Math 365 Partial solutions to Exam 2

1. (a) No. Counterexample: n=6. (There are many other counterexamples possible.)
(b) If $12|n$, then $2|n$ and $6|n$. True.

2. (a) $5 \cdot 4 \cdot 3 \cdot 2 = 120$
(b) $3 \cdot 2 \cdot 3 \cdot 2 = 36$

3. For this problem, a Venn diagram is very helpful, and alternative methods of solution can be based on a diagram. The solution here is based on known relations among cardinalities of sets.
(a) Let A denote the set of survey respondents owning an American car, and let J denote the set of those owning a Japanese car. Then $n(A \cup J) = 110 - 12 = 98$, and so

$$\begin{aligned} n(A \cup J) &= n(A) + n(J) - n(A \cap J) \\ 98 &= 73 + 39 - n(A \cap J) \end{aligned}$$

and it follows that $n(A \cap J) = 14$.
(b) Let G denote the set of survey respondents owning a German car. From the given information, we have $n(A \cup J \cup G) = 107$, $n(A \cap G) = 8$, and $n(J \cap G) = 0$, and from this it also follows that $n(A \cap J \cap G) = 0$. So

$$\begin{aligned} n(A \cup J \cup G) &= n(A) + n(J) + n(G) - n(A \cap J) - n(A \cap G) - n(J \cap G) + n(A \cap J \cap G) \\ 107 &= 73 + 39 + n(G) - 14 - 8 - 0 + 0 \end{aligned}$$

and from this it follows that $n(G) = 17$.

4. (a) Same: $224 \cdot 5 = 112 \cdot 2 \cdot 5 = 112 \cdot 10$
(b) Not the same: $14,800 - 99 = (14,800 + 1) - (99 + 1) = 14,801 - 100$, which is not the same as $14,799 - 100$ (in each expression, 100 is subtracted from a number, but those numbers are different).

5.

$$\begin{array}{ll} 3 \quad \underline{2,5,8} & \qquad 4 \quad \underline{0,2,4,6,8} \\ \\ 9 \quad \underline{5} & \qquad 11 \quad \underline{7} \end{array}$$

6.

97 is prime (check it is not divisible by 2, 3, 5, 7, and this suffices, as $\sqrt{97} < \sqrt{100} = 10$, and these are all the prime numbers less than 10)

187 is composite (as it is divisible by 11)

$2^7 - 1$ is prime (it is equal to 127, and again one can check divisibility by primes less than or equal to $\sqrt{127} < \sqrt{144} = 12$)

$19 \cdot 23 + 23 \cdot 89$ is composite (since 23 is a factor of each term, it is a factor of the number, specifically, the number is equal to $23 \cdot (19 + 89) = 23 \cdot 108$)

$19! + 17$ is composite (Since $19! = 19 \cdot 18 \cdot 17 \cdot 16 \cdots 3 \cdot 2 \cdot 1$, the number 17 is a factor of each term of the number, specifically, the number is equal to $17 \cdot (19 \cdot 18 \cdot 16 \cdots 3 \cdot 2 \cdot 1 + 1)$)

7. (a)

$$\begin{aligned} 91 &= 35 \cdot 2 + 21 \\ 35 &= 21 \cdot 1 + 14 \\ 21 &= 14 \cdot 1 + 7 \\ 14 &= 7 \cdot 2 \end{aligned}$$

So $\text{GCD}(91, 35) = 7$ (the last nonzero remainder above)

(b) $\text{LCM}(91, 35) = \dfrac{91 \cdot 35}{7} = \dfrac{91 \cdot 5 \cdot 7}{7} = 91 \cdot 5 = 455$

8. (a) True

(b) False. Counterexample: $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, $C = \{3, 4\}$. Then $A \cap B = \{3\} = A \cap C$, but $B \neq C$. (Many other counterexamples are possible.)

(c) False. Counterexample: $n = 21$ (n is divisible by 3, but each digit of n is not)

(d) False. Counterexample: $a = 2, b = 4, d = 8$ (then d divides $2 \cdot 4 = 8$ but d does not divide 2 nor 4)

(e) True.