under no copyright protection than under complete copyright protection.

The proposition is the polar opposite of the result in the benchmark Proposition 1 with a monopolist firm. The proposition establishes that under conditions specified in (18), in equilibrium, firms choose no copyright protection. This result also implies that there are conditions in which copyright protection may not be desirable even when there are no network effects and it is costless for firms to impose copyright protection. This points to the fact that some of our intuition from the monopoly situation may not hold true when we consider competition. The proposition also shows that in situations where firms are better off when they do not serve a price-sensitive segment, but are unable to commit to not serving (because of competitive reasons), the presence of piracy can solve the commitment problem. In other words, firms can use weak copyright enforcement as a coordination device to reduce price competition and make higher profits.

It is also important to note that the proposition is derived under the assumption that copyright enforcement is costless. In practice, this is not the case. Indeed, perfect copyright protection is likely to be prohibitively costly. This suggests that our results could be stronger if we allow for costly copyright protection. For example, consider the case where the cost of setting $\alpha_1$ for firm 1 is given by $-\xi \cdot \ln(\alpha_1)$. Note that this term is $\propto$ for $\alpha_1 = 0$ and $0$ for $\alpha_1 = 1$. Although it is difficult to analytically find the equilibrium $\alpha$, in this case, we can do so numerically. Suppose $\theta \sim U(0,1)$, $\xi = 0.1$, and $\beta = 1$. In this case, both firms are better off setting $\alpha = 1$ for $\delta \leq 0.81$, which is substantially weaker than the condition $\delta < 1/3$ implied by (18). Even for $\delta = 0.9$, firms set $\alpha_1 = 0.34$, while at $\delta = 1$, they set $\alpha_1 = 0.27$. In other words, in this case, if the firms were to ignore the strategic effect (and behave as if $\delta = 1$), then the firms would set
more stringent copyright protection. Thus although Proposition 4 shows that no copyright protection is beneficial to firms when (18) is satisfied, the strategic effect impacts the equilibrium copyright protection at all levels of \( \delta \). Thus the key insights in this paper do not require the full force of the assumption that (18) holds.\(^{13}\)

5. Model Extensions

In this section, we discuss the implications of relaxing some of the assumptions in our model. This helps us better assess the generality of our results and better understand the contexts in which our results apply.

5.1. Network Externalities

The base model assumes that there are no network effects. Because prior research has shown that piracy can only be beneficial when network effects are strong, it is useful to examine whether the same intuition holds in our framework. To accomplish this, we extend our basic formulation to allow for network externalities. In particular, we assume that consumers also derive a utility from the size of the network, which is the number of consumers using a product. We assume that the products are incompatible, so that the network effects are specific to each product. Let the surplus that a consumer at location \( \theta \) in the noncopier segment gets from buying product 1 be given by

\[
CS_1(\theta, \theta) = R_1(\theta) + \gamma z_1' - p_1,
\]

where \( R_1(\theta) \) is as defined before \( p_1 \) is the price, and \( \gamma z_1' \) represents the network effects.\(^{14}\) In particular, \( z_1' \) is the expected network size for product 1 and \( \gamma \) represents the intensity of network effects. Thus, as \( \gamma \) increases, network effects become more important. Note that the expected network size consists not only of sales of the product but also copies of the product. Thus, consumers who copy the product exert a positive externality on the utility of other consumers.

The consumer surplus for product 2 is analogously defined. The consumer in the noncopier segment who is indifferent between product 1 and product 2 is indexed by \( \theta_i \) and is given by

\[
\theta_i = \frac{1 + (q_1 - q_2) - (p_1 - p_2) + \gamma (z_1' - z_2')}{2}.
\]

Analogously, the consumer in the potential copier segment who is indifferent between the two firms is indexed by \( \theta_j \) and is given by\(^{15}\)

\[
\theta_j = \frac{\delta + (q_1 - q_2) + p_2 - p_1 + \gamma (z_1' - z_2')}{2\delta}.
\]

The consumers who copy also care about the network benefits and would take the expected network size into account while deciding which product to copy. The consumer who copies and is indifferent between copying 1 and 2 is indexed by \( \theta_k \), which is given by

\[
\theta_k = \frac{\delta + (q_1 - q_2) + \gamma (z_1' - z_2')}{2\delta}.
\]

To proceed, we make a simplifying assumption that firms choose \( \alpha_i \in [0, 1] \). This assumption restricts the strategy space in the first stage of the game and makes the analysis tractable. If we relax the assumption, closed-form solutions are difficult to obtain. However, as we discuss later, numerical simulations reveal that the basic nature of results is unchanged with this alternate and more complicated formulation. If both firms were to allow copying, then the network size for firm 1’s product, i.e., \( z_1 \) is given by

\[
z_1 = F(\theta_i) + F(\theta_j).
\]

Note that the \( \theta_i \)s in the above equation are a function of \( z_1' \) and \( z_2' \). We also impose the rational expectations condition

\[
\begin{align*}
z_1' &= z_1, \\
z_2' &= z_2.
\end{align*}
\]

As before, we assume that the market is fully covered. To solve this, we make some further simplifying assumptions. First, we assume that \( f(\cdot) \) is uniform with range \((0, 1)\) and \( \beta = 1 \). To ensure that a unique and stable rational expectations equilibrium exists, we also assume that \( \gamma < \delta/(1 + \delta) \). This condition ensures that the network effects are not so large that small changes in network sizes can lead to large shifts in consumer demand, leading to multiple equilibria in which only one of the firm survives. Also, we focus on equilibrium in which consumers expect that copiers would copy rather than buy (which we will show holds in equilibrium). Finally, in case of multiple equilibria, we assume that firms coordinate to the Pareto-dominant equilibrium.

\(^{13}\)The results on social welfare do depend on the assumption that copying is costless for consumers. If the cost of copying is greater than the distribution and production costs (which are also assumed to be zero in our framework), then the social welfare results will be weaker. However, if consumers’ cost of copying is not higher than the combined production and distribution costs, our result holds.

\(^{14}\)Note that we are modeling direct network effects as is common in literature. Other types of network effects include indirect network effects (e.g., Church and Gandal 1992) and cross-market network effects (e.g., Chen and Xie 2007).

\(^{15}\)In our formulation, we assume that noncopiers and potential copiers value network effects equally because there is no evidence to suggest that the valuations would differ. However, if we assume that the potential copiers value network benefits at \( \lambda_1 z_1' \), where \( 0 < \lambda < 1 \), then the results are similar.
As before, we first solve the last stage of the game in which firms make pricing decisions given the qualities $q_1$ and $q_2$. The firms solve for the equilibrium qualities given $P_1(q_1, q_2)$ and $P_2(q_1, q_2)$. Our interest in the analysis is to examine how network effects impact the results of the previous section. We have the following result:

**Proposition 5.** For $(-5 + 4\sqrt{2})/7 < \delta \leq \sqrt{8}/9 \approx 0.94$, $\gamma \geq \gamma_2$, and large $k$, in equilibrium, one firm allows copying and the other firm does not, where

$$
\gamma_2 = \left(\frac{7\delta + 15 - \sqrt{49\delta^2 + 224 - 1748}}{32}\right). \quad (28)
$$

Furthermore, the firm that allows copying produces a higher quality product, charges a higher price, and makes higher profits.\(^{16}\) However, if $\gamma \in (\gamma_1, \gamma_2)$, then both firms allow copying, where

$$
\gamma_1 = \frac{3(2 + 5\delta - \sqrt{12 + 48 + \delta^2})}{16(1 + \delta)}. \quad (29)
$$

The result thus shows that as the level of network effect increases, it is possible that some firms may choose to use stronger copyright protection. This result is in contrast to the standard argument in the literature, where network effects invariably discourage firms from using strong copyright protection. Furthermore, the result shows that while the firms are a priori symmetric, in equilibrium, they end up making different profits because of the strategic copyright protection choices.

To understand this result, first consider the case when $\delta = 1$ and there are no network effects. In this case, as we discussed before, piracy is only harmful in that it takes away sales from the potential copier segment. As $\gamma$ increases, a firm may find that piracy also leads to increased network size, and therefore increased ability to charge a higher price from the noncopier segment. This makes piracy attractive. Of course, since firms are symmetric and the market is of fixed size, this leads to more intense price competition, and we have a prisoner’s dilemma in which both firms end up allowing piracy, although not allowing piracy may be better. This is consistent with prior research, which has shown that firms allow more copying as network effects become stronger. Now, let us consider the case when $\delta$ is low. In this case, piracy reduces price competition, and as we have established before, can make it attractive for firms to allow some copying. However, note that an increase in $\gamma$ also makes piracy a less effective tool in reducing price competition. For lower values of $\gamma$, the utility enhancing effects are more important and firms prefer to have weaker copyright protection. However, for larger values of $\gamma$, the second effect becomes more important, and if one firm allows copying, it becomes attractive for the other firm to restrict copying, and therefore reduce price competition. For example, consider the case when $\gamma = 0.25$, the firms switch to an asymmetric solution. If firms were to set $\alpha = 1$, then they would each charge a price of $p_1^* = 0.5$ and make profits of 0.25. However, in this case, at least one firm would benefit by switching to $\alpha = 0$. This allows both the firms to increase their prices. The firm that allows copying (say, firm 1) sets $p_1^* = 0.83$, while the other firm sets $p_2^* = 0.67$. The profits of firm 1 go up to 0.463, while firm 2’s profits rise to 0.296. Note that the firm that imposes copyright protection is at a disadvantage relative to firm 1, because to reduce competition, it leaves the copier segment to firm 1. This enables firm 2 to charge higher prices (because it has a higher installed base) and this, in turn, enables firm 2 to charge higher prices.

The proposition thus shows an interesting role of copyright enforcement in the presence of network effects. Recall that absent network effects, weaker copyright protection, enables firms to commit to charge higher prices. Therefore, in this context, weaker copyright protection serves as a coordination mechanism to reduce price competition. In the presence of strong network effects, however, the role of copyright protection changes. In this case, weaker copyright protection leads to even more intense price competition. In this situation, stronger copyright protection enables firms to coordinate to reduce price competition.

Figure 1 shows how firms’ equilibrium copyright strategy changes as a function of $\delta$ and $\gamma$. Note that for large values of $\delta$, large network effects lead to weaker copyright enforcement. This is consistent with prior research. However, as $\delta$ decreases, and the strategic effect of copying becomes more important, we see that an increase in $\gamma$ can lead to higher levels of copyright enforcement. Also, as is intuitive, the range of $\gamma$ for which the asymmetric effect holds is larger for smaller values of $\delta$.

Although the result shows that there is an asymmetric equilibrium, we need to consider the possibility that this result may be an artifact of our assumption that firms choose only two levels of $\alpha$. The more general case is more complicated and difficult to analyze using closed-form expressions.\(^{17}\) Nevertheless, we conducted numerical simulations using a discrete version of the game in which each firm

\(^{16}\) In case, $\gamma_2 > \delta/(1 + \delta)$, then the asymmetric solution will not exist.

\(^{17}\) An additional complexity is that we need to check for each parameter values whether consumers who can copy will copy or buy under the rational expectations framework.
was allowed to choose among 101 levels of $\alpha$ from 0 to 1 in increments of 0.01. We calculated the equilibrium profits for each of these 10,201 combinations and numerically searched for the pure strategy Nash equilibrium of this large discrete game. For $\delta = 1/2, \beta = 1,$ and $k$ large, we find that at $\gamma = 0.19$, both firms set $\alpha^* = 1$. However, as $\gamma$ increases, one of the firms (say, firm 2) sets $\alpha_2^* = 0$ at $\gamma = 0.2$, while the other firm continues to set $\alpha_1^* = 1$.\footnote{We also considered the case where there is a cost of copyright protection given by $-0.01 \ln(\alpha + 0.01)$ (where the constant 0.01 is used under the log term to avoid infinities in the numerical simulation). Again, we find that one firm continues to set $\alpha^*_1 = 1$, while the other firm switches from $\alpha_2 = 1$ at $\gamma = 0.20$ to lower levels, so that at $\gamma = 0.32$, $\alpha_2^* = 0.19$.} Thus the main insight of the analysis continues to hold even when we allow firms to choose multiple levels of $\alpha$. It is also important to note that our analysis assumed that the market demand is fixed. In this situation, network effects tend to intensify price competition. Our analysis does not incorporate a beneficial aspect of piracy in the presence of network effects, i.e., the market expansion effect because of increased network size. In the next section, we analyze whether our results are still valid when we consider scenarios where the total market is not fixed and the presence of network effects can increase the size of the market.

### 5.2. Market Is Not Fully Covered

In our base model, we assumed that the market is sufficiently competitive, so that the full market is covered and the total market volume does not depend on firms’ prices. It is possible that the market demand could be influenced by the firms’ prices. We can incorporate this by postulating a linear demand function of the form

$$D_n(p_1, p_2) = \frac{1 + q_1 - q_2 - p_1 + \eta p_2}{2}, \quad (30)$$

where $D_n(\cdot)$ is the demand from the noncopier segment and $0 \leq \eta \leq 1$. The demand from the copier segment is analogously defined as

$$D_c(p_1, p_2) = (1 - \alpha)\left[\frac{\delta + \delta(q_1 - q_2) - p_1 + \eta p_2}{2\delta}\right]. \quad (31)$$

Note that the analysis reduces to the base case when $\eta = 1$ (see Equations (12) and (13)). Also, the total demand is a decreasing function of $p_1$ and $p_2$. The remainder of the analysis is as before. We have the following result:

**Proposition 6.** For large $k$, if $\delta \in (\delta^*, 1/3)$, then in equilibrium, both firms allow copying, where

$$\delta^* = \frac{-\eta^2 - 4 + \sqrt{2\eta^2 + 4 - 4\eta}}{12 - 2\eta^2 - 4\eta}. \quad (32)$$

Thus the proposition shows that the main result still holds even in situations where the market is not fully covered as long as $\eta$ is not too small. In other words, as long as firms are sufficiently competitive, our main results hold. Also, note that for $\eta = 0$, $\delta^*$ reduces to $1/3$ and the firm never benefits by allowing piracy. This is consistent with our result since $\eta = 0$ implies that the firm is a monopolist. Furthermore, as $\eta \to 1$, the results are stronger. This is intuitive because the full market coverage assumes strong competitive interaction and the results depend on the ability of piracy to reduce the level of competition.

Now, we consider the case when the market is not fully covered and there are network effects. In our formulation with constant demand, network effects tended to increase competition. However, as is well established in the literature, network effects can increase the size of the market. We incorporate this in our formulation to examine whether our results would still obtain in this modified framework. To model this, we assume that the demand from the noncopier segment is given by

$$D_n(p_1, p_2) = \frac{1 + q_1 - q_2 - p_1 + \eta p_2 + (z_1^* - \eta z_2^*)}{2}, \quad (33)$$

where $0 \leq \eta \leq 1$. Note that the demand reduces to the constant demand formulation when $\eta = 1$. On the other hand, when $\eta = 0$, the firms are effectively monopolists.

The demand from the potential copier segments, when they are unable to copy (which is a fraction $(1 - \alpha_1)(1 - \alpha_2)$), is analogously defined as

$$D_c(p_1, p_2) = (1 - \alpha_1)(1 - \alpha_2)\left[\frac{\delta + \delta(q_1 - q_2) - p_1 + \eta p_2 + z_1^* - \eta z_2^*}{2\delta}\right]. \quad (34)$$

Note that if $\eta = 1$ and $\gamma = 0$, we have the base case. If $\eta = 1$ and $\gamma > 0$, we have the case analyzed
in Proposition 5. Also, because copying allows consumers to get a product at zero effective price, piracy will lead to increased market size. This, in turn, can benefit both firms because the increased network size increases the price that consumers who purchase are willing to pay. Thus, this generalization allows the beneficial market expansion effect of network effects. We have the following proposition:

**Proposition 7.** Let $\delta = 1/2$, $\eta = 0.8$, then for large $k$ if $\gamma > 0.27$, in equilibrium, one firm allows copying and the other firm does not.

The proposition thus shows that even in situations in which the market is not fully covered and the presence of network effects can increase the total market size, our results are valid. The proposition is derived for specific values of $\delta$ and $\eta$, but we can easily determine the critical $\gamma$ for other parameter combinations. Figure 2 plots the equilibrium outcomes for different values of $\gamma$ and $\eta$ at $\delta = 1/2$. We see that the range in which an increase in $\gamma$ can lead to a firm choosing more stringent copyright protection, is lower as $\eta$ increases. This is intuitive, since in the extreme case when $\eta = 0$ and the firms are effectively monopolists, network externalities always makes copyright enforcement less attractive. However, as the level of competition ($\eta$) increases, this effect is tempered by the other effects that we have previously discussed. Consequently, as $\eta$ increases, we are more likely to find that an increase in network effects can make copyright enforcement more attractive. Also, note the figure plots the equilibria for $\delta = 1/2$. For lower values of $\delta$, the region in which the asymmetric solution holds increases. This is also intuitive because at lower values of $\delta$, the effects identified in the paper are stronger. Taken together, the results of this section suggest that as markets become more mature and there is limited potential for growth, then the effects identified in this paper will become stronger.

**Figure 2** Equilibrium Copyright Enforcement When Market Is Not Fully Covered

<table>
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<th>$\gamma$</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
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<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
</tr>
</tbody>
</table>

*Note. $\delta = 1/2$."

6. Conclusion

This paper examines how the presence of competition changes the effect of piracy on firms’ prices, profits, and R&D levels. Furthermore, we wished to examine how these effects impact firms’ incentives to enforce copyright laws. Toward this goal, we first developed a model in which there were no network effects and the market is completely covered. In a monopoly setting in such situations, a firm will always be hurt by higher levels of piracy. However, once we consider a duopoly, the results change. Our results suggest that copying by more price-sensitive consumers can enable the firms to credibly charge higher prices. This leads to a reduction in price competition. Furthermore, this positive effect of piracy on firms’ profits can sometimes outweigh the negative impact because of lost sales. We also show that piracy can sometimes lead to an increase in innovation and improve social welfare. We establish conditions under which firms may choose weak copyright protection even in mature markets with no network externalities, such as those for entertainment products. These results suggest that as markets for entertainment products mature in developing countries with large income disparities, such as China and India, firms may find it beneficial not to impose strict copyright protection.\[19\]

We also extend our model to consider copyright protection in the presence of network effects. Our results show that the role of copyright protection in reducing price competition is substantially different in the presence of strong network externalities. In particular, we find that when network effects are strong, one firm may choose to enforce copyright protection so as to reduce price competition. Our results suggest that in maturing markets for products with large network effects, it may be beneficial for some firms to impose weak copyright protection. Thus, in contrast to previous research, our results show that the presence of strong network effects is neither a necessary nor sufficient condition for firms to choose weak copyright protection.

In developing the theoretical model, we made several assumptions and future research can examine the implications of relaxing these assumptions. For instance, we do not allow for the possibility that firms could price discriminate between the two segments using different product lines. Future research can examine how our results change if the firms could

\[19\] We have also extended our model to examine how firm asymmetries can impact their decision to enforce copyright protection. We find that large firms, such as Microsoft, will find it beneficial to more strictly enforce copyright enforcement. Details are available in Technical Appendix B at http://mktsci.pubs.informs.org.
produce a cheaper version of its product for segment 2.\textsuperscript{20} Our paper examined a single-period model in which both the pirates and the legal purchasers of the software enter the market at the same time. It can be argued that some consumers need to buy before the pirates can copy the product. For example, in a two-period model in which piracy is allowed in only the second period, one could examine how piracy might affect the adoption decisions of consumers. Our results would suggest that absent network effects, piracy would lead to higher prices in the second period. This would lead to rational consumers buying early, and therefore piracy can accelerate product diffusion in the first period even when there are no network effects.\textsuperscript{21} Future research can further examine how piracy would affect diffusion of digital products even in cases where network effects are not significant. Our model also does not allow for the possibility that over time piracy can affect the incentives for noncopiers. If these consumers do not pirate because they find piracy to be morally objectionable, it is plausible that as piracy becomes more prevalent, such consumers may also consider pirating. Clearly, a firm that is maximizing long-term profits will need to take this effect into account, while deciding on its copyright protection policy.\textsuperscript{22} Future research can examine how such dynamic aspects affect firms’ incentives to enforce copyright protection.

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**Appendix**

**Proof of Proposition 1.** Using the Envelope theorem, we have

\[
\frac{d\Pi_1}{d\alpha} = \begin{cases} 
-\beta p_1 & \text{if } p_1 \leq \delta(v + q_1) - \delta \\
-\beta p_1 F\left(\frac{\delta(v + q_1) - p_1}{\delta}\right) & \text{otherwise}
\end{cases}
\]  

(35)

which is weakly negative. Thus the firm will choose \(\alpha = 0\).

To see the second part, note that if \(v\) is sufficiently large, then the firm’s profit function is given by

\[\Pi_1 = [1 + \beta(1 - \alpha)]p_1^* - kq_1^*/2.\]

(36)

Since the market is fully covered, \(p_1^* = \delta(v + q_1) - \delta\). Therefore, we have

\[\Pi_1 = [\delta(v + q_1) - \delta][1 + \beta(1 - \alpha)] - kq_1^*/2.\]

(37)

Therefore

\[q_1^* = \frac{\delta[1 + \beta(1 - \alpha)]}{k}.\]

(38)

Thus \(q_1^*\) is decreasing in \(\alpha\). Social welfare is defined by

\[SW = \int (v + q_1^* - \theta)f(\theta)d\theta + \int \beta(\delta(v + q_1^*) - \delta\theta)f(\theta)d\theta - kq_1^*/2.\]

(39)

It is easily shown that

\[\frac{\partial SW}{\partial \alpha} = [1 - \delta + \alpha, \beta \delta]\frac{\partial q_1^*}{\partial \alpha} < 0,\]

(40)

where the inequality follows since \(q_1^*\) is decreasing in \(\alpha\) and \(0 < \delta < 1\).

**Proof of Proposition 2.** The relevant first-order condition is

\[-f(\theta_2^*) - \beta(1 - \alpha_1)(1 - \alpha_2)f(\theta_1^*)\]

\[+ \frac{[F(\theta_2^*) + \beta(1 - \alpha_1)(1 - \alpha_2)f(\theta_1^*)]}{\delta} = 0.\]

(41)

If \(q_1 = q_2\), then the symmetric solution would imply that \(\theta_1^* = \theta_2^* = 1/2\) and \(F(1/2) = 1/2\) for a symmetric distribution. Therefore we have

\[-f(1/2)\]

\[\frac{1 + \beta(1 - \alpha_2)^2}{\delta}p_1 + \frac{1}{2} + \frac{\beta(1 - \alpha_2)^2}{2} = 0.\]

(42)

This implies that

\[p_1^* = \frac{\delta[1 + \beta(1 - \alpha_2)^2]}{f(1/2)[\beta + \beta(1 - \alpha_2)^2]}\]

(43)

Differentiating, we have

\[\frac{\partial p_1^*}{\partial \alpha} = \frac{2\beta(1 - \alpha)(1 - \alpha^2)}{f(1/2)[\delta + \beta(1 - \alpha_2)^2]} > 0.\]

(44)
Using the Envelope theorem for the third-stage profit function, after simplification we get

\[
\frac{d\Pi_{13}}{d\alpha} = \frac{2(1-\alpha)\beta p_1^*}{2(\delta + \beta(1-\alpha)^2)} [1-2\delta - \beta(1-\alpha)^2],
\]

which is greater than zero only if

\[
\alpha > 1 - \sqrt{\frac{1-2\delta}{\beta}}.
\]

Note that this implies that the firm profits are decreasing in \(\alpha\) for small values and increasing after the critical value defined in (46) is reached. Because the cost of enforcement is zero, the firms’ maximum profits as a function of \(\alpha\) can be determined by comparing the two extreme points, i.e., \(\alpha = 0\) and \(\alpha = 1\). We have

\[
\Pi_{13}(\alpha = 0) = \frac{\delta(1+\beta)^2}{2f(1/2)(\delta + \beta)}. \tag{47}
\]

Similarly,

\[
\Pi_{13}(\alpha = 1) = \frac{1}{2f(1/2)}. \tag{48}
\]

Thus, firms’ profits are higher under \(\alpha = 1\) if \(\delta < 1/(2 + \beta)\), which completes the proof. □

**Proof of Proposition 3.** In this case, we need to consider the third-stage decision by allowing for the possibility that the second-stage qualities are different. For notational simplicity, denote \(f(\theta_i^*) = f_1\) and \(f(\theta_i^*) = f_2\). Given that \(\alpha_1 = \alpha_2 = \alpha\), in the second stage of the game, the firms decide on qualities. We need to determine the impact of \(q_1\) and \(q_2\) on the third-stage profit function, where we allow for the possibility that \(q_1 \neq q_2\).

Using the Envelope theorem, we have

\[
\frac{d\Pi_{13}}{dq_1} = \frac{\partial \Pi_{13}}{\partial q_1} + \frac{\partial \Pi_{13}}{\partial p_2} \frac{\partial p_2}{\partial q_1} - kq_1 = 0. \tag{50}
\]

We have

\[
\frac{\partial \Pi_{13}}{\partial q_1} = \frac{f_1 + \beta(1-\alpha)^2 f_2 p_1}{2}. \tag{51}
\]

Also,

\[
\frac{\partial \Pi_{13}}{\partial p_2} = \frac{f_1 \delta + \beta(1-\alpha)^2 f_2 p_1}{2\beta}. \tag{52}
\]

Thus the relevant first-order condition for firm 1 in the second stage is given by

\[
\frac{f_1 + \beta(1-\alpha)^2 f_2 p_1^*}{2} + \frac{f_1 \delta + \beta(1-\alpha)^2 f_2 p_1^*}{2\beta} \frac{\partial p_2}{\partial q_1} - kq_1^* = 0. \tag{53}
\]

We therefore need to evaluate \(\partial p_2^*/\partial q_1\). Using the implicit function theorem

\[
\frac{\partial p_2^*}{\partial q_1} = \left[ \frac{\partial^2 \Pi_{13}}{\partial p_2 \partial q_1} \right]^{-1} \left( \frac{\partial^2 \Pi_{13}}{\partial p_1^2} - \frac{\partial^2 \Pi_{13}}{\partial p_1 \partial q_1} \right), \tag{54}
\]

where

\[
|J| = \frac{\partial^2 \Pi_{13}}{\partial p_2^2} - \frac{\partial^2 \Pi_{13}}{\partial p_1 \partial q_1} - \frac{\partial^2 \Pi_{13}}{\partial p_1^2} \tag{55}
\]

We have

\[
\frac{\partial \Pi_{13}}{\partial p_1} = \left[ \frac{-f_1}{2} - \frac{\beta(1-\alpha)^2 f_1}{2\delta} \right] p_1 + \left[ f_1 + \beta(1-\alpha)^2 f_2 \right]. \tag{56}
\]

Since we are focusing on symmetric solution, \(f_1 = f_2 = f(1/2)\). Also, \(f(1/2) = 0\). To see this, assume it is not true. If \(f(1/2) > 0\), then by continuity for sufficiently small \(\epsilon\), \(f(1/2 + \epsilon) < f(1/2 - \epsilon)\) violating symmetry. An analogous argument works for \(f(1/2) < 0\) case. Thus \(f(1/2) = 0\). Thus under symmetry, after simplification, we have

\[
\frac{\partial^2 \Pi_{13}}{\partial p_1^2} = -\frac{f(1/2)}{\delta} \frac{\delta + \beta(1-\alpha)^2}{\delta}. \tag{57}
\]

Also,

\[
\frac{\partial^2 \Pi_{13}}{\partial p_1 \partial p_2} = \frac{f(1/2)}{2\delta} \left[ \delta + \beta(1-\alpha)^2 \right]. \tag{58}
\]

Similarly, we have

\[
\frac{\partial^2 \Pi_{13}}{\partial p_2^2} = \frac{f(1/2)}{2} \left[ 1 + \beta(1-\alpha)^2 \right]. \tag{59}
\]

Analogously, we have

\[
\frac{\partial^2 \Pi_{23}}{\partial p_1^2} = -\frac{f(1/2)}{\delta} \frac{\delta + \beta(1-\alpha)^2}{\delta}. \tag{60}
\]

Also,

\[
\frac{\partial^2 \Pi_{23}}{\partial p_1 \partial p_2} = \frac{f(1/2)}{2\delta} \left[ \delta + \beta(1-\alpha)^2 \right]. \tag{61}
\]

Thus the first-order condition (50) becomes

\[
A_2 p_1^* + A_1 p_1^* \left( \frac{-A_2}{3A_1} \right) - kq_1^* = 0, \tag{67}
\]

which reduces to

\[
q_1^* = \frac{2A_2 p_1^*}{3k} = \frac{\delta [1 + \beta(1-\alpha)^2]}{3k [\delta + \beta(1-\alpha)^2]} \tag{68}
\]

Differentiating, we have

\[
\frac{\partial q_1^*}{\partial \alpha} = \frac{-2\beta \delta(1-\alpha)(1 + \beta(1-\alpha)^2)[2\delta - 1 + \beta(1-\alpha)^2]}{3k [\delta + \beta(1-\alpha)^2]^2}. \tag{69}
\]

Thus, in a symmetric solution

\[
|J| = 4A_2^2 - A_1^2 = 3A_1^2 > 0. \tag{64}
\]

Also, define

\[
A_2 = \frac{f(1/2)(1 + \beta(1-\alpha)^2)}{2}. \tag{65}
\]

Thus, using (54) and (64), it follows that

\[
\frac{\partial p_2^*}{\partial q_1} = -2A_1 A_2 + A_1 A_2 = -A_2. \tag{66}
\]

Thus the first-order condition (67) becomes

\[
A_2 p_1^* + A_1 p_1^* \left( \frac{-A_2}{3A_1} \right) = 0, \tag{67}
\]

which reduces to

\[
q_1^* = \frac{2A_2 p_1^*}{3k} = \frac{\delta [1 + \beta(1-\alpha)^2]}{3k [\delta + \beta(1-\alpha)^2]} \tag{68}
\]
Thus, \( q_1^* \) increases as \( \alpha \) increases if
\[
\alpha > 1 - \sqrt{\frac{1-2\delta}{\beta}}. \tag{70}
\]
To calculate the equilibrium profits, we can plug the equilibrium qualities back into the profit equation. We have
\[
\Pi_{12} = \frac{\delta[1 + \beta(1-\alpha)^2]}{f(1/2)[\delta + \beta(1-\alpha)^2]} \cdot \frac{1+\beta(1-\alpha)^2}{2} - \frac{1}{18k} \left[ \frac{\delta[1 + \beta(1-\alpha)^2]^2}{\delta + \beta(1-\alpha)^2} \right]^2. \tag{71}
\]
We know from previous proposition that if (46) holds, then the first term is higher for \( \alpha = 1 \) than for any other \( \alpha \). Since, as \( k \) increases, the second term becomes arbitrarily small, the result follows. \( \square \)

**Proof of Proposition 4.** Assume that the equilibrium is \((\alpha_1^*, \alpha_2^*)\). In this case, a proportion \( \beta_1 \alpha_1^* \) consumers can copy both products, \( \beta(1-\alpha_1^*)(1-\alpha_2^*) \) consumers can copy neither, \( \beta(1-\alpha_1^*) \alpha_2^* \) can copy only 2 and \( \beta \alpha_1^*(1-\alpha_2^*) \) can copy only 1. Since by assumption copiers prefer to copy, the profit function in the third stage for the firm are
\[
\Pi_{13} = p_1[F(\theta_1^*) + \beta(1-\alpha_1^*)(1-\alpha_2^*)F(\theta_1^*)] \tag{72}
\]
\[
\Pi_{23} = p_2[1-F(\theta_2^*) + \beta(1-\alpha_1^*)(1-\alpha_2^*)(1-F(\theta_2^*)]. \tag{73}
\]
Let \( \Pi_{13}(q_1, q_2, \alpha_1^*, \alpha_2^*) \) and \( \Pi_{23}(q_2, q_1, \alpha_1^*, \alpha_2^*) \) represent the equilibrium profit functions in the last stage. First, note that the profits only depend on \((1-\alpha_1^*)(1-\alpha_2^*)\), which we can denote by the parameter \( \zeta \). Then the profit functions are symmetric in the second stage, and therefore there must be a symmetric equilibrium in quality even if \( \alpha_1^* \neq \alpha_2^* \). This, in turn, implies that there must be a symmetric equilibrium in prices. Then from (71), the profits for both firms will be given by
\[
\Pi_{12} = \frac{\delta[1 + \beta \zeta]}{f(1/2)[\delta + \beta \zeta]} \cdot \frac{1+\beta \zeta}{2} - \frac{1}{18k} \left[ \frac{\delta[1 + \beta \zeta]^2}{\delta + \beta \zeta} \right]^2. \tag{74}
\]
If (46) holds, then we know that \( \Pi_{12} \) is maximized at \( \zeta = 0 \), i.e., when \((1-\alpha_1^*)(1-\alpha_2^*) = 0 \). Therefore, at least one firm must choose \( \alpha_1^* = 1 \), but then the other firm weakly prefers to set \( \alpha = 1 \), which completes the proof.

Now, we investigate the impact on social welfare. Note that piracy affects prices but in the symmetric case, does not impact the final consumer choices. Because prices are transfers from one group (consumers) to another (firms), these have no impact on social welfare. Social welfare is only affected by the impact of copying on innovation levels. The social welfare function, when there are two products, is given by
\[
SW = 2 \int_0^{1/2} (v + q - \theta)f(\theta) \, d\theta
+ \beta \int_0^{1/2} (\delta(v+q) - \delta\theta)f(\theta) \, d\theta - \frac{2kq^2}{2}. \tag{75}
\]
First, we establish the socially optimal qualities. Differentiating
\[
(1+\beta \delta) - 2kq_{SW} = 0. \tag{76}
\]
Note that the first-order condition implies that the social welfare function is concave in \( q \) and is increasing at \( q = 0 \). The socially optimal quality is given by
\[
q_{SW} = \frac{1+\beta \delta}{2k}. \tag{77}
\]
We know that
\[
q^* = \frac{\delta(1+\beta \zeta)^2}{3k(\delta + \beta \zeta)}. \tag{78}
\]
Therefore
\[
\frac{\partial q^*}{\partial \zeta} = \frac{\delta\beta(1+\beta \zeta)(2\delta + \beta \zeta - 1)}{3k(\delta + \beta \zeta)^2}. \tag{79}
\]
Note that \( q^* \) is decreasing at \( \zeta = 0 \) if \( \delta < 1/(2+\beta) \) and is increasing if \( \zeta > (1-2\delta)/\beta \). Since \( \zeta \) varies from 0 to 1, to find the maximum value of \( q^* \) over \( \zeta \), we only need to look at the extreme points, i.e., \( \zeta = 0 \) and \( \zeta = 1 \).

We have
\[
q(\zeta = 0) - q(\zeta = 1) = \frac{\beta(1-\delta)}{3k(\delta + \beta)} > 0. \tag{80}
\]
Also,
\[
q_{SW} - q(\zeta = 0) = \frac{1+\beta \delta}{6k} > 0. \tag{81}
\]
The result then follows since the welfare function is concave and by the ordering \( q(\zeta = 1) < q(\zeta = 0) < q_{SW} \). \( \square \)

**Proof of Proposition 5.** We will analyze the three possibilities: both firms allow copying, both do not allow copying, and only one firm allows copying.

### Both Firms Allow Copying

The network size for firm 1 is
\[
z_1 = \theta_1^* + \theta_2^*. \tag{82}
\]
The rational expectations condition is
\[
z_1^* = z_1. \tag{83}
\]
Analogous condition exists for firm 2. Also, note that the full-coverage assumption implies that
\[
z_1 + z_2 = 2. \tag{84}
\]
By solving, we get under the rational expectations equilibrium
\[
z_1 = \frac{-2\gamma \delta + 2\delta \delta_1 - \delta \delta_1 + \delta \delta_2 + 2\delta - 2\delta \delta_2 - 2\gamma}{2(-\delta + \gamma \delta + \gamma)}. \tag{85}
\]
By solving the game as usual, we have
\[
\Pi_{12}(\alpha_1 = 1) = (9k^2 \delta - 2\gamma \delta^2 + 9 \gamma k^2 + 2\gamma \delta - \gamma^2 - 9k \gamma^2 \delta - 18k \gamma \delta - \delta^2 + 2\gamma^2 \delta + 9k \gamma^2 - \gamma^2 \delta^2) \cdot (18k(\delta - \gamma)^2)^{-1}. \tag{86}
\]
### Both Firms Do Not Allow Copying

Using the approach as before, we get
\[
\Pi_{12}(\alpha_1 = 0) = \frac{2(-4\delta^2 + 9k \delta^2 - 9k \gamma - 18k \gamma \delta + 9k \delta - 9k \gamma \delta^2)}{9k(\delta + 1)^2}. \tag{87}
\]
Note, however, that we need to check in this case that both firms would choose to serve both the segments. It can be shown that this is true as long as $\delta > (5 - 4\sqrt{2})/7 \approx 0.0938$.

**Only One Firm Allows Copying**

Without loss of generality, assume that firm 1 allows copying and firm 2 does not. The analysis is similar to that before. After straightforward calculations, we find that the equilibrium qualities are

$$q_1^* = \frac{9k - 6k\gamma - 2}{3k(9k(1 - \gamma) - 2)},$$

$$q_2^* = \frac{9k - 12k\gamma - 2}{3k(9k(1 - \gamma) - 2)}.$$  \hspace{1cm} (88)

Note that

$$q_1^* - q_2^* = \frac{2\gamma}{9k\gamma + 2 - 9k} > 0.$$  \hspace{1cm} (90)

Also,

$$p_1^* - p_2^* = \frac{2(q_1^* - q_2^*) + 2\gamma}{3},$$

which where the second inequality follows since $(q_1^* - q_2^*) > 0$. This proves one part of the proposition. By substituting the equilibrium qualities in the profit functions, we get after simplification

$$\Pi_{12}(\alpha_1 = 1, \alpha_2 = 0) = \frac{(9k - 1 - 9k\gamma)(6k\gamma + 2 - 9k)^2}{18k(9k\gamma + 2 - 9k)^2},$$  \hspace{1cm} (92)

$$\Pi_{22}(\alpha_1 = 1, \alpha_2 = 0) = \frac{(9k - 1 - 9k\gamma)(12k\gamma + 2 - 9k)^2}{18k(9k\gamma + 2 - 9k)^2}. $$  \hspace{1cm} (93)

Note that

$$\Pi_{12}^* - \Pi_{22}^* = \frac{2\gamma(9k - 1 - 9k\gamma)}{3(9k - 9k\gamma - 2)} > 0,$$  \hspace{1cm} (94)

which shows another part of the proposition.

To complete the proof, we need to establish that the asymmetric equilibrium holds under the conditions specified. Also, we need to check that the condition that copiers copy rather than buy holds. It can be shown that if $\delta \leq 0.94$, then copiers would prefer to copy.

Now, we prove that under the conditions specified in the proposition, the asymmetric equilibrium is valid. We need to show that neither firm would like to deviate. We first consider firm 1. If it deviates and restricts copying, then the profits are given by (87). If the difference is positive, then the firm does not deviate. After simplification and taking the limit as $k \to \infty$, this reduces to

$$\Delta_1 = \frac{-3(32\gamma^2 + 32\gamma\delta - 24\gamma - 60\delta\gamma - 9 + 27\delta)}{18(1 - \gamma)(1 + \delta)}. $$  \hspace{1cm} (95)

For the asymmetric equilibrium to hold, we need $\Delta_1 > 0$. It can be shown that if $\gamma > \gamma_1$, then $\Delta_1 > 0$, where

$$\gamma_1 = \frac{3(2 + 5\delta - \sqrt{12 + 4\delta + \delta^2})}{16(1 + \delta)}. $$  \hspace{1cm} (96)

Now, consider firm 2’s incentives to deviate. If it does, then the profits it obtains are given by (86). If the difference $\Delta_2 > 0$, then firm 2 will not deviate and the equilibrium would hold, where $\Delta_2$ reduces to, after simplification

$$\Delta_2 = \frac{\gamma(-16\gamma^2 + 7\delta\gamma + 15\gamma - 6\delta)}{18(1 - \gamma)(\delta - \gamma)}. $$  \hspace{1cm} (97)

Again, we can show that $\gamma > \gamma_2$, then $\Delta_2 > 0$, where

$$\gamma_2 = \frac{7\delta + 15 - \sqrt{49\delta^2 - 174\delta + 225}}{32}. $$  \hspace{1cm} (98)

Also, $\gamma_1 < \gamma_2$ for the relevant region. Thus we have established that if $\gamma < \gamma_2$, then the asymmetric equilibrium holds. This also establishes that the symmetric solution will not hold because at least one firm benefits by deviating.

Now, consider the case when $\gamma \in (\gamma_1, \gamma_2)$. In this case, the asymmetric solution does not hold because firm 2 would like to deviate and allow copying. However, firm 1 would not deviate and would also allow copying. Therefore, in this case, both firms allow copying. This completes the proof. $\square$

**Proof of Proposition 6.** In this case, we solve the game as usual and obtain

$$\Pi_{12}(\alpha_1 = 0) = \frac{2\delta}{(2 - \eta)(1 + \delta)}. $$  \hspace{1cm} (99)

It can be shown that both firms would choose to serve both the segments if $\delta > \delta^*$, where

$$\delta^* = \frac{-\eta^2 + 4\sqrt{2}\eta^2 + 4 - 4\eta}{12 - \eta^2 - 4\eta}. $$  \hspace{1cm} (100)

Now, consider the case when $\alpha_1 = 1$. We have

$$\Pi_{11}(\alpha_1 = 1) = \frac{1}{2(2 - \eta^2)}. $$  \hspace{1cm} (101)

Finally, consider the asymmetric equilibrium. Assume without loss of generality that firm 1 allows copying. Also, we assume that copiers prefer to copy rather than buy 2’s product. The profits for both firms in this case will be the same as the case when $\alpha_1 = 1$. We also find that in this situation, no copier would purchase product 2. With this, it immediately follows that the asymmetric solution is weakly dominated by the solution in which both firms allow copying. To see which equilibrium dominates, we compare the profits in (99) and (101). The difference in profits is given by

$$\Pi_{11}(\alpha_1 = 1) - \Pi_{11}(\alpha_1 = 0) = \frac{1 - 3\delta}{2(1 + \delta)(2 - \eta^2)} < 0. $$  \hspace{1cm} (102)

Thus, if $\delta \in (\delta^*, 1/3)$, it is an equilibrium for both firms to set $\alpha = 1$. $\square$

**Proof of Proposition 7.** We start the analysis by considering the case when both firms do not allow copying. In this case, we have

$$\Pi_{22}(\alpha_i = 0) = \frac{25(10 - 27\gamma)(50 - 27\gamma)}{27(9\gamma - 10)^2(10 - 3\gamma)}. $$  \hspace{1cm} (103)

As we did in the proofs of Propositions 5 and 6, we also check whether a firm can deviate and offer the product only to the noncopier segments, and find that under the conditions specified in the proposition, the firm will not deviate and the equilibrium profits calculated are valid.
The analysis for the case when both firms allow copying leads to
\[ \Pi_1^*(\alpha_i = 1) = \frac{25(27\gamma - 10)(\gamma - 10)^2(27\gamma^2 - 109\gamma + 50)}{144(3\gamma - 10)(27\gamma^2 - 115\gamma + 50)^2}. \] (104)

Now, consider the asymmetric case in which firm 1 allows copying and firm 2 does not. We will assume that copiers copy product 1 rather than copy 2. We also check this condition and find that it holds under the conditions in the proposition. By solving as usual, the equilibrium profits for the firms in the asymmetric case are
\[ \Pi_1^* = \frac{(50 - 9\gamma)(350 - 1,900\gamma - 495\gamma^2 + 81\gamma^3)}{36(-10 + 9\gamma)(9\gamma - 70)^2(3\gamma - 10)^2(\gamma - 10)}. \] (105)
\[ \Pi_2^* = \frac{25(50 - 9\gamma)(117\gamma^2 - 920\gamma + 700)}{36(-10 + 9\gamma)(3\gamma - 10)^2(3\gamma - 10)^2(\gamma - 10)}. \] (106)

Now, we examine whether firm 1 can benefit by deviating and not allowing copying. The profits in this case are given by (103). We find that if \( \gamma > 0.128 \), then firm 1 will not deviate. Also, we need to examine whether firm 2 can benefit by allowing copying. The profits in this case are given by (104). We find that if \( \gamma > 0.27 \), then firm 2 will not deviate. Thus, if \( \gamma > 0.27 \), the asymmetric equilibrium holds, which completes the proof. \( \square \)

References