

Math 365 Exam 2
October 19, 2012
S. Witherspoon

Name _____

There are 8 questions, for a total of 100 points. Point values are written beside each question. *No calculators allowed. Show your work for full credit.*

1. [10 points] Sonja wants to compute $9 \cdot 19$ in her head. Use mental mathematics to find the answer, and write down each step in the process as it could be explained to Sonja.

2. Consider the following proposition about all whole numbers m and n .

p : If m is even and n is even, then mn is even.

(a) [5] Is p true? If not, give a counterexample.

(b) [5] State the *converse* of p . Is it true? If not, give a counterexample.

(c) [5] State the *contrapositive* of p . Is it true? If not, give a counterexample.

3. How many one-to-one correspondences are there between the sets $\{a, b, c, d, e\}$ and $\{1, 2, 3, 4, 5\}$ if

(a) [5] in each correspondence, a must correspond to 1?

(b) [5] in each correspondence, a must correspond to an even number and b must correspond to an odd number?

4. [15] **Short answer.** For each, answer the following question, filling in the blank with “yes” or “no”. Is the number 142,296 divisible by

2 _____

3 _____

4 _____

5 _____

6 _____

7 _____

8 _____

9 _____

10 _____

11 _____

5. Fifty students majoring in biology, chemistry or physics were given a survey. There were 6 students double-majoring in biology and chemistry. No students double-majored in biology and physics, and there were no triple majors. There were 22 students majoring in biology, 12 in chemistry, and 24 in physics (these numbers include double majors).

(a) [5] Draw a Venn diagram to illustrate the information given.

(b) [5] How many students double-majored in chemistry and physics?

(c) [5] How many students majored *only* in chemistry (i.e. not including double majors)?

6. [10] For each of the following numbers, state whether it is prime or composite.

101

14,641

$2^7 - 1$

$5 \cdot 7 \cdot 11 + 11 \cdot 13 \cdot 17$

$15! + 7$

7. [10] A movie theater starts two movies at 7 pm. The first movie lasts 75 minutes, and the second movie lasts 90 minutes. If these movies run continuously all night, what time will it be when they start at the same time again?

8. [15] **True/False and Counterexample.** For each statement, tell whether it is true or false. If it is false, give a counterexample.

(a) For all sets A and B , $(A - B) \cup B = A$.

(b) For all sets A and B , if $A - B = \emptyset$, then $A \subseteq B$.

(c) For all whole numbers a and b , if $\text{GCD}(a, b) = 2$, then a is even and b is even.

(d) For all whole numbers a and b , if a is even and b is even, then $\text{GCD}(a, b) = 2$.

(e) For all whole numbers a , $\text{GCD}(a, 2a) = 2a$ and $\text{LCM}(a, 2a) = a$.